Hindawi Publishing Corporation
Advances in Acoustics and Vibration
Volume 2009, Article ID 694290, 13 pages
doi:10.1155/2009/694290

Research Article

Sequentially Adapted Parallel Feedforward Active Noise Control of Noisy Sinusoidal Signals

Govind Kannan,[1] Issa M. S. Panahi,[1] and Richard W. Briggs[2]

[1] *Department of Electrical Engineering, University of Texas at Dallas, 800 W. Campbell Rd., Richardson, TX 75080, USA*
[2] *Department of Radiology, University of Texas Southwestern Medical Center, 5323 Harry Hines Blvd, Dallas, TX 75390, USA*

Correspondence should be addressed to Govind Kannan, govind@student.utdallas.edu

Received 12 September 2008; Revised 31 January 2009; Accepted 7 April 2009

Recommended by Lars Hakansson

A large class of acoustic noise sources has an underlying periodic process that generates a periodic noise component, and thus their acoustic noise can in general be modeled as the sum of a periodic signal and a randomly fluctuating signal (usually a broadband background noise). Active control of periodic noise (i.e., for a mixture of sinusoids) is more effective than that of random noise. For mixtures of sinusoids in a background broadband random noise, conventional FXLMS-based single filter method does not reach the maximum achievable Noise Attenuation Level ($\mathrm{NAL}_{\max}$). In this paper, an alternative approach is taken and the idea of a parallel active noise control (ANC) architecture for cancelling mixtures of periodic and random signals is presented. The proposed ANC system separates the noise into periodic and random components and generates corresponding antinoises via separate noise cancelling filters, and tends to reach $\mathrm{NAL}_{\max}$ consistently. The derivation of $\mathrm{NAL}_{\max}$ is presented. Both the separation and noise cancellation are based on adaptive filtering. Experimental results verify the analytical development by showing superior performance of the proposed method, over the single-filter approach, for several cases of sinusoids in white noise.

1. Introduction

Active noise control (ANC) is a technique of cancelling acoustic noise by generating an appropriate antinoise signal using loudspeakers, and directing it towards the region where noise cancellation is required. Rapid progress in digital signal processors (DSPs) and sensor technologies as well as new real-time adaptive control algorithm designs [1–3] has opened the door to new approaches in ANC. Active control of sinusoidal signals has been an important research and development topic [4] and can lead to different system architectures [5] and design simplifications [6]. The standard approach to realize an ANC system is to employ the FXLMS algorithm in feedforward configuration [4, 7] as portrayed in Figure 1. $P(z)$ denotes the primary path and $S(z)$ denotes the secondary path. From Figure 1, it follows that the exact solution to the noise cancelling filter (the control filter) $W(z)$ is given by

$$W_0(z) = \frac{P(z)}{S(z)}. \tag{1}$$

Since the stable transfer function $S(z)$ is in general non-minimum phase, $W_0(z)$ should in general be a stable but noncausal system, and hence unrealizable unless $x(n)$ is perfectly predictable. The noncausality restriction no longer applies to a periodic signal since its past and future values can be determined from the knowledge of the period or alternatively from the frequencies, amplitudes, and phases of the constituent sinusoids. Hence a periodic noise signal can ideally be cancelled perfectly by designing an appropriate (predictive) cancelling filter in the ANC system. For linearly modeled ANC system with stationary signals, the causality restriction places an upper limit on the achievable noise attenuation level. The noise attenuation level (NAL) is defined as the ratio of noise power before cancellation to the noise power after cancellation. Computation of the maximum achievable NAL for some given stationary noise process is derived in this paper. The maximum achievable NAL is denoted by $\mathrm{NAL}_{\max}$. For an additive mixture of periodic and random signals, the $\mathrm{NAL}_{\max}$ can be calculated by assuming that the periodic component is cancelled completely. However experience with the FXLMS algorithm, using single cancelling filter, for additive mixtures of periodic and random signals shows that the NAL achieved is usually less than the NAL_{max}. In this paper an ANC structure for such mixtures, that can achieve performances much closer to NAL_{max}, is introduced. The notion of separating the periodic and random noise components and generating corresponding anti-noise components is developed. Due to the simultaneous and additive way in which the two anti-noise components are generated and combined, the proposed method is termed as parallel feedforward ANC. It should be noted that in this paper, the adaptation scheme is sequential, that is, the two noise cancelling filters are computed by a two-step adaptation procedure.

This idea, which was initially introduced in [8] in an empirical fashion, is given a rigorous analytical flavor with supporting experimental results in this paper. The proposed separation-based parallel ANC system has three stages: the first stage estimates the sinusoidal parameters and separates the periodic part from the random part of the noise signal using adaptive noise cancelling technique. The second and third stages operate simultaneously, generating the corresponding anti-noise signals for periodic part and random part, respectively. Experimental results using different mixtures of noise signals show improved performance of the proposed ANC method.

Active noise control of a mixture of sinusoids with or without harmonic relation is a well-studied topic. An indepth study of the convergence properties of the FXLMS algorithm when the noise to be controlled is a harmonically related mixture of sinusoids is presented in [9]. The idea of using a parallel bank of control filters, each handling a subset of the sinusoids is presented and analyzed. The more general case of active noise control of nonharmonically related sinusiods is analysed in a novel manner in [10] by employing a characteristic equation derived from an equivalent state-space model. A faster converging version of the FXLMS algorithm is also derived. A root-locus-based convergence analysis of an FXLMS-based ANC system for nonharmonically related sinusoids is done in [11]. A phase tuning approach based on an integrative internal model principle is developed in [12] for active noise control of sinusoidal disturbances. It should be noted that the methods developed in [9–12] are applicable to pure sinusoidal mixtures without any broadband random noise.

The purpose of this paper is to introduce a new active noise control (ANC) architecture for effective active noise control of sinusoidal mixtures in a random background noise. The key idea is that for sinusoidal signals in random noise, it is possible to separate the sinusoidal and random parts and generate separate anti-noise signals in a parallel fashion. Since periodic signals can be expressed as a mixture of sinusoids, the proposed method is also applicable to periodic signals with an additive random noise. The effectiveness of noise attenuation is expressed interms of the Noise Attenuation Level (NAL), which measures the relative level of attenuated noise with respect to the actual noise. The notion of, and an exact formula for, maximum achievable Noise Attenuation Level (NAL_{max}) for sinusoidal signals in white noise is presented. The proposed ANC system design is shown to achieve performances close to NAL_{max}. The key ideas communicated are as follows:

(1) Given a feedforward ANC setup (one reference microphone and one error microphone setup) and a stationary acoustic noise input, there exists a maximum achievable noise attenuation level (NAL_{max}) if linear estimation with an MMSE criterion is employed. The method to calculate this upper limit is developed for sinusoidal mixtures in random noise.

(2) In the active noise control of a mixture of sinusoids and a random noise with a single noise cancelling filter (or control filter), the adaptive algorithm finds an optimum filter that is neither equal to the optimum filter when sinusoids alone are present nor equal to the optimum filter when random noise alone is present. Alternatively, assuming that the sinusoidal and random parts can be separated, better cancellation can be achieved by formulating two mean square errors (MSEs) criteria-one for the sinusoidal component and the other for the random component.

(3) For the periodic signal part, perfect cancellation can be achieved by equalizing the nonminimum phase secondary path with a noncausal equalizer in a causal manner since for periodic signals, the future values can be determined from the knowledge of the period or alternatively from the frequencies, amplitudes, and phases of the constituent sinusoids. For the random part the best cancellation achieveable is given by NAL_{max}.

(4) For a mixture of sinusoids and random noise, it is possible to separate the periodic and random components and have a parallel ANC architecture. This architecture results in two separate control filters, each of which is optimum for the respective component and hence the overall system can reach NAL_{max} better than the conventional single-filter configuration. It seems that such an approach has not been considered in the literature so far.

The paper is organized as follows. The notational framework and various definitions are introduced in "Notations and Definitions" section, towards the end of the paper. Section 2 introduces the proposed parallel feed forward ANC architecture. Section 3 develops the methodology to calculate NAL_{max}. Section 4 presents and discusses the experimental results. Concluding remarks are made in Section 5. The reader is advised to peruse the "Notations and Definitions" section before proceeding to read Sections 2 and 3.

2. The Parallel Feedforward ANC Method (PFANC)

2.1. Motivation. The key idea of this paper is that for sinusoidal plus random noise mixtures, separating the sinusoids and random noise and generating antinoise signals

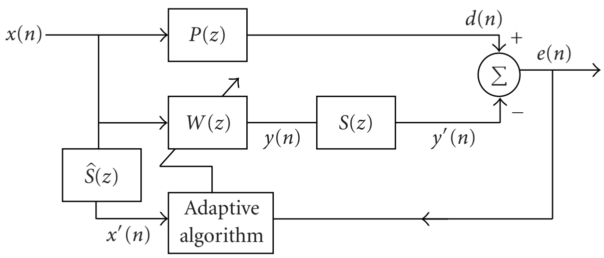

Figure 1: The feedforward FXLMS algorithm. $W(z)$ generates an appropriate anti-noise so as to minimise $e(n)$ in a mean square sense.

separately improves the performance of the ANC system. The motivation for this approach lies in the observation that the optimum control (cancelling) filters for the periodic part and the random part are not the same. Assuming perfect separation of $x(n)$ into $q(n)$ and $\nu(n)$, and separate generation of anti-noise signals for the periodic and random parts, the expression for the composite anti-noise signal generated by the ANC system is

$$y'(n) = -\Big(w_q(n) * q(n) + w_\nu(n) * \nu(n)\Big) * s(n), \tag{2}$$

where $w_q(n)$ and $w_\nu(n)$ are the noise cancelling filters of $q(n)$ and $\nu(n)$, respectively, and $*$ denotes the convolution operation.

The best noise cancellation is achieved when $W_q(\omega)$ and $W_\nu(\omega)$ are optimum in the MMSE sense. The superscript $\circ$ denotes the optimality of any solution in the rest of the manuscript. From (1), it is obvious that for the tones at ω_k, the optimum filter is any $W_q^\circ(\omega)$ such that

$$W_q^\circ(\omega_k) = \frac{P(\omega_k)}{S(\omega_k)} = W_0(\omega_k). \tag{3}$$

For the random part $\nu(n)$, the optimum solution $W_\nu^\circ(\omega)$ is the causal Wiener filter (whose exact form is shown later in (51)), approximating the noncausal function $P(\omega)/S(\omega)$.

Let $e_{\text{pr}}(n)$ denote the residual noise for the proposed parallel anti-noise generation approach. Then assuming perfect separation of $x(n)$ into $q(n)$ and $\nu(n)$, $e_{\text{pr}}(n)$ is given by

$$e_{\text{pr}}(n) = d(n) - w_q(n) * q(n) * s(n) - w_\nu(n) * \nu(n) * s(n). \tag{4}$$

Since $d(n)$ can be written as the sum of $d_q(n)$ and $d_\nu(n)$, (4) can be written as

$$\begin{aligned} e_{\text{pr}}(n) &= \Big(d_q(n) - w_q(n) * q(n) * s(n)\Big) \\ &\quad + (d_\nu(n) - w_\nu(n) * \nu(n) * s(n)) \\ &= e_q(n) + e_\nu(n). \end{aligned} \tag{5}$$

Thus $e_{\text{pr}}(n)$ consists of two components: $e_q(n)$ from the sinusoidal component and $e_\nu(n)$ from the random component. It should be noted that when $W_q(\omega) = W_q^\circ(\omega)$, there is perfect cancellation of $q(n)$, that is, $e_q^\circ(n) = 0$. The best cancellation of $\nu(n)$ occurs when $W_\nu(\omega) = W_\nu^\circ(\omega)$ with the corresponding minimum error $e_\nu^\circ(n)$. Since $e_q(n)$ and $e_\nu(n)$ are statistically independent, the MMSE is

$$\mathrm{E}\Big\{e_{\text{pr}}^{\circ 2}(n)\Big\} = \mathrm{E}\Big\{e_q^{\circ 2}(n)\Big\} + \mathrm{E}\Big\{e_\nu^{\circ 2}(n)\Big\} = \mathrm{E}\Big\{e_\nu^{\circ 2}(n)\Big\}, \tag{6}$$

where $\mathrm{E}\{\cdot\}$ is the expectation operator.

Let $e_{\text{sgl}}(n)$ denote the residual noise for the single-filter approach. The antinoise is given by

$$y'(n) = -w_{\text{sgl}}(n) * (q(n) + \nu(n)) * s(n). \tag{7}$$

The expression for $e_{\text{sgl}}(n)$ is then

$$e_{\text{sgl}}(n) = d(n) - w_{\text{sgl}}(n) * (q(n) + \nu(n)) * s(n). \tag{8}$$

The best cancellation (in the MMSE sense) is achieved by minimising $\mathrm{E}\{e_{\text{sgl}}^2(n)\}$ with respect to $w_{\text{sgl}}(n)$. Let the MMSE solution be $w_{\text{sgl}}^\circ(n)$ with the corresponding error $e_{\text{sgl}}^\circ(n)$

$$\begin{aligned} e_{\text{sgl}}^\circ(n) &= d(n) - w_{\text{sgl}}^\circ(n) * (q(n) + \nu(n)) * s(n) \\ &= \Big(d_q(n) - w_{\text{sgl}}^\circ(n) * q(n) * s(n)\Big) \\ &\quad + \Big(d_\nu(n) - w_{\text{sgl}}^\circ(n) * \nu(n) * s(n)\Big). \end{aligned} \tag{9}$$

In general, the MMSE solution, $w_{\text{sgl}}^\circ(n)$ will be different from both $w_q^\circ(n)$ and $w_\nu^\circ(n)$. Since only $w_q^\circ(n)$ will result in an error sequence $e_{\text{sgl}}(n)$ with no tonal component, $w_{\text{sgl}}^\circ(n)$ will always result in a residual tonal component in $e_{\text{sgl}}^\circ(n)$. The residual tonal component is denoted by $e_q^{\text{res}}(n)$ and the residual random noise component is denoted by $e_\nu^{\text{res}}(n)$. Thus (9) can be written as

$$e_{\text{sgl}}^\circ(n) = e_q^{\text{res}}(n) + e_\nu^{\text{res}}(n), \tag{10}$$

where $e_q^{\text{res}}(n) = d_q(n) - w_{\text{sgl}}^\circ(n) * q(n) * s(n)$ and $e_\nu^{\text{res}}(n) = d_\nu(n) - w_{\text{sgl}}^\circ(n) * \nu(n) * s(n)$.

The expression for MMSE owing to statistical independence is then

$$\mathrm{E}\Big\{e_{\text{sgl}}^{\circ 2}(n)\Big\} = \mathrm{E}\Big\{e_q^{\text{res}^2}(n)\Big\} + \mathrm{E}\Big\{e_\nu^{\text{res}^2}(n)\Big\}. \tag{11}$$

It should be recalled that the best attenuation of the random component is attained when $w_{\text{sgl}}^\circ(n)$ equals $w_\nu^\circ(n)$. Since it is not the case in general, it follows that

$$\mathrm{E}\Big\{e_\nu^{\text{res}^2}(n)\Big\} \geq \mathrm{E}\Big\{e_\nu^{\circ 2}(n)\Big\}. \tag{12}$$

Since $\mathrm{E}\{e_\nu^{\text{res}^2}(n)\} \geq \mathrm{E}\{e_\nu^{\circ 2}(n)\}$ and $\mathrm{E}\{e_q^{\text{res}^2}(n)\}$ are positive quantities, it follows that

$$\mathrm{E}\Big\{e_{\text{sgl}}^{\circ 2}(n)\Big\} = \mathrm{E}\Big\{e_q^{\text{res}^2}(n)\Big\} + \mathrm{E}\Big\{e_\nu^{\text{res}^2}(n)\Big\} \geq \mathrm{E}\Big\{e_\nu^{\circ 2}(n)\Big\}. \tag{13}$$

Since from (6), the MMSE for the proposed method $\mathrm{E}\{e_{\text{pr}}^{\circ 2}(n)\} = \mathrm{E}\{e_\nu^{\circ 2}(n)\}$, it follows from (13) that

$$\mathrm{E}\Big\{e_{\text{pr}}^{\circ 2}(n)\Big\} \leq \mathrm{E}\Big\{e_{\text{sgl}}^{\circ 2}(n)\Big\}. \tag{14}$$

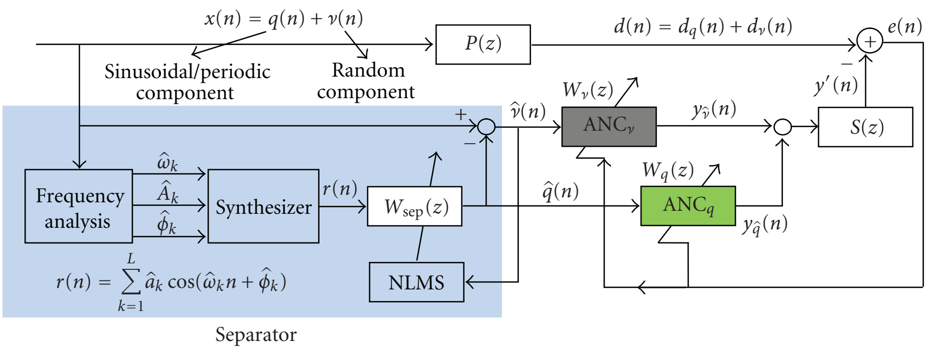

FIGURE 2: The proposed scheme: active noise control employing separation of sinusoidal and random parts of the reference signal.

Thus the MMSE of the proposed method is always lesser than the MMSE of single-filter method.

It should also be noted that when a single adaptive filter is used, the adaptive algorithm tends to cancel the dominating signal component, that is, the tones more effectively. As a result, the random component may be attenuated to a lesser degree, unaffected, or even reinforced. In fact, the adaptive filter in the single-filter case shows a non-Wiener behavior [13], the analysis of which is nontrivial. In contrast, the proposed ANC structure offers a simple means of enhancing performance.

2.2. Parallel Feedforward ANC (PFANC): Problem Formulation. The parallel feedforward ANC is formulated, as two subproblems, as follows.

(1) Given an acoustic noise signal $x(n) = q(n) + \nu(n)$, devise a method of separating $x(n)$ into $q(n)$ and $\nu(n)$, where $q(n)$ is periodic, and $\nu(n)$ is random.

(2) Design an ANC system that has two parallel adaptive noise cancelling filters $W_q(z)$ and $W_\nu(z)$ for generating antinoise signals separately. It should be noted that, for analytical purposes, we consider a very short convergence time and hence drop the time indices that should accompany $W_q(z)$ and $W_\nu(z)$, owing to their time-varying (adaptive) nature. These two cancelling filters are parts of ANC_q and ANC_ν, respectively, (see Figure 2). The overall system will monitor the error microphone output $e(n)$ to adjust the weights of these cancelling filters.

In the proposed approach, the first sub-problem is addressed in three steps: a frequency analyzer block which estimates the sinusoidal frequencies, a synthesizer block which generates a reference signal based on the estimated frequencies, and an adaptive noise canceller block which separates $x(n)$ into $q(n)$ and $\nu(n)$. The second sub-problem is solved by setting up two FXLMS-based ANC systems. The method is elaborated in Figure 2. A more detailed explanation of the proposed approach is presented in subsequent sections.

2.3. PFANC Method. The proposed PFANC method can be divided into three major parts.

(1) Estimation of sinusoidal parameters

(2) Separation of sinusoidal and random components of the noise signal.

(3) Generation of antinoise signals corresponding to the sinusoidal and random signal components.

2.3.1. Estimation of Sinusoidal Parameters. This part estimates the sinusoidal parameters from the measured input noise signal $x(n)$. Let M samples of $x(n)$ be observed. The estimation problem can then be stated as

Given that $x(n)$ can be modeled as

$$x(n) = \sum_{k=1}^{L} a_k \cos(\omega_k n + \phi_k) + \nu(n) \tag{15}$$

and given M observed samples of $x(n)$, estimate ω_k, a_k, ϕ_k. The determination of L is implicit in the problem.

The above estimation problem is an important and a well-studied one [14]. Since there are many methods available, and since this is an important step in the proposed scheme, the tools and techniques that were found to be particularly effective are described. The first step is detecting the number L of sinusoids, which is essentially an order estimation problem. The next step is estimation of ω_k, a_k, ϕ_k. An information-theoretic approach [15] for the first step and a subspace-based high resolution parametric approach (ESPRIT) [14] for the second step are prescribed.

Detection of L. The first step is the detection of L, the number of sinusoids, which amounts to order estimation. The gist of all the order estimation methods is defining an information measure as a function of the number of sinusoids. The number of sinusoids that best explains the observations, minimizes the information measure. Out of the host of order estimation techniques available [16], the method described in [15] is particularly effective for the sinusoids in noise scenario. The information measure is based on the Minimum Description Length (MDL) principle introduced in [17]. The reasons for adopting this method are twofold: (1) the method has been shown to be consistent, that is, it asymptotically approaches the optimum MAP rule [15, 17] and (2) there is no need to choose a penalty function [15].

The following quantities need to be defined, before introducing the technique.

In general for any quantity H, $\hat{\text{H}}$ denotes the estimated value of the quantity whereas the uncapped H denotes the true value. For example, if $\boldsymbol{\omega}$ is the vector of frequencies. then $\hat{\boldsymbol{\omega}}$ is the vector of estimated frequencies. Let us define the vectors $\boldsymbol{\omega} = [\omega_1 \cdots \omega_L]^T$, $\mathbf{a} = [a_1 \cdots a_L]^T$, $\boldsymbol{\varphi} = [\phi_1 \cdots \phi_L]^T$, $\mathbf{x}(n) = [x(n) \cdots x(n-m+1)]^T$, and matrix

$$\mathbf{R} = E\left\{\mathbf{x}(n)\mathbf{x}^H(n)\right\}. \tag{16}$$

$\mathbf{R}$ is the $m \times m$, that is, order-m covariance matrix of the observations $x(n)$. Superscripts H and T denote Hermitian and transpose operations, respectively.

However in practice, the covariance matrix $\mathbf{R}$ of order m is calculated from M observations ($M > m$) of $x(n)$, that is, from $\{x(1), x(2), x(3), \ldots, x(M)\}$. The observations data length M should be greater than the covariance matrix order m so that an accurate estimate of $\mathbf{R}$ can be computed. The expression for the sample covariance matrix $\hat{\mathbf{R}}$ is [14]

$$\hat{\mathbf{R}} = \frac{1}{M}\sum_{t=m}^{M} \mathbf{x}_t \mathbf{x}_t^H \quad m \times m, \tag{17}$$

where,

$$\mathbf{x}_t = [x(t) \cdots x(t-m+1)]^T. \tag{18}$$

The eigen-structure of $\hat{\mathbf{R}}$ has all the information about the angular frequencies ω_k. Additionally let $\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_{2L}, \ldots, \hat{\lambda}_m$ be the eigenvalues of $\hat{\mathbf{R}}$ sorted in decreasing magnitude. The number of sinusoids is then determined from the l (for $2L$ complex exponentials) that minimizes [15]

$$\begin{aligned}\text{MDL}(l) = {} & M(m-l)\log\left(\frac{(1/(m-l))\sum_{i=l+1}^{m}\hat{\lambda}_i}{\prod_{i=l+1}^{m}\hat{\lambda}_i^{1/(m-l)}}\right) \\ & + \frac{1}{2}l(2m-l)\log M.\end{aligned} \tag{19}$$

The number of sinusoids L can be found from $L = l_{\min}/2$, since for L real sinusoids there are $2L$ complex exponentials.

The minimum is found by computing the MDL(l) for values of l ranging from $1, 2, \ldots, m-1$. The eigenvalues are computed from an eigenvalue decomposition of $\hat{\mathbf{R}}$. An example plot of MDL for $l = 1, 2, \ldots, m-1$ is shown in Figure 4.

Estimation of $\boldsymbol{\omega}, \mathbf{a}, \boldsymbol{\varphi}$. The estimation of $\boldsymbol{\omega}, \mathbf{a}, \boldsymbol{\varphi}$ can be done using various ways like MUSIC, ESPRIT, FFT, and so forth, [14]. The method which has been shown to have very good statistical performance [14] is the nonlinear least squares (NLS) method in which the optimal values of $\boldsymbol{\omega}, \mathbf{a}, \boldsymbol{\varphi}$ minimizes the least squares measure,

$$\text{f}(\boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\varphi}) = \sum_{n=1}^{M}\left| x(n) - \sum_{k=1}^{L} a_k \cos(\omega_k n + \phi_k)\right|^2. \tag{20}$$

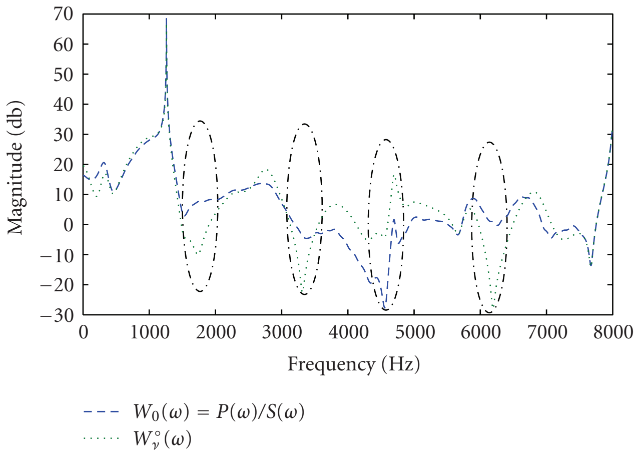

FIGURE 3: The optimum filter $W_\nu^\circ(\omega)$ for the random noise can be way-off when compared to $W_0(\omega)$, that is, $W_\nu^\circ(\omega_k) \neq W_0(\omega_k)$. The dashed circled regions show frequencies where $W_\nu^\circ(\omega)$ deviates considerably from $W_0(\omega)$. Presence of tones in these regions will affect the performance of single-filter ANC methods.

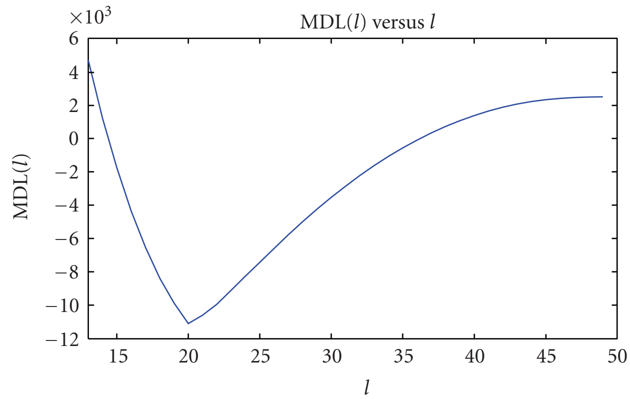

FIGURE 4: Plot of MDL as a function of l computed from (19). $x(n)$is a mixture of 10 (real) sinusoids in white noise where the SNR is $\eta = 0$ dB. It is apparent from the figure that the minimum occurs at $l = 2L = 20$.

In fact, for Gaussian noise scenarios the NLS is the best (in terms of minimum variance) estimator that achieves the Cramer-Rao bound. The estimation accuracy of NLS is given by

$$\text{var}(\hat{\omega}_k) = \frac{6}{M^3 \eta_k}. \tag{21}$$

For ANC problems, M could be allowed to be reasonably large and as will be shown later, the PFANC method demonstrates superior performance for reasonable SNR values ($\eta >$ 10 dB). For example, for $\eta_k = 10$ dB and $M = 10000$, var($\hat{\omega}_k$) will be of the order of 10^{-8}. Thus, very precise estimates can be expected from the NLS method. The drawback is that the minimization in (20) is a complex multidimensional search problem and could be computationally intractable. This drawback can be addressed by using the subspace methods, whose estimation performance is close to the best performance shown in (21) and is computationally efficient [14]. Out of the multitude of subspace methods (like MUSIC, ESPRIT, and Min-Norm), ESPRIT has been shown to have superior statistical performance [14, 18]. Moreover, ESPRIT does not give spurious frequencies once the correct order L is determined using (19).

ESPRIT Calculations. For completeness of discussion, the steps in frequency estimation using the ESPRIT method are as follows.

(1) Calculate the sample covariance matrix using (17).

(2) Calculate the eigenvalue decomposition of $\widehat{\mathbf{R}}$

$$\widehat{\mathbf{R}} = \widehat{\mathbf{\Omega}}\widehat{\mathbf{\Lambda}}\widehat{\mathbf{\Omega}}^{H}, \tag{22}$$

where, $\widehat{\mathbf{\Lambda}} = \operatorname{diag}(\widehat{\lambda}_1, \widehat{\lambda}_2, \ldots, \widehat{\lambda}_m)$.

(3) Let $\widehat{\mathbf{\Omega}}$ be the matrix composed of the eigenvectors corresponding to the eigenvalues $\widehat{\lambda}_1, \widehat{\lambda}_2, \ldots, \widehat{\lambda}_{2L}$,

$$\widehat{\mathbf{\Omega}} = [\widehat{\boldsymbol{\sigma}}_1, \widehat{\boldsymbol{\sigma}}_2, \ldots, \widehat{\boldsymbol{\sigma}}_{2L}]. \tag{23}$$

Define

$$\begin{aligned} \widehat{\mathbf{\Omega}}_1 &= \begin{bmatrix} \mathbf{I}_{m-1} & 0 \end{bmatrix} \widehat{\mathbf{\Omega}} \quad (m-1) \times 2L, \\ \widehat{\mathbf{\Omega}}_2 &= \begin{bmatrix} 0 & \mathbf{I}_{m-1} \end{bmatrix} \widehat{\mathbf{\Omega}} \quad (m-1) \times 2L. \end{aligned} \tag{24}$$

(4) The frequencies are estimated by solving the system of linear equations

$$\widehat{\mathbf{\Omega}}_1 \widehat{\mathbf{\Theta}} \approx \widehat{\mathbf{\Omega}}_2 \tag{25}$$

whose least squares solution is given by

$$\widehat{\mathbf{\Theta}} = \left(\widehat{\mathbf{\Omega}}_1^{H} \widehat{\mathbf{\Omega}}_1\right)^{-1} \widehat{\mathbf{\Omega}}_1^{H} \widehat{\mathbf{\Omega}}_2. \tag{26}$$

The argument of the complex eigenvalues of $\widehat{\mathbf{\Theta}}$ gives the frequencies ω_k. Let δ_k be the eigenvalue of $\widehat{\mathbf{\Theta}}$. Then the frequency estimates are

$$\widehat{\omega}_k = -\arg(\delta_k). \tag{27}$$

The amplitudes and phases can be estimated as

$$\widehat{\boldsymbol{\beta}} = \left(\mathbf{G}^{H}\mathbf{G}\right)^{-1} \mathbf{G}^{H}\mathbf{x}, \tag{28}$$

where $\widehat{\boldsymbol{\beta}} = [\widehat{a}_1 e^{j\widehat{\phi}_1} \cdots \widehat{a}_L e^{j\widehat{\phi}_L}]^{T}$, $\mathbf{x} = [x(1) \cdots x(M_0)]^{T}$. The matrix $\mathbf{G}$ is calculated from the estimated frequencies $\widehat{\omega}_k$'s according to

$$\mathbf{G} = \begin{bmatrix} e^{i\widehat{\omega}_1} & \cdots & e^{i\widehat{\omega}_L} \\ \vdots & & \vdots \\ e^{iM_0\widehat{\omega}_1} & \cdots & e^{iM_0\widehat{\omega}_L} \end{bmatrix}, \tag{29}$$

M_0 indicates that the first M_0 samples of the data frame $\mathbf{x}$ are used for ESPRIT calculations.

The estimated frequencies are then used to generate a reference periodic signal $r(n)$ shown in Figure 2 using

$$r(n) = \sum_{k=1}^{L} \widehat{a}_k \cos\left(\widehat{\omega}_k n + \widehat{\phi}_k\right). \tag{30}$$

The estimates $\widehat{a}_k$ and $\widehat{\phi}_k$ can be further improved, by estimating $q(n)$ from $r(n)$, by using the adaptive cancellation technique as explained in the next section.

2.3.2. Signal Separation. This block essentially estimates $\widehat{q}(n)$ from $r(n)$ in (30) by trying to cancel $q(n)$ from $x(n) = q(n) + v(n)$ (i.e., by estimating $\widehat{q}(n)$ from $r(n)$). This step adaptively finetunes the parameters $\widehat{a}_k, \widehat{\phi}_k$ from (30) to their actual values and also separates $x(n)$ into $\widehat{q}(n)$ and $\widehat{v}(n)$. It can be easily seen from Figure 2 that after convergence, the error signal is $\widehat{v}(n)$ and the cancelling signal becomes $y(n) \approx -\widehat{q}(n) \Rightarrow \widehat{q}(n) \approx -y(n)$, thus achieving separation. The update equation for the weight vector $\mathbf{w}_{\text{sep}}(n+1)$ is given by

$$\mathbf{w}_{\text{sep}}(n+1) = \mathbf{w}_{\text{sep}}(n) + \frac{\mu \widehat{v}(n)\mathbf{r}(n)}{\mathbf{r}^{T}(n)\mathbf{r}(n)}. \tag{31}$$

An initial portion of the reference signal $x(n)$ is used to estimate the tones and achieve component separation. The separated signal components will be denoted by $\widehat{q}(n)$ and $\widehat{v}(n)$ to indicate that they are estimates. For high η, the estimates are highly accurate. By incorporating the fact that the effect of an arbitrary transfer function on a sinusoid can be modeled by two taps [4, 9], computationally efficient implementations of the above separation scheme can be derived.

2.3.3. Generation of Antinoise Signal. This block generates the antinoise (i.e., the cancelling) signal in two steps via two parallel ANC systems, ANC_q and ANC_v. ANC_q uses $\widehat{q}(n)$ as the reference and generates an antinoise $y_q(n)$ which attempts to cancel $d_q(n)$. $y_q(n)$, after passing through $S(z)$ becomes $y'_q(n)$. The error signal $e_q(n)$ is given by

$$e_q(n) = d(n) - y'_{\widehat{q}}(n) = d_v(n) + \left(d_q(n) - y'_{\widehat{q}}(n)\right), \tag{32}$$

where $d(n)$ denotes the desired noise to be cancelled and $y_{\widehat{q}}(n)$ denotes the anti-noise generated by ANC_q as shown in Figure 2. The adaptive filter $W_q(z)$ and $S(z)$ linearly operate on the periodic signal $\widehat{q}(n)$ to produce $y'_{\widehat{q}}(n)$ which lies in the same subspace as that of $\widehat{q}(n)$. Since $d_v(n)$ is uncorrelated with $d_q(n)$, convergence in this case essentially means that $(d_q(n) - y'_{\widehat{q}}(n))$ is minimised in the mean square sense. Thus after convergence, from (32)

$$e_q(n) \approx d_v(n). \tag{33}$$

The update equation for the weight vector $\mathbf{w}_q(n+1)$ is given by

$$\mathbf{w}_q(n+1) = \mathbf{w}_q(n) + \frac{\mu\, e_q(n)\widehat{\mathbf{q}}'(n)}{\widehat{\mathbf{q}}'^{T}(n)\widehat{\mathbf{q}}'(n)}, \tag{34}$$

where $\hat{\mathbf{q}}'(n)$ is the signal vector obtained after filtering $\hat{\mathbf{q}}(n)$ with $\hat{S}(z)$. After convergence is attained in ANC_q, the adaptation is stopped and $\mathbf{w}_q(n+1)$ is fixed to the steady-state value. The system ANC_q continues to generate antinoise which cancels $d_q(n)$. The error microphone's output is $d_\nu(n)$. The next step is to switch on ANC_ν whose reference input is the separated random part $\hat{\nu}(n)$ and error input is $e_\nu(n)$ which it tries to minimize by generating the antinoise signal $y_{\hat{\nu}}(n)$.

The update equation for the weight vector $\mathbf{w}_\nu(n+1)$ is given by

$$\mathbf{w}_\nu(n+1) = \mathbf{w}_\nu(n) + \frac{\mu\, e_\nu(n)\hat{\boldsymbol{\nu}}'(n)}{\hat{\boldsymbol{\nu}}'^T(n)\hat{\boldsymbol{\nu}}'(n)}, \tag{35}$$

where $\hat{\boldsymbol{\nu}}'(n)$ is the signal obtained after filtering $\hat{\boldsymbol{\nu}}(n)$ with $\hat{S}(z)$. After convergence the error is $e_\nu(n)$ and should not contain any component of the periodic noise as long as the acoustic system is stationary. Since we have access to only one error sequence, the procedure for finding the optimum $W_q(z)$ and $W_\nu(z)$ is sequential. However, this sequential adaptive tuning procedure can be repeated when the error sequence energy goes above a prespecified threshold, since the changes in the acoustic system will result in an increase in the energy of the error sequence.

It should be noted that the adaptation methods presented here for the implementation of ANC_q and ANC_ν are based on simple FXLMS for the sake of simplicity and quick experimentation. For the tonal active noise control part ANC_q, the analysis and methods described in [9–12] can be incorporated. The proposed method can also be extended to accommodate multiple reference microphones in which case the reduced reference signal set procedure developed in [19] can lessen the controller dimension and computational complexity.

3. Calculation of NAL_{max}

The goal of this section is to sketch the method for calculating NAL_{max}. NAL_{max} is obtained, assuming that the adaptive ANC system has converged and that the adaptive ANC system is linear time-invariant (stationary), as shown in Figure 5(a). First the NAL_{max} for wide sense stationary random noise is derived and then expanded to accommodate sinusoidal mixtures in a wide sense stationary random noise. This derivation is important since it gives an insight into how well the proposed method performs when compared to conventional ANC structure with a single cancelling filter. NAL_{max} calculation is also very useful during actual real-time implementation of an ANC system. Based on a few basic measurements on an actual real-time ANC setup, the computation of NAL_{max} gives an indication of the expected performance. This comes in handy when experimenting with real-time ANC systems like the acoustic laboratory shown in Figure 9. Once the maximum limit is known, the sources of error in an underperforming system can be analyzed.

In the three subsections that follow, the expressions for NAL_{max} are derived for three cases: when the acoustic noise is (i) wide-sense stationary, (ii) a sum of sinusoidal signals, and (iii) a mixture of sinusoids and a wide-sense stationary random noise, that is, combination of (i) and (ii).

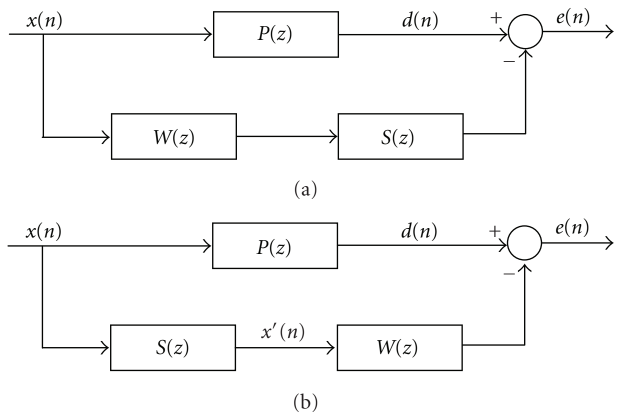

Figure 5: (a) Feedforward ANC system (b) interchanging the $W(z)$ and $S(z)$ blocks owing to their linear time-invariant nature.

3.1. $\textbf{NAL}_{\textbf{max}}$ *for Wide-Sense Stationary Signals.* In linear estimation problems involving stationary signals, Wiener filtering provides the best estimate of a linear time-invariant (LTI) system based on (minimum mean square error) MMSE criterion [20, 21]. In a linear stationary ANC system as shown in Figure 5(a), the transfer function $W(z) = P(z)/S(z)$ often has poles outside the unit circle, that is, when $S(z)$ is nonminimum-phase. In such cases, a stable noncausal Wiener filter is the best estimate of $W(z) = P(z)/S(z)$. An expression for MMSE using *noncausal* Wiener filter is given in [4, 19]. However if $x(n)$ is not perfectly predictable, a noncausal Wiener filter is not realizable. The realizeable MMSE estimate is then the *causal* and stable Wiener filter that approximates $P(z)/S(z)$.

In order to derive a measurable expression for NAL_{max}, let $W_c^o(z)$ be the causal stable IIR Wiener filter that best approximates $P(z)/S(z)$:

$$W_c^o(z) \approx \frac{P(z)}{S(z)}, \tag{36}$$

with the superscript and subscript denoting the optimality and causality, respectively, of the solution.

The expression for $W_c^o(z)$ is obtained as follows:

Let $\Psi_{uv}(z)$ be the cross-power spectral density of two jointly stationary processes $u(n)$ and $v(n)$. The expression for $\Psi_{uv}(z)$ is

$$\Psi_{uv}(z) = \sum_{n=-\infty}^{n=+\infty} R_{uv}(n) z^{-n}, \tag{37}$$

where $R_{uv}(n)$ denotes the cross-covariance sequence. $\Psi_{uu}(z)$ denotes the power spectral density of a stationary process $u(n)$ and can be expressed using (37) after appropriate substitution.

Assuming the signals in the system have real positive (power spectrum density) PSD satisfying Paley-Wiener condition [21], the stable causal Wiener solution is given in

[20, 22]. Referring to Figure 5(b) the causal Wiener solution is

$$W_c^\circ(z) = \frac{1}{\Psi_{x'}(z)}\left\{\frac{\Psi_{dx'}(z)}{\Psi_{x'}(z^{-1})}\right\}_+, \quad (38)$$

where $\{\cdot\}_+$ denotes the causal part of the quantity inside. The subscript c stands for causality. $\Psi_{x'}(z)$ is the minimum-phase and causal spectral factor of $\Psi_{x'x'}(z)$, that is,

$$\Psi_{x'x'}(z) = \Psi_{x'}(z)\Psi_{x'}(z^{-1}). \quad (39)$$

The next few steps are devoted to the computation of $\Psi_{dx'}(z)$ and $\Psi_{x'}(z)$ in terms of $P(z)$, $S(z)$ and the input PSD $\Psi_{xx}(z)$. The transfer function that maps $x(n)$ to $x'(n)$ is $S(z)$ and the transfer function that maps $x'(n)$ to $d(n)$ is $P(z)/S(z)$. Thus using the theory of linear filtering of random processes [21, 23] $\Psi_{dx'}(z)$ is related to $\Psi_{xx}(z)$ as follows:

$$\begin{aligned}\Psi_{dx'}(z) &= \frac{P(z)}{S(z)}\Psi_{x'x'}(z)\\ &= \frac{P(z)}{S(z)}\left(S(z)S(z^{-1})\Psi_{xx}(z)\right)\\ &= S(z^{-1})P(z)\Psi_{xx}(z).\end{aligned} \quad (40)$$

It should be noted that

$$\begin{aligned}\Psi_{x'x'}(z) &= S(z)S(z^{-1})\Psi_{xx}(z)\\ &= S(z)S(z^{-1})\Psi_x(z)\Psi_x(z^{-1}),\end{aligned} \quad (41)$$

where, the spectral factorization of $\Psi_{xx}(z)$ is

$$\Psi_{xx}(z) = \Psi_x(z)\Psi_x(z^{-1}), \quad (42)$$

$\Psi_x(z)$ is minimum-phase and causal. $S(z)$ can be decomposed into $S_{\min}(z)$ and $S_{\max}(z)$ as follows:

$$S(z) = S_{\min}(z)S_{\max}(z). \quad (43)$$

$S_{\max}(z)$ is a polynomial that contains only the zeros outside the unit circle and is responsible for the noncausality of $P(z)/S(z)$. Hence it follows that $S_{\min}(z)$ is a minimum phase, stable and rational function. Thus (41) becomes

$$\Psi_{x'x'}(z) = \underbrace{S_{\min}(z)S_{\max}(z)\Psi_x(z)}_{\text{causal}}\underbrace{S_{\min}(z^{-1})S_{\max}(z^{-1})\Psi_x(z^{-1})}_{\text{noncausal}}, \quad (44)$$

$S_{\max}(z^{-1})$ is a polynomial of positive powers of z and hence is noncausal. The causal $S_{\max,c}(z)$ can be expressed as

$$S_{\max,c}(z) = z^{-r}S_{\max}(z^{-1}), \quad (45)$$

where r is the order of $S_{\max}(z^{-1})$.

It should be noted that any stable transfer function can also be written as a product of a stable minimum-phase and a stable all-pass system. In this case

$$\begin{aligned}S(z) &= S_{\min}(z)\, z^{-r}S_{\max}(z^{-1})\frac{S_{\max}(z)}{z^{-r}S_{\max}(z^{-1})}\\ &= S_{\min}(z)S_{\max,c}(z^{-1})\frac{S_{\max}(z)}{S_{\max,c}(z)}.\end{aligned} \quad (46)$$

By definition, $S_{\min}(z)S_{\max,c}(z^{-1})$ is a minimum-phase system and will be denoted by $S_{\mathrm{MIN}}(z)$.

$S_{\max}(z)/S_{\max,c}(z)$ is an all-pass transfer function and will be denoted as $S_{\mathrm{AP}}(z)$. It should be noted that the inverse of $S_{\mathrm{AP}}(z)$ is $S_{\mathrm{AP}}(z^{-1})$. The above calculations lead to the familiar minimum-phase, all-pass decomposition [22]

$$S(z) = S_{\mathrm{MIN}}(z)S_{\mathrm{AP}}(z). \quad (47)$$

Equation (44) can now be written as

$$\begin{aligned}&\Psi_{x'}(z)\Psi_{x'}(z^{-1})\\ &= \underbrace{\left[S_{\min}(z)S_{\max,c}(z)\Psi_x(z)\right]}_{\text{min-phase,causal}}\underbrace{\left[S_{\min}(z^{-1})S_{\max,c}(z^{-1})\Psi_x(z^{-1})\right]}_{\text{nonmin-phase,noncausal}}.\end{aligned} \quad (48)$$

The minimum-phase and causal $\Psi_{x'}(z)$ is computed as

$$\Psi_{x'}(z) = S_{\min}(z)S_{\max,c}(z)\Psi_x(z). \quad (49)$$

Substituting (49) and (40) in (38)

$$\begin{aligned}W_c^\circ(z) &= \frac{1}{S_{\min}(z)S_{\max,c}(z)\Psi_x(z)}\\ &\quad\times\left\{P(z)\cdot\Psi_x(z)\cdot\frac{S_{\max,c}(z)}{S_{\max}(z)}\right\}_+\end{aligned} \quad (50)$$

which can be written as

$$\begin{aligned}W_c^\circ(z) &= \frac{1}{S_{\mathrm{MIN}}(z)\Psi_x(z)}\\ &\quad\times\left\{P(z)\cdot\Psi_x(z)\cdot S_{\mathrm{AP}}(z^{-1})\right\}_+\end{aligned} \quad (51)$$

and the corresponding output error of the ANC system is computed by

$$\begin{aligned}e_c^\circ(n) &= d(n) - y'(n)\\ &= p(n) * x(n) - w_c^\circ(n) * s(n) * x(n),\end{aligned} \quad (52)$$

where $*$ denotes the convolution operation.

When using numerical software, it should be noted that even though the form of $W_c^\circ(z)$ in (51) is IIR, it should be considered as a very long FIR filter. The calculation of (51) can be done entirely in the frequency domain by using numerical software like MATLAB. The minimum-phase, maximum-phase decomposition can be done employing cepstral techniques [22]. The causal part can be found by taking the (Inverse Fast Fourier Transform) IFFT of the quantity inside the bracket and discarding the negative index terms. The maximum possible NAL, that is, the upper bound, can then be computed using

$$\mathrm{NAL}_{\max,x}^{\mathrm{random}} = 20\log_{10}\left(\frac{\|(d(m))\|_2}{\|(e_c^\circ(m))\|_2}\right) \quad (53)$$

for

$$n - \frac{L_f}{2} < m \le n + \frac{L_f}{2}, \quad (54)$$

where L_f is the frame length and n the frame position and $\|\cdot\|_2$ denotes the 2-norm operation.

It should be noted that $\mathrm{NAL}_{\max,x}^{\mathrm{random}}$ given in (53) can be computed as long as the spectral factorization of (42) is valid.

3.2. **NAL**$_{\max}$ *for a Mixture of Sinusoids with No Random Noise.* When $x(n)$ contains sinusoidal components with no random noise, the power spectral density $\Psi_{xx}(z)$ of $x(n)$ becomes singular. Hence the technique developed in Section 3.1 cannot be applied. The way out of this difficulty is by recognising that for a pure mixture of sinusoids or a purely periodic signal without any background random noise, the effect of nonminimum phase secondary path can be perfectly equalised. Hence perfect cancellation can be acheived. The nonminimum phase nature essentially translates to a noncausal equaliser which can be realized for purely periodic signals because the future values of the signal are known by indexing the past, that is, by knowing one past period of the signal. For example placing a (noncausal) equalizer $H_{\text{eq}}(z) = S_{\text{AP}}(z^{-1})$ as shown in Figure 6 can perfectly equalize a nonminimum phase secondary path. As a result, the effective secondary path becomes the minimum-phase $S_{\text{MIN}}(z)$, which can be exactly equalized since it is perfectly invertible. Factorizing $S(z)$ as $S_{\text{MIN}}(z)S_{\text{AP}}(z)$, the equalizer $H_{\text{eq}}(z)$ is obtained as

$$H_{\text{eq}}(z) = \sum_{k=0}^{L_S-1} s_{\text{AP}}(-n)z^{-k} \approx S_{\text{AP}}(z^{-1}), \tag{55}$$

where L_S is the order of $S_{\text{AP}}(z)$ (after sufficient truncation) when expressed in FIR form. Thus $h_{\text{eq}}(n) = s_{\text{AP}}(-n)$. The equalization operations are summarized as

$$\begin{aligned} x_{\text{eq}}(n) &= \sum_{k=1}^{L_S-1} h_{\text{eq}}(k)x(n+k) \\ &= \sum_{k=1}^{L_S-1} h_{\text{eq}}(k)x\Big(n+k-mN_p\Big), \end{aligned} \tag{56}$$

where N_p is the period and $m = \lfloor k/N_p \rfloor + 1$. Here $\lfloor \cdot \rfloor$ denotes the integer floor. For cases for which $N_p \geq L_S$,

$$x_{\text{eq}}(n) = \sum_{k=0}^{L_S-1} h_{\text{eq}}(k)x\Big(n+k-N_p\Big). \tag{57}$$

Thus in theory, a pure mixture of sinusoids can be perfectly cancelled. Thus arbitrarily high noise attenuation level can be reached, that is, $\text{NAL}_{\max}^{\text{sinusoids}} \rightarrow \infty$.

3.3. **NAL**$_{\max}$ *for Mixture of Sinusoids in Wide-Sense Stationary Random Noise.* When $x(n)$ contains sinusoidal components in addition to the wide-sense stationary random noise, the upper bound becomes a function of η and the $\text{NAL}_{\max}$ associated with the random noise part. The derivation below is fundamentally based on the fact that every stationary sequence can be expressed as a sum of a perfectly predictable sequence and a stochastic sequence which are orthogonal to each other (Wold's Decomposition) [24]. In this case the sinusoidal components constitute the perfectly predictable part and the background broadband noise component constitutes the stochastic part.

The noise attenuation level Γ_x by definition is given by

$$\Gamma_x = \frac{\xi_d}{\xi_e} = \frac{\xi_d}{\xi_{e_q} + \xi_{e_v}}, \tag{58}$$

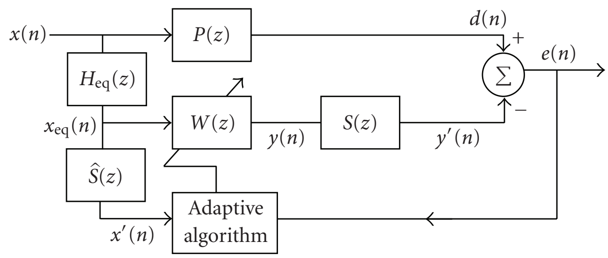

Figure 6: Perfect nonminimum phase equalization for periodic signals. $\text{H}_{\text{eq}}(z)$ can be noncausal since the input $x(n)$ is periodic.

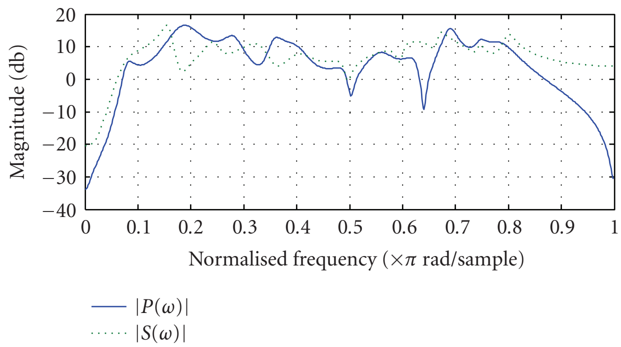

Figure 7: Magnitude responses of the primary and secondary paths used in the simulation.

where ξ_d is the power of $d(n)$, ξ_e is the power of $e(n)$, ξ_{e_q} is the power of the tonal component (after convergence) of $e(n)$, and ξ_{e_v} is the power of the random component (after convergence) of $e(n)$.

Due to linearity, $d(n) = p(n) * (x(n)) = p(n) * (q(n) + v(n)) = d_q(n) + d_v(n)$. Since the deterministic component $q(n)$ and the random component $v(n)$ are independent,

$$\xi_{d_x} = \xi_{d_q} + \xi_{d_v}, \tag{59}$$

$$\Gamma_x = \frac{\xi_{d_q} + \xi_{d_v}}{\xi_{e_q} + \xi_{e_v}}. \tag{60}$$

The performance gain due to the proposed separation-based parallel feedforward method can be understood from (60). By using a dedicated cancelling filter, the periodic part can be almost perfectly canceled, that is, $\xi_{e_q} \approx 0$. Perfect cancellation of the periodic part is not possible with conventional single filter ANC. Hence the denominator of (60) is always higher for the conventional method. On the assumption that the periodic part can be perfectly cancelled, (60) becomes

$$\Gamma_x \approx \frac{\xi_{d_q} + \xi_{d_v}}{\xi_{e_v}}, \tag{61}$$

Γ_x in (61) defines an upper bound for the ANC system since it is the maximum NAL possible under the assumptions of perfect separation and perfect cancellation of the periodic part $q(n)$. Thus for a mixture of periodic tones in random noise, the maximum achievable NAL, that is, the $\text{NAL}_{\max,x}^{\text{mixture}}$ is given by Γ_x. Denoting the NAL of an ANC system to a

Table 1: Performance comparison of the proposed separation-based parallel ANC and conventional single-filter ANC for different tones and SNR combination. Γ_x (dB) is the maximum possible NAL.

Number	Number of sinusoids, L	η (dB)	Proposed PFANC NAL (dB)	Single-Filter ANC NAL (dB)	$\text{NAL}_{\max,x}^{\text{mixture}}$ from (62)
1	1	10	25.57	24.61	26.38
2	1	20	33.69	20.16	36.33
3	1	30	44.56	25.97	46.24
4	1	40	53.03	35.60	56.24
5	2	10	31.83	33.21	34.20
6	2	20	42.88	32.20	44.09
7	2	30	52.34	35.91	54.05
8	2	40	58.00	44.64	64.00
9	3	10	31.96	31.86	33.00
10	3	20	41.99	30.69	43.00
11	3	30	50.03	35.34	53.80
12	3	40	55.86	44.43	64.00
13	4	10	31.54	31.43	33.00
14	4	20	40.36	30.5	42.67
15	4	30	48.32	35.49	53.64
16	4	40	58.40	44.35	63.00
21	7	10	31.45	30.05	31.72
22	7	20	41.48	41.87	42.05
23	7	30	49.03	44.65	54.02
24	7	40	57.40	47.65	62.95
17	8	10	31.45	30.64	30.64
18	8	20	44.40	42.14	43.25
19	8	30	42.14	29.52	46.79
20	8	40	55.40	47.06	63.00

random noise input by $\Gamma_\nu = \xi_{d_\nu}/\xi_{e_\nu}$, the expression in (61) can be written as

$$\begin{aligned}\text{NAL}_{\max,x}^{\text{mixture}} = \Gamma_x &= \left(\frac{\xi_{d_\nu}}{\xi_{e_\nu}} + \frac{\xi_{d_\nu}}{\xi_{e_\nu}}\frac{\xi_{d_q}}{\xi_{d_\nu}}\right) \\ &= \Gamma_\nu + \Gamma_\nu\frac{\xi_{d_q}}{\xi_{d_\nu}} = \Gamma_\nu\left(1+\frac{\xi_{d_q}}{\xi_{d_\nu}}\right).\end{aligned} \tag{62}$$

The above expression generalizes the form of maximum achievable NAL derived for stationary random noise in Section 3.1 to stationary random noise along with sinusoids. It should be noted that Γ_ν of (62) is calculated from (51)–(53) after setting $\Psi_{xx}(z) = \Psi_{\nu\nu}(z)$, that is, $\Gamma_\nu = \text{NAL}_{\max,\text{v}}^{\text{random}}$. To see the effect of the primary path $P(z)$ it should be noticed that

$$\Gamma_x = \Gamma_\nu\left(1+\frac{\sum_{k=1}^{L}\left(A_k^2/2\right)|P(e^{j\omega_k})|^2}{\xi_\nu\left((1/2\pi)\int_{-\pi}^{+\pi}|P(e^{j\omega})|^2 d\omega\right)}\right), \tag{63}$$

where ω_k's are the frequencies of the sinusoids in $q(n) = \sum_{k=1}^{L} a_k\cos(\omega_k n + \phi_k)$.

In case of a single tone ($L = 1$),

$$\Gamma_x = \Gamma_\nu\left(1+\eta_1\frac{|P(e^{j\omega_1})|^2}{\left((1/2\pi)\int_{-\pi}^{+\pi}|P(e^{j\omega})|^2 d\omega\right)}\right), \tag{64}$$

and for

$$\alpha_1 = \frac{|P(e^{j\omega_1})|^2}{\left((1/2\pi)\int_{-\pi}^{+\pi}|P(e^{j\omega})|^2 d\omega\right)}. \tag{65}$$

The expression becomes

$$\Gamma_x = \Gamma_\nu(1+\eta_1\alpha_1). \tag{66}$$

For the parallel ANC system, (66) can be written as

$$\text{NAL}_{\max,x}^{\text{mixture}} = \text{NAL}_{\max,\nu}^{\text{random}}(1+\eta_1\alpha_1). \tag{67}$$

That is, the improvement in NAL of a mixture of periodic and random noise signals is greater than that of the performance of a purely random noise by a factor of $(1 + \eta_1\alpha_1)$. If we assume $\eta_1\alpha_1 \gg 1$, then $\Gamma_x \approx \Gamma_\nu\eta_1\alpha_1$. It seems that this way of characterizing maximum achievable performances of ANC system is new. Furthermore, α_1 is low when there are deep notches in $P(e^{j\omega})$ at the frequency ω_1, in which case the relative position of reference microphone should be changed.

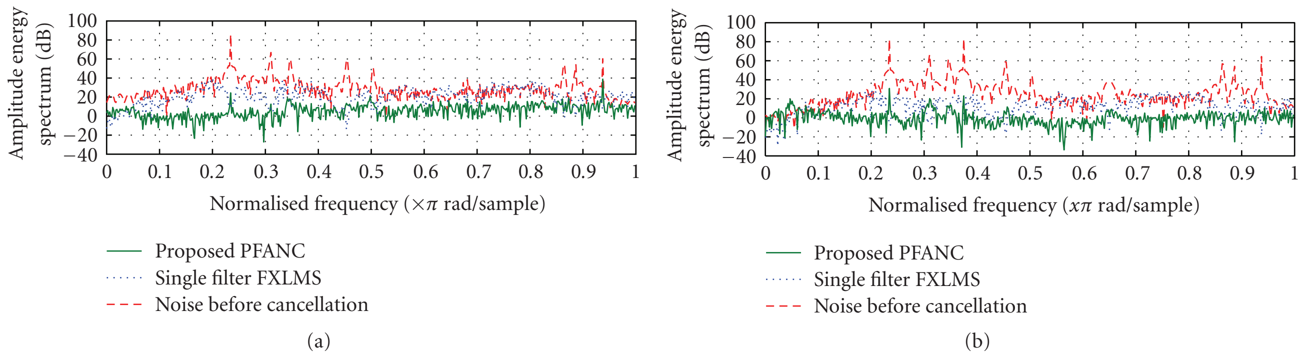

Figure 8: (a) Comparison of the conventional FXLMS with single filter and the proposed PFANC for 8 tones in white noise. (b) Comparison of the conventional FXLMS with single filter and the proposed PFANC for 10 tones in white noise.

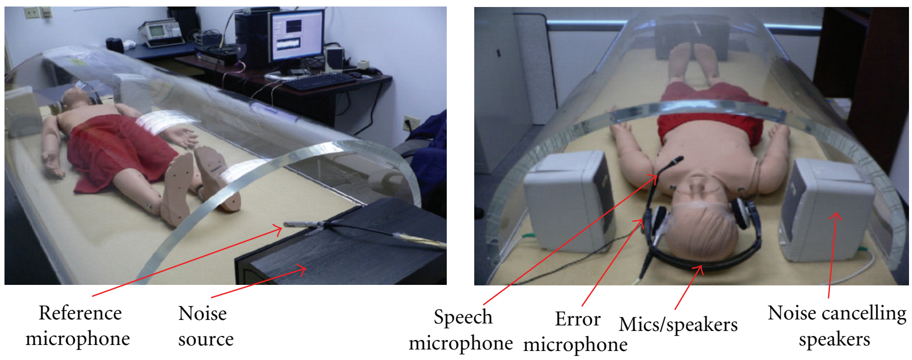

Figure 9: The real-time Active Noise Control setup. The transparent semicircular structure mimics the MRI cavity. Recorded noise from actual MRI room is used to drive the speakers to recreate MRI environment. The area around the manikin's ears is where cancellation is desired.

As a summary, the procedure described in the paper is as follows.

Step 1. Given a feedforward ANC set up as shown in Figure 9, record $x(n)$ and estimate $P(z)$ and $S(z)$. The same ANC setup can be used for both measurements. The primary path $P(z)$ can be measured switching off the anti-noise generation (i.e., $y(n) = 0$) and by recording the measurements $x(n)$ from the reference microphone and $d(n)$ from error microphone, and estimating the transfer function that maps $x(n)$ to $d(n)$. For measurement of $S(z)$, methods given in [4] can be employed.

Step 2. If $x(n)$ does not have tones use (51) through (53) to calculate $\text{NAL}_{\max}$. The conventional single-filter approach is sufficient for this case. Additionally the optimum Wiener filter can also be calculated from (51) and can be used to initialize the adaptive filter $W(z)$.

Step 3. If $x(n)$ has tones along with random noise, use (67) to calculate the $\text{NAL}_{\max}$. This step needs calculation of the power spectral density of $\nu(n)$ which can be generated from $x(n)$ using the adaptive noise cancellation-based separation that was developed in Sections 2.3.1 and 2.3.2.

Step 4. Implement the PFANC scheme. ANC_q requires the calculation of the period N_p from the autocorrelation sequence of $x(n)$ and the calculation of $H_{\text{eq}}(z)$ from (55). Additionally the optimum Wiener filter can also be calculated from (51) and can be used to initialize the adaptive filter $W_\nu(z)$.

4. Experimental Results

Simulation of the proposed method, using FXLMS for adaptation, was implemented by synthesizing $x(n) = q(n) + \nu(n)$ for different SNR values η and different linear combinations of multiple sinusoids. Single and multitone signals of various frequencies were synthesized and $\nu(n)$ of varying power was added. The $P(z)$ and $S(z)$ used in these experiments were obtained from [4] and characterize real life acoustic transfer functions (Figure 7).

Each experiment was done on a 100000 sample long $x(n)$ generated at a sampling rate of 16 kHz. The first 10000 samples were used for the frequency estimation part, that is, determination of L, ω_k, a_k, ϕ_k .The next 20000 samples were used to train the adaptive noise canceller that is, to separate $q(n)$ and $\nu(n)$ as discussed in Section 2.3.2. The rest of the samples were used to generate antinoises via ANC_q and ANC_ν. The performance of the proposed method is compared with the conventional FXLMS-based ANC [4]. The results are shown in Table 1. The last column shows the maximum achievable noise attenuation level $\mathrm{NAL}_{\max,x}^{\text{mixture}}$ using (62). $\mathrm{NAL}_{\max,\nu}^{\text{random}}$ was found to be 19.58 dB for the $P(z)$ and $S(z)$ used. It can be seen that the NAL obtained by the proposed method is closer to $\mathrm{NAL}_{\max,x}^{\text{mixture}}$ than that obtained by the conventional single-filter method. For low SNR values (like 10 dB) the performance improvement is less noticeable when compared to higher SNR values. For higher SNR values excellent performance of the proposed method can be observed and thus the proposed method targets high SNR applications. It should also be noticed that perfect separation and cancellation of the periodic part may not be achieved in practice resulting in maximum NAL being less than that given by (62).

Experiments 21 through 24 were carried out using a signal derived from functional magnetic resonance imaging (fMRI) acoustic noise [25, 26]. fMRI acoustic noise has SNR's > 20 dB and hence amenable to the proposed approach. Seven significant tones from a recorded fMRI acoustic noise were estimated and the sinusoidal component $q(n)$ was created from the seven tones. White noise was then added with different SNR values. Figures 8(a) and 8(b) present a comparison of noise cancellation performance between the conventional single-filter method and the proposed PFANC for an 8 tones case and a 10 tones case, respectively. As it can be seen, PFANC considerably reduces the background noise in addition to sinusoidal noise.

5. Conclusion

In this paper the focus has been on active noise control of acoustic signals that have a periodic/harmonic structure along with a random noise. The key idea stems from the observation that the periodic component and the random component may need different noise cancelling filters for the best performance. This leads to the idea of *separation* of the periodic and random components and the proposed *parallel* form of ANC, in which the reference input is separated into periodic and random components and separate anti-noise signals are parallely generated. A novel adaptive filtering-based implementation of signal separation and anti-noise generation is developed and simulated. A simple but an insightful formula is derived to quantify performance. The method is shown to work very well with single and multi-tone signals in white noise.

Some important and recurring signals, blocks, and notation that are used in the paper are listed in the Notations and Definitions section. Other definitions are made as they are introduced.

Notations and Definitions

ξ: It gives the signal energy. For a signal $g(n)$, the energy is given by $\xi_g = \mathrm{E}\{g^2(n)\}$, where $\mathrm{E}\{.\}$ is the expectation operator.

f: The subscript "w" for any quantity f denotes the component of f, corresponding to the subscripted variable.

$x(n)$: The version of acoustic noise recorded by a reference microphone placed somewhere in the noisy area. This measurement is known as the reference signal in ANC literature.

$q(n)$: The periodic component of noise which can be written as a sum of sinusoids, that is, $q(n) = \sum_{k=1}^{L} a_k \cos(\omega_k n + \phi_k)$.

$\nu(n)$: The random noise component. It is assumed to be zero-mean and stationary, where $x(n) = q(n) + \nu(n)$.

N_p: The period of the periodic component. N_p can be measured from the autocorrelation sequence of $x(n)$.

η: The (signal-to-noise ratio) SNR of periodic component to the random component, that is, $\eta = \xi_q/\xi_\nu = \sum_{k=1}^{L} a_k^2/2\xi_\nu$.

η_k: The individual SNR of sinusoidal components, that is, $\eta_k = a_k^2/2\xi_\nu$. It follows that $\eta = \sum_{k=1}^{L} \eta_k$.

$e(n)$: The measurement from error microphone. The cancellation is achieved around the error microphone. The error signal corresponding to a particular component will have a subscript to denote that. For example, the component of error signal corresponding to $q(n)$ will be denoted as $e_q(n)$ and its power will be ξ_{e_q}.

$P(z)$: The stable linear transfer function representing the *primary path*, which is the path through the acoustic cavity between reference and error microphones. The corresponding impulse response is $p(n)$.

$S(z)$: The stable linear transfer function representing the *secondary path*. $S(z)$ models system functions of microphones, digital and analog circuits, the cancelling loudspeaker, and the acoustic gaps between the loudspeaker and error microphone. $S(z)$ is stable and assumed to have no zeros on the unit circle.

$\hat{S}(z)$: An estimate of $S(z)$, which is placed in the reference signal path. In all the ensuing discussions, $\hat{S}(z)$ is considered as an accurate model, that is, $\hat{S}(z)=S(z)$.

$d(n)$: The noise signal after passing through $P(z)$ representing the noise signal to be canceled, that is, the *desired signal*. It should be noted that $d(n) = d_q(n) + d_\nu(n)$.

$W(z)$: The main adaptive noise cancelling (control) filter which generates the appropriate anti-noise signal using the error and reference input signals.

$y(n)$: The control signal generated by $W(z)$. $y(n)$ gets shaped into $y'(n)$ when it reaches the error microphone due to the effect of the secondary path $S(z)$.

NAL_{max}: The maximum achieveable Noise Attenuation Level possible for a given ANC. algorithm/strategy. The NAL_{max} for an input reference signal $g(n)$ will be denoted as $NAL_{max,g}$

Γ_g: Noise Attenuation Level (NAL) of an ANC system which has the reference input $g(n)$, where $\Gamma_g = \xi_d/\xi_e$. Depending on the context, Γ_g can also indicate the maximum achievable NAL for the given reference signal $g(n)$.

Acknowledgments

This study was supported by the VA IDIQ contract number VA549-P-0027 awarded and administered by the Dallas, TX VA Medical Center. The content of this paper does not necessarily reflect the position or the policy of the U.S. government, and no official endorsement should be inferred.

References

[1] M. Pawelczyk, "Adaptive noise control algorithms for active headrest system," *Control Engineering Practice*, vol. 12, no. 9, pp. 1101–1112, 2004.

[2] M. O. Tokhi, M. A. Hossain, and M. H. Shahed, *Parallel Computing for Real-Time Signal Processing and Control*, Springer, London, UK, 2003.

[3] M. A. Hossain and M. O. Tokhi, "Real-time design constraints in implementing active vibration control algorithms," *International Journal of Automation and Computing*, vol. 3, no. 3, pp. 252–262, 2006.

[4] S. M. Kuo and D. R. Morgan, *Active Noise Control Systems: Algorithms and DSP Implementation*, John Willey & Sons, New York, NY, USA, 1996.

[5] S. Johansson, S. Nordebo, and I. Claesson, "Convergence analysis of a twin-reference complex least-mean-squares algorithm," *IEEE Transactions on Speech and Audio Processing*, vol. 10, no. 4, pp. 213–221, 2002.

[6] M. Pawelczyk, "Feedforward algorithms with simplified plant model for active noise control," *Journal of Sound and Vibration*, vol. 255, no. 1, pp. 77–95, 2002.

[7] M. O. Tokhi and S. M. Veres, *Active Sound and Vibration Control: Theory and Applications*, Institution of Electrical Engineers, London, UK, 2002.

[8] G. Kannan, A. A. Milani, and I. M. S. Panahi, "Active noise control of noisy periodic signals using signal separation," in *Proceedings of IEEE International Conference on Acoustic, Speech, and Signal Processing (ICASSP '08)*, pp. 1617–1620, Las Vegas, Nev, USA, March-April 2008.

[9] S. M. Kuo and A. B. Puvvala, "Effects of frequency separation in periodic active noise control systems," *IEEE Transactions on Audio, Speech and Language Processing*, vol. 14, no. 5, pp. 1857–1866, 2006.

[10] Y. Hinamoto and H. Sakai, "Analysis of the filtered-X LMS algorithm and a related new algorithm for active control of multitonal noise," *IEEE Transactions on Audio, Speech and Language Processing*, vol. 14, no. 1, pp. 123–130, 2006.

[11] L. Vicente and E. Masgrau, "Novel FxLMS convergence condition with deterministic reference," *IEEE Transactions on Signal Processing*, vol. 54, no. 10, pp. 3768–3774, 2006.

[12] Y. Zhang, P. G. Mehta, R. R. Bitmead, and C. R. Johnson Jr., "Direct adaptive control for tonal disturbance rejection," in *Proceedings of the IEEE American Control Conference*, vol. 3, pp. 1480–1482, Philadelphia, Pa, USA, June 1998.

[13] N. J. Bershad and J. C. M. Bermudez, "Sinusoidal interference rejection analysis of an LMS adaptive feedforward controller with a noisy periodic reference," *IEEE Transactions on Signal Processing*, vol. 46, no. 5, pp. 1298–1313, 1998.

[14] P. Stoica and R. Moses, *Introduction to Spectral Analysis*, Prentice-Hall, Upper Saddle River, NJ, USA, 1997.

[15] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, no. 2, pp. 387–392, 1985.

[16] P. Stoica and Y. Selén, "Model-order selection: a review of information criterion rules," *IEEE Signal Processing Magazine*, vol. 21, no. 4, pp. 36–47, 2004.

[17] J. Rissanen, "Modeling by shortest data description," *Automatica*, vol. 14, no. 5, pp. 465–471, 1978.

[18] P. Stoica and T. Söderström, "Statistical analysis of MUSIC and subspace rotation estimates of sinusoidal frequencies," *IEEE Transactions on Signal Processing*, vol. 39, no. 8, pp. 1836–1847, 1991.

[19] W. Dehandschutter, J. van Herbruggen, J. Swevers, and P. Sas, "Real-time enhancement of reference signals for feedforward control of random noise due to multiple uncorrelated sources," *IEEE Transactions on Signal Processing*, vol. 46, no. 1, pp. 59–69, 1998.

[20] M. H. Hayes III, *Statistical Digital Signal Processing and Modeling*, John Willey & Sons, New York, NY, USA, 1996.

[21] A. Papoulis and S. U. Pillai, *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, New York, NY, USA, 4th edition, 2001.

[22] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing*, Prentice-Hall, Upper Saddle River, NJ, USA, 4th edition, 2006.

[23] S. Elliott, *Signal Processing for Active Control*, Academic Press, New York, NY, USA, 2001.

[24] A. Papoulis, "Levinson's algorithm, Wold's decomposition, and spectral estimation," *SIAM Review*, vol. 27, no. 3, pp. 405–441, 1985.

[25] A. A. Milani, G. Kannan, I. M. S. Panahi, R. Briggs, and K. Gopinath, "Weight stacking analysis of delayless subband adaptive algorithms for fMRI active noise cancellation," in *Proceedings of the IEEE Dallas Engineering in Medicine and Biology Workshop (DEMBS '07)*, pp. 134–137, Dallas, Tex, USA, November 2007.

[26] V. R. Ramachandran, G. Kannan, A. A. Milani, and I. M. S. Panahi, "Speech enhancement in functional MRI environment using adaptive sub-band algorithms," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP '07)*, vol. 2, pp. 341–344, Honolulu, Hawaii, USA, April 2007.


Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
International Journal of Distributed Sensor Networks
Advances in Civil Engineering
Hindawi
Submit your manuscripts at
http://www.hindawi.com
Journal of Control Science and Engineering
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration