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*Research Article*

# Comparative Risk Aversion under Background Risk Revisited

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This paper determines a new sufficient condition of the (von Neumann-Morgenstern) utility function that preserves comparative risk aversion under background risk. It is the single crossing condition of risk aversion. Because this condition requires monotonicity in the local sense, it may satisfy the U-shaped risk aversion observed in the recent empirical literature.

## 1. Introduction

Pratt [1] and Arrow [2] introduced the notion of risk aversion and its associated order in the expected utility framework. These are represented by concavity and the degree of concavity of the (von Neumann-Morgenstern) utility function. Comparative risk aversion, which is the order of risk aversion, has intuitive and reasonable properties in many decision-making problems that have appeared in economics and finance. For example, in typical portfolio problems, more risk averse investors hold less risky and more risk-free assets. In this example, investors face single risk. However, it is natural that investors face other risk which cannot traded in asset markets, for example, human capital risk cannot be traded in asset markets. In other situations, we face risk which investors cannot control and trade. This risk is called background risk. Optimal decision problems are rather complex by the presence of background risk. Over the past three decades, many researchers have examined how background risk influences optimal decisions in economics and finance. (Gollier [3] provided an excellent survey of this topic.) Since comparative risk aversion may be different with and without background risk, comparative risk aversion too weal to compare optimal decisions in the presence of background risk. This leads to the following question on this topic: "what conditions guarantee that comparative risk aversion is preserved in the presence of background risk?" This paper provides a new answer to this question.

Important contributions to this question are from Kihlstrom et al. [4], Nachman [5], and Pratt [6]. The first two studies obtained a sufficient condition for the preservation of comparative risk aversion in the presence of background risk. As in the case of additive and multiplicative background risk, these sufficient conditions are decreasing absolute risk aversion (DARA) and decreasing relative risk aversion (DRRA), respectively. Pratt [6] established a necessary and sufficient condition for the preservation of comparative risk aversion in the presence of background risk. This paper proposes a new sufficient condition, which is the single crossing condition of risk aversion. The motivation for our analysis has arisen from recent developments. From an empirical viewpoint, Jackwerth [7] and Aït-Sahalia and Lo [8] observed U-shaped absolute and relative risk aversion using options data. Since the U-shaped risk aversion is decreasing in low wealth and increasing in high wealth, DARA and DRRA determined by Kihlstrom et al. [4] and Nachman [5] are inconsistent with this type of risk aversion Because it is difficult to imagine the shapes of risk aversion to satisfy conditions determined by Pratt [6], we cannot determine whether Pratt's conditions satisfy this empirical finding or not. The condition proposed in this paper may be consistent with this observation, because our condition requires monotonicity of risk aversion in the local sense. From a theoretical viewpoint, Jewitt [9] and Athey [10] proposed a new comparative static technique using the concept of log-supermodularity. We derive a new sufficient condition by applying this technique.

The organization of the paper is as follows. In Section 2, we provide some preliminary discussion for the analysis. In Section 3, we determine the condition on utility function that preserves (reserves) comparative risk aversion under background risk and compare our result with the previous studies. In Section 4, we provide the condition in the case that the payoff function has additive and multiplicative forms and discuss the implications of our result for recent empirical findings. In the last section, we make concluding remarks. Some technical issues are presented in the appendices.

## 2. Preliminaries

Because our setting is basically identical to that of Nachman [5], we borrow his notation. Let us consider utility function $u : \mathbb{R} \to \mathbb{R}$ of a decision maker (DM). The utility function $u$ is strictly increasing, and the higher order derivatives required in the analysis are assumed to exist. We note that the DM is not necessarily risk averse, that is, concavity of the utility function is not required for the analysis. Let us consider payoff function $g : X \times Y \to \mathbb{R}, X, Y \subseteq \mathbb{R}$. The payoff function $g$ is strictly increasing function of $x$. $x \in X$ is a realization of a decision variable and $y \in Y$ is a realization of an exogenous variable. The exogenous risk $\tilde{y}$, called background risk, is a random variable with probability density function (PDF) $m$ defined over support $Y$. The capital letter $M$ stands for cumulative distribution function (CDF) associated with PDF $m$. For example, let us consider a financial market with one risk-free asset and one risky asset. In this economy, endogenous risk is the market portfolio, and exogenous risk is nontraded labor income risk (Weil, [11]).

Let us define the derived utility function as

$$v(x) := \int_Y u(g(x,s))m(s)ds. \tag{1}$$

The derivatives of the derived utility function are written as follows:

$$\begin{aligned} v'(x) &= \int_Y u'(g(x,s))g_x(x,s)m(s)ds, \\ v''(x) &= \int_Y \{u''(g(x,s))(g_x(x,s))^2 \\ &\quad + u'(g(x,s))g_{xx}(x,s)\}m(s)ds, \end{aligned} \tag{2}$$

where prime denotes derivatives, and $g_x$ and $g_{xx}$ denote the first- and second-order partial derivatives of $g$ with respect to $x$. The derived utility function $v$ is also a strictly increasing function by the above assumptions, $g_x > 0$ and $u' > 0$.

Let us define the function

$$h(x,y) := g_x(x,y)\mathcal{A}(g(x,y);u) + \mathcal{A}(x,y;g), \tag{3}$$

where $\mathcal{A}(\cdot;u) := -u''/u'$ and $\mathcal{A}(\cdot;g) := -g_{xx}/g_x$. Recall that $\mathcal{A}(u)$ is the Arrow-Pratt absolute risk aversion of the utility function $u$ (Pratt [1] and Arrow [2]). Using the function

$$\hat{m}(y;u) := \frac{g_x(x,y)u(g(x,y))m(y)}{\int_Y g_x(x,s)u'(g(x,y))m(s)ds}, \tag{4}$$

the Arrow-Pratt absolute risk aversion of the utility function $u$ under background risk, or equivalently that of the utility function $v$, can be rewritten as

$$\mathcal{A}(x;v) = -\frac{v''(x)}{v'(x)} = \int_Y h(x,s)\hat{m}(s;u)ds. \tag{5}$$

The derivation of (5) is in Appendix A. We note that the function $\widehat{M}(y;u) := \int^y \hat{m}(s;u)ds$ can be viewed as the CDF defined over support $Y$, because $\hat{m}(y;u) > 0$ for all $y \in Y$ and $\int_Y \hat{m}(s;u)ds = 1$.

## 3. Main Result

Pratt [1] and Arrow [2] introduced the notion of comparative risk aversion defined as follows: $u_1$ is more risk averse than $u_2$ if $\mathcal{A}(x;u_1) = -u_1''(x)/u_1'(x) \geq -u_2''(x)/u_2'(x) = \mathcal{A}(x;u_2)$. We denote this as $u_1 \geq_A u_2$. The aim of the paper is to determine a new sufficient condition that guarantees the preservation of comparative risk aversion under background risk, $u_1 \geq_A u_2 \Rightarrow v_1 \geq_A v_2$.

*3.1. Theorem.* Before providing the theorem, we define the notion of the single crossing condition: $H : Y \to \mathbb{R}$ satisfies the single crossing condition at $y_i$ from above (below), $\exists y_i$, for all $y, (y - y_i)\, H(y) \leq 0((y - y_i)\, H(y) \geq 0)$. Figure 1 describes an example of function satisfying the single crossing condition from above. We also define $y_1, y_2 \in Y$ as follows: there exists $y_i \in Y$ such that $h_i(x,y_i) = \int_Y h_i(x,s)\hat{m}(s;u_i)ds (= \mathcal{A}(x;v_i))$ for $i = 1, 2$. The following theorem is our main result.

**Theorem 1.** *Suppose that $g(x,y)$ is an increasing function of $y$ for all $y \in Y$. Suppose also that either of $H_1(y) := h_1(x,y) - h_1(x,y_1)$ or $H_2(y) := h_2(x,y) - h_2(x,y_2)$ satisfies the single crossing condition from above (below) $\exists y_1$, for all $y, (y - y_1)\, H_1(y) \leq 0$ or $\exists y_2$, for all $y, (y - y_2)\, H_2(y) \leq 0$ ($\exists y_1$, for all $y, (y - y_1)\, H_1(y) \geq 0$ or $\exists y_2$, for all $y, (y - y_2)\, H_2(y) \geq 0$). If $u_1 \geq_A u_2$, then $v_1 \geq_A (\leq_A) v_2$.*

*3.2. Two Lemmas.* For the preparation of the proof, we provide the following two lemmas. A similar result to the first lemma was obtained by Osaki [12], Ohnishi and Osaki [13], and others in various contexts. We note that Pratt [6] obtained a similar result and gave a different proof for the result of Kihlstrom et al. [4], using the property of stochastic dominance. Before providing the first lemma, we define monotone likelihood ratio dominance (MLRD). For the sake of simplicity, we assume that two random variables $\tilde{y}_1$ and $\tilde{y}_2$ with CDF $M(1)$ and $M(2)$ have the same support $Y$.

*Definition 1.* $M(2)$ dominates $M(1)$ in the sense of MLRD if $m(y;2)/m(y;1)$ is increasing in $y \in Y$. We denote this as $M(1) \leq_{\text{MLRD}} M(2)$.

**Lemma 1.** *$u_1 \geq_A u_2$ if and only if $M(u_1) \leq_{MLRD} M(u_2)$.*

The proof of Lemma 1 is presented in Appendix B.

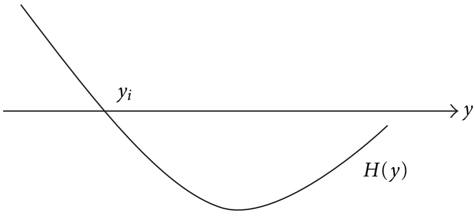


Figure 1

Lemma 2 is known as the variation diminishing property. Hence, we give the following lemma without a proof. It was presented by Karlin and Novikoff [14] and Karlin [15]. Jewitt [9] and Athey [10] discussed some economic applications. Before providing the lemma, we define log-supermodularity as follows: $m : Y \times \Theta \to \mathbb{R}$ is log-supermodular with respect to $y$ and $i$ if for all $y_L \le y_H, m(y_L, 2)m(y_H, 1) \le m(y_L, 1)m(y_H, 2)$. (Log-supermodularity is also called $TP_2$ property.) If we let $M(i)$ to denote CDF, log-supermodularity is equivalent to $M(1) \le_{\text{MLRD}} M(2)$. (See, e.g., Gollier [3].)

**Lemma 2.** *Let us assume that $H : \mathbb{R} \to \mathbb{R}$ satisfies the single crossing condition at $y_i$ from above (below): $\exists y_i$, for all $y, (y - y_i)\ H(y) \le 0((y - y_i)\ H(y) \ge 0)$. If a function $m(y; i) : Y \times I \to \mathbb{R}$ is log-supermodular for $y$ and i, then $\int_Y H(s)m(s; 1)dt = 0$ implies $\int_Y H(s)m(s; 2)dt \le (\ge)0$.*

*3.3. Proof.* In this subsection, we provide a proof of Theorem 1. Because the logic is similar, we only provide the proof for the following statement.

*Suppose that $H_1(y) = h_1(x, y) - h_1(x, y_1)$ satisfies the single crossing condition at $y_1$ from above: $\exists y_1$, for all $y, (y - y_1)\ H_1(y) \le 0$. If $u_1 \ge_A u_2$, then $v_1 \ge_A v_2$.*

From Lemma 1,

$$u_1 \ge_A u_2 \Longleftrightarrow \widehat{M}(u_1) \le_{\text{MLRD}} \widehat{M}(u_2). \tag{6}$$

This is equivalent to that $\hat{m}(y, u_i)$ is log-supermodular. Therefore, we have the following inequality:

$$\begin{aligned} 0 &= \int_Y \{h_1(x, s) - h_1(x, y_1)\} \hat{m}(s; u_1) ds \\ &\ge \int_Y \{h_1(x, s) - h_1(x, y_1)\} \hat{m}(s; u_2) ds \\ &\ge \int_Y \{h_2(x, s) - h_1(x, y_1)\} \hat{m}(s; u_2) ds. \end{aligned} \tag{7}$$

The first inequality comes from the variation diminishing property (Lemma 2). It follows that by direct calculation:

$$\begin{aligned} &h_1(x, y) - h_2(x, y) \\ &\quad = g_x(x, y)\{\mathcal{A}(g(x, y); u_1) - \mathcal{A}(g(x, y); u_2)\}. \end{aligned} \tag{8}$$

Hence, $\mathcal{A}(g(x, y); u_1) \ge \mathcal{A}(g(x, y); u_2)$ is equivalent to $h_1(x, y) \ge h_2(x, y)$. Therefore, we obtain the second in-equality. The above inequality can be rewritten as

$$\begin{aligned} &0 \ge \int_Y \{h_2(x, s) - h_1(x, y_1)\} \hat{m}(s; u_2) ds \\ &\Longleftrightarrow h_1(x, y_1) \ge \int_Y h_2(x, s) \hat{m}(s; u_2) ds \\ &\Longleftrightarrow \mathcal{A}(x; v_1) \ge \mathcal{A}(x; v_2). \end{aligned} \tag{9}$$

The last equivalence is from $\int_Y h_1(x, s) \hat{m}(s; u_1) ds = h_1(x, y_1) = \mathcal{A}(x; v_1)$ which is defined in Section 3.1. This completes the proof.

*3.4. Remarks.* We close this section with a comment on the relationship between our analysis and Nachman's [5] analysis. He determined another sufficient condition that guarantees comparative risk aversion under background risk. First, we review his analysis. Nachman [16] proved that if $u_1 \ge_A u_2$, then $\widehat{M}(u_2)$ dominates $\widehat{M}(u_1)$ in the sense of first-order stochastic dominance (FSD), $\widehat{M}(y; u_2) \le \widehat{M}(y; u_1)$ for all $y \in Y$. Let us consider random variables $\tilde{y}_1$ and $\tilde{y}_2$ with CDF $M(1)$ and $M(2)$. It is well known that the following two conditions are equivalent:

(i) $M(2)$ dominates $M(1)$ in the sense of FSD;

(ii) $\mathbb{E}[g(\tilde{y}_1)] \le \mathbb{E}[g(\tilde{y}_2)]$ for every increasing function $g$;

(see, e.g., Gollier [3] and Müller and Stoyan [17]). Using this property, we obtain that if $u_1 \ge_A u_2$ and either of $h_1(x, y)$ or $h_2(x, y)$ is a decreasing function of $y$, then $v_1 \ge_A v_2$.

Because monotone functions imply functions that satisfy the single crossing condition, the condition determined in this paper is weaker than that determined by Nachman [5]. The reason why our condition is weaker than Nachman's condition is that our analysis uses a stronger stochastic dominance than his analysis, that is, MLRD is a stronger stochastic dominance than FSD.

## 4. Specific Forms

In this section, we consider two specific forms of background risk: additive and multiplicative. We specify additive background risk which has the additive payoff function $g(x, y) = x + y$ and multiplicative background risk which has the multiplicative payoff function $g(x, y) = xy$. In the multiplicative case, we restrict $x$ and $y$ to be positive, $x, y \ge 0$. Applying our main result, we can easily determine a condition that guarantees the preservation of comparative risk aversion in the presence of additive and multiplicative background risk. We also discuss some implications for the U-shaped absolute and relative risk aversion observed in recent empirical literature.

*4.1. Additive Background Risk.* Over the past three decades, many studies considered the effects of additive background risk, which has the additive payoff function, $g(x, y) = x + y$.

We examine which conditions on utility functions that guarantee $v_1 \geq_A v_2$ under additive background risk using Theorem 1. As in the case of additive background risk, we have $h(x, y) = \mathcal{A}(x + y; u)$. Thus, the Arrow-Pratt absolute risk aversion of the utility function $u$ under additive background risk, that is, that of the utility function $v$, is given by

$$\mathcal{A}(x; v) = \int_Y \mathcal{A}(x + s; u)\widehat{m}(s; u)ds. \tag{10}$$

We define $y_1, y_2 \in Y$ such that $y_i = \int_Y \mathcal{A}(x + s; u_i)\widehat{m}(s; u)ds (= \mathcal{A}(x; v_i))$ for $i = 1, 2$. We obtain the following corollary by applying Theorem 1.

**Corollary 1.** *Suppose that $g(x, y)$ has an additive form, $g(x, y) = x + y$. Suppose also that either of $\mathcal{A}(x + y; u_1) - \mathcal{A}(x + y_1; u_1)$ or $\mathcal{A}(x + y; u_2) - \mathcal{A}(x + y_2; u_2)$ satisfies the single crossing condition from above (below). If $u_1 \geq_A u_2$, then $v_1 \geq_A (\leq_A) v_2$.*

*4.2. Multiplicative Background Risk.* In a recent paper, Franke et al. [18] considered the effects of multiplicative background risk on risk aversion, where multiplicative risk has the multiplicative payoff function, $g(x, y) = xy$. First, we define relative risk aversion, $\mathcal{R}(x; u) := -u''(x)x/u'(x)$. In the case of multiplicative background risk, we have $h(x, y) = \mathcal{R}(xy; u)$. Thus, the Arrow-Pratt absolute risk aversion of the utility function $u$ under multiplicative background risk, that is, that of the derived utility function $v$, is given by

$$\mathcal{A}(x; v) = \int_Y \mathcal{R}(xs; u)\widehat{m}(s; u)ds. \tag{11}$$

We also define $y_1, y_2 \in Y$ such that $y_i = \int_Y \mathcal{R}(xs; u_i)\widehat{m}(s; u)ds (= \mathcal{A}(x; v_i))$ for $i = 1, 2$. We obtain the following corollary by a discussion similar to that in the previous subsection. (Because both the endogenous and exogenous risks are defined over positive region, relative risk aversion is equal to absolute risk aversion.)

**Corollary 2.** *Suppose that $g(x, y)$ has a multiplicative form, $g(x, y) = xy$. Suppose also that either of functions $\mathcal{R}(xy; u_1) - \mathcal{R}(xy_1; u_1)$ or $\mathcal{R}(xy; u_2) - \mathcal{R}(xy_2; u_2)$ satisfies the single crossing condition from above (below). If $u_1 \geq_A u_2$, then $v_1 \geq_A (\leq_A) v_2$.*

*4.3. Implications.* Monotone functions are functions that satisfy the single crossing condition. Hence, Corollaries 1 and 2 hold under DARA and DRRA. Thus, corollaries can be viewed as a generalization of Kihlstrom et al. [4], and Nachman [5]. This generalization is important not only from a technical viewpoint, but also from a empirical viewpoint. Jackwerth [7] and Aït-Sahalia and Lo [8] observed U-shaped absolute and relative risk aversion, respectively. This observation means that DARA and DRRA do not include any predictions to guarantee the preservation of risk aversion, $u_1 \geq_A u_2 \Rightarrow v_1 \geq_A v_2$. On the other hand, when risk aversion satisfies the single crossing condition from above, we do not require that risk aversion is decreasing in a global sense. In other words, risk aversion is only decreasing in the neighborhood of the single crossing point. These corollaries may imply the existence of utility functions in the following manner:

(i) they preserve comparative risk aversion under additive (multiplicative) background risk;

(ii) they are consistent with recent empirical findings, for example, absolute (relative) risk aversion is decreasing in low wealth and increasing in high wealth.

## 5. Concluding Remarks

In this paper, we determined a new sufficient condition for utility functions that preserve comparative risk aversion under background risk. The condition determined by Kihlstrom et al. [4], and Nachman [5] requires the monotonicity in the global sense. Our condition, on the other hand, requires it only in the local sense. This generalization does not only have theoretical but also has empirical importance because recent empirical literature has observed the U-shaped risk aversion using options data.

## Appendices

## A. Derivation of (5)

$$\begin{aligned}
\mathcal{A}(x; v) &= -\frac{v''(x)}{v'(x)} \\
&= -\frac{\int_Y \left\{ (g_x(x,s))^2 u''(g(x,s)) + g_{xx}(x,s)u'(g(x,s)) \right\} m(s)ds}{\int_Y g_x(x,s)u'(g(x,s))m(s)ds} \\
&= \int_Y \frac{-\left\{ (g_x(x,s))^2 u''(g(x,s)) + g_{xx}(x,s)u'(g(x,s)) \right\}}{g_x(x,s)u'(g(x,s))} \\
&\quad \times \frac{g_x(x,s)u'(g(x,s))m(s)}{\int_Y g_x(x,s)u'(g(x,s))m(s)ds} ds \\
&= \int_Y \left[ g_x(x,s)\left\{ -\frac{u''(g(x,s))}{u'(g(x,s))} \right\} + \left\{ -\frac{g_{xx}(x,s)}{g_x(x,s)} \right\} \right] \\
&\quad \times \frac{g_x(x,s)u'(g(x,s))m(s)}{\int_Y g_x(x,s)u'(g(x,s))m(s)ds} ds \\
&= \int_Y \{ g_x(x,s)\mathcal{A}(g(x,s;u)) + \mathcal{A}(x,s;g) \} \widehat{m}(s)ds \\
&= \int_Y h(x,s)\widehat{m}(s;u)ds.
\end{aligned} \tag{A.1}$$

## B. Proof of Lemma 1

It follows from a straightforward calculation that

$$\frac{\partial}{\partial y}\left(\frac{u_2'(g(x,y))}{u_1'(g(x,y))}\right) = \frac{g_y(x,y)}{\{u_1'(g(x,y))\}^2}\left(u_1'(g(x,y))u_2''(g(x,y)) - u_1''(g(x,y))u_2'(g(x,y))\right). \tag{B.1}$$

Because $g_y(x,y)/\{u_1'(g(x,y))\}^2 \geq 0$,

$$\operatorname{sgn}\left\{\frac{\partial}{\partial y}\left(\frac{u_2'(g(x,y))}{u_1'(g(x,y))}\right)\right\} = \operatorname{sgn}\{\mathcal{A}(g(x,y);u_1) - \mathcal{A}(g(x,y);u_2)\}. \tag{B.2}$$

Therefore, $\mathcal{A}(g(x,y);u_1) \geq \mathcal{A}(g(x,y);u_2)$ is equivalent to $u_2'(g(x,y))/u_1'(g(x,y))$ is an increasing function of $y$, or equivalently

$$\frac{u_2'(g(x,y))}{u_1'(g(x,y))} \leq \frac{u_2'(g(x,z))}{u_1'(g(x,z))} \tag{B.3}$$

for all $y, z \in Y$ with $y \leq z$. On the other hand, we have that

$$\begin{aligned}
&\frac{\hat{m}(y;u_2)}{\hat{m}(y;u_1)} \leq \frac{\hat{m}(z;u_2)}{\hat{m}(z;u_1)} \\
&\Longleftrightarrow \frac{g_x(x,y)u_2'(g(x,y))m(y)}{g_x(x,y)u_1'(g(x,y))m(y)} \leq \frac{g_x(x,z)u_2'(g(x,z))m(z)}{g_x(x,z)u_1'(g(x,z))m(z)} \\
&\Longleftrightarrow \frac{u_2'(g(x,y))}{u_1'(g(x,y))} \leq \frac{u_2'(g(x,z))}{u_1'(g(x,z))}.
\end{aligned} \tag{B.4}$$

Combining the above two discussions, we obtain the following:

$$\mathcal{A}(g(x,y);u_1) \geq \mathcal{A}(g(x,y);u_2) \Longleftrightarrow \frac{\hat{m}(y;u_2)}{\hat{m}(y;u_1)} \leq \frac{\hat{m}(z;u_2)}{\hat{m}(z;u_1)}, \quad \forall y,z \in Y \quad \text{with } y \leq z. \tag{B.5}$$

Hence, we complete the proof.

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