

THE EFFECT OF MAGNETIC SUSCEPTIBILITY ON THE MOTION OF PARTICLES IN A FERROHYDROSTATIC SEPARATOR*

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Separation in magnetic fluids is a sink-and-float technique based on the generalised Archimedes law whereby, in addition to the conventional force of gravity, a magnetically induced force acts on the fluid. Since this density-based process takes place in a non-uniform magnetic field, magnetic properties of the material to be separated also play a role. The objective of this study is to derive an equation of motion of a non-magnetic particle in a magnetised magnetic fluid and to solve this equation analytically so that particle trajectories could be derived. Furthermore, the magnetic susceptibility of magnetisable particles is taken into account by means of the effective density of the particle. This effective density is the sum of the physical (true) density of the particle and the density associated with the interaction of the particle with the external non-homogeneous magnetic field. A criterion for the accuracy of separation in the form of threshold mass magnetic susceptibility is obtained. This criterion is then verified experimentally.

Keywords: Magnetic fluid; Ferrohydrostatic separation; Magnetic susceptibility; Apparent density

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INTRODUCTION

Equation of Motion for Non-Magnetic Particle

A ferrohydrostatic separator features a stationary magnetic fluid, housed in a separation chamber placed between pole-pieces of a magnet. A particle suspended in a ferrofluid is acted upon by the force of gravity and by two buoyancy forces. The first is the classical Archimedes gravity-related force, and the other is the magnetically induced buoyancy force due to the magnetic 'weight' of the magnetic fluid as introduced by Rosensweig [1].

These forces acting on a non-magnetic particle of radius b (small enough to assume that the magnetic field gradient is uniform within the particle), volume V_p and density ρ_p immersed in a magnetic fluid of density ρ_f , dynamic viscosity η and saturation magnetisation M_f in the presence of an external magnetic field of gradient ∇H can be written as follows (see Fig. 1):

(i) Force of gravity:

$$\vec{F}_G = \rho_p V_p \vec{g} \quad (1)$$

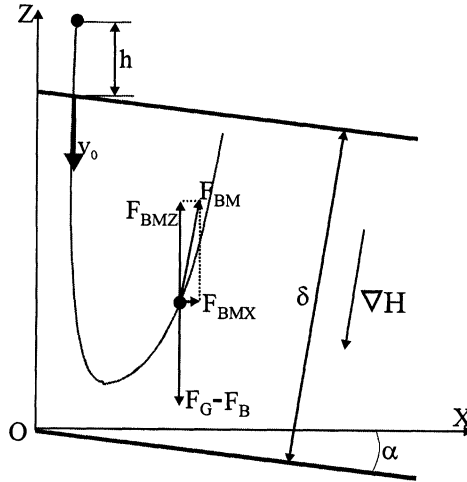


FIGURE 1 Section through the magnetic fluid bridge of depth δ , inclined with an angle α .

(ii) Buoyancy force:

$$\vec{F}_B = \rho_f V_p \vec{g} \quad (2)$$

(iii) The magnetically derived buoyancy force:

$$\vec{F}_{BM} = \mu_0 V_p M_f \vec{\nabla} H \quad (3)$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic permeability of vacuum, and M_f is the saturation magnetisation of the ferrofluid.

(iv) Drag force (written for the regime where Stokes Law is applicable and with the magnetic fluid at rest):

$$\vec{F}_D = 6\pi\eta b \vec{v}_p \quad (4)$$

In equilibrium the total force of the particle is equal to zero and the so-called apparent density of the ferrofluid can be written as:

$$\rho_{ap} = \rho_f + \frac{\mu_0 M_f \nabla H}{g} \cos \alpha \quad (5)$$

α being the angle of the ferrofluid along the horizontal direction and g is the gravitational acceleration.

The equations of motion of a particle can be written as:

$$\begin{cases} \frac{d^2 x}{dt^2} = \frac{dv_x}{dt} = \frac{1}{\rho_p V_p} (F_{Bx} + F_{Dx}) \\ \frac{d^2 z}{dt^2} = \frac{dv_z}{dt} = \frac{1}{\rho_p V_p} (F_{Bz} + F_{Dz} - G_a) \end{cases} \Rightarrow \begin{cases} \frac{dv_x}{dt} + K v_x = W \\ \frac{dv_z}{dt} + K v_z = W' \end{cases} \quad (6)$$

where the variables K , W and W' have the following forms:

$$K = \frac{6\pi\eta b}{\rho_p V_p}, \quad W = g \frac{\rho_{ap} - \rho_f}{\rho_p} \sin \alpha, \quad W' = g \frac{\rho_{ap} \cos \alpha - \rho_p + 2\rho_f \sin^2(\alpha/2)}{\rho_p} \quad (7)$$

The initial conditions for Eq. (6) are:

$$\text{for } t = 0 \rightarrow \begin{cases} v_x = 0 \\ v_z = v_0 \end{cases}.$$

If a particle drops into the ferrofluid from height h , then $v_0 = -\sqrt{2gh}$.

Differential Eq. (6) can be solved analytically and the solutions are:

$$\begin{cases} \frac{dx}{dt} = v_x = \frac{W}{K}(1 - e^{-Kt}) \\ \frac{dz}{dt} = v_z = v_0 e^{-Kt} + \frac{W'}{K}(1 - e^{-Kt}) \end{cases} \quad (8)$$

The initial conditions for Eqs. (8) are:

$$t = 0 \rightarrow \begin{cases} x = 0 \\ z = \delta \end{cases}.$$

Equation (8) with these initial conditions give the following solutions:

$$\begin{cases} x = \frac{W}{K}t - \frac{W}{K^2}(1 - e^{-Kt}) \\ z = \delta + \left(\frac{v_0}{K} - \frac{W'}{K^2}\right)(1 - e^{-Kt}) + \frac{W'}{K}t \end{cases} \quad (9)$$

Equations (9) represent parametric equations for particle trajectories, which can be obtained if the time t is eliminated. It is a difficult task, which can be overcome by choosing different values for time t and by calculating pairs of co-ordinates (x, z) . There are few interesting situations which are discussed in the following:

- When $\alpha = 0$, $h = 0$ and $\rho_{ap} = \rho_p$ then $v_0 = 0$ and, from Eq. (7), $W = 0$ and $W' = 0$. Therefore $v_x = 0$ and $v_z = 0$. Thus, in this particular situation, the particle is at rest on the fluid surface.
- When $h = 0$ then $v_0 = 0$ and, from Eq. (8), $v_x/v_z = W/W'$ which is a constant for a given set of parameters including the density of the fluid, the density of the particle, apparent density and angle α . In this particular case, the trajectory of the particle is a line (see Fig. 2).

Figure 2 depicts trajectories of non-magnetic particles (of diameter 3 mm) of different densities, given in kg/m^3 ; the other parameters being: the apparent density of the ferrofluid $\rho_{ap} = 3500 \text{ kg/m}^3$, the ferrofluid density $\rho_f = 990 \text{ kg/m}^3$, the ferrofluid viscosity $\eta = 1.65 \times 10^{-3} \text{ kg/m/s}$, the angle $\alpha = 2^\circ$ and drop height $h = 3 \text{ mm}$.

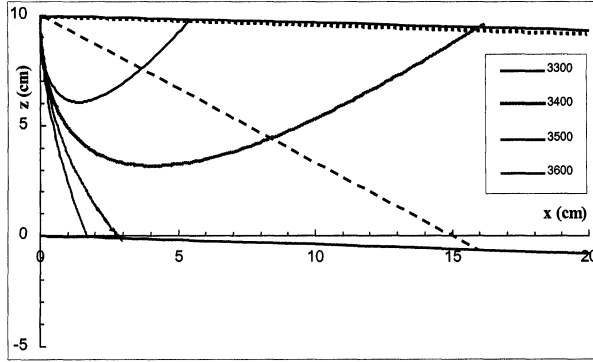


FIGURE 2 Trajectories of non-magnetic particles of different densities. $\rho_{ap} = 3500 \text{ kg/m}^3$ for $h = 3 \text{ mm}$. The dashed straight lines represent the trajectories of particle of $\rho_p = 3500$ and 3550 kg/m^3 for $h = 0$.

The Equation of Motion of a Magnetic Particle

In this case an additional force acts on a particle as a consequence of its non-zero magnetic susceptibility and its interaction with the external non-homogeneous magnetic field. If we assume that the magnetic field is uniform inside the particle, then this force can be written, as:

$$F_M = \mu_0 V_p \rho_p \chi_p H \nabla H \quad (10)$$

where χ_p is the mass magnetic susceptibility of the particle.

As in the case of the apparent density of the ferrofluid, the effective density of a particle can be assumed to be a sum of its physical density and the density associated with the magnetic force:

$$\rho_{eff} = \rho_p \left(1 + \frac{\mu_0 \chi_p H \nabla H}{g} \cos \alpha \right) \quad (11)$$

Equation (11) can be used to define density increment by which the effective density of a particle increases as a result of its interaction with the non-uniform magnetic field:

$$\Delta \rho(\%) = \frac{\rho_{eff} - \rho_p}{\rho_p} \cdot 100 = \frac{\mu_0 \chi_p H \nabla H}{g} \cdot 100 \quad (12)$$

It is clear from Eqs. (11) and (12) that the effective density of the particle and also the density increment due to the magnetic susceptibility depend on the position of the particle within the ferrofluid, as a result of the fact that the magnetic field strength H varies along the vertical.

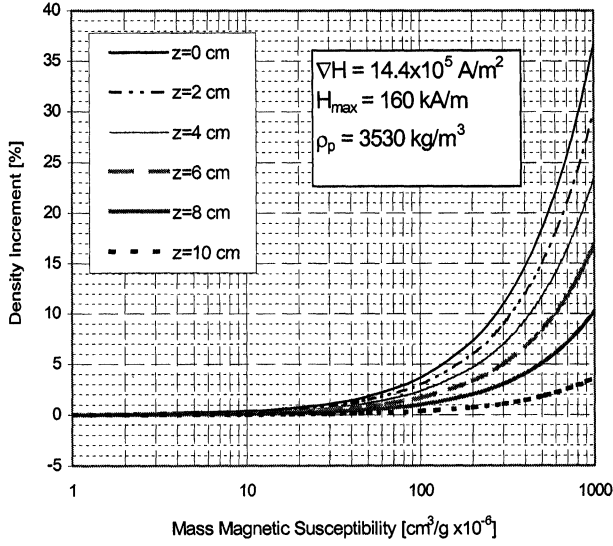
The following trends can be observed in Fig. 3:

- (a) Variation of the density increment with the magnetic susceptibility of the particle at different heights measured from the bottom of the magnetic fluid for the fixed gradient $\nabla H = 14.4 \times 10^5 \text{ A/m}^2$ ($\cong 180 \text{ Oe/cm}$) and magnetic field strength at the bottom of the ferrofluid pool $H_{\max} = 160 \text{ kA/m}$ ($\cong 2 \text{ kOe}$), and
- (b) The same variation but in this case at the fixed height $z = 10 \text{ cm}$ (position at which particles enter the fluid) and different $H_{\max} = 160, 240, 320$ and 400 kA/m ($\cong 2, 3, 4$ and 5 kOe).

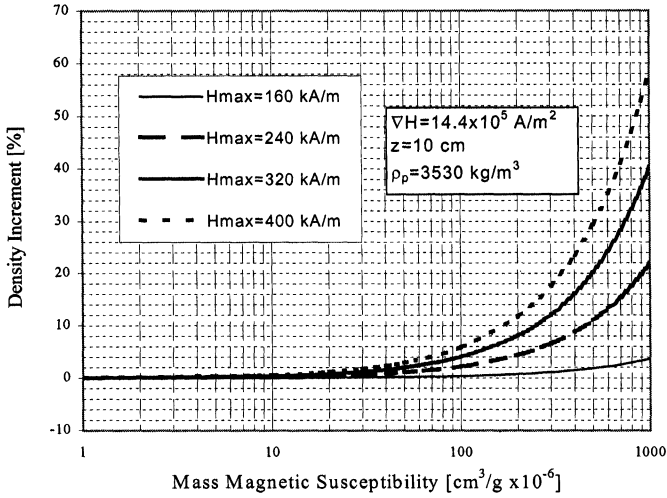
It can be seen in Fig. 3a that the density increment is greater at the bottom of the chamber where the magnetic field intensity is maximum. In practical terms it means that particles that were, at the point of entry into the ferrofluid, denser than the apparent density of the fluid, will keep sinking at an increased rate as they move through the fluid. The sink product of separation will thus contain not only particles the physical density of which is higher than the apparent density of the ferrofluid, but also particles less dense, with appreciable contribution from the magnetic susceptibility (the second term on the right-hand side of Eq. (11)). On the other hand, the float product will comprise particles that are truly less dense than the apparent density of the ferrofluid, even when the magnetic contribution is taken into account.

It can also be seen in Fig. 3b that in order to reduce the effect of the magnetic susceptibility on the accuracy of separation, the lowest possible magnetic field strength should be used, compatible with the required magnetic field gradient.

An easy way to obtain a trajectory of a magnetic particle is to consider, in Eq. (6), the effective density (Eq. (11)) instead of the physical density (ρ_p) of the particle, calculated using the mean magnetic field intensity ($\bar{H} = (H_{\max} + H_{\min})/2$) within the ferrofluid. In this way, the parametric equations for the trajectories remain the same as those in Eq. (8). Several trajectories of magnetic particles of different mass magnetic susceptibilities (in $\text{cm}^3/\text{g} \times 10^{-6}$) are shown in Fig. 4. The parameters are the same as in Fig. 2, the density of particle



a)



b)

FIGURE 3 Density increment vs. mass magnetic susceptibility; (a) At different distances from the bottom of the ferrofluid pool; (b) At different maximum magnetic fields.

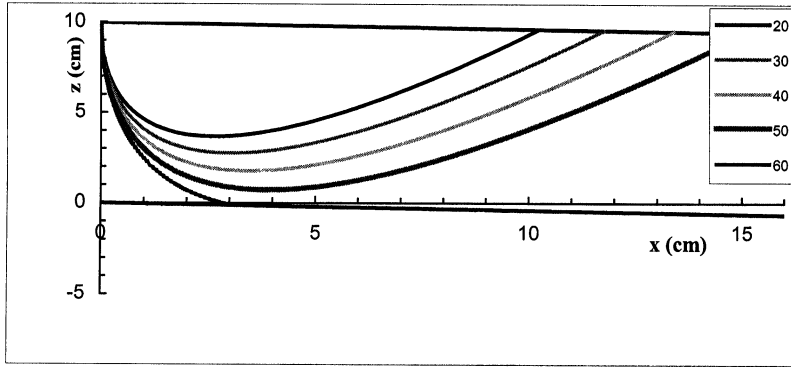


FIGURE 4 Trajectories of magnetic particles of different susceptibilities. $\rho_{ap} = 3500 \text{ kg/m}^3$, $\rho_p = 3300 \text{ kg/m}^3$.

being 3300 kg/m^3 , the magnetic field gradient is $\nabla H = 12 \times 10^5 \text{ A/m}^2$ ($\cong 150 \text{ Oe/cm}$) and the average magnetic field intensity is $\bar{H} = 100 \text{ kA/m}$ ($\cong 1.25 \text{ kOe}$).

As a comparison between Fig. 2 and Fig. 4 it can be observed that, under the same operational conditions, the non-magnetic particles of density 3300 kg/m^3 will float, while magnetic particles of mass magnetic susceptibility equal to $60 \times 10^{-6} \text{ cm}^3/\text{g}$ will sink.

At this stage, using the effective density of the particle, a criterion for selectivity of the separation can be introduced, taking into account that if $\rho_{ap} > \rho_{eff}$ a particle will float and if $\rho_{ap} \leq \rho_{eff}$ a particle will sink.

By inserting the expression of effective density given by Eq. (11) in the static equation, $\rho_{ap} = \rho_{eff}$, threshold mass magnetic susceptibility is given by:

$$\chi_{p\text{thres}} = \frac{g}{\mu_0 \bar{H} \nabla H \cos \alpha} \left(\frac{\rho_{ap}}{\rho_p} - 1 \right) \quad (13)$$

If the mass magnetic susceptibility of the particle is smaller than this threshold mass magnetic susceptibility ($\chi_p < \chi_{p\text{thres}}$) then the particle will float, while the particle will sink if the inverse inequality applies. It transpires from Eq. (13) that the threshold mass magnetic susceptibility does not depend on particle size.

EXPERIMENTAL

The experimental work was carried out in order to investigate the following issues:

- Existence of a threshold mass magnetic susceptibility for different apparent densities of the ferrofluid, and
- Whether this threshold mass magnetic susceptibility depends on particle size.

Magnetic tracers of well-defined magnetic susceptibility were used in the tests. These tracers were 4, 6, 8 and 10 mm cubes with mass magnetic susceptibilities of 20, 40, 60, 100, 200 and $300 \times 10^{-6} \text{ cm}^3/\text{g}$ and density of 3530 kg/m^3 .

The experiments were conducted by passing magnetic tracers of the same size and the same mass magnetic susceptibility through a ferrohydrostatic separator at a fixed apparent density of the ferrofluid. Then, keeping the apparent density of the ferrofluid and the size of the particles constant, the mass susceptibility was varied. The above steps were repeated for all sizes and for the following apparent densities of the ferrofluid: 3565, 3600, 3650, 3700, 3750, 3800, 3850 and 3900 kg/m^3 . For each fixed apparent density of the fluid the threshold mass magnetic susceptibilities was observed and then compared with theoretical values calculated from Eq. (13).

RESULTS AND DISCUSSION

Typical results, for one fixed apparent density, are also presented in Fig. 5. The graph shows the number of particles, which either float or sink. Columns No. 1, 2, 3, 4, 5, 6 and 7 represent magnetic susceptibilities 20, 40, 60, 80, 100, 200 and $300 \times 10^{-6} \text{ cm}^3/\text{g}$, respectively. Within each column there is a set of bars representing particle sizes 4, 6, 8 and 10 mm. Different patterns of the bars indicate which particles of a given size either floated or sank. For instance, column 4 (magnetic susceptibility $80 \times 10^{-6} \text{ cm}^3/\text{g}$) shows that all five 4 mm and 6 mm tracers floated while one out of five 8 mm and 10 mm tracers sank. For all graphs the density of the tracers was 3530 g/cm^3 .

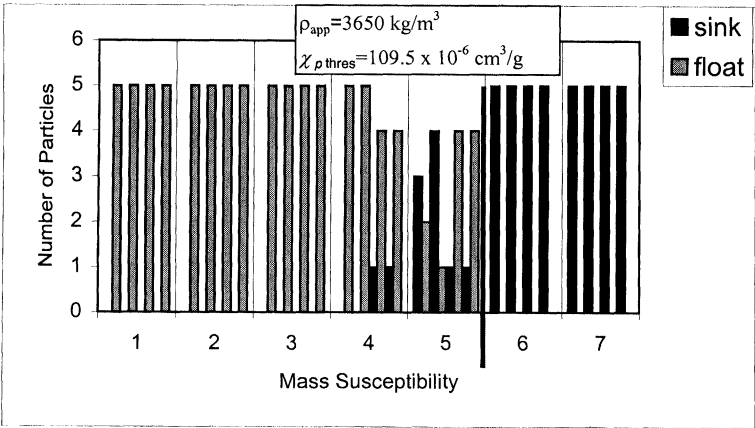


FIGURE 5 Experimental results and the theoretical value of $\chi_{p\text{thres}}$ (bold vertical line).

TABLE I Experimental and theoretical values of $\chi_{p\text{thres}}$

ρ_{app} (kg/m ³)	3565	3600	3650	3700	3750	3800	3850	3900	3950
$\chi_{p\text{thres}}$ (exp.) (cm ³ /g × 10 ⁻⁶)	> 60	> 60	> 100	> 100	> 100	> 200	> 200	> 300	> 300
$\chi_{p\text{thres}}$ (theoretical) form Eq. (13) (cm ³ /g × 10 ⁻⁶)	36.1	66.85	109.5	148.8	178.9	225.5	260	292.4	319.4

For the best agreement between experiment and theory the columns to the left from the threshold mass magnetic susceptibility (the bold vertical line) must contain only particles which float while the columns on right-hand side should consist only of particles which sink. It can be observed from Fig. 5 that, for that specific apparent density, it happened for 8 and 10 mm particles only.

In order to compare the experimental results with theoretical predictions it is necessary to calculate the values for $\chi_{p\text{thres}}$ from Eq. (13). The comparison is presented in Table I.

It was observed that there is a threshold mass magnetic susceptibility for all apparent densities. Although the theoretical value for $\chi_{p\text{thres}}$, calculated using Eq. (13), does not depend on the size of particles, the experimental results show that the best agreement between theory and experiment is observed for 6 mm tracers.

For other size the behaviour is not consistent. It can be explained by the following argument: the apparent density of the ferrofluid is not rigorously constant and a variation of 10 kg/m^3 is translated into a variation of $10\text{--}20 \times 10^{-6} \text{ cm}^3/\text{g}$ in $\chi_{p\text{thres}}$. It can be observed when the theoretical threshold mass susceptibility has the value close to the magnetic susceptibility of the tracers. For instance, for the apparent density of ferrofluid equal to 3650 kg/m^3 , the theoretical value of $\chi_{p\text{thres}}$ is $109.5 \times 10^{-6} \text{ cm}^3/\text{g}$ and it can be shown from the experimental results that the tracers with magnetic susceptibility equal to $100 \times 10^{-6} \text{ cm}^3/\text{g}$ either float or sink.

The same observation was made for apparent densities of 3565 and 3600 kg/m^3 . Theoretical values for $\chi_{p\text{thres}}$, calculated using criterion (13) are in good agreement with experimental values for all apparent densities, except for the values corresponding to the apparent densities equal to 3565 and 3850 kg/m^3 .

CONCLUSIONS

The effect of the magnetic susceptibility of the particle was taken into account by means of the effective density of the particle. The effective density of the particle having a non-zero magnetic susceptibility is greater than the real density and depends on the position of the particle in ferrofluid. Using this effective density, a threshold mass magnetic susceptibility was obtained.

The experimental work has proved that:

- there is a threshold mass magnetic susceptibility for all sizes of particles;
- the threshold mass magnetic susceptibility is not independent of the size of the particles;
- the best agreement between experimental and theoretical results was observed for 6 mm tracers.

Reference

- [1] R.E. Rosensweig (1966). Fluidmagnetic Buoyancy. *AIAA Journal*, **4**(10), 1751–1758.