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Research Article

Photonic Crystal Waveguide Weakly Interacting with Multiple Off-Channel Resonant Features Formed of Kerr Nonlinear Dielectric Media

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A theoretical study is presented of guided modes of a photonic crystal waveguide for cases in which they interact with multiple bound electromagnetic modes localized on off-channel impurity features of Kerr nonlinear media. The interest is on the properties of resonant scattering and optical bistability exhibited by the system and the coherent scattering of the guided modes due to their simultaneous resonant interactions with multiple bound modes. In first study, two off-channel features on opposite sides of a photonic crystal waveguide are made of different Kerr nonlinear dielectric media. In second study, an off-channel feature is composed of two neighboring sites having different Kerr dielectric properties. In addition to numerical results a number of analytical results are presented providing simple explanations of the quantitative behaviors of the systems. A relationship of these systems to forms of electromagnetic-induced transparency and modifications of waveguide dispersion relations is discussed.

1. INTRODUCTION

Recently there has been a renewed interest in resonant features in optical systems [1–4]. Some of this comes from proposals of mechanisms for the generation of slow light and for the establishment of conditions associated with electromagnetic-induced transparency [1, 2]. Here the alignment of different frequency resonances in an optical medium is used to set the dispersion relation of slow optical modes or to contribute to an effective dielectric constant of the medium as a whole. Properties of the system are manipulated by light of one frequency to affect the propagation of optical modes of a second frequency. Another focus of the study of resonances has been on applications in photonic crystal systems for multiplexing optical modes of different frequencies from photonic crystal waveguides [3, 4], for the generation of intense localized electromagnetic fields with which to investigate nonlinear dielectric properties [5–11], or for the modulation of guided modes at one frequency by those of another [12, 13]. The design of systems for these later applications is based on the resonant interaction of guided modes with modes bound and localized on off-channel impurity features [3–17], and the entire system is composed of linear dielectric media [3, 4] or could have Kerr nonlinear media [6, 7, 13, 14] on the off-channel features. A common aspect in the above examples is the transfer at resonant scattering of significant amounts of energy from optical or guided modes to excited states of atoms in a medium or to off-channel features in systems of photonic crystal waveguides. In this paper we will extend the studies of resonant scattering of photonic crystal waveguide modes to treat new types of multiple off-channel features composed of Kerr media sites. New waveguide transmission effects will be shown to arise, mediated by multiple scattering resonances of waveguide modes due to their interactions with multiple frequency bound modes on off-channel features. Some discussion of resonant photonic crystal waveguide systems that exhibit a type of induced transparency will be given as well as suggestions made of methods for the modification of waveguide dispersion relations through resonant interactions.

The reader is reminded that a photonic crystal is a periodic array of dielectric media [18–35] having electromagnetic modes that are Bloch waves with a frequency spectrum separated into a series of pass and stop bands. Propagating modes only exist at pass band frequencies and no modes propagate through the photonic crystal at stop band frequencies. Impurities are introduced into the system by changing the dielectric material within a localized region of the photonic crystal. For properly chosen impurity materials, bound electromagnetic modes are localized at stop band frequencies about the impurity media. Generalizing the ideas of impurity features, more complex structures, exhibiting a variety of different properties, can be created within photonic crystals. For example, a waveguide is formed in a photonic crystal by embedding an infinite line of translationally invariant media in the photonic crystal lattice so that it binds modes at stop band frequencies. These guided modes propagate along the channel formed by the line of changed dielectric media.

In the following, waveguides and impurities are treated for two-dimensional photonic crystals [21, 22, 36–40], designed as a system of parallel axis dielectric cylinders that are arrayed on a square lattice. The electromagnetic modes of interest propagate in the plane of the Bravais lattice with electric field vectors polarized parallel to the cylinder axes. Systems of this type have been a focus of much experimental and theoretical efforts [18–40] employing a wide variety of approaches. The theoretical approach used in this paper is based on a difference equation formulation for fields in the waveguide channels and off-channel features of a two-dimensional waveguide [36–40]. The difference equations are obtained from an exact integral equation formulation for the fields and have been used in a number of studies on waveguides and impurities in which numerical results (illustrating some of the behaviors found in networks of photonic crystal waveguides and impurity features) have been generated for a particular realization of the square lattice system described in [6, 37, 38]. Alternative approaches are numerical computer simulations, and the reader is referred to the well-known literature for a discussion of these [18–40].

In the present paper the focus is on the resonant interaction of waveguide modes with modes on off-channel features. This was first studied for linear dielectric media by Noda et al. [3, 4] with the objective of using the resonant interaction between waveguide modes and localized bound modes on a single off-channel site to download energy from the waveguide. Here energy is removed from the photonic crystal waveguide at the off-channel single impurity site and then taken from the photonic crystal as a whole. The coupling between the guided mode and the off-channel site is weak so that guided modes with frequencies off-resonance are not affected by the off-channel site. The characteristic frequency of the impurity mode on the off-channel site is set by the size and dielectric constant of the site so that by placing different impurity sites at intervals along the waveguide channel an efficient means of multiplexing from the waveguide channel is achieved. Both experimental and theoretical (computer simulation) studies were presented by Noda et al. for systems formed entirely of linear dielectric media, and good agreement between simulation and experimental data was found. Following these studies, a great number of simulation studies on the same and similar types of resonant systems formed of linear dielectric media were made [41–46]. (Note: We only list some representative works, many more can be found in the literature.)

Later extensions of the theoretical studies to treat waveguides interacting with off-channel sites formed of Kerr nonlinear media were made by Cowan and Young [14] and by McGurn [7, 13]. These involve analytical methods. The transmission coefficient of a single waveguide mode scattering from Kerr off-channel features was shown to exhibit optical bistability properties arising from the nonlinearity of the off-channel site. Specifically, multiple valued solutions for the transmission exist, and the transmission observed in the system depends on its history of electromagnetic interactions. The optical bistability is similar to the transmission bistability found for light at normal incidence on a slab of Kerr medium [47–53]. In the work of McGurn [7] considerations were carried to more complex off-channel features than single site impurities. These included multiple sites supporting resonantly excited intrinsic localized modes, and single and multiple sites connecting to a second semi-infinite waveguide. In addition, considerations of the interaction of two modes at different frequencies with some of the off-channel Kerr features were made in [13]. Here the field dependence of the dielectric properties of the Kerr media allows two different frequency waveguide modes to interact with one another by a field modulation of the nonlinear dielectric material [12, 13]. The modulation is enhanced in frequency regions at which localized modes on the off-channel features are resonantly excited by one or both of the waveguide modes. As a result one mode can be used to switch on or off the other mode's propagation in the channel or to impress an amplitude modulation on the other. In addition, recently there has been some experimental studies on related systems involving interactions mediated by Kerr nonlinear media of guided modes at two different frequencies [12].

The present paper is an extension of the work on nonlinear off-channel features to treat the interaction of guided modes with multiple localized off-channel modes. The multiple localized modes occur at different frequencies but are taken to be close together in frequency so that a guided mode experiences coherent scattering arising from its simultaneous interaction with the resulting closely spaced multiple resonances. In one study guided modes weakly interact with different single site features on opposite sides of the waveguide channel. In a second, guided modes weakly interact with an off-channel feature that can support multiple bound modes occurring at different frequencies. In some cases multiple resonant interactions with bound localized modes lead to a type of effect in the guided mode transmission coefficient that is reminiscent of electromagnetic-induced transparency. Resonant scattering effects also can modify the dispersion relation of light in the waveguide.

The order of this paper is as follows: in Section 2 a brief description of the difference equation approach is given and numerical values of the parameters used in this paper are explained. In Section 3 the problem of a waveguide weakly interacting with two single off-channel Kerr sites on opposite sides of a waveguide is discussed. The problem of a waveguide weakly interacting with an off-channel Kerr feature composed of two sites is also treated. The off-channel sites are shown to support multiple localized bound state modes at different frequencies that can be tuned to exhibit a variety of resonant effects. A number of analytic and numerical results illustrate the bistability properties of the fields and transmission coefficients and resonant effects in the system. In Section 4 some discussions of waveguides weakly interacting with periodic off-channel features are given and conclusions are presented.

2. BRIEF REVIEW OF DIFFERENCE EQUATIONS FOR WAVEGUIDE MODES

We consider a two-dimensional photonic crystal composed of parallel axis dielectric cylinders arrayed on a square lattice of lattice constant, a_0 [36–38]. Electromagnetic modes propagate in the plane of the square lattice with electric fields polarized parallel to the cylinder axes. With this polarization the electric field modes are Bloch waves satisfying a Helmholtz equation for the periodic dielectric constant of the photonic crystal.

Impurities and waveguides are introduced into the photonic crystal by changing the dielectric properties of the photonic crystal. The changes in the dielectric of the photonic crystal, $\epsilon(\vec{r}_\parallel)$, as a function of position are specified by the function of position $\delta\epsilon(\vec{r}_\parallel)$ so that the total dielectric of the system is $\epsilon(\vec{r}_\parallel)+\delta\epsilon(\vec{r}_\parallel)$. For an impurity or waveguide mode with a frequency ω in the stop band of the photonic crystal [37–39],

$$E(\vec{r}_\parallel) = \int d^2 r'_\parallel G(\vec{r}_\parallel, \vec{r'}_\parallel \mid \omega)\delta\epsilon(\vec{r'}_\parallel)\frac{\omega^2}{c^2}E(\vec{r'}_\parallel) \tag{1}$$

is an exact integral equation whose solutions are the impurity mode fields, $E(\vec{r}_\parallel)$. Equation (1) is obtained from the Helmholtz equation for the total dielectric $\epsilon(\vec{r}_\parallel)+\delta\epsilon(\vec{r}_\parallel)$, and $G(\vec{r}_\parallel, \vec{r'}_\parallel|\omega)$ is the Green function of the Helmholtz equation describing the propagation of electromagnetic modes in the bulk of the photonic crystal. In general $\delta\epsilon(\vec{r}_\parallel)$ is taken to be a type of step function in space that is nonzero only in the region of the replacement materials. In regions of linear dielectric impurity material it is a constant denoted as $\delta\epsilon_{00}$. A convenient way of looking into (1) is to consider it as an integral equation eigenvalue problem for the eigenvectors $E(\vec{r}_\parallel)$ and eigenvalues $\delta\epsilon_{00}$. Specifying ω in a stop band of the photonic crystal gives the values of $\delta\epsilon_{00}$ needed to support impurity modes at that frequency and the wave functions of the impurity modes.

Equation (1) can also be used to study Kerr nonlinear impurity media. For these materials $\delta\epsilon(\vec{r}_\parallel) = \delta\epsilon_{00}(1+\lambda|E(\vec{r}_\parallel)|^2)$ so that the impurity or waveguide media depends on the intensity of the applied electric field [39, 40]. The resulting problem is no longer a simple eigenvalue problem, but a problem in which $E(\vec{r}_\parallel)$ and $\delta\epsilon_{00}$ must be solved for self-consistently at a given value of ω specified in a stop band of the photonic crystal.

We will be interested in impurities and waveguides formed by cylinder replacement in the photonic crystal. The replacement cylinders are made in whole or in part of a different type of dielectric media from the cylinders in the bulk photonic crystal. For certain types of cylinder replacements, (1) for the waveguide modes reduces to a set of difference equations. For example, this occurs when the field, $E(\vec{r}_\parallel)$, changes slowly over the region of nonzero $\delta\epsilon(\vec{r}_\parallel)$ in each separate replacement cylinder along the waveguide channel. The resulting difference equations, obtained from (1), for a waveguide along the x-axis of a square lattice photonic crystal are then [6, 7, 36, 37]

$$E_{n,0} = \gamma\left[aE_{n,0} + b\left(E_{n+1,0} + E_{n-1,0}\right)\right]. \tag{2}$$

Here the replacement cylinder sites along the x-axis are labeled $(n, 0)$, for an infinite set of consecutive integers n, and $E_{n,0}$ is the field in the $(n, 0)$ site. The factor γ is proportional to the dielectric contrast $\delta\epsilon_{00}$, and the couplings a and b are obtained from (1) as averages of the Green functions over the same replacement channel site and between closest neighboring replacement channel sites, respectively. The couplings a and b depend on the frequency ω and the geometric properties of the bulk photonic crystal. As a simple example, note that substituting a plane wave form $E_{n,0} \propto e^{ikn}$ in (2) gives the dispersion [37]

$$1 = \gamma[a + 2b\cos k] \tag{3}$$

relating a, b, γ, and k for an infinite waveguide. The reader is referred to [6, 7, 36–40] for a detailed discussion of $\delta\epsilon_{00}$, γ, a, and b. (Here we just note that $\gamma = \int d^2 r_\parallel \delta\epsilon(\vec{r}_\parallel)$, $a = \int d^2 r_\parallel G(0, \vec{r}_\parallel)\delta\epsilon(\vec{r}_\parallel)/\gamma$, and $b = \int d^2 r_\parallel G(a_w\hat{i}, \vec{r}_\parallel)\delta\epsilon(\vec{r}_\parallel)/\gamma$ where the integrals are taken over a primitive lattice cell of the waveguide, i.e., located at the origin of coordinates and a_w in the lattice constant of the waveguide.) For the case of a Kerr nonlinear interaction (1) gives the nonlinear difference equations

$$\begin{aligned} E_{n,0} = \gamma\Big\{&a\left(1+\lambda\left|E_{n,0}\right|^2\right)E_{n,0} \\ &+ b\Big[\left(1+\lambda\left|E_{n+1,0}\right|^2\right)E_{n+1,0} \\ &\quad+ \left(1+\lambda\left|E_{n-1,0}\right|^2\right)E_{n-1,0}\Big]\Big\}. \end{aligned} \tag{4}$$

These reduce in the limit that $\lambda = 0$ to (2) for the waveguide of linear media.

In the solutions given later, a and b are evaluated at a frequency, ω, in the stop band of a specific realization of a square lattice photonic crystal studied elsewhere [7, 37, 38]. The numerical values of a, b, and ω are taken from [7, 37, 38] for the particular bulk photonic crystal that was used in the studies presented in [6, 7, 36–40]. These have frequency $\omega a_0/2\pi c = 0.440$ with wavenumber $k = 2.5$ and the relevant parameters of a and b for our considerations give the ratio $b/a = 0.0869$. This ratio is all that is needed to generate the data presented later in the figures of this paper. The reader is referred to the above cited works (and in particular [7, 37, 38]) for the details of the photonic crystal geometry and the generation of the numerical values of the a's and b's.

3. MULTIPLE OFF-CHANNEL FEATURES AND OFF-CHANNEL FEATURES WITH MULTIPLE RESONANCES

In this section problems involving the resonant interaction of waveguide modes with localized modes bound on multiple off-channel features formed of Kerr nonlinear media and the resonant transmission of guided modes as they scatter from multiple off-channel bound modes are treated. These are generalizations of the problem treated in [7] of a guided mode interacting with a single off-channel site of Kerr nonlinear media and the problem treated in [13] of two guided modes at different frequencies interacting with a single off-channel site of Kerr nonlinear media. In the following, comparisons are made of our present results with those from the earlier studies. The off-channel features in these studies are all taken to be weakly interacting with the waveguide modes. They are weakly interacting in the sense that the off-channel features are far enough from the waveguide channel that the modes in the channel only interact with those on the off-channel features at a scattering resonance between the two modes. When the system is outside of the small parameter space that gives resonant scattering, there is little or no interaction between the waveguide modes and the modes on the off-channel features.

3.1. *Multiple off-channel features*

The first system consists of a waveguide of linear dielectric media that weakly interacts with two different off-channel single sites formed from Kerr nonlinear media. The two Kerr sites are of different media so that each site supports its own localized bound mode, and the two modes occur at different frequencies. The two off-channel sites are on opposite sides of the waveguide channel and weakly interact with the same waveguide channel site. For a schematic of the system, the reader is referred to Figure 1(a).

The difference equations for the system in Figure 1(a) are obtained from (1) and (2). Along the bulk of the waveguide channel [7]

$$E_{n,0} = \gamma\left[aE_{n,0} + b\left(E_{n+1,0} + E_{n-1,0}\right)\right] \tag{5}$$

for $|n| > 0$, and the $(0,0)$ channel site is weakly coupled to the two off-channel Kerr sites so that

$$\begin{aligned} E_{0,0} = {} & \gamma\left[aE_{0,0} + b\left(E_{1,0} + E_{-1,0}\right)\right] \\ & + c\left[\gamma_p\left(1 + \lambda\left|E_{0,m}\right|^2\right)E_{0,m} + \gamma_p'\left(1 + \lambda\left|E_{0,-m}\right|^2\right)E_{0,-m}\right]. \end{aligned} \tag{6}$$

In (6), the off-channel sites are at $(0, m)$ and $(0, -m)$ for m a positive integer, c describes the weak coupling of the off-channel sites to the waveguide, and γ_p and γ_p' denote the γ values in (4) for the two different Kerr nonlinear sites. (Note that the notation γ, γ_p, and γ_p' is introduced to distinguish between the values of these parameters in the linear dielectric media of the waveguide (2) and the Kerr nonlinear dielectric media of the two different off-channel sites (4).) For the Kerr sites at $(0, m)$ and $(0, -m)$

$$E_{0,m} = \gamma_p a\left(1 + \lambda\left|E_{0,m}\right|^2\right)E_{0,m} + \gamma c E_{0,0}, \tag{7}$$

$$E_{0,-m} = \gamma_p' a\left(1 + \lambda\left|E_{0,-m}\right|^2\right)E_{0,-m} + \gamma c E_{0,0}. \tag{8}$$

The transmission coefficient of a guided mode incident from infinity onto the Kerr features is calculated using the methods in [7]. The transmission coefficient, T, is given by

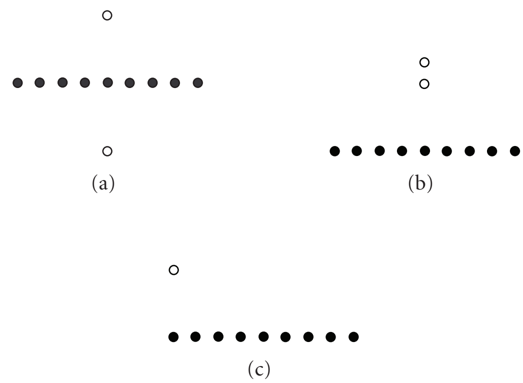

Figure 1: Schematic plots. (a) A straight infinitely long waveguide of linear dielectric media (closed circles) and two off-channel Kerr impurities (open circles). The two off-channel sites can be of different Kerr media. (b) A straight infinitely long waveguide of linear dielectric media (closed circles) and a cluster of two off-channel Kerr impurity sites (open circles). The two off-channel sites are of different media. (c) The basis of a periodic waveguide. The closed circles form the main waveguide channel of linear dielectric media and the open circles represent off-channel Kerr sites. In the absence of the off-channel sites the resulting waveguide would be infinite and uniform with the lattice constant of the channel sites equal to the nearest neighbor separation of the closed circles. Note that in all figures only the waveguide channel and off-channel impurity sites are shown. The dielectric cylinders forming the bulk of the photonic crystal are not shown. See [7, 37] for more details regarding the geometry of the systems used to provide the numerical illustrations of the theory presented in this paper.

$$T = \frac{4\sin^2 k}{4\sin^2 k + \left[(c/b)\left(1 + 2(b/a)\cos k\right)\left(r - \gamma c + r_1 - \gamma c\right)\right]^2}, \tag{9}$$

where r is a solution of

$$0 = \lambda|t|^2|r|^2 r + \left(1 - \frac{1}{\gamma_p a}\right)r + \frac{\gamma c}{\gamma_p a} \tag{10}$$

and r_1 is a solution of

$$0 = \lambda|t|^2\left|r_1\right|^2 r_1 + \left(1 - \frac{1}{\gamma_p' a}\right)r_1 + \frac{\gamma c}{\gamma_p' a}. \tag{11}$$

Here k is the wavenumber of the guided mode for a plane wave form $E_{n,0} \propto e^{ikn}$ in the waveguide channel, t is the amplitude of the transmitted wave, and from (3) $\gamma a = 1/(1 + 2(b/a)\cos k)$ for the linear media waveguide. In the limit that $\gamma_p' = 0$ (9) through (11) reduce to the solution given in (7) of [7] for a single off-channel Kerr site.

Figure 2 presents results for the guided mode transmission in the system of Figure 1(a). To facilitate a comparison with the results from [7] for a single off-channel site, the parameters a, b, c, t, and λ were taken from [7]. A focus of our presentation is on the variety and types of behavior the system can display. The values of the parameters a and b were

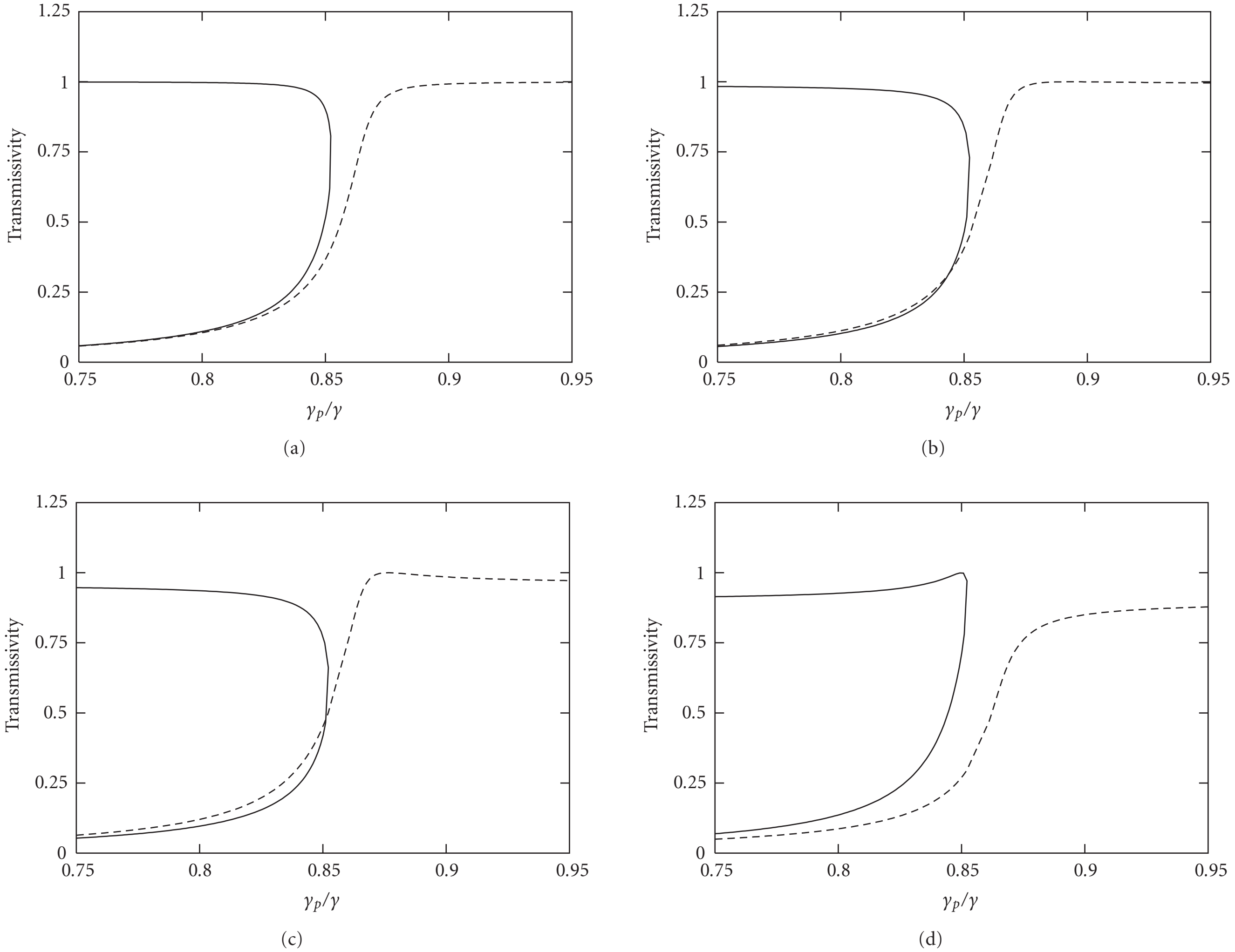

FIGURE 2: Plot of the transmission coefficient of the guided mode versus γ_p/γ. (a) The system of a waveguide weakly interacting with a single off-channel site (i.e., the system in Figure 1(a) with the lower open circle absent). (b) The system in Figure 1(a) for $\gamma'_p/\gamma = 0.830$. (c) The system in Figure 1(a) for $\gamma'_p/\gamma = 0.845$. (d) The system in Figure 1(a) for $\gamma'_p/\gamma = 0.870$. The two branches of the bistable transmission are indicated by different line styles.

quoted in Section 2, the parameter c is such that $c/a = 0.02$, and $\lambda|t|^2 = 0.00025$. In Figure 2(a) results are shown from [7] for the case in which only one off-channel site is present. The plot shows the transmission coefficient of a guided mode with $\omega a_0/2\pi c = 0.440$ and $k = 2.5$ plotted versus γ_p/γ, where the equations of the system are obtained from (5) through (7) above, taking $\gamma'_p = 0$. The nonlinearity of the off-channel site most affects the system when the transmission coefficient departs significantly from unity. As seen in the plot, the resonant scattering of the guided mode with the bound state mode occurs in the linear limit at $\gamma_p/\gamma \approx 0.861$, and below this resonance the system exhibits bistability. In Figure 2(b) results are shown for the transmission coefficient versus γ_p/γ for the case in which the off-channel site at $(0, -m)$, described by γ'_p, is set to $\gamma'_p/\gamma = 0.830$. This is below the resonance at $\gamma_p/\gamma \approx 0.861$. For this parameterization the field on the $(0, -m)$ site has three solutions, and the solution used to make the plot is that with the smallest absolute field magnitude. This corresponds to the curve closest to unity in Figure 2(a). The presence of the small field on the $(0, -m)$ site mainly affects the left branch of the guided mode transmission coefficient, sliding it down on the plot so that it crosses over the curve forming the right branch of the transmission coefficient. In Figure 2(c) results are shown for the system in Figure 2(b), but now for the case in which $\gamma'_p/\gamma = 0.845$. This ratio of γ'_p/γ is below and closer to the resonance at $\gamma_p/\gamma \approx 0.861$ and the mode associated with the off-channel site is on the same branch of modal solutions as that in Figure 2(b). As a consequence of the increased proximity of the two resonances the transmission is enhanced to unity at $\gamma_p/\gamma = 0.877$. In Figure 2(d) results are shown for the system as in Figure 2(b) for the case in which $\gamma'_p/\gamma = 0.870$. This is above the resonance feature at $\gamma_p/\gamma = 0.861$. Now the field on the $(0, -m)$ site has only one solution. The effects of the $(0, -m)$ site are observed in both branches of the transmission coefficient curves. While the right branch of the curve is shifted downward, it is interesting to see that the left branch is both shifted and distorted in such a manner that the transmission of the guided mode is enhanced toward unity near $\gamma_p/\gamma = 0.849$. The interaction of the two nonlinear resonances leads to an enhancement of the guided mode transmission in selected regions near its resonant interaction with the $(0, m)$ site. This is an example of two resonances in the system that act collectively to enhance the over all transmission of a guided mode along the waveguide.

3.2. Single off-channel features that support multiple localized bound modes

Next consider an off-channel feature that supports two localized bound state modes at different frequencies. The feature consists of two neighboring sites formed of different impurity media, and there is a weak interaction with the waveguide leading to resonant scattering of guided modes with the multiple set of modes (i.e., at different frequencies) on the off-channel feature. The reader is referred to Figure 1(b) for a schematic representation of the system.

The bulk of the waveguide is again described by (5), but at the $(0, 0)$ channel site in place of (6) we have

$$E_{0,0} = \gamma[aE_{0,0} + b(E_{1,0} + E_{-1,0})] + \gamma_p c(1 + \lambda |E_{0,m}|^2)E_{0,m}. \tag{12}$$

In addition (7) and (8) are replaced by

$$\begin{aligned} E_{0,m} &= \gamma_p a(1 + \lambda |E_{o,m}|^2)E_{0,m} + \gamma c E_{0,0} \\ &\quad + \gamma'_p b(1 + \lambda' |E_{0,m+1}|^2)E_{0,m+1}, \end{aligned} \tag{13}$$

$$\begin{aligned} E_{0,m+1} &= \gamma'_p a(1 + \lambda' |E_{0,m+1}|^2)E_{0,m+1} \\ &\quad + \gamma_p b(1 + \lambda |E_{0,m}|^2)E_{0,m} \end{aligned} \tag{14}$$

for the two off-channel sites at $(0, m)$ and $(0, m + 1)$.

A simplification can be made in (12) through (14) by taking $\lambda' = 0$. This removes the nonlinearity from the $(0, m+1)$ site while retaining the nonlinearity on the $(0, m)$ site. It does not affect the qualitative behaviors observed in the results presented below. In the following, the focus will be on the nonlinear resonance at the $(0, m)$ site as affected by the $(0, m + 1)$ site, composed of different media from $(0, m)$ and the rest of the photonic crystal waveguide. Solving (5) and (12) through (14) in this limit for a guided mode incident from infinity onto the off-channel features, the transmission coefficient of the guided mode is given by

$$T = \frac{4\sin^2 k}{4\sin^2 k + [(c/b)(1 + 2(b/a)\cos k)(1/f)(r - \gamma c)]^2}, \tag{15}$$

where

$$f = 1 + \frac{\gamma'_p \gamma_p b^2}{\gamma_p a(1 - \gamma'_p a)}. \tag{16}$$

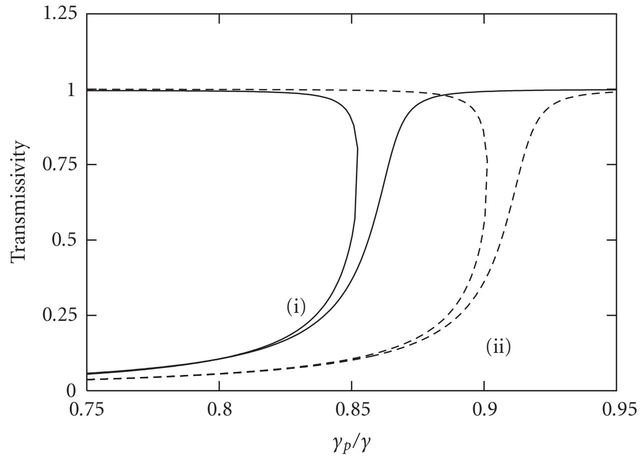

Figure 3: Plot of the transmission coefficient of the guided mode versus γ_p/γ for the system in Figure 1(b). Curves labeled (i) are from Figure 2(b) in [7] and (ii) are for $\gamma'_p = \gamma$.

Here k is the wavenumber of the guide mode, and r is a solution of

$$0 = \lambda |t|^2 |r|^2 r + \left(1 - \frac{1}{\gamma_p a f}\right) r + \frac{\gamma c}{\gamma_p a f}, \tag{17}$$

where t is the amplitude of the transmitted part of the guided mode. Notice that in the case that $f = 1$ (15) through (17) reduce to the single site limit discussed in [7]. From (16) it is seen that f differs significantly from one only near the resonance conditions of the $(0, m + 1)$ site, that is, for $\gamma'_p a \approx 1$. In the limit that f diverges at this resonance, the transmission coefficient becomes unity. An additional interesting limit occurs at $\gamma'_p a = 1/[1 - b^2/a^2]$. Here $f = 0$ so that $T = 0$.

In Figure 3, results are presented for the transmission coefficient versus γ_p/γ for $\lambda |t|^2 = 0.00025$, $k = 2.5$, $c/a = 0.02$, and $\lambda' |t|^2 = 0$. This illustrates the behavior of the transmission coefficient in the presence of modes modified by the impurity material on the $(0, m + 1)$ site. One set of curves are the results in Figure 2(b) of [7] for the transmission of the guided mode when there is only one off-channel site, that is, only the $(0, m)$ site. The other curves are for the case in which there is a second off-channel site present with $\gamma'_p = \gamma$. The presence of the second site is seen, in this instance, to shift the resonance upward. A general idea of the effects of γ'_p on the resonant transmission of the guided modes can be obtained by plotting the values of γ_p/γ at which the guided mode is fifty percent transmitted versus γ'_p/γ. This occurs for two values of γ_p/γ (one above and one below the resonance) and gives an idea as to the shift of the resonance with changing γ'_p/γ. In Figure 4(a) a plot of these values is shown, where the upper values are indicated by x's and the lower values by +'s. At $\gamma'_p/\gamma = 0.861$ (i.e., $\gamma'_p a = 1$) a single off-channel site at $(0, m + 1)$ itself supports a bound state. The presence of this is observed in the plot as a type of asymptotic resonant behavior near $\gamma'_p/\gamma = 0.861$. Figure 4(b) shows the values of f versus γ'_p/γ associated with the plot in Figure 4(a). Again the neighborhood of $\gamma'_p/\gamma = 0.861$ displays a $\gamma'_p a = 1$ resonance in f.

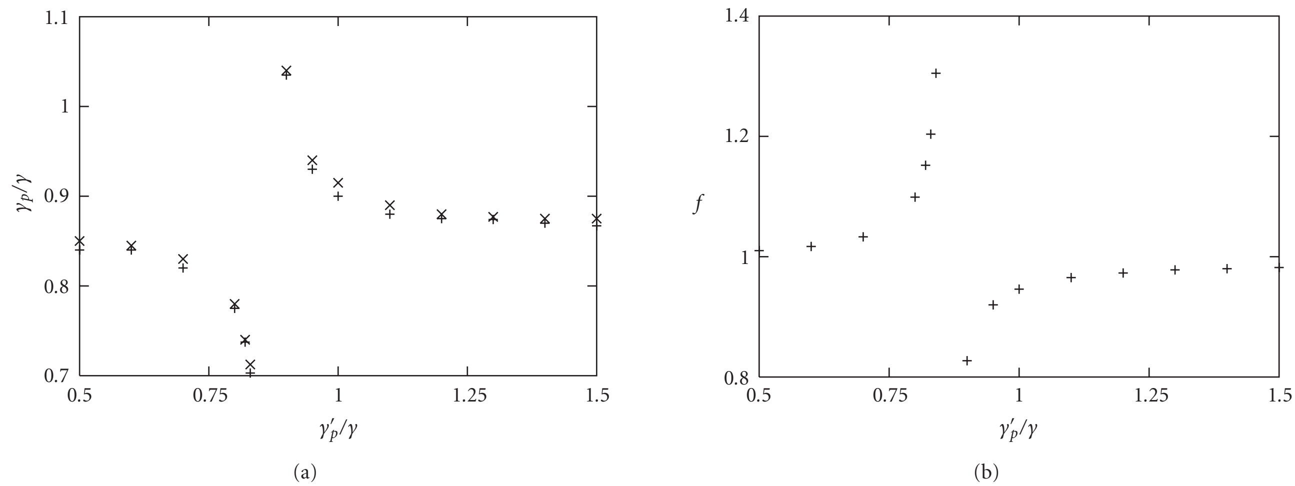

FIGURE 4: (a) Plot of γ_p/γ versus γ'_p/γ showing for each γ'_p/γ the two solutions of γ_p/γ at which the transmission is 50%. The upper solution is denoted by x and the lower solution by $+$. (b) Plot of f versus γ'_p/γ.

4. PERIODIC SYSTEMS AND CONCLUSIONS

The above results can be generalized to treat waveguides with periodic off-channel features. Consider a single off-channel site weakly coupled to a waveguide of linear dielectric media and replicated at fixed intervals along the waveguide channel. A schematic of the basis of the periodic system is given in Figure 1(c) and consists of n waveguide channel sites and one off-channel Kerr site. The bulk of the waveguide channel is described by (2) excepted at the periodically occurring coupling sites of the off-channel features, labeled $ln+1$ where l runs over consecutive integers. At these sites

$$\begin{aligned} E_{ln+1,0} &= \gamma\left[aE_{ln+1,0} + b\left(E_{ln+2,0} + E_{ln,0}\right)\right] \\ &\quad + \gamma_p c\left(1 + \lambda\left|E_{ln+1,m}\right|^2\right)E_{ln+1,m}, \\ E_{ln+1,m} &= \gamma_p a\left(1 + \lambda\left|E_{ln+1,m}\right|^2\right)E_{ln+1,m} + \gamma c E_{ln+1,0}. \end{aligned} \tag{18}$$

Here m is an integer labeling the vertical position of the off-channel sites, and the other notation is as in the previous sections. Upon removing the off-channel site, then, the system reverts to the infinite waveguide described by (2).

We look for solutions of the difference equations in the form of Bloch waves of wave vector k such that the Kerr dielectric is constant throughout the periodicity of the lattice. Labeling the field amplitudes at the black basis sites in Figure 1(c) from left to right as $d_1, d_2, \dots, d_n$, and the off-channel Kerr site by e we find

$$d_j = G_j(k,\omega)\frac{c}{a}\left(e - \gamma c d_1\right), \tag{19}$$

$$0 = \gamma_p a\lambda|e|^2 e + \left(\gamma_p a - 1\right)e + \gamma c d_1, \tag{20}$$

where $j = 1, 2, 3, \dots, n$. These equations determine the site amplitudes e and d_j and the values of γ_p for which solutions exist for specified γ, k, and ω. In (19)

$$G_p(k,\omega) = \frac{1}{n}\sum_{r=1}^{n}\frac{e^{i(p-1)(k+2\pi r/n)}}{1 - \gamma a - 2\gamma b\cos(k + 2\pi r/n)}, \tag{21}$$

where $k = 2\pi s/Nn$ for s an integer is the wave vector of the Bloch wave, and N is the number of primitive lattice cells in the system. The ω dependence of $G_p(k,\omega)$ enters through the ω dependence of a and b.

In the linear limit (i.e., $\lambda = 0$), (19) through (21) are easily treated. The values of γ_p needed to support waves with wave vector k and frequency ω for a specified γ are obtained from the algebraic conditions for a solution of (19) and (20) and are given by

$$\gamma_p a = \left[1 + \frac{c}{a}\gamma c G_1(k,\omega)\right]^{-1}. \tag{22}$$

Upon setting ω in a stop band and choosing k and γ, (22) gives γ_p for a Bloch waveguide mode propagating along the periodic waveguide. It is interesting to note from (21) and (22) that the properties of the system are particularly susceptible to changes in γ for γ near the poles of the terms comprising the sum over n in (21). In these regions small changes in the system parameters show up as large modulations of the Bloch modes in the waveguide.

A similar analysis to that for the linear media system can be made for the fully nonlinear equations. In this analysis the field amplitude on the off-channel site, e, is determined as a solution of

$$\lambda|e|^2 e = -\frac{1}{\gamma_p a}\left[\gamma_p a - \left(1 + \frac{c^2}{a}\gamma G_1(k,\omega)\right)^{-1}\right]e. \tag{23}$$

This again determines γ_p for solutions of e and d_j in (19) and (20) to exist with given k, ω, and γ. In the region of weak nonlinearity the nonlinear solutions can be treated as perturbations of the linear media limit. These are of particular interest for γ near the above discussed poles. Near these poles it is possible to tune the modal dispersion relations by adjusting the field intensity of the Block modes. The change in dielectric due to the Kerr field dependences of the dielectric media shows up as a change in the effective value of γ.

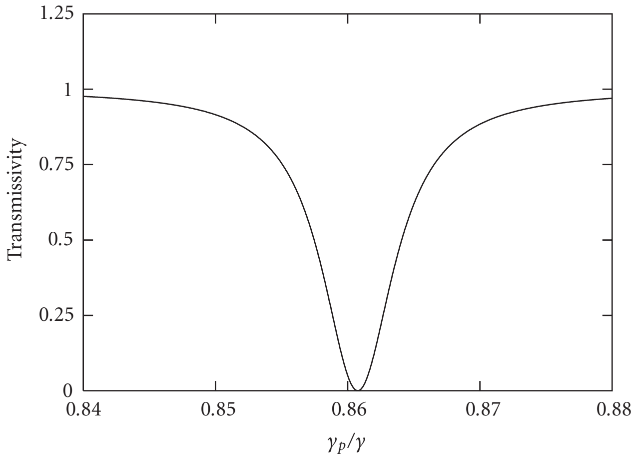

Figure 5: Plot of the system in Figure 2(a) for the case in which $\lambda = 0$, that is, an off-channel site of linear dielectric media.

In conclusion, a theory is presented illustrating some of the effects that can be developed in guided mode transmissions in which the guided modes interact with multiple off-channel localized modes occurring at different frequencies. The off-channel features are of Kerr nonlinear media so that the effects are complicated due to the nonlinearity of the interacting systems. Small shifts in the bound modes are found, in certain circumstances, to enhance the transmission of the guided modes within the waveguide channel. This is a form of induced transparency through the interaction of the guided mode with multiple resonances in the system. In the case where an off-channel feature binds multiple modes, the guided mode transmission resonance can be tuned by varying the proximity of the multiple modes. Periodic arrays of off-channel features formed of Kerr nonlinear media can be used to adjust the dispersion relations of waveguide modes. These adjustments arise from the field dependent dielectric properties of the off-channel sites which can give strong enhancements near the poles in (21).

The theory that we have used in our studies is based on a difference equations formulation which is essentially exact within the limits discussed in Section 2. Alternative methodologies for treating photonic crystals and photonic crystal waveguides are (1) finite-difference time-domain computer simulations which write Maxwell's equations in terms of difference equations in space and time, solving the resulting set of equations by computer [54–56], (2) transfer matrix formulations which solve the electrodynamic boundary value equations using matrix techniques [57], and (3) finite element boundary methods which are based on the numerical application of variational principles [58]. These later methods (i.e., listed 1, 2, 3 above) are numerical in implementation and thus are subject to numerical errors. Examples of their use can be found in many of the papers cited in Section 1.

The results of the difference equation method used in this paper are generally found to be in qualitative agreement with results based on alternative simulation methodologies, when applied to similar geometries of photonic crystal waveguides interacting with off-channel impurity features. (Note that often only a qualitative comparison can be made between results from two different methods because the regions of validity of the different methods are different.) For example, in the case of a waveguide interacting with a single off-channel impurity site formed of linear dielectric media (i.e., the $\lambda = 0$ limit of the results in Figure 2(a)) a single dip is observed in the transmission at resonance. A plot of the transmission of the system in Figure 2(a) for $\lambda = 0$ is shown in Figure 5. The single dip in transmission in Figure 5 is qualitatively the same as the transmission dip observed by Noda et al. in their computer simulation data presented in [3, 4] for an off-channel linear media dielectric impurity interacting with photonic crystal waveguide modes propagating in a photonic crystal slab. The origins of the effects in the two similar systems is the same, that is, resonant scattering of the guided mode with the mode bound to the impurity site. The geometric parameters in the two systems are different (one is a two-dimensional photonic crystal and the other is a photonic crystal slab), but the two-dimensional patterning in the two systems is found to display similar types of resonant effects. (Note: It is emphasized that the two-dimensional photonic crystal formed of infinite dielectric cylinders is different from the photonic crystal slab studied by Noda et al. The photonic crystal slab, which is often studied by experimentalists, can have lossy guided and defect modes [59]. In spite of this the off-channel resonant effects have been observed both theoretically and experimentally in the photonic crystal slab geometry [3, 4] and are similar to those found in the two-dimensional photonic crystal formed of infinite cylinders.) Likewise, for the modes of the photonic crystal waveguide interacting with a single Kerr nonlinear media off-channel impurity site, that is, the system studied in Figure 2(a). The qualitative bistability (i.e., region of multiple solutions for the transmission coefficients) exhibited in the transmission versus γ_p/γ in Figure 2(a) is the same as that found in the results for the transmission presented in the work of Cowen and Young [14] and more recently in some studies of mechanical systems [60] and other optical systems [61–63]. The origins of the bistability in these three different nonlinear systems with similar geometries is the same, that is, resonant interactions and nonlinearity. As an example of another generalized optical system, the work of Miroshnichenko et al. [62] treated the transmission of waveguide modes past a single site of Kerr nonlinear media in which the single site has interactions with many sites along the waveguide channel. This system generalizes the Kerr single site problems in which the Kerr site interacts with only a single site on the waveguide. As one of the properties treated in the paper, a Fano resonance in the transmission and a region of bistable transmission are exhibited by the system that are qualitatively similar to those observed in [7, 14], and Figure 2(a). These comparisons lead us to conclude that the results for the systems presented in this paper will be not only useful for the systems studied in this paper but generalize to similar optical and mechanical systems that will be treated in the future by the other methods discussed above. We hope that this paper will stimulate such studies.

It is interesting also to note that there have been some recent experiments on systems consisting of waveguides interacting with Kerr nonlinear site impurities [12, 64]. In the experiments in [12] the Kerr impurity is located within the waveguide channel, and it is used to cause the interaction of waveguide modes with two different frequencies. This allows the two modes to affect each others transmission down the waveguide. In [64] the system again involves the interaction of modes with two different frequencies but is not based on photonic crystal technology. It is hoped that with the successful experiments on the systems in [12, 64] and the results presented in this paper will encourage people to experimentally study the systems proposed in this paper.

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