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Research Article

Nonlinear Dynamics of Sea Clutter

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We review experimental evidence for the nonlinearity of sea clutter and the role of the z-parameter or Mann-Whitney rank-sum statistic in quantifying this nonlinear behavior in the context of a hybrid AM/FM model for sea clutter, viewed as a cyclostationary process. An independent theoretical derivation of the stochastic dynamics of radar scattering in a sea clutter environment, in terms of a pair of coupled stochastic differential equations for the received envelope and radar cross-section (RCS), enables the identification of nonlinearity in terms of the shape parameter for the RCS. We are led to conclude that, from both experimental and theoretical points of view, the dynamics of sea clutter are nonlinear with a consistent degree of nonlinearity that is determined by the sea state.

1. INTRODUCTION

Haykin et al. [1] advocated a state-space formalism for the processing of radar signals in the presence of sea clutter (i.e., radar backscatter from an ocean surface). Such a model not only accounts for the temporal dimension of sea clutter in an explicit manner but also for its statistical characterization. Basic to this formalism is whether the underlying dynamics of sea clutter are linear or nonlinear.

In the detailed experimental study reported in Haykin et al., [1] it was also demonstrated that sea clutter is a nonlinear dynamic process, with the degree of nonlinearity increasing as the "sea state" becomes higher. The conclusion reached on the nonlinearity of sea clutter was based on two premises, using real-life data collected with an instrument-quality coherent radar system.

(1) The characterization of sea clutter embodies two forms of continuous-wave modulation:

(i) amplitude modulation (AM), which is linear, and

(ii) frequency modulation (FM), which is nonlinear.

The latter phenomenon is responsible for the nonlinearity of sea clutter.

(2) The z-parameter, denoting the Mann-Whitney rank-sum statistic, is less than the special value -3, which is a strong indicator of nonlinearity.

With regards to point 1, it is also noteworthy that in another study that focussed on the spectral characterization of sea clutter using the Loève transform [2], it was discovered for the first time that sea clutter is a cyclostationary process. Cyclostationarity is ordinarily associated with modulation. But knowing that sea clutter is cyclostationary, it does not tell us the type of modulation involved in the characterization of its waveform.

In this paper, we expand on the characterization of sea clutter as a nonlinear dynamic process, using a principled theoretical approach. In particular, the approach is rooted in stochastic differential equation (SDE) theory. The issue of the dynamics of radar scattering in a sea clutter environment has been addressed in the literature independently from both theoretical and experimental points of view. Perhaps most notably in the former case, Field & Tough [3, 4] develop a theoretical basis for the dynamics which is demonstrated to agree with experimental data to a remarkable degree of accuracy. In the latter case, Haykin et al. [1] study experimental data to motivate a line of argument leading to the conclusion that sea clutter is inherently nonlinear (and indeed possibly chaotic). In the current paper, we bring these two independent lines of development together in a consistent way in order to establish the nonlinear nature of sea clutter from both physical and mathematical viewpoints. More precisely, the scattering dynamics can be derived from first principles in terms of a pair of stochastic differential equations (SDEs) for the received envelope and the radar cross-section (RCS) that feature a nonlinear coupling and encode the statistical character of the sea state in terms of a certain "shape parameter." Examination of the differentiable parts in this system of SDEs reveals a corresponding "noise-free skeleton," that is, a nonlinear vector process, with a degree of nonlinearity dependent on the shape parameter in a manner consistent with that shown experimentally by Haykin and coworkers. This significant development affirms the case for the nonlinear character of radar sea clutter.

The paper is organized as follows. Section 2 provides a summary of the experimental study that led to the formulation of a hybrid AM/FM model, and the conclusion that sea clutter is a nonlinear dynamic process. Section 3 summarizes the essential ingredients of SDE theory necessary for the basic interpretation of the SDE dynamics of radar sea clutter. In Section 4 we apply this formalism to establish the nonlinear character of the stochastic dynamics of the vector process consisting of the radar cross-section (RCS) and resultant back-scattered amplitude or "received envelope." This is achieved from first principles via an extended random walk model. The extent of the nonlinearity in the resulting SDE description is quantified in terms of a certain "shape parameter" (the relative variance in the RCS, minus one) that encodes the sea state. We conclude in Section 5 with a discussion of the interplay between the two independent lines of enquiry that lead to the common conclusions concerning the nonlinear character of radar sea clutter. We also indicate how our results may suggest which types of experiments to perform to further substantiate and enhance the theoretical framework, and discuss future prospects for the investigation of chaotic dynamics.

We refer the reader also to the recent book by Haykin [5], where the experimental results of the current paper are mentioned in the broader context of adaptive radar signal processing.

2. THE HYBRID AM/FM MODEL OF SEA CLUTTER

In an independent study reported in Gini & Greco [6], sea clutter was viewed as a fast "speckle" process multiplied by a "texture" component that represents the slowly varying power level of the sea clutter signal; such a model is perceptually satisfying. This is known as the K-distribution model and is widely used in the literature. It is the model that we will be concerned with in our dynamical description of sea clutter throughout the paper. The slow variation of the sea clutter power level was attributed to the large ocean waves passing through the observed ocean patch. The speckle was modeled as a stationary compound complex Gaussian process, and the texture was modeled as a harmonic process.

Inspired by the Gino-Greco model of sea clutter, Haykin and coworkers carried out an extensive physical study of sea clutter collected by the instrument quality coherent IPIX radar, where the radar data were recorded on the East Coast of Canada [1]. In that paper, it was demonstrated that amplitude modulation and frequency modulation play important roles in the waveform description of sea clutter. The hybrid AM/FM model of sea clutter has been substantiated further in Greco & Gini [7].

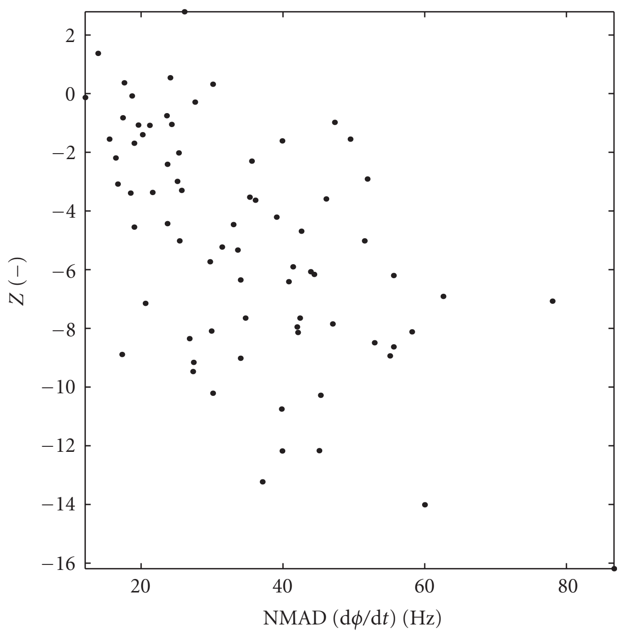

FIGURE 1: Z value versus NMAD ($\dot{\phi}$), computed for 78 data sets measured by the IPIX radar at various experimental conditions.

To explain the physical presence of modulation in sea clutter, we observe that when a large wave passes across a patch of the ocean surface, it will first accelerate and then decelerate the water's motion on the ocean surface. The continuous tilting of the ocean surface by the waves gives rise to amplitude modulation.

Moreover, the ocean wave will cause a cyclic motion of the instantaneous velocity of scatterers on the ocean surface, thereby giving rise to frequency modulation as another characteristic of the sea clutter waveform. When the mean velocity of the scatterers is high at a given instant of time, then the spectral spread (i.e., the bandwidth occupied by the frequency modulation) around that mean is correspondingly high, which is in perfect accord with modulation theory.

It is well known that, unlike amplitude modulation, frequency modulation is a nonlinear process [8]. Therefore, the presence of frequency modulation in the physical behavior of the sea clutter waveform leads us to hypothesize that sea clutter is a nonlinear dynamic process. To validate this hypothesis, Haykin et al. use 78 different coherent radar data sets to compute the z-parameter, which denotes the Mann-Whitney rank-sum statistic [9]. The results of this test are reproduced in Figure 1, where the z-parameter is plotted against the spectral width modulation.

A value of z less than -3 is considered to be a strong reason for rejecting the null hypothesis that the sea clutter data under test can be described by linearly correlated noise. In Figure 1, we clearly see that the large majority of the experimental points lie below $z = -3$. Those points were in actual fact representative of high sea states. Based on these experimental results, Haykin et al. concluded that sea clutter is indeed a nonlinear dynamic process, with the degree of nonlinearity increasing with increasing sea state.

3. ELEMENTS OF SDE THEORY

Stochastic differential equation (SDE) theory has significant implications for statistical signal processing. It has recently proven successful in this context in the application to radar sea clutter [3, 4, 10]. In a more general physical context, including optical propagation, the stochastic calculus has led to substantial new theoretical developments in the subject of electromagnetic scattering from random media [11–13].

More recently, SDE techniques have been applied to wireless channel modelling [14] to include the effects of phase fluctuations in multipath reception. The fact that similar techniques are applicable to both the radar backscattering and wireless propagation problems stems from the fact that each is multipath in nature, with the only essential difference being that for radar the receiver and transmitter are colocated. This latter feature, however, does not affect the structure of the mathematical model used to describe the resulting amplitude signal.

In this paper, we will consider the RCS and received envelope processes to evolve according to the dynamics governed by a stochastic differential equation (SDE). In the context of the radar cross-section, such dynamics arise from taking the continuum limit of a generic population dynamic model for the (discrete) number of component scatterers. For the scattered radiation, the origin of the SDE dynamics lies in the behavior of the component phases which are taken to evolve in time according to a Wiener process W_t [15] on a suitable (Rayleigh) timescale. Thus, as we will see explicitly in Section 3, we are able to represent the essential ingredients of the radar back-scatter temporally, in the form of a set of continuous time SDEs, the basic mathematics of which we now introduce. Consider an arbitrary continuous time stochastic process, say q_t, which evolves in time according to

$$\mathrm{d}q_t = b_t\mathrm{d}t + \sigma_t\mathrm{d}W_t. \tag{1}$$

Herein, b_t is a random process referred to as the "drift," and represents the ordinary time derivative of the process q_t in the case that σ vanishes. The quantity σ_t, on the other hand, is the amplitude of the noise or fluctuating part of q_t, in general a random process, and referred to as the "stochastic volatility" of q_t. In the cases we study, it will become apparent that $b_t = b(t, q_t)$ and $\sigma_t = \sigma(t, q_t)$ for some specific functions b, σ, and accordingly the process q_t is called a "diffusion."

In contrast to the part of $\mathrm{d}q_t$ containing b_t, the σ_t term contributes an essential part to q_t that is not differentiable, in the ordinary sense that $\mathrm{d}\mathrm{d}t$ is well-defined. Nevertheless, the (Ito) stochastic differential of q_t can be well defined.

In the engineering physics literature, one is perhaps more familiar with the "Langevin" equation for the time derivative

$$\frac{\mathrm{d}q}{\mathrm{d}t} = b_t + \sigma_t\Gamma_t \tag{2}$$

in which Γ_t is the familiar white noise process and Γ_t has the autocorrelation property $\langle\Gamma_t\Gamma_{t'} = \delta(t - t')\rangle$. For our purposes, it will be sufficient to understand and interpret from the dynamical equations for the RCS and the received radar amplitude that, in a discrete-time setting,

$$\delta q_{t_i} = b_{t_i}\delta t + \sigma_{t_i}n_{t_i}\delta t^{1/2}, \tag{3}$$

where $\{t_i\}$ is a discrete set of observation times, $\delta t = t_{i+1} - t_i$, and $\{n_{t_i}\}$ are a collection of independent $\mathcal{N}(0, 1)$ random variables. Thus, in terms of the Wiener process, we make the discrete time identification $\delta W_{t_i} = n_{t_i}\delta t^{1/2}$. Moving from (2) to (3), the same drift and volatility coefficients become sampled at this discrete set of times.

Then the above properties of q and its time derivative are evident. The essential distinguishing feature of the Ito stochastic differential is that it refers to an integration of (3) in which the volatility is to be evaluated at the left most point of each time subinterval (see [15] for a detailed rigourous account).

The essence of the approach taken is therefore to postulate the exact dynamics in continuous time, and then sample at a discrete set of times corresponding to the physical measurements. This procedure is inevitably more precise than an attempt at a model that is fundamentally discrete time in nature, since the physical observables are not quantized in time.

4. NONLINEAR DYNAMICS FROM SDE THEORY

We will assume the (dynamical extension of the) random walk model for the resultant back-scattered amplitude or "received envelope"

$$\mathcal{E}_t^{(N)} = \sum_{j=1}^{N} \overbrace{a_j \exp[i\varphi_t^{(j)}]}^{s^{(j)}} \tag{4}$$

with (fluctuating) population size N, random phasor step $s^{(j)}$, "form factors" a_j, and component phases $\varphi^{(j)}$, wherein the collection $\{N, a_j, \varphi^{(j)}\}$ is assumed to be mutually independent. Our basic dynamical assumption is that the component phases $\varphi_t^{(j)}$ evolve according to a Wiener process on a suitable (Rayleigh) timescale, that is, that $\mathrm{d}\varphi_t^{(j)} = \mathcal{B}\mathrm{d}W_t^{(j)}$, which relation serves to define the constant $\mathcal{B}$.

The key result of relevance to our discussion is obtained by taking the (Ito) stochastic differential of (4), in the limit that N, the number of component scatterers becomes large. Accordingly we introduce the normalized amplitude process $\Psi_t = \mathcal{E}_t^{(N)}/N^{1/2}$, and a continuous valued RCS x_t via $N = \overline{N}x$, where $\overline{N}$ denotes the mean of the discrete scattering population size. In terms of these quantities, we now provide the following coupled stochastic dynamics of the RCS and scattered amplitude/received envelope. (We can express $\Psi_t = I_t + jQ_t$ ($j = \sqrt{-1}$), the familiar sum of its "in-phase" and "quadrature phase" components.)

Proposition 1. *The dynamics of the RCS and received envelope for radar sea clutter, with shape parameter $\nu = \alpha - 1$, are* *given by the following set of nonlinearly coupled SDEs:*

$$\mathrm{d}x_t = \mathcal{A}(\alpha - x_t)\mathrm{d}t + (2\mathcal{A}x_t)^{1/2}\mathrm{d}W_t^{(x)}, \tag{5}$$

$$\frac{\mathrm{d}\Psi_t}{\Psi_t} = \left[\mathcal{A}\left(\frac{2(\alpha - x_t) - 1}{4x_t}\right) - \frac{1}{2}\mathcal{B}\right]\mathrm{d}t + \left(\frac{\mathcal{A}}{2x_t}\right)^{1/2}\mathrm{d}W_t^{(x)} + \frac{\mathcal{B}^{1/2}}{\gamma_t}\mathrm{d}\xi_t, \tag{6}$$

in which γ_t is a unit power Rayleigh process, whose dynamics are obtained by setting x_t equal to a constant of unity and $\mathcal{A} = 0$ in the above system.

This result pertains to (the simulation of) sea clutter from a generic radar system.

Thus, in terms of the familiar K-distribution model for sea clutter, the fast-speckle component is represented by γ_t (or its modulus squared) which is multiplied by a "texture" component, the RCS x_t, according to the product representation $\Psi_t = x_t^{1/2}\gamma_t$. Incidentally, the separation of the radar scattering process into the RCS and received amplitude (or intensity) components in this manner is introduced in a statistical context in Jakeman [16], Jakeman & Tough [17] and developed in a stochastic dynamical context in Field & Tough [3, 4]. (The original proof of this result appears as [4, Proposition 2.1], and we will omit the details of this mathematical derivation which are outside the scope of this paper.)

It is beneficial at this point, in relation to the above proposition, to explain the roles of the various quantities that occur in more familiar radar terminology. The shape parameter ν used in the SDE model is the same as that familiar from the standard K-distribution statistical model of sea clutter. The quantity Ψ is the total radar backscattered *amplitude*, or received envelope, incorporating both speckle and texture components; its modulus squared is equal to the total backscattered intensity, that is taken to be K-distributed. The RCS or texture component is represented by the correlated process x_t.

Thus, the nonlinear SDE for Ψ_t is derived theoretically from first principles beginning with the random walk model for the scattered electric field under the assumption of a uniform phase distribution. (The assumption of a uniform phase distribution can be relaxed, and a corresponding detailed dynamical description in terms of SDEs has been given in [11].) An immediate consequence of this dynamical equation is the "noise-free skeleton", obtained by setting the volatility coefficients of the fluctuating Wiener terms, that is, those containing W_t, equal to zero. Accordingly, the randomness of the process is eliminated and the residual dynamics are deterministic and differentiable. Physically, this corresponds to an evolution conditioned on the current state of the system and then averaged over an ensemble. (In other words, for an Ito process q_t with SDE $\mathrm{d}q_t = b_t\mathrm{d}t + \sigma_t\mathrm{d}W_t$, the ensemble average evolution is determined by $\mathbf{E}|_\mathbf{t}[\mathrm{d}q_t] = b_t\mathrm{d}t$, where $\mathbf{E}|_\mathbf{t}$ denotes the expectation conditional on information at time t.) The concept of the residual noise-free part is explored further below.

This set of coupled stochastic dynamical equations is manifestly *nonlinear* by virtue of the reciprocal term in x_t appearing in the amplitude equation, and only reduces to linear dynamics in the special case that $\mathcal{A}$ vanishes, that is, the scattering cross-section is constant (Rayleigh scattering). It turns out that a natural quantifier of this nonlinearity, in the context of the SDE model for K-distributed noise, is the parameter α appearing in the coupled system of Proposition 1, as discussed below and illustrated in Figure 2.

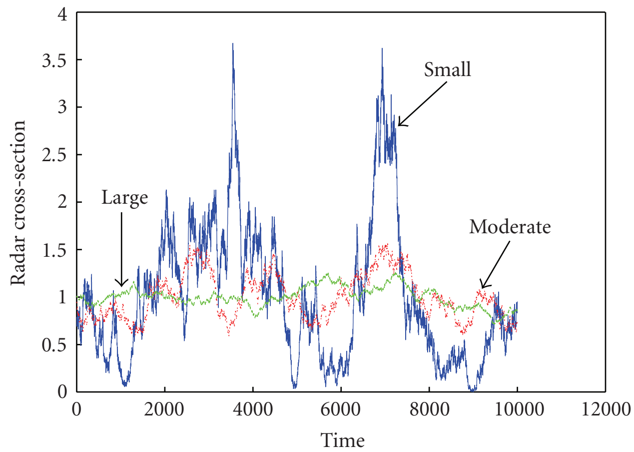

Figure 2: Normalized RCS time-series for low/moderate/high values of the shape parameter; simulated data with $\mathcal{A}\delta t = 0.001$, $\alpha = 1, 10, 100$.

4.1. Radar parameters

In the present context, it is worth remarking on some of the key salient features of the SDE theory, in relation to the sensitivity analysis of sea clutter to certain radar parameters. Most notably, this kind of description is illuminating in respect of the following issues.

4.1.1. Correlation

The constants $\mathcal{A}, \mathcal{B}$ in (5), (6) have the physical dimension of frequency, so that their reciprocals represent correlation timescales for the RCS modulation (texture) and unit power Rayleigh (speckle) components, respectively. The constant $\mathcal{B}$ is electromagnetic in origin with a value $\mathcal{B} \sim c|\mathbf{k}|$, where $\mathbf{k}$ is the wave vector of the carrier and c is the speed of light. In radar situations, the illuminating radiation is such that $\mathcal{A} \ll \mathcal{B}$, with the value of $\mathcal{A}$ being determined as an intrinsic property of the statistics of the scattering surface, independent of the electromagnetic wave. Accordingly, in radar, the correlation time for the RCS is much longer than that of the Rayleigh speckle (cf. also the discussion of amplitude and frequency modulation in Section 5). The pulse frequency of the radar is the reciprocal of δt in the discrete implementation of the coupled system of Proposition 1, and is assumed small compared to the Rayleigh correlation timescale $\mathcal{B}^{-1}$, which amounts to the dimensionless criterion $\mathcal{B}\delta t \ll 1$.

4.1.2. Superposition

In light of the SDE theory, we may argue that the SDE of sea clutter is independent of the amplitude profile of a transmitted pulse, provided the transmit energy is maintained constant. This property, which is derived explicitly in Field [12], is related to the fact that the form factors (i.e., the amplitude weightings) in (4) may be taken as a unity for an asymptotically large population (cf. also [17] where the emergent statistical properties are independent of the choice of form factors).

For a radar pulse of constant amplitude, suppose that the two halves of the pulse have transmit frequencies ω_1 and ω_2. Then we may consider the correlation between the SDE of sea clutter for the two portions as frequency ω_1 increases relative to ω_2. The transmit frequencies are proportional to the Rayleigh constant $\mathcal{B}$ appearing in (6) ($\mathcal{B}\sim c|\mathbf{k}|$, as explained above), and the relationship between the two SDEs, for the two different transmit frequencies, is through (6). The two terms involving the constant $\mathcal{A}$ are the same for both SDEs. On the other hand, the terms involving the Rayleigh constant have different $\mathcal{B}$ values corresponding to the two transmit frequencies. Nevertheless, on physical grounds, the two complex Wiener processes ξ_t for each transmit frequency should be considered perfectly correlated. The reason for this correlation is that the physical origin of the component phase fluctuations $\phi^{(j)}$ is (microscopic) Doppler—the Doppler frequency ratio ω_1/ω_2 is a function of the radial velocity of the jth member of the population, so the micro-Doppler phase shift scales with the transmit frequency; the ξ_t process is the same for any transmit frequency (assuming that these are transmitted simultaneously) as this depends only on the behavior of the component scatterer.

In a similar fashion, consider the simultaneous transmission of two pulses of constant amplitude, with two different frequencies as above, and the resulting SDE of sea clutter received by a common antenna. Since Maxwell's equations of electromagnetism are linear, the resulting Ψ is a linear superposition $\Psi = \kappa_1^{1/2}\Psi^{(1)} + \kappa_2^{1/2}\Psi^{(2)}$, where κ_1, κ_2 are the relative intensities of the two transmit waveforms, normalized so that $\kappa_1 + \kappa_2 = 1$, and $\Psi^{(i)}$ are the constituent complex amplitude processes, both satisfying the SDE (6), with different Rayleigh constants $\mathcal{B}$ corresponding to the two transmit frequencies. Since the beams are simultaneous, the ξ processes are perfectly correlated, with the remaining parts of (6) involving the constant $\mathcal{A}$, the same for both transmit frequencies. Thus, the nonlinear dynamics do not infringe the principle of superposition inherent in Maxwell's equations. (It is necessary to assume here that the scattering populations $N^{(1)}$ and $N^{(2)}$ pertaining to the different transmit frequencies are equal.)

4.1.3. Sea state and polarization

Next, consider the two different copolarizations "HH" and "VV." The SDE theory conveniently represents the spikiness in the RCS of sea clutter due to "HH," versus the noise-like character due to "VV," as follows.

The cross-section SDE (5) emerges as a large N limit of an underlying discrete-valued model for the scattering population, the so-called birth-death-immigration (BDI) model [18], in which α occurs as the ratio of the immigration and birth rates. A property of the continuum limit of this population model, as represented by the SDE (5), is that the distribution of x_t is (univariate) gamma, with parameter α. As a consequence, since the distribution of the modulus amplitude for a given value of the RCS is Rayleigh (as follows from (4) for fixed N), the intensity emerges as being K-distributed (also parameterized by α). Thus, the BDI population model is appropriate to an RCS that generates K-distributed data. Now, for this gamma distribution, we have $\mathrm{Var}[x] = \mathbf{E}[x] = \alpha$. So the absolute magnitude of fluctuations in the RCS, that give rise to the K-distribution for the intensity (as opposed to the Rayleigh "noise-like" distribution), becomes more appreciable as α increases. However, the appropriate theoretical measure of "spikiness" is the *relative* variance R given by

$$R = \frac{\mathrm{Var}[x]}{(\mathbf{E}[x])^2}, \tag{7}$$

($\mathbf{E}$ denotes the expectation functional) which is the physical parameter of interest since it is dimensionless and invariant under rescaling of the RCS. In the case of K-scattering that we consider, R is equal to $1/\alpha$, and therefore the horizontal copolarization "HH" has small α, with larger α for vertical copolarization "VV." The SDE theory explains that if the ratio α of the immigration to birth rates is small, then the sea clutter possesses spikes. It is therefore a natural mathematical, as opposed to a detailed phenomenological, way of encoding this physical property of the sea surface. (However, the SDE theory does not explain why for "HH" polarization one should expect the population to behave this way, the phenomenological reasons for which we do not describe here.) Correspondingly, there are two different K-distributions for the intensity, indexed by different values of the shape parameter $\nu = \alpha - 1$, for the respective polarizations, where α is the SDE parameter appearing in Proposition 1.

The situation as regards the extent of the temporal fluctuations in the RCS for low/moderate/high values of the shape parameter is illustrated in Figure 2, which has been generated independently via a direct numerical integration of (5) according to (3), for various values of the shape encoding parameter α. The figure demonstrates the extreme departures from the mean value for large R, which represents in physical terms sea spikes or glints in the scattering surface. As the sea state settles down to a low value, the (normalized) RCS has small fluctuations away from its mean (unity), so that there is no significant modulation of the Rayleigh scattering time series—in other words, the scattering is of constant local power. We remark also that spikiness should also be more apparent at low grazing angles, represented by corresponding small values of the shape parameter.

As the sea state diminishes, correspondingly in terms of the SDE dynamics, the parameter $\alpha \to \infty$ and the relative variance in the RCS tend to zero. Thus in Figure 2 the nonlinear term becomes less pronounced. Accordingly, as we have seen in Section 2, so does the degree of nonlinearity as measured by the z-parameter, which further substantiates the experimental findings reported in Haykin et al. [1].

Our analysis therefore establishes the precise relationship between the radar shape parameter, its statistical interpretation, and the dynamical SDE theory, via the explicit appearance of α in the coupled system of Proposition 1.

5. DISCUSSION

We have described a detailed analysis of radar sea clutter data, whose primary purpose is to address the presence of nonlinearity, from real experimental data. A natural quantifier for this nonlinearity is the z-parameter or Mann-Whitney rank-sum statistic, which has been successfully applied in the context of a hybrid AM/FM model for sea clutter. The SDE dynamical model of radar sea clutter has also been verified previously to a remarkable degree of accuracy, in terms of real experimental data (see [3, Section 4(b)]). Moreover, an independent theoretical account for such a model was provided in Field & Tough [4], and has served as the basis for other significant developments [11, 12]. As we have seen in Section 4, this stochastic dynamic behavior is inherently nonlinear, due to the broader timescale fluctuations in the RCS. The extent of nonlinearity arises naturally in the SDE description through the relative variance or shape parameter, which encodes the sea state. Thus, from an SDE dynamical perspective, the nonlinear character of radar sea clutter is firmly established, both theoretically and experimentally.

Calculation of the z-parameter is from real data *containing* noise, the latter being akin to the stochastic fluctuating terms present in (5), (6). However, z has the stochastic element removed, that is, it is not a random variable. Accordingly, some ensemble averaging takes place in the calculation of z, and for this purpose, the statistical properties of ergodicity and stationarity are assumed, legitimate over realistic short timescales. In terms of the parameter $\mathcal{A}$ of (5), such timescales are short enough that the assumption of constant $\mathcal{A}$ is valid. Nevertheless, they should be long enough (of the order of $\mathcal{A}^{-1}$) for the fluctuations in the RCS (or equivalently, as we elucidate below, the frequency modulation effect) to be appreciable so that nonlinearity can indeed be detected.

From an engineering physics perspective, the dynamics of sea clutter are perhaps more naturally viewed in terms of amplitude (AM) and frequency modulation (FM). Studies have indicated that the degree of nonlinearity is governed by the extent of FM which, in turn, is more noticeable for higher sea states (i.e., the shape parameter ν is large). To relate this further to the SDE description of Section 4, it is convenient to view the resultant amplitude process Ψ_t in the product representation $\Psi = x^{1/2}y$, in which x is the RCS and y is a unit power Rayleigh process. Then, the AM consists of the fluctuations of y_t (Rayleigh "speckle") which is "frequency" modulated by the RCS process x_t over a much broader timescale. The FM/AM contributions therefore have characteristic frequencies determined by $\mathcal{A}$, $\mathcal{B}$, respectively. With zero FM, that is, x_t constant, the dynamics of the resultant amplitude are rendered linear, according to Proposition 1.

It is worth emphasizing that the roles of α and $\mathcal{A}$, $\mathcal{B}$ are essentially different, both theoretically and in terms of their radar significance. The parameter α determines the associated gamma distribution for the RCS and corresponding intensity K-distribution, and provides a scale invariant measure of the spikiness of the backscatter. On the other hand, the frequency constants $\mathcal{A}$ and $\mathcal{B}$ set the fluctuation timescales of the respective texture and speckle processes, and thus leave the (asymptotic) statistics invariant. In terms of Figure 2, the adjustment of $\mathcal{A}$ can be considered as an amplitude preserving dilation of the time series along the temporal axis, with smaller values of $\mathcal{A}$ yielding longer duration between peaks in the texture component.

Observe that, whereas the FM/AM characteristics of the received envelope do not map to unique stochastic dynamics, conversely the SDE description allows for explicit extraction of both the FM/AM constituents (see [12]), and therefore the SDE description is more fundamental than the spectral one. Indeed, given the SDE dynamics, we are able to extract all higher order statistical information through the propagators obtained as solutions of the associated Fokker-Planck equations [4, 19]. In this way, the SDE description of sea clutter should be viewed as the most complete dynamical description, which preserves the inherent randomness in the physical processes involved.

We have seen that independent lines of enquiry, from theoretical and experimental perspectives, lead to the common conclusion that radar sea clutter is nonlinear over appreciable timescales such that the temporal variation in the RCS is significant. The degree of nonlinearity is determined by the sea state, which is represented by a certain "shape parameter" ν that features in the SDE for the RCS. (More precisely, $\nu = \alpha - 1$, where α is the parameter in the SDE for the RCS, and $\alpha \geq 0$ arises from the parameters in the scattering population model.) Consistently, the nonlinearity is also determined by the extent of frequency modulation which, in terms of real experimental data, has been quantified in terms of a certain z-parameter representing the Mann-Whitney rank-sum statistic.

From a theoretical point of view, the deterministic part of the stochastic dynamics (5), (6) is nonlinear, and is augmented with the addition of fluctuating Wiener terms in the description of real experimental data, which is inherently noisy (cf. the discussion of chaos surrounding [20, Figure 2], and also [21, 22].) We recommend that further studies be made on the noise-free skeleton of the coupled system (5), (6), which is manifestly nonlinear, to establish the existence or otherwise of an underlying deterministic chaotic behavior. If chaos is present, then this system of nonlinearly coupled SDEs is an instance of "stochastic chaos." We remark in this respect that the presence of the Wiener fluctuating terms in the system has the effect of stabilizing the system, so that any chaotic behavior may no longer be observable experimentally. These issues will be pursued in a subsequent paper.

It is worth emphasizing again that the SDE theory of sea clutter is experimentally valid, in its own terms (see

[3, Section 4(b)]), and has also succeeded in practical applications, such as radar anomaly detection, to a remarkable degree of accuracy. The theory also provides a way of generating synthetic data, over which we have direct control, in terms of its dependence on the sea state. Thus, in principle, we could measure the z-parameter for a data set simulated using SDEs, for which the shape parameter is known, and thereby develop the precise relationship between the z-parameter and shape parameter quantifiers of nonlinearity. It may, indeed, also be possible to relate the two parameters in purely theoretical terms. We can also generate data with and without noise, which forms the basis for further experiment. We suggest that these two lines of enquiry could form the basis of future developments in the investigation of the nonlinear properties of radar scattering dynamics.

The novelty of the current paper can be summarized as follows. In Haykin's paper, it was experimentally demonstrated that sea clutter becomes increasingly nonlinear as the sea state increases. In this new paper, for the first time, theoretical justification of this important result has been presented based on the earlier results of Field.

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