

## Research Article

# Noninvasive Model Independent Noise Control with Adaptive Feedback Cancellation

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An active noise control (ANC) system is model dependent/independent if its controller transfer function is dependent/independent on initial estimates of path models in a sound field. Since parameters of path models in a sound field will change when boundary conditions of the sound field change, model-independent ANC systems (MIANC) are able to tolerate variations of boundary conditions in sound fields and more reliable than model-dependent counterparts. A possible way to implement MIANC systems is online path modeling. Many such systems require invasive probing signals (persistent excitations) to obtain accurate estimates of path models. In this study, a noninvasive MIANC system is proposed. It uses online path estimates to cancel feedback, recover reference signal, and optimize a stable controller in the minimum  $H_2$  norm sense, without any forms of persistent excitations. Theoretical analysis and experimental results are presented to demonstrate the stable control performance of the proposed system.

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## 1. INTRODUCTION

Most active noise control (ANC) systems are model dependent. Let  $\hat{P}(z)$  and  $\hat{S}(z)$  denote estimates of primary and secondary path transfer functions  $P(z)$  and  $S(z)$ . Either  $\hat{S}(z)$  or both  $\hat{P}(z)$  and  $\hat{S}(z)$  must be obtained by initial system identification for model-dependent ANC systems. Controller transfer function  $C(z)$  of a model-dependent ANC system is either designed by minimizing  $\|\hat{P}(z) + \hat{S}(z)C(z)\|$ , or adapted with the aid of  $\hat{S}(z)$  [1, 2]. If estimates  $\hat{P}(z)$  and  $\hat{S}(z)$  contain too much error, a model-dependent ANC system may generate constructive instead of destructive interference. This is mathematically equivalent to  $\|P(z) + S(z)C(z)\| > \|P(z)\|$  even if  $\|\hat{P}(z) + \hat{S}(z)C(z)\|$  is minimized. If phase error in  $\hat{S}(z)$  exceeds  $90^\circ$  in some frequency, an ANC system adapted by the filtered- $X$  least mean square (FXLMS) algorithm may become unstable [3–5]. An operator of a model-dependent ANC system must have the knowledge and skill to obtain accurate estimates of path models by initial system identification for each individual application.

During the operation of an ANC system, changes of environmental or boundary conditions may cause significant

changes to path transfer functions  $P(z)$  and  $S(z)$ . Since a model-dependent ANC system only remembers initial path estimates  $\hat{P}(z)$  and  $\hat{S}(z)$ , variation of  $P(z)$  and  $S(z)$  may cause mismatch with initial estimates  $\hat{P}(z)$  and  $\hat{S}(z)$  to degrade ANC performance. In cases of severe mismatch between path transfer functions and their initial estimates, a model-dependent ANC system may generate constructive instead of destructive interference, or even become unstable.

Model-independent ANC (MIANC) systems depend on online path modeling or invariant properties of sound fields to update or design controllers [6–8]. These systems avoid initial path modeling and are adaptive to variations of environmental or boundary conditions of sound fields. Many adaptive MIANC systems require invasive persistent excitations to obtain accurate path estimates and ensure closed-loop stability [6, 7, 9, 10]. Noninvasive MIANC systems are able to ensure closed-loop stability without persistent excitations, which are possible by a recently developed algorithm, known as orthogonal adaptation [11, 12], if the primary noise signal is directly available as the reference signal.

In many real applications, the primary noise signal is not necessarily available and the reference signal must be recovered from the sound field [1, 2]. When an ANC system is

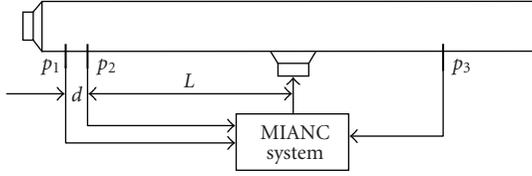


FIGURE 1: Configuration of the proposed MIANC system.

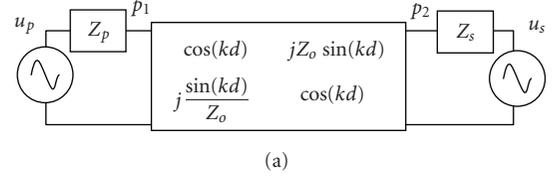
active, a measured signal is a linear combination of primary and secondary signals. Feedback of ANC signal in the measurement is mathematically modeled by a feedback transfer function  $F(z)$  from the controller to the reference sensor. Accurate estimation of  $F(z)$  is as important as accurate estimation of  $P(z)$  or  $S(z)$  [9, 13]. A complete noninvasive MIANC (CNMIANC) system must be able to suppress the noise signal without injecting probing signals for online modeling of  $P(z)$ ,  $S(z)$ , and  $F(z)$ . Most available methods for adaptive feedback cancellation require persistent excitations [9, 13]. In this study, a new method is presented for adaptive feedback cancellation without persistent excitations.

It was proposed to use a pair of sensors to measure pressure signals in ducts, from which traveling waves are resolved [14, 15]. The outbound wave could be used directly as the reference signal without cancelling feedback signals if an infinite-impulse-response (IIR) controller could be implemented accurately [14, 15]. In reality, it is very difficult to implement a stable ideal IIR ANC controller [16]. Most practical ANC systems use finite-impulse-response (FIR) controllers. The outbound wave in a duct is a linear combination of primary noise and reflected version of feedback signal. Instead of using the outbound wave directly as the reference, the least mean square (LMS) algorithm is applied in this study to cancel feedback signals in the outbound wave before using it as the reference. Orthogonal adaptation is combined with the proposed ANC configuration to implement a CNMIANC system. Experimental result is presented to demonstrate the performance of the CNMIANC system.

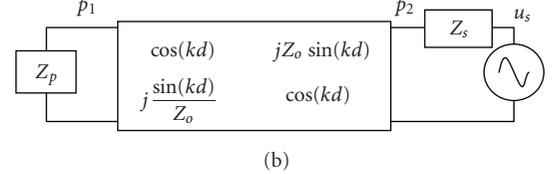
## 2. SYSTEM CONFIGURATION AND MODEL

Figure 1 illustrates the configuration of the proposed ANC system. The primary source is represented by the upstream speaker and the secondary source is the midstream speaker. Cross-sectional area of the duct is small enough such that sound field in the duct can be modeled by a 1D sound field in the frequency range of interest. Three microphone sensors are installed in the duct, measuring signals  $p_1$ ,  $p_2$ , and  $p_3$ , respectively. Since the primary noise signal is not available to the ANC system, the reference signal is recovered from  $p_1$  and  $p_2$ , while  $p_3$  is the error signal to be minimized by the ANC system.

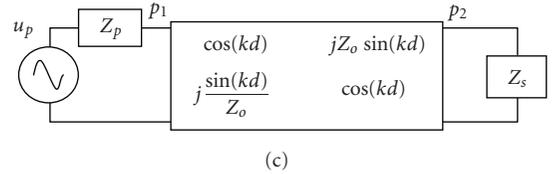
Let  $d$  denote the axial distance between  $p_1$  and  $p_2$ . The acoustical two-port theory [16, 17] has been applied by many ANC researchers for the design and analysis of ANC systems. It is adopted here as an analytical tool. An equivalent acoustical circuit is shown in Figure 2 to model the two-microphone system. The upstream part, from the primary source to loca-



(a)



(b)



(c)

FIGURE 2: (a) Acoustical two-port circuit in the duct system, (b) contribution by controller, and (c) contribution by primary source.

tion of  $p_1$ , is equivalent to an acoustical source with strength  $u_p$  and impedance  $Z_p$ . The downstream part, from location of  $p_2$  to the outlet, is represented by another acoustical source with strength  $u_s$  and impedance  $Z_s$ . Characteristic impedance of the duct is represented by  $Z_o$ .

The linear system theory allows one to solve  $p_1$  and  $p_2$  in Figure 2(a) by focusing on acoustical circuits of Figures 2(b) and 2(c) before adding two solutions together as the final solution of Figure 2(a). For the case of  $u_p = 0$ , which is represented by Figure 2(b), one obtains

$$\begin{aligned} p_2|_{u_p=0} &= \left[ \cos(kd) + j \frac{Z_o}{Z_p} \sin(kd) \right] p_1|_{u_p=0}, \\ u_s - p_2|_{u_p=0} &= \left[ \frac{Z_s}{Z_p} \cos(kd) + j \frac{Z_s}{Z_o} \sin(kd) \right] p_1|_{u_p=0}, \end{aligned} \quad (1)$$

where  $k$  is the wave number. One can solve, from (1),

$$p_2|_{u_p=0} = \frac{Z_o [Z_p \cos(kd) + j Z_o \sin(kd)]}{(Z_s + Z_p) Z_o \cos(kd) + j (Z_s Z_p + Z_o^2) \sin(kd)} u_s, \quad (2)$$

$$p_1|_{u_p=0} = \frac{Z_o Z_p}{(Z_s + Z_p) Z_o \cos(kd) + j (Z_s Z_p + Z_o^2) \sin(kd)} u_s. \quad (3)$$

Similarly, for the case of  $u_s = 0$ , which is represented by Figure 2(c), one obtains

$$\begin{aligned} p_1|_{u_s=0} &= \left[ \cos(kd) + j \frac{Z_o}{Z_p} \sin(kd) \right] p_2|_{u_s=0}, \\ u_p - p_1|_{u_s=0} &= \left[ \frac{Z_p}{Z_s} \cos(kd) + j \frac{Z_p}{Z_o} \sin(kd) \right] p_2|_{u_s=0}, \end{aligned} \quad (4)$$

from which one can solve

$$p_1|_{u_s=0} = \frac{Z_o[Z_s \cos(kd) + jZ_o \sin(kd)]}{(Z_s + Z_p)Z_o \cos(kd) + j(Z_s Z_p + Z_o^2) \sin(kd)} u_p, \quad (5)$$

$$p_2|_{u_s=0} = \frac{Z_o Z_s}{(Z_s + Z_p)Z_o \cos(kd) + j(Z_s Z_p + Z_o^2) \sin(kd)} u_p. \quad (6)$$

Adding (2) and (6), one may write

$$\begin{aligned} p_2 &= p_2|_{u_p=0} + p_2|_{u_s=0} \\ &= \frac{Z_o[Z_p \cos(kd) + jZ_o \sin(kd)]u_s + Z_o Z_s u_p}{(Z_s + Z_p)Z_o \cos(kd) + j(Z_s Z_p + Z_o^2) \sin(kd)}. \end{aligned} \quad (7)$$

The same method is applicable to (3) and (5) for

$$\begin{aligned} p_1 &= p_1|_{u_p=0} + p_1|_{u_s=0} \\ &= \frac{Z_o[Z_s \cos(kd) + jZ_o \sin(kd)]u_p + Z_o Z_p u_s}{(Z_s + Z_p)Z_o \cos(kd) + j(Z_s Z_p + Z_o^2) \sin(kd)}. \end{aligned} \quad (8)$$

The next step is to use complex factor  $\alpha = Z_o / ((Z_s + Z_p)Z_o \cos(kd) + j(Z_s Z_p + Z_o^2) \sin(kd))$  to simplify (7) and (8). The results read

$$\begin{aligned} p_2 &= \alpha \{ [Z_p \cos(kd) + jZ_o \sin(kd)]u_s + Z_s u_p \}, \\ p_1 &= \alpha \{ [Z_s \cos(kd) + jZ_o \sin(kd)]u_p + Z_p u_s \}. \end{aligned} \quad (9)$$

Since  $\cos(kd) = 0.5(e^{jkd} + e^{-jkd})$  and  $j\sin(kd) = 0.5(e^{jkd} - e^{-jkd})$ , (9) can be written as

$$\begin{aligned} p_2 &= \alpha \left\{ \left[ \frac{Z_p + Z_o}{2} e^{jkd} + \frac{Z_p - Z_o}{2} e^{-jkd} \right] u_s \right. \\ &\quad \left. + \left[ \frac{Z_s + Z_o}{2} + \frac{Z_s - Z_o}{2} \right] u_p \right\}, \\ p_1 &= \alpha \left\{ \left[ \frac{Z_s + Z_o}{2} e^{jkd} + \frac{Z_s - Z_o}{2} e^{-jkd} \right] u_p \right. \\ &\quad \left. + \left[ \frac{Z_p + Z_o}{2} + \frac{Z_p - Z_o}{2} \right] u_s \right\}. \end{aligned} \quad (10)$$

Let

$$w_i = \alpha \left\{ \frac{Z_s - Z_o}{2} u_p + \frac{Z_p + Z_o}{2} e^{jkd} u_s \right\} \quad (11)$$

$$w_o = \alpha \left\{ \frac{Z_p - Z_o}{2} u_s + \frac{Z_s + Z_o}{2} e^{jkd} u_p \right\} \quad (12)$$

represent the in- and outbound waves in the duct. By comparing (10) with (11) and (12), one can see that (10) are equivalent to

$$p_2 = w_i + w_o e^{-jkd}, \quad p_1 = w_i e^{-jkd} + w_o. \quad (13)$$

The in- and outbound waves can be resolved from  $p_1$  and  $p_2$  via

$$\begin{aligned} \begin{bmatrix} w_i \\ w_o \end{bmatrix} &= \begin{bmatrix} e^{-jkd} & 1 \\ 1 & e^{-jkd} \end{bmatrix}^{-1} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \\ &= \frac{1}{1 - e^{-2jkd}} \begin{bmatrix} -e^{-jkd} & 1 \\ 1 & -e^{-jkd} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}. \end{aligned} \quad (14)$$

In a digital implementation of ANC system, it is recommended to select sampling interval  $\delta t$  such that its product with sound speed  $c$  satisfies  $c\delta t = d$ . As a result, the delay operator  $\exp(-jkd) = z^{-1}$  becomes an exact one-sample delay for discrete-time ANC systems.

### 3. FEEDBACK CANCELLATION

It is indicated by (12) that the outbound wave contains feedback from  $u_s$  that must be cancelled to recover the reference signal. Let  $R_1 = (Z_p - Z_o)/(Z_p + Z_o)$  denote the upstream reflection coefficient. By multiplying  $e^{-jkd} R_1$  to (11), one obtains

$$e^{-jkd} R_1 w_i = \alpha \left\{ \frac{(Z_s - Z_o)(Z_p - Z_o)}{2(Z_p + Z_o)} e^{-jkd} u_p + \frac{Z_p - Z_o}{2} u_s \right\}. \quad (15)$$

A subtraction of (15) from (12) enables one to write

$$w_o - e^{-jkd} R_1 w_i = n, \quad (16)$$

where

$$n = \frac{\alpha [(Z_s + Z_o)(Z_p + Z_o)e^{jkd} - (Z_s - Z_o)(Z_p - Z_o)e^{-jkd}]}{2(Z_p + Z_o)} u_p \quad (17)$$

is only contributed by the primary source  $u_p$ .

Using  $\cos(kd) = 0.5(e^{jkd} + e^{-jkd})$  and  $j\sin(kd) = 0.5(e^{jkd} - e^{-jkd})$ , one can see that the common denominator of  $p_1$ ,  $p_2$ , and all transfer functions in the duct is

$$\begin{aligned} (Z_s + Z_p)Z_o \cos(kd) + j(Z_s Z_p + Z_o^2) \sin(kd) \\ = 0.5(Z_s + Z_o)(Z_p + Z_o)e^{jkd} - 0.5(Z_s - Z_o)(Z_p - Z_o)e^{-jkd}. \end{aligned} \quad (18)$$

Substituting (18) into the definition of  $\alpha$  (immediately after (8)), one obtains

$$2Z_o = \alpha [(Z_s + Z_o)(Z_p + Z_o)e^{jkd} - (Z_s - Z_o)(Z_p - Z_o)e^{-jkd}]. \quad (19)$$

A further substitution of (19) into (17) leads to

$$n = \frac{Z_o u_p}{Z_p + Z_o}. \quad (20)$$

This is the reference signal to be recovered by the proposed ANC system.

A question to be answered here is why not recovering the reference signal from a pressure signal such as  $p_1$ . The hint is (8) that may be expressed as  $p_1 = F(j\omega)u_s + B(j\omega)u_p$ . In view of (8), the acoustical feedback transfer function is

$$F(j\omega) = \frac{Z_o Z_p}{(Z_s + Z_p)Z_o \cos(kd) + j(Z_s Z_p + Z_o^2) \sin(kd)}. \quad (21)$$

Since  $F(j\omega)$  is a transfer function with resonant poles, it has an infinite impulse response (IIR). In many ANC systems, a

finite-impulse-response (FIR) filter  $\hat{F}(j\omega)$  is used to approximate  $F(j\omega)$ . This means inevitable approximation errors in the first place.

Besides, all transfer functions in a duct are sensitive to values of  $Z_o$ ,  $Z_s$ , and  $Z_p$ . In particular,  $Z_s$  is the impedance of the entire downstream segment from the location of  $p_2$  to the duct outlet. Objects moving near the duct outlet could cause changes of  $Z_s$ . A fracture in any downstream part may also cause a significant change to  $Z_s$  as well. If initial estimate  $\hat{F}(j\omega)$  is remembered by an ANC system, it is a stability issue how significant will  $F(j\omega) - \hat{F}(j\omega)$  turn out as a result of a small variation of  $Z_s$ . An indicative answer might be

$$\frac{\partial}{\partial Z_s} F(j\omega) = \frac{-Z_o Z_p [Z_o \cos(kd) + j Z_p \sin(kd)]}{[(Z_s + Z_p) Z_o \cos(kd) + j (Z_s Z_p + Z_o^2) \sin(kd)]^2}. \quad (22)$$

The common denominator of  $p_1$ ,  $p_2$ , and all transfer functions in the duct has an alternative form in (18), which is equivalent to

$$\begin{aligned} & 0.5(Z_s + Z_o)(Z_p + Z_o)e^{jkd} - 0.5(Z_s - Z_o)(Z_p - Z_o)e^{-jkd} \\ &= 0.5(Z_s + Z_o)(Z_p + Z_o)e^{jkd} \left[ 1 - \frac{(Z_s - Z_o)(Z_p - Z_o)}{(Z_s + Z_o)(Z_p + Z_o)} e^{-2jkd} \right] \\ &= 0.5(Z_s + Z_o)(Z_p + Z_o)e^{jkd} [1 - R_1 R_2 e^{-2jkd}], \end{aligned} \quad (23)$$

where  $R_2 = (Z_s - Z_o)/(Z_s + Z_o)$  represents the downstream reflection coefficient.

Since resonant frequencies of the duct are roots of the common denominator, it is suggested by (22) and (23) that all transfer functions in the duct, including the feedback transfer function  $F(j\omega)$ , are sensitive to variance of  $Z_s$  at the resonant peaks. The stronger the resonance, the more sensitive of transfer functions with respect to  $Z_s$ . If an ANC system recovers the reference signal from a pressure signal like  $p_1$ , a small online variation of  $Z_s$  may cause a significant mismatch between  $F(j\omega)$  and initial estimate  $\hat{F}(z)$ . As a result, closed-loop stability is sensitive to possible variation of  $Z_s$ .

If the reference signal is recovered from traveling waves with (16), the situation will be different. In a discrete-time implementation, one may rewrite (16) to  $n(z) = w_o - F_w(z)w_i$ , where the acoustical feedback transfer function is a delayed version of upstream reflection coefficient  $F_w(z) = z^{-1}R_1(z)$ . Here,  $R_1 = (Z_p - Z_o)/(Z_p + Z_o)$  is only sensitive to  $Z_p$  and  $Z_o$ . Characteristic impedance  $Z_o$  is a real constant depending on sound speed and cross-sectional area between  $p_1$  and  $p_2$ . It seldom changes significantly in online ANC operations. As for  $Z_p$ , it is the impedance of the upstream portion from the primary source to the location of  $p_1$ . In most applications,  $p_1$  and  $p_2$  are measured as close as possible to the primary source. Impedance  $Z_p$  is deeply hidden in a very short segment of the duct. Its variation, if any, would be certainly not as significant as that of  $Z_s$ .

No matter how significant are the possible variations of  $Z_p$  or  $Z_o$ , the passive upstream reflection always has a limited magnitude  $|R_1| < 1$ . For each pair of fixed  $Z_p$  and  $Z_o$ ,

$|R_1(j\omega)|$  does not have sharp peaks or dips as a function of  $\omega$ . In many cases,  $|R_1|$  is constant for a pair of fixed  $Z_p$  and  $Z_o$ . Let  $X(j\omega)$  denote the Fourier transform of  $x(t)$ , then  $X(j\omega) = L\{x(t)\}$  and  $x(t) = L^{-1}\{X(j\omega)\}$  share many similar properties. For example, if  $x(t)$  is a low-frequency function of  $t$ , then the bandwidth of  $X(j\omega)$  is narrow in terms of  $\omega$ . Similarly, if  $X(j\omega)$  is a ‘‘low-frequency’’ function of  $\omega$ , then the time duration of  $x(t)$  is short (a narrow bandwidth in terms of  $t$ ). The fact that  $|R_1|$  is a ‘‘low-frequency’’ function of  $\omega$  for each pair of fixed  $Z_p$  and  $Z_o$  implies short impulse responses of  $R_1(z)$ . It is, therefore, reasonable to assume that  $R_1(z) = \sum_{k=0}^m r_k z^{-k}$  can be approximated by a FIR transfer function with negligible errors (Assumption A1). If both  $Z_p$  and  $Z_o$  are constant,  $R_1$  is a single constant. Resonant effects in the duct are hidden in wave signals  $w_i$  and  $w_o$  without affecting  $R_1$ . This is a major difference between recovering the reference signal from traveling waves and recovering the reference signal from a pressure signal.

Even if an estimate of  $F_w(z)$  is obtained by initial identification, it is less likely that online variations of environmental or boundary conditions could cause significant mismatch between  $F_w(z)$  and its initial estimate. The resultant ANC system is semimodel independent if its reference signal is recovered with (16) in combination with a MIANC adaptation algorithm such as orthogonal adaptation.

#### 4. COMPLETE NONINVASIVE MIANC

Noninvasive model-independent feedback cancellation is possible by applying LMS to (16). With assumption A1, online estimate of the feedback transfer function is represented by polynomial

$$\hat{R}(z) = \sum_{k=0}^m \hat{r}_k(t) z^{-k}, \quad (24)$$

where  $r_k(t)$  is the  $k$ th coefficient for the  $t$ th sample. An estimated version of (16) would be

$$\hat{n} = w_o - z^{-1} \hat{R}(z) w_i, \quad (25)$$

which has a discrete-time domain expression,

$$\hat{n}(t) = w_o(t) - \sum_{k=0}^m \hat{r}_k(t) w_i(t - k - 1). \quad (26)$$

Coefficients of  $\hat{R}(z) = \sum_{k=0}^m \hat{r}_k(t) z^{-k}$  are updated with the LMS algorithm as follows:

$$\hat{r}_k(t+1) = \hat{r}_k(t) + \mu \hat{n}(t) w_i(t - k - 1), \quad (27)$$

where  $\mu > 0$  is a small constant representing the LMS step size. Since  $R_1(z) = \sum_{k=0}^m r_k z^{-k}$  by assumption A1, the discrete-time domain version of (16) is

$$n(t) = w_o(t) - \sum_{k=0}^m r_k w_i(t - k - 1). \quad (28)$$

Subtracting (28) from (27), one obtains

$$\begin{aligned}\hat{n}(t) - n(t) &= \sum_{k=0}^m [r_k - \hat{r}_k(t)] w_i(t-k-1) \\ &= \sum_{k=0}^m \Delta r_k(t) w_i(t-k-1),\end{aligned}\quad (29)$$

where  $\Delta r_k(t) = r_k - \hat{r}_k(t)$  is the estimation error of  $r_k$ . Let  $\Delta r = [\Delta r_0, \Delta r_1, \dots, \Delta r_m]^T$  and let  $\hat{\omega}_i(t) = [w_i(t-1), w_i(t-2), \dots, w_i(t-m-1)]^T$ . It is possible to express (29) in an inner product

$$\hat{n}(t) - n(t) = \Delta r^T \hat{\omega}_i(t). \quad (30)$$

Estimation residues of LMS algorithms are usually expressed as inner products like (30). It has been proven that the LMS algorithm is able to drive the convergence of these inner products towards zero.

If the primary noise signal  $u_p$  was available, mathematical model of the error signal may be expressed in the discrete-time  $z$ -transform domain as  $e(z) = P(z)u_p(z) + S(z)u_s(z)$ , where the actuation signal would be synthesized as  $u_s(z) = C(z)u_p(z)$ . Since  $u_p$  is actually not available, the ANC system has to recover  $\hat{n}(z)$  from the outbound wave and then synthesize  $u_s(z) = C(z)\hat{n}(z)$  instead. After the convergence of  $\hat{n}(z) \rightarrow n(z)$ , one may express the mathematical model of the error signal to

$$e(z) = \left\{ P(z) \left[ 1 + \frac{Z_p}{Z_o} \right] + S(z)C(z) \right\} n(z), \quad (31)$$

where (20) has been substituted. Let  $H(z) = P(z)[1 + Z_p/Z_o]$ , then (31) becomes

$$e(z) = [H(z) + S(z)C(z)]n(z). \quad (32)$$

It is mathematically equivalent to another ANC system whose primary source is available to the controller as  $n(z)$ , with primary path transfer function  $H(z)$  and secondary path transfer function  $S(z)$ . Orthogonal adaptation is readily applicable to (32) to implement a noninvasive MIANC system.

It is assumed that  $H(z)$  and  $S(z)$  can be approximated by FIR filters with negligible errors (Assumption A2). Let  $h^T = [h_0 \ h_1 \ \dots \ h_m]$  and  $s^T = [s_0 \ s_1 \ \dots \ s_m]$  denote coefficients of  $H(z)$  and  $S(z)$ , respectively, the discrete-time domain version of  $e(z) = H(z)n(z) + S(z)u_s(z)$  is a discrete-time convolution:

$$e(t) = \sum_{k=0}^m h_k n(t-k) - \sum_{k=0}^m s_k u_s(t-k), \quad (33)$$

where  $e(t)$ ,  $n(t)$ , and  $u_s(t)$  denote samples of  $e(z)$ ,  $n(z)$ , and  $u_s(z)$ , respectively. Introducing coefficient vector  $\theta^T = [h^T \ s^T]$  and regression vector  $\phi_t = [n(t) \ n(t-1) \ \dots \ n(t-m), u_s(t) \ u_s(t-1) \ \dots \ u_s(t-m)]^T$ , one may rewrite (33) to

$$e(t) = \theta^T \phi_t. \quad (34)$$

Let  $\hat{H}(z)$  and  $\hat{S}(z)$  denote online estimates of  $H(z)$  and  $S(z)$ . Path estimates  $\hat{H}(z)$  and  $\hat{S}(z)$  are obtained by minimizing estimation error as follows:

$$e(z) = e(z) - \hat{H}(z)n(z) - \hat{S}(z)u_s(z) = \Delta H(z)n(z) + \Delta S(z)u_s(z), \quad (35)$$

where  $\Delta H(z) = H(z) - \hat{H}(z)$  and  $\Delta S(z) = S(z) - \hat{S}(z)$  are online modeling errors. Let  $\hat{\theta}^T = [\hat{h}^T \ \hat{s}^T]$  denote online estimate of  $\theta^T = [h^T \ s^T]$ , then  $\hat{h}^T = [\hat{h}_0 \ \hat{h}_1 \ \dots \ \hat{h}_m]$  and  $\hat{s}^T = [\hat{s}_0 \ \hat{s}_1 \ \dots \ \hat{s}_m]$  represent the coefficients of  $\hat{H}(z)$  and  $\hat{S}(z)$ , respectively. Similar to the equivalence between (34) and  $e(z) = H(z)n(z) + S(z)u_s(z)$ , (35) has a discrete-time domain equivalence

$$\varepsilon_t = e(t) - \hat{\theta}^T \phi_t = \Delta \hat{\theta}^T \phi_t, \quad (36)$$

where  $\Delta \theta = \theta - \hat{\theta}$  is the online coefficient error vector. The entire CNMIANC system performs three online tasks that are mathematically represented by the minimization of three inner products. The first is inner product given in (30); the second one is given in (36); and the third one is  $\hat{\theta}^T \phi_t$ .

Equations (30) and (36) contain estimation errors  $\Delta r$  and  $\Delta \theta$ . Most available estimation algorithms, such as LMS and the recursive least squares (RLS), are very capable of driving inner products like (30) and (36) towards zero, or at least minimizing their magnitudes [18]. A difficult problem is how to force  $\Delta r \rightarrow 0$  and  $\Delta \theta \rightarrow 0$ . Available solutions inject significant levels of ‘‘persistent excitations’’ (invasive probing signals) to the estimation system [6, 7, 9, 10, 13]. A unique feature of the proposed CNMIANC is no persistent excitations. The system works well without requiring  $\Delta r \rightarrow 0$  and  $\Delta \theta \rightarrow 0$ .

For (30), minimizing the inner product in the right-hand side implies convergence of  $\hat{n} \rightarrow n$  in the left-hand side. It would be great if  $\Delta r \rightarrow 0$  as well. Otherwise,  $\Delta r$  may just converge to a FIR filter that filters out  $w_i$  from  $w_o$ . On the other hand, minimizing the inner product in (36) only implies  $\varepsilon_t \rightarrow 0$ . The question is what does it further imply? One may consider the equivalence between (34) and  $e(z) = H(z)n(z) + S(z)u_s(z)$ , which holds if one replaces  $\theta^T = [h^T \ s^T]$ ,  $H(z)$ , and  $S(z)$  with respective estimates  $\hat{\theta}^T = [\hat{h}^T \ \hat{s}^T]$ ,  $\hat{H}(z)$ , and  $\hat{S}(z)$ . The equivalence is now between forcing  $\hat{\theta}^T \phi_t \approx 0$  and forcing

$$\hat{H}(z)n(z) + \hat{S}(z)u_s(z) = [\hat{H}(z) + \hat{S}(z)C(z)]n(z) \approx 0. \quad (37)$$

The CNMIANC system uses online estimates of  $\hat{H}(z)$  and  $\hat{S}(z)$  to solve  $C(z)$  that minimizes  $\|\hat{H}(z) + \hat{S}(z)C(z)\|_2$ . This is equivalent to forcing  $\hat{\theta}^T \phi_t \approx 0$ . One can obtain

$$\|e\| = \|\varepsilon_t + \hat{\theta}^T \phi_t\| \leq \|\varepsilon_t\| + \|\hat{\theta}^T \phi_t\| \quad (38)$$

by adding  $\hat{\theta}^T \phi_t$  to both sides of (36). As the CNMIANC system drives  $\varepsilon_t = \Delta \hat{\theta}^T \phi_t \rightarrow 0$  and forces  $|\hat{\theta}^T \phi_t| \approx 0$  ultimately, it implies ultimate convergence of  $\|e\| \rightarrow 0$  even though  $\Delta \theta$  does not necessarily converge to zero [11, 12].

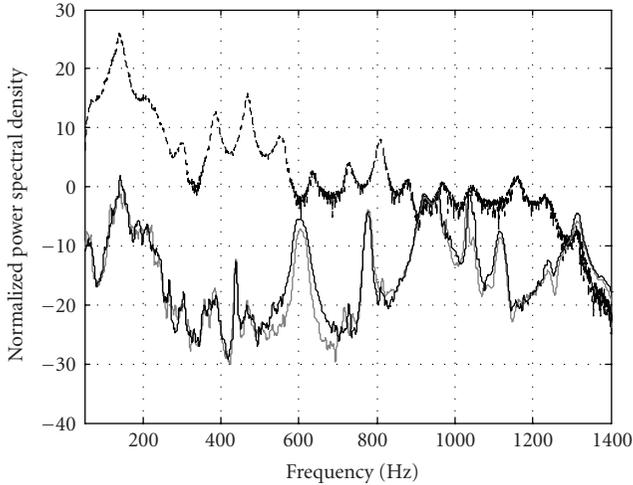


FIGURE 3: Normalized PSDs of  $e(t)$  for (a) uncontrolled case (dashed-black), (b) controlled case with  $u_p(t)$  available (solid-gray), and (c) controlled case with recovered  $\hat{n}(t)$  (solid-black).

## 5. EXPERIMENTAL VERIFICATION

A CNMIANC system was implemented and tested in an experiment, with a configuration shown in Figure 1. Cross-sectional area of the duct was  $12 \times 15 \text{ cm}^2$ . Two microphones were placed 30 cm downstream from the primary speaker with a space of  $d = 10 \text{ cm}$  between  $p_1$  and  $p_2$ . The distance between  $p_2$  and the secondary speaker is represented by  $L$  in Figure 1. To guarantee a causal ANC system, the value of  $L$  must satisfy  $L > 2d$  such that the outbound wave is at least two samples ahead of sound propagation in duct. The sampling interval of the controller was 0.29 millisecond with a sampling frequency of 3.448 Hz, which satisfies  $d = c\delta t$  with  $c = 344 \text{ m/s}$  and  $\exp(jkd) = z$ . The cutoff frequency of antialias filters was chosen to be 1200 Hz. The in- and outbound waves were recovered from pressure signals with (14). The reference signal was recovered with (25). Coefficients of  $\hat{R}(z)$  were adapted with (27). Another online modeling process used (34) to obtain coefficients of  $\hat{H}(z)$  and  $\hat{S}(z)$ . The ANC transfer function was solved by online minimization of  $\|\hat{H}(z) + \hat{S}(z)C(z)\|_2$ . The CNMIANC system was implemented in a dSPACE 1103 board.

Error signal  $e(t)$  and primary noise  $u_p(t)$  were collected as vectors  $e$  and  $u_p$  for three cases. In case 1, there was no control action. In case 2,  $u_p(t)$  was available as the reference signal for an ANC system to suppress noise in the duct. In case 3,  $u_p(t)$  was not available and the CNMIANC system had to recover  $\hat{n}(t)$  from  $p_1$  and  $p_2$  for controller synthesis. For each respective case, power spectral densities (PSD's) of  $e(t)$  and  $u_p(t)$  were computed with a MATLAB command called "pmtm()". Computational results are denoted as vectors  $P_e = \text{pmtm}(e)$  and  $P_p = \text{pmtm}(u_p)$ , where argument vectors  $e$  and  $u_p$  represent measurement samples of  $e(t)$  and  $u_p(t)$ . The normalized PSD of  $e(t)$  was calculated as  $P_{ne} = 10\log(P_e/P_p)$  for all three cases.

Shown in Figure 3 are normalized PSD of  $e(t)$  for the three cases. For case 1, normalized PSD of  $e(t)$  is represented

by the dashed-black curve. For case 2, normalized PSD of  $e(t)$  is plotted with the solid-gray curve. For case 3, normalized PSD of  $e(t)$  is represented by the solid-black curve. Both ANC systems were able to suppress noise with good control performance as seen in Figure 3. The proposed CNMIANC has slightly worse performance since its reference was the recovered signal  $\hat{n}(t)$  instead of the true primary source  $u_p(t)$ . This is a small price to pay in case  $u_p(t)$  is not available to the ANC system. The proposed CNMIANC system was stable and able to recover the reference and suppress noise without any persistent excitations.

The CNMIANC system was robust with respect to sudden parameter change in the duct. In the experiment, the duct outlet was changed from completely open to completely closed. Such a sudden change shifted all resonant frequencies in the duct. Path transfer functions also changed suddenly. The CNMIANC system remained stable and converged very quickly.

## 6. CONCLUSIONS

The primary source is not necessarily available as the reference signal for ANC systems in all practical applications. When the primary source is not available, the ANC system must recover the reference signal from a sound field to which ANC is applied. Feedback cancellation is an important issue in ANC systems that recover reference signals from sound fields. In most MIANC systems, persistent excitations are required for online modeling of feedback path model and adaptive feedback cancellation [9, 13]. In this study, a CNMIANC system is proposed that recovers reference signal from traveling waves without persistent excitations. The corresponding feedback path model is the upstream reflection coefficient and hence closer to an FIR filter than pressure feedback transfer functions (IIR path models in resonant ducts). Theoretical analysis and experimental results are presented to demonstrate the stable operation of the proposed CNMIANC system.

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