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Research Article

The Kerr Nonlinearity in an N-Type Four-Level System via Spontaneously Generated Coherence

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The Kerr nonlinearity is investigated in an N-type four-level atomic system whose both supper levels are degenerated or nearly degenerate. It is found that the spontaneously generated coherence as a result of the degenerate levels can change the Kerr nonlinearity dramatically depending on the angle between two spontaneous channels. In addition, we discover that the probe field makes its own contribution to the nonlinearity in comparison to indifference in general three-level systems.

In the coherent interaction between light fields and atoms quantum coherence or interference gives rise to a series of interesting effects such as coherent population trapping (CPT) [1], electromagnetically induced transparency (EIT) [2–4], and electromagnetically induced absorption (EIA) [5–8]. Recent years, spontaneously generated coherence (SGC) [9, 10] brought about by degenerate or nearly degenerate levels attracted a lot of interests. The SGC refers to the interference between spontaneous emission paths. The existence of such interference relies on the angle between two dipole transitions. With the exception of theoretical investigation which revealed that SGC leads to some interesting results it was also experimentally ascertained that the SGC plays a key role in charged quantum dots, which has potential value on element in quantum information network [11–14].

As is widely accepted, the third-order nonlinearity in optical media plays a crucial role in the nonlinear optics. Quantum coherence or interference can easily enhance the nonlinearity of optical media regardless of the condition of weak-light [15–21]. On account of the rarity of the SGC, the relationship between nonlinearity and SGC was seldom studied apart from a recent theoretical investigation carried out onto the Kerr nonlinearity of general three-level systems [22]. We focus our interest on the Kerr nonlinearity with regard to the SGC in an N-type four-level system.

The N-type four-level system with supper levels degenerate is shown in Figure 1. The levels $|2\rangle$ and $|4\rangle$ are excited levels and nearly degenerate, while the levels $|1\rangle$ and $|3\rangle$ are ground levels. A coherent field w_S drives the transition $|3\rangle - |4\rangle$ with the Rabi frequency Ω_S, and another field w_C couples the transition $|3\rangle - |2\rangle$ with the Rabi frequency Ω_C. The transitions $|3\rangle - |2\rangle$ and $|3\rangle - |4\rangle$ are simultaneously monitored by the probe field w_P with the Rabi frequency ε. The excited level $|4\rangle$ has a spontaneous emission rate γ_3, whereas the excited level $|2\rangle$ has the two γ_1 and γ_2 corresponding to lower levels $|1\rangle$ and $|3\rangle$, respectively.

The Hamiltonian of the above system can be written as $H = H_0 + H_I$, where

$$\begin{aligned} H_0 &= -\hbar\Delta_C|1\rangle\langle 1| - \hbar\delta|3\rangle\langle 3| - \hbar(\delta - \Delta_S)|4\rangle\langle 4|, \\ H_I &= -\hbar\Omega_C|2\rangle\langle 1| - \hbar\varepsilon|2\rangle\langle 3| - \hbar\Omega_S|4\rangle\langle 3| + H.c. \end{aligned} \tag{1}$$

In (1), $\Delta_C = w_{21} - w_C$, $\Delta_S = w_{43} - w_S$, and $\delta = w_{23} - w_P$. According to the Hamiltonian, the density matrix equations governed by master equation can be written as

$$\begin{aligned} \dot{\rho}_{11} &= i\Omega_C^*\rho_{21} - i\Omega_C\rho_{12} + 2\gamma_1\rho_{22}, \\ \dot{\rho}_{21} &= -(i\Delta_C + \gamma_1 + \gamma_2)\rho_{21} + i\Omega_C(\rho_{11} - \rho_{22}) \\ &\quad + i\varepsilon\rho_{31} - p\sqrt{\gamma_2\gamma_3}\rho_{41}, \end{aligned}$$

$$
\begin{aligned}
\dot{\rho}_{31} &= i(\Delta_P - \Delta_C)\rho_{31} + i\varepsilon^*\rho_{21} - i\Omega_C\rho_{32} + i\Omega_S^*\rho_{41},\\
\dot{\rho}_{41} &= \left[i(\Delta_P - \Delta_S - \Delta_C) - \gamma_3\right]\rho_{41}\\
&\quad - i\Omega_C\rho_{42} + i\Omega_S\rho_{31} - p\sqrt{\gamma_2\gamma_3}\rho_{21},\\
\dot{\rho}_{22} &= i\Omega_C\rho_{12} - i\Omega_C^*\rho_{21} + i\varepsilon\rho_{32} - i\Omega_P^*\rho_{23}\\
&\quad - 2(\gamma_1 + \gamma_2)\rho_{22} - p\sqrt{\gamma_2\gamma_3}(\rho_{42} + \rho_{24}),\\
\dot{\rho}_{32} &= \left[i\Delta_P - (\gamma_1 + \gamma_2)\right]\rho_{32} - i\Omega_C^*\rho_{31}\\
&\quad + i\varepsilon^*(\rho_{22} - \rho_{33}) + i\Omega_S^*\rho_{42} - p\sqrt{\gamma_2\gamma_3}\rho_{34},\\
\dot{\rho}_{42} &= \left[i(\Delta_P - \Delta_S) - (\gamma_1 + \gamma_2 + \gamma_3)\right]\rho_{42} - i\Omega_C^*\rho_{41}\\
&\quad - i\varepsilon^*\rho_{43} + i\Omega_S\rho_{32} - p\sqrt{\gamma_2\gamma_3}(\rho_{22} + \rho_{44}),\\
\dot{\rho}_{33} &= i\varepsilon^*\rho_{23} - i\varepsilon\rho_{32} + i\Omega_S^*\rho_{43} - i\Omega_S\rho_{34} + 2(\gamma_1 + \gamma_2)\rho_{22}\\
&\quad + 2\gamma_3\rho_{44} + 2p\sqrt{\gamma_2\gamma_3}(\rho_{42} + \rho_{24}),\\
\dot{\rho}_{43} &= -(i\Delta_S + \gamma_3)\rho_{43} - i\varepsilon\rho_{42} + i\Omega_S(\rho_{33} - \rho_{44}) - p\sqrt{\gamma_2\gamma_3}\rho_{23},\\
\dot{\rho}_{44} &= i\Omega_S\rho_{34} - i\Omega_S^*\rho_{43} - 2(\gamma_1 + \gamma_2)\rho_{44} - p\sqrt{\gamma_2\gamma_3}(\rho_{24} + \rho_{42}).
\end{aligned}
\tag{2}
$$

The above density elements observe the relations $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$ and $\rho_{ij} = \rho_{ji}^*$. The term $p\sqrt{\gamma_2\gamma_3}$ stands for the SGC which is resulted from the cross-coupling between two spontaneous emission channels. The parameter p is defined as $p = |\vec{\mu}_{23} \cdot \vec{\mu}_{43}| \exp(i\theta)/(|\vec{\mu}_{23}||\vec{\mu}_{43}|) = \cos\theta$ where θ denotes the angle between the dipole matrix elements $\vec{\mu}_{23}$ and $\vec{\mu}_{43}$.

Our analysis is based on the steady-state solutions of the density matrix equations; therefore we set $\dot{\rho}_{ij} = 0$. However, actually; to give the accurate solution of every matrix element is unrealistic, so we have to turn to some techniques or parameters. We assume that $\rho_{11} + \rho_{22} = C_1$ and $\rho_{33} + \rho_{44} = C_2$ where C_1 and C_2 are constants satisfying the relations $C_1 + C_2 = 1$, $0 \le C_1 \le 1$, and $0 \le C_2 \le 1$. On condition that the drive field w_C and the signal field w_S are much stronger than the probe field w_P, we use nth-order of perturbation theory on ε. The first- and third-order susceptibilities are governed by the expressions $\chi^{(1)} = 2N|\mu_{23}|^2\rho_{23}^{(1)}/\varepsilon_0\hbar\varepsilon$ and $\chi^{(3)} = 2N|\mu_{23}|^4\rho_{23}^{(3)}/3\varepsilon_0\hbar^3\varepsilon^3$, where N represents the number of atoms in an ensemble, ε_0 is the vacuum dielectric constant, and $\hbar$ is the Plank constant. The Kerr nonlinearity corresponds to the refractive part of the third-order susceptibility.

The calculated linear absorption, dispersion, nonlinearity, and Kerr nonlinearity as a function of the probe detuning δ with the different SGCs are shown in Figure 2. For the sake of simplicity, we scale all Rabi frequencies and offsets by the decay rate γ_3 and order $\gamma_3 = \Gamma$. From the figure, we can see that when $p = 0$, the general linear absorption, dispersion, nonlinear absorption, and Kerr nonlinearity of the N-type four-level system occur. In this case, we calculate and find that the maximal Kerr nonlinearity is about 300 times larger than that of the maximal dispersion. When $p = -1$, it is obvious that the Kerr nonlinearity is greatly enhanced and in the mean

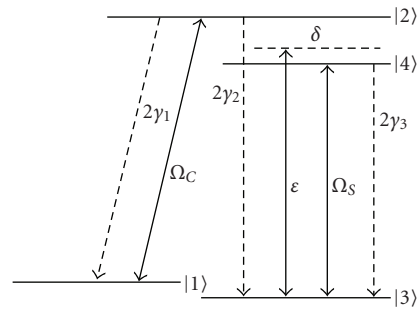

Figure 1: The N-type four-level atomic system with supper levels nearly degenerate.

while the EIT window evidently broadens; moreover this, the maximal Kerr nonlinearity completely enters the EIT window and at this time it is 2080 times as large as that of the dispersion. It illustrates that both enhanced Kerr nonlinearity and negligible atomic absorption can be met simultaneously in via SGC. When $p = 1$, it means that the dipole matrix elements $\vec{\mu}_{23}$ and $\vec{\mu}_{43}$ are parallel; the graph showsthat the linear absorption, dispersion, nonlinear absorption, and Kerr nonlinearity are exactly adverse to the case $p = -1$ in the direction of Y-axis. What we need to emphasize here is that although the maximal Kerr nonlinearity is smaller than that of $p = -1$, the ratio of the maximal Kerr nonlinearity to the maximal dispersion largely enhances to be 13050.

From the above discussion, we know that the SGC has changed the Kerr nonlinearity. Now we focus on the relationship between the maximal Kerr nonlinearity and the SGC, as it is shown in Figure 3 that the maximal Kerr nonlinearity is a function of p. From this figure, we can find that when p is near the point of zero, which means that the orthogonalization between the dipole elements $\vec{\mu}_{23}$ and $\vec{\mu}_{43}$, the maximal nonlinearity has its least value. As the absolute value of p enhances, the nonlinearity shows a relative small increase. However, When p gets to the values 1 or -1, it exhibits a rapid increase and obtains the largest value at the points of 1 and -1. As regards the reason for the difference between values at $p = 1$ or $p = -1$, we temporarily cannot give sound interpretation.

From [22] we learn that in general three-level systems, the amplitude of the probe field has no connection with Kerr nonlinearity; in the case of the present system, however, we find that the Rabi frequency of the probe field changes the Kerr nonlinearity. In Figure 4 we show the maximal Kerr nonlinearity as a function of ε from 0.001 to 0.01 with different constants C_2. It can be seen that when $C_2 = 0$ which implies that the N-type four-level system decays to a two-level system according to the definition about C_2, the maximal Kerr nonlinearity keeps even, that is to say, the nonlinearity has nothing to do with the probe field. When $C_2 = 0.1$ or $C_2 = 0.2$, their results are similar to each other. Under this condition, in comparison with the first condition, N-type four-level system comes into being; as a consequence the Rabi frequency of the probe field makes its own contribution to the nonlinearity. From

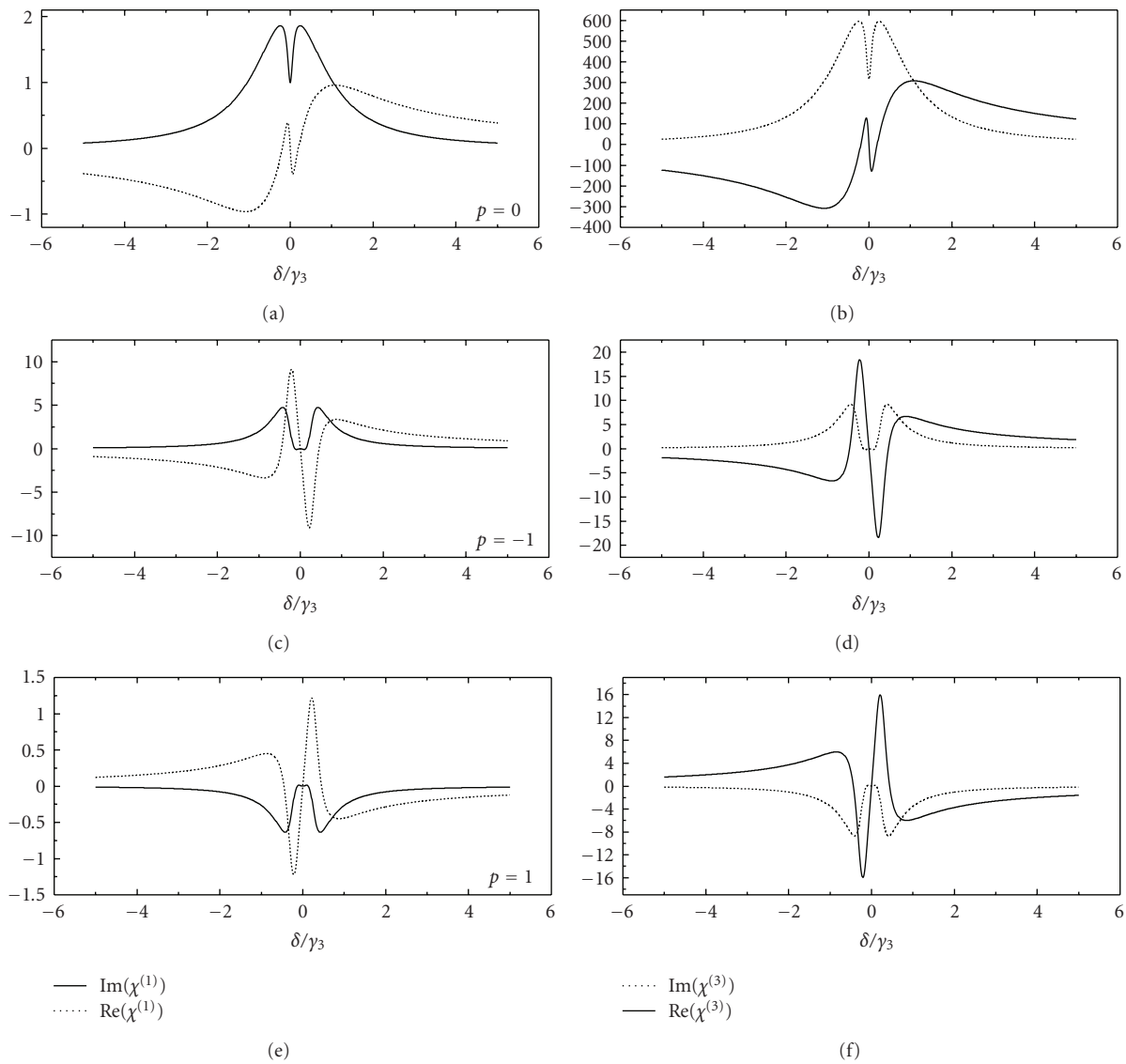

Figure 2: Linear absorption $\mathrm{Im}(\chi^{(1)})$, dispersion $\mathrm{Re}(\chi^{(1)})$, nonlinear absorption $\mathrm{Im}(\chi^{(3)})$, and Kerr nonlinearity $\mathrm{Re}(\chi^{(3)})$ as a function of the probe detuning δ with the different p. (a), (c), and (e) show the linear absorptions (solid curves) and dispersions (dashed curves); (b), (d), and (f) show the nonlinear absorptions (dashed curves) and Kerr nonlinearities (solid curves). The couples of (a) and (b), (c) and (d), and (e) and (f) are in response to different parameters $p = 0, -1$, and 1, respectively. The other parameters are set as $\gamma_1 = \gamma_2 = 0.5\gamma_3$, $\Omega_C = \Omega_S = 0.2\gamma_3$, $\Omega_P = 0.01\gamma_3$, $\Delta_C = \Delta_S = 0$, and $C_2 = 0.1$.

the figure, we can find that the maximal Kerr nonlinearity enhances intensely with the decrease of the Rabi frequency of the probe field, especially at the point of $\varepsilon = 0$, whereas it in the case of $C_2 = 0.2$ is more intense than that in the case of $C_2 = 0.1$. The more C_2, the larger the influence of the SGC on the nonlinearity. When $\varepsilon = 0$, the Kerr nonlinearity rises to an infinity in response to the mere existence of the vacuum field. The interesting effect possibly gives us an effective approach to attain giant Kerr nonlinearities.

In summary, we have discussed the Kerr nonlinearity in the N-type four-level system with degenerate supper levels. The results revealed that the SGC can enhance the Kerr nonlinearity, especially when the angle between spontaneous emission channels tends to be zero or π. At the same time, because of the existence of the SGC, and because the complexity of the system strengthens and becomes quite different from that of general three-level systems, we find that the Rabi frequency of the probe field as a part of two-photon process also makes its own effort to the Kerr nonlinearity; the result shows the smaller the Rabi frequency is, the larger the Kerr nonlinearity is.

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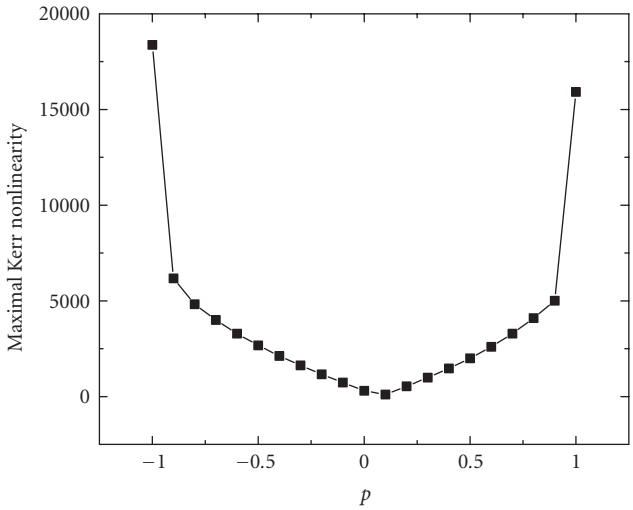

Figure 3: The maximal Kerr nonlinearity as a function of p. Other parameters are the same as those in Figure 2.

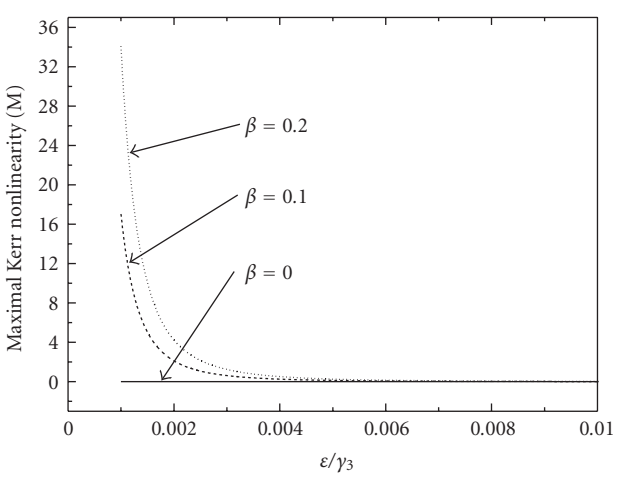

Figure 4: The maximal Kerr nonlinearity as a function of the Rabi frequency ε with different constants C_2. The SGC is given by $p = 1$. Other parameters are the same as those in Figure 2.

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