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Research Article

Pseudo-TE/TM Waves in a Self-Dual Chiral Medium

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Using successively the conventional Gibbs and the differential-form formulations of Maxwell's equations, we analyze the electromagnetic fields in a self-dual chiral material (invariance under the exchanges $[(\mathbf{E} \Leftrightarrow -\mathbf{H}), (\mathbf{D} \Leftrightarrow \mathbf{B}), (\varepsilon \Leftrightarrow -\mu)]$. We prove that such a medium supports pseudo-TE/TM waves (only a component is null with two relations between four of the five other components). An interesting example is supplied by the Courant-Hilbert quasi-undistorted progressing waves: their form does not change along propagation; only their amplitude could change.

1. Introduction

A self dual chiral medium is invariant under the duality transformations [1, 2]:

$$\mathbf{E} \Longrightarrow -\mathbf{H}, \quad \mathbf{H} \Longrightarrow -\mathbf{E}, \quad \mathbf{D} \Longrightarrow \mathbf{B}, \quad \mathbf{B} \Longrightarrow \mathbf{D}, \\ \mu \Longrightarrow -\varepsilon, \quad \varepsilon \Longrightarrow -\mu, \quad \xi \Longrightarrow \xi \tag{1}$$

in which **E**, **B**, **D**, and **H** are the components of the electromagnetic field and $\varepsilon \cdot \mu \cdot \xi$ constant permittivity, permeability, and chirality parameters. A simple illustration is supplied by the Post constitutive relations [3] used in this work, $\varepsilon \cdot \mu$ are real and ξ pure imaginary ($\xi^2 < 0$):

$$\mathbf{B} = \mu\mathbf{H} - \xi\mathbf{E}, \qquad \mathbf{D} = \varepsilon\mathbf{E} + \xi\mathbf{H}. \tag{2}$$

We analyze the behaviour of electromagnetic fields in this self-dual medium using successively the Gibbs conventional and the differential-form formulations of Maxwell's equations. These last ones, in a isotropic homogeneous medium, have TE and TM waves as particular solutions when the fields do not depend on one coordinate x, y, z. We prove here the existence of pseudo-TE/TM waves, pseudo meaning that only one of their components is null while two relations depending on ε, μ, ξ exist between four of the five other components. An example is supplied by quasi-undistorted free fields of the Courant-Hilbert type [4].

2. Gibbs Conventional Formulation

Taking (2) into account, the Maxwell equations with $\partial_\tau = 1/c\partial_t$

$$\nabla \wedge \mathbf{E} + \partial_\tau(\mu\mathbf{H} - \xi\mathbf{E}) = 0, \quad \nabla \wedge \mathbf{H} - \partial_\tau(\varepsilon\mathbf{E} + \xi\mathbf{H}) = 0, \\ \nabla \cdot (\mu\mathbf{H} - \xi\mathbf{E}) = 0, \quad \nabla \cdot (\varepsilon\mathbf{E} + \xi\mathbf{H}) = 0 \tag{3}$$

have the explicit form when $\partial_z\mathbf{E} = 0$, $\partial_z\mathbf{H} = 0$:

$$\partial_y E_z + \partial_\tau(\mu H_x - \xi E_x) = 0, \tag{4a}$$

$$\partial_x E_z - \partial_\tau\left(\mu H_y - \xi E_y\right) = 0, \tag{4b}$$

$$\partial_x E_y - \partial_y E_x + \partial_\tau(\mu H_z - \xi E_z) = 0, \tag{4c}$$

$$\partial_x(\mu H_x - \xi E_x) + \partial_y\left(\mu H_y - \xi E_y\right) + \partial_z(\mu H_z - \xi E_z) = 0, \tag{4d}$$

$$\partial_y E_z - \partial_\tau(\varepsilon E_x + \xi H_x) = 0, \tag{5a}$$

$$\partial_x E_z + \partial_\tau\left(\varepsilon E_y + \xi H_y\right) = 0, \tag{5b}$$

$$\partial_x H_y - \partial_y H_x - \partial_\tau(\varepsilon E_z + \xi H_z) = 0, \tag{5c}$$

$$\partial_x(\varepsilon E_x + \xi H_x) + \partial_y\left(\varepsilon E_y + \xi H_y\right) + \partial_z(\varepsilon E_z + \xi H_z) = 0. \tag{5d}$$

We are interested in the solutions of these equations when E_z or $H_z = 0$.

2.1. Pseudo-TE Waves ($E_z = 0$). Let us write (4c)

$$\partial_x\left(E_y + \frac{\xi H_y}{\varepsilon}\right) - \partial_y\left(E_x + \frac{\xi H_x}{\varepsilon}\right) - \frac{\xi}{\varepsilon}\left(\partial_x H_y - \partial_y H_x\right) + \partial_\tau(\mu H_z - \xi E_z) = 0. \tag{6}$$

Substituting (5c) into the third term of (6) with $E_z = 0$ this equation becomes

$$\partial_x\left(\varepsilon E_y + \xi H_y\right) - \partial_y(\varepsilon E_x + \xi H_x) + (n^2 - \xi^2)\partial_\tau H_z = 0, \quad n^2 = \varepsilon\mu, \tag{7}$$

while according to (4a), (4b), $E_z = 0$ implies

$$\mu H_{x,y} - \xi E_{x,y} = 0 \tag{8}$$

so that (4d) is satisfied. Now writing (5a), (5b) $\varepsilon E_x + \xi H_x = \partial_\tau^{-1}\partial_y H_z$, $\varepsilon E_y + \xi Hy = -\partial_\tau^{-1}\partial_x H_z$, and substituting these expressions into (7) gives the wave equation satisfied by H_z ($\xi^2 < 0$):

$$\left[\partial_x^2 + \partial_y^2 - (n^2 - \xi^2)\partial_\tau^2\right]H_z = 0. \tag{9}$$

Finally, taking into account (8), (5a) and (5b) determine $E_{x,y}$ in terms of H_z:

$$\mu\partial_y H_z - (n^2 + \xi^2)\partial_\tau E_x = 0, \qquad \mu\partial_x H_z + (n^2 + \xi^2)\partial_\tau E_y = 0 \tag{10}$$

so that $\partial_x E_x + \partial_y E_y = 0$ which implies (5d) according to (8).

2.2. Pseudo-TM Waves ($H_z = 0$). Proceeding similarly, we write (5c)

$$\partial_x\left(H_y - \frac{\xi E_y}{\mu}\right) - \partial_y\left(H_x - \frac{\xi E_x}{\mu}\right) + \frac{\xi}{\mu}\left(\partial_x E_y - \partial_y E_x\right) - \partial_\tau(\varepsilon E_z + \xi H_z) = 0. \tag{11}$$

Substituting (4c) into the third term of (11) gives with $H_z = 0$

$$\partial_x\left(\mu H_y - \xi E_y\right) - \partial_y(\mu H_x - \xi E_x) + (n^2 - \xi^2)\partial_\tau E_z = 0, \tag{12}$$

while according to (5a) and (5b), $H_z = 0$ implies in agreement with (5d) that

$$\varepsilon H_{x,y} + \xi H_{x,y} = 0. \tag{13}$$

Now, substituting (4a) and (4b) into (12) written as $\mu H_x - \xi E_x = -\partial_\tau^{-1}\partial_y E_z$, $\mu H_y - \xi E_y = \partial_\tau^{-1}\partial_x E_z$ gives the wave equation satisfied by E_z:

$$\left[\partial_x^2 + \partial_y^2 - (n^2 - \xi^2)\partial_\tau^2\right]E_z = 0, \tag{14}$$

while according to (13), (4a) and (4b) supplying $H_{x,y}$ in terms of E_z become

$$\varepsilon\partial_y E_z + (n^2 + \xi^2)\partial_\tau H_x = 0, \qquad \varepsilon\partial_x E_z - (n^2 + \xi^2)\partial_\tau H_y = 0 \tag{15}$$

so that $\partial_x H_x + \partial_y H_y = 0$ imply (4d).

2.3. Quasi Distortion Free Solutions. Harmonic plane waves are the most elementary solutions of the wave equations (9) and (14) but the Courant-Hilbert quasi-undistorted progressing waves [4] are of some interest. The form of these waves, defined on characteristic surfaces (wave fronts), does not change along their propagation but their amplitude is not constant. To prove that (9) and (14) have such solutions, we introduce the variables

$$u = (n^2 - \xi^2)^{-1/2}\tau + y, \qquad v = (n^2 - \xi^2)^{-1/2}\tau - y, \tag{16}$$

and, these wave equations become

$$\left(\partial_x^2 - 4\partial_u\partial_v\right)\Psi(u, v, x) = 0 \tag{17}$$

with quasi-undistorted solutions [5, 6]:

$$\Psi(u, v, x) = (u + ia)^{-1/2}\exp\left[ikv - ikx^2(u + ia)^{-1}\right], \tag{18}$$

and, it is easily noticed that the wave front

$$\Omega = v - x^2(u + ia)^{-1} \tag{19}$$

is a solution of the characteristic partial differential equation ($\xi^2 < 0$):

$$(\partial_x\Omega)^2 + \left(\partial_y\Omega\right)^2 - (n^2 - \xi^2)(\partial_\tau\Omega)^2 = 0. \tag{20}$$

Substituting (18) into (16) gives $\{E_x, E_y\}$ in terms of H_z from which are obtained $\{H_x, H_y\}$ according to (8). A similar result follows from (18) substituted into (15), just exchanging $\{\mu, \xi\}$ into $\{\varepsilon, -\xi\}$ (see (13)) and the roles of $E_{x,y}$, $H_{x,y}$ in the previous statement.

Now according to (16) $\partial_y = \partial_u - \partial_v$ so that (10) with $m = (n^2 - \xi^2)^{-1/2}$, $p^2 = (n^2 + \xi^2)^{-1}$, and $H_z = \Psi$ gives

$$E_x = \mu\int_{-\infty}^{\infty} d\tau\, p^2(\partial_u - \partial_v)\Psi, \qquad E_y = -\mu\int_{-\infty}^{\infty} d\tau\, p^2\partial_x\Psi. \tag{21}$$

Similar expressions are obtained from (15) for the components $H_{x,y}$ of the pseudo-TM waves. The roles of x, y may be changed giving with $u' = (n^2 - \xi^2)^{-1/2}\tau + x$, $v' = (n^2 - \xi^2)^{-1/2}\tau - x$ the quasi-undistorted waves:

$$\Psi(u', v', y) = (u' + ia)^{-1/2}\exp\left[ikv' - iky^2(u' + ia)^{-1}\right]. \tag{22}$$

The fields (18) and (22) represent 1D-modulated Gaussian beams [7].

3. Differential—Form Formulation

The three-dimensional differential-form formulation of Maxwell's equations is [1], in absence of charge and current, with the exterior derivative operator $d = dx\partial_x + dy\partial_y + dz\partial_z$ and $\tau = ct$:

$$\text{(a) } d \wedge \underline{E} + \partial_\tau \underline{B} = 0, \qquad \text{(b) } d \wedge \underline{B} = 0,$$
$$\text{(a) } d \wedge \underline{H} - \partial_\tau \underline{D} = 0, \qquad \text{(b) } d \wedge \underline{D} = 0. \tag{23}$$

In these equations $\underline{E}$, $\underline{H}$ are the 1-forms:

$$\{\underline{E}, \underline{H}\} = \{E_x, H_x\}dx + \left\{E_y, H_y\right\}dy + \{E_z, H_z\}dz, \quad (24a)$$

and $\underline{B}$, $\underline{D}$ the 2-forms:

$$\{\underline{B}, \underline{D}\} = \{B_x, D_x\}(dy \wedge dz) + \left\{B_y, D_y\right\}(dz \wedge dx) + \{B_z, D_z\}(dx \wedge dy) + . \quad (24b)$$

Now, let $^*h$ be the Hodge star operator, then

$$^*h(dx, dy, dz) \Longrightarrow (dy \wedge dz, dz \wedge dx, dx \wedge dy), \quad (25)$$

from which we get the permittivity, permeability, and chirality operators:

$$\varepsilon = \varepsilon\, ^*h, \qquad \mu = \mu\, ^*h, \qquad \xi = \xi\, ^*h, \quad (26a)$$

so that in a self-dual chiral medium, the 2-forms $\underline{D}$, $\underline{B}$ become

$$\underline{D} = \varepsilon\underline{E} + \xi\underline{H}, \qquad \underline{B} = \mu\underline{H} - \xi\underline{E}, \quad (27)$$

and the coefficients of the differentials in (24b) are

$$D_{x,y,z} = \varepsilon E_{x,y,z} + \xi H_{x,y,z}, \qquad B_{x,y,z} = \mu H_{x,y,z} - \xi E_{x,y,z}. \quad (28a)$$

Substituting (24a) and (24b) into (23), this set of Maxwell's equations becomes

$$\begin{aligned}&\left(\partial_y E_z - \partial_z E_y + \partial_\tau B_x\right)(dy \wedge dz)\\ &\quad + (\partial_z E_x - \partial_x E_z + \partial_\tau By)(dz \wedge dx)\\ &\quad + \left(\partial_x E_y - \partial_y E_x + \partial_\tau B_z\right)(dx \wedge dy) = 0,\end{aligned} \quad (29a)$$

$$\begin{aligned}&\left(\partial_y H_z - \partial_z H_y \text{-} \partial_\tau D_x\right)(dy \wedge dz)\\ &\quad + \left(\partial_z H_x - \partial_x H_z \text{-} \partial_\tau D_y\right)(dz \wedge dx)\\ &\quad + \left(\partial_x H_y - \partial_y H_x \text{-} \partial_\tau D_z\right)(dx \wedge dy) = 0.\end{aligned} \quad (29b)$$

Finally, a simple calculation gives for the second set (23) of Maxwell's equations:

$$\left[\partial_x\{B_x, D_x\} + \partial_y\left\{B_y, D_y\right\} + \partial_z\{B_z, D_z\}\right](dx \wedge dy \wedge dz) = 0. \quad (30)$$

3.1. Wave Equations for Fields. We get according to (23) and (27)

$$\begin{aligned}&\partial_\tau^{-1}(d \wedge \underline{E}) + \mu\underline{H} - \xi\underline{E} = 0,\\ &\partial_\tau^{-1}(d \wedge \underline{H}) - \varepsilon\underline{E} + \xi\underline{H} = 0,\end{aligned} \quad (31)$$

and, eliminating $\underline{\mathrm{H}}$ from (31) supplies the differential-form formulation of the wave equation $W_e\underline{E} = 0$ for the E-field. A simple calculation gives after multiplication by ∂_τ^2

$$W_e\underline{E} = (\varepsilon + \xi^2\mu^{-1})\partial_\tau{}^2\underline{E} - 2\xi\mu^{-1}\partial_\tau(d \wedge \underline{E}) + d \wedge \mu^{-1}d \wedge \underline{E}. \quad (32)$$

The first two terms of this expression become using the Hodge star operators ε, ξ and its inverse μ^{-1}:

$$\begin{aligned}&(\varepsilon + \xi^2\mu^{-1})\partial_\tau{}^2\underline{E}\\ &\quad = \left(\varepsilon + \frac{\xi^2}{\mu}\right)\partial_\tau{}^2\\ &\qquad \times \left[E_x(dy \wedge dz) + E_y(dz \wedge dx) + E_z(dx \wedge dy)\right],\\ &\xi\mu^{-1}\partial_\tau(d \wedge \underline{E})\\ &\quad = \frac{\xi}{\mu}\,\partial_\tau\Big[\left(\partial_y E_z - \partial_z E_y\right)(dy \wedge dz)\\ &\qquad + (\partial_z E_x - \partial_x E_z)(dz \wedge dx)\\ &\qquad + \left(\partial_x E_y - \partial_y E_x\right)(dx \wedge dy)\Big].\end{aligned} \quad (33)$$

In the last term of (32), we have, still using the inverse Hodge star operator μ^{-1},

$$\begin{aligned}\mu^{-1}d \wedge \underline{E} = \frac{1}{\mu}\Big[&\left(\partial_y E_z - \partial_z E_y\right)dx + (\partial_z E_x - \partial_x E_z)dy\\ &+ \left(\partial_x E_y - \partial_y E_x\right)dz\Big],\end{aligned} \quad (34)$$

so that Δ and $\nabla.$ are the Laplacian and the divergence operators, and we get

$$\begin{aligned}d \wedge \mu^{-1}d \wedge \underline{E} = \frac{1}{\mu}[&(-\Delta E_x + \partial_x\nabla \cdot \mathbf{E})(dy \wedge dz)\\ &+ \left(-\Delta E_y + \partial_y\nabla \cdot \mathbf{E}\right)(dz \wedge dx)\\ &+ (-\Delta E_z + \partial_z\nabla \cdot \mathbf{E})(dx \wedge dy)].\end{aligned} \quad (35)$$

Then, substituting (33) and (35), multiplied by μ into (32) gives with $n^2 = \varepsilon\mu$

$$\begin{aligned}&W_e\underline{E}\\ &\quad = \Big[(n^2 + \xi^2)\partial_\tau{}^2\, E_x - 2\xi\partial_\tau\left(\partial_y E_z - \partial_z E_y\right)\\ &\qquad - \Delta E_x + \partial_x\nabla \cdot \mathbf{E}\Big](dy \wedge dz)\\ &\qquad + \Big[(n^2 + \xi^2)\partial_\tau{}^2 E_y - 2\xi\partial_\tau(\partial_z E_x - \partial_x E_z)\\ &\qquad - \Delta E_y + \partial_y\nabla \cdot \mathbf{E}\Big](dz \wedge dx)\\ &\qquad + \Big[(n^2 + \xi^2)\partial_\tau{}^2 E_z - 2\xi\partial_\tau\left(\partial_x E_y - \partial_y E_x\right)\\ &\qquad - \Delta E_z + \partial_z\nabla \cdot \mathbf{E}\Big](dx \wedge dy),\end{aligned} \quad (36)$$

and, substituting (29a) into (36), we get finally

$$W_e\underline{E} = \left[(n^2+\xi^2)\partial_\tau{}^2 E_x + 2\xi\partial_\tau{}^2 B_x - \Delta E_x + \partial_x\nabla\cdot\mathbf{E}\right] \times (dy\wedge dz) + \left[(n^2+\xi^2)\partial_\tau{}^2 E_y + 2\xi\partial_\tau{}^2 B_y - \Delta E_y + \partial_y\nabla\cdot\mathbf{E}\right] \times (dz\wedge dx) + \left[(n^2+\xi^2)\partial_\tau{}^2 E_z + 2\xi\partial_\tau{}^2 B_z - \Delta E_z + \partial_z\nabla\cdot\mathbf{E}\right] \times (dx\wedge dy). \quad (37)$$

Similarly, eliminating $\underline{E}$ from (31) gives the wave equation $W_h\underline{H} = 0$ with

$$W_h\underline{H} = \left[(n^2+\xi^2)\partial_\tau{}^2 H_x + 2\xi\partial_\tau{}^2 D_x - \Delta H_x + \partial_x\nabla\cdot\mathbf{H}\right] \times (dy\wedge dz) + \left[(n^2+\xi^2)\partial_\tau{}^2 H_y + 2\xi\partial_\tau{}^2 D_y - \Delta H_y + \partial_y\nabla\cdot\mathbf{H}\right] \times (dz\wedge dx) + \left[(n^2+\xi^2)\partial_\tau{}^2 H_z + 2\xi\partial_\tau{}^2 D_z - \Delta H_z + \partial_z\nabla\cdot\mathbf{H}\right] \times (dx\wedge dy). \quad (38)$$

These equations will be used by imposing that one of the three terms in (37) and (38) is null, that is, one component of the E and H fields is solution of a wave equation, the other two components being obtained from Maxwell's equations.

3.2. Pseudo-TE/TM Fields. To get the pseudo-TM fields we suppose that the coefficient of the $dx \wedge dy$ term in (37) is null which gives

$$(n^2+\xi^2)\partial_\tau{}^2 E_z + 2\xi\partial_\tau{}^2 B_z - \Delta E_z + \partial_z\nabla\cdot\mathbf{E} = 0. \quad (39)$$

In addition, these fields must satisfy the conditions

$$\partial_z\mathbf{E} = 0, \qquad \partial_z\mathbf{H} = 0, \qquad H_z = 0, \quad (40)$$

and then, (39) reduces to the wave equation (14) satisfied by E_z. To get the other components, we further assume $D_x = D_y = 0$ so that

$$E_{x,y} = -\frac{\xi H_{x,y}}{\varepsilon}, \qquad B_{x,y} = \frac{(n^2+\xi^2)H_{x,y}}{\varepsilon}. \quad (41)$$

Then, the first two terms of the Maxwell equations (29b) are null and, we are left with

$$\left(\partial_x H_y - \partial_y H_x - \varepsilon\partial_\tau E_z\right)(dx\wedge y) = 0, \quad (42a)$$

while we get from (29a), still taking into accont (40),(41)

$$\left[\varepsilon\partial_y E_z + (n^2+\xi^2)\partial_\tau H_x\right](dy\wedge dz) - \left[\varepsilon\partial_x E_z - (n^2+\xi^2)\partial_\tau H_y\right](dz\wedge dx) + \xi\left(\partial_y H_x - \partial_x H_y - \varepsilon\partial_\tau E_z\right)(dx\wedge y) = 0. \quad (42b)$$

The set (42a), (42b) supplies two equations to determine the components H_x, H_y in terms of the solutions E_z of the wave equation (14). For the pseudo-TE fields, making null the coefficient of $dx \wedge dy$ term in (38) null and imposing (40) with $E_z = 0$ instead of $H_z = 0$, supply the wave equation (9) satisfied by H_z. We further assume $B_{x,y} = 0$ so that

$$H_{x,y} = \frac{\xi E_{x,y}}{\mu}, \qquad D_{x,y} = \frac{(n^2+\xi^2)E_{x,y}}{\mu}. \quad (43)$$

Then the first two terms of (29a) are null and one is left with

$$\left(\partial_x E_y - \partial_y E_x + \mu\partial_\tau H_z\right)(dxd\wedge y) = 0, \quad (44)$$

while we get from (29b)

$$\left[\mu\partial_y H_z - (n^2+\xi^2)\partial_\tau E_x\right](dy\wedge dz) - \left[\mu\partial_x H_z - (n^2+\xi^2)\partial_\tau E_y\right](dz\wedge dx) + \xi\left(\partial_x E_y - \partial_y E_x - \mu\partial_\tau H_z\right)(dx\wedge y) = 0. \quad (45)$$

Thus, we obtain a set of two equations (44), (45) to determine $E_{x,y}$ once known the solution H_z of the wave equation (9).

The integration of these 2-forms has to be performed on 2D-manifolds with a suitable numerical technique such as finite elements [7] using a judicious choice of test functions among which the Whitney forms [8] have a particular interest. Many works have been devoted to this integration problem [8–10] where further references can be found.

4. Discussion

Two topics emerge from this work. The existence of pseudo-TE/TM waves in self media allows to make a comparison between the conventional and the differential-form formulations of Maxwell's equations. Let us limit this discussion to TE waves (similar conclusions hold valid for pseudo-TM waves). In both formalisms, we start with the component H_z satisfying in some domain of R^3 the wave equation (9) with proper boundary conditions. Then, since, according to (8), $H_{x,y}$ are obtained at once from $E_{x,y}$, we are left to get these last two components in terms of H_z.

In the conventional formalism, this requirement leads, according to (21), to perform the numerical integration of the expressions:

$$\int_{-\infty}^{\infty} d\tau (n^2+\xi^2)^{-1}\partial_y H_z, \qquad \int_{-\infty}^{\infty} d\tau (n^2+\xi^2)^{-1}\partial_x H_z, \quad (46)$$

which is a rather ordinary business.

On the other hand, in the differential-form formalism,we have to cope with the differential equations (44), (45) whose integration on a 2D-manifold M requires an important work as just mentioned at the end of Section 3. So, an interesting question is to investigate when a formalism outpaces the other one, taking into account accuracy and computation time.

The second topics concern the existence in self-dual media of quasi-undistorted progressing waves. These fields carry on an infinite energy but, using the finite aperture approximation for diffraction, we may obtain such fields with a finite energy, able, in principle, to be launched in the physical space [11]. Then, they could be used in communications [12] and to generate beams of directed energy (electromagnetic bullets) of interest in laser and radar technologies [13].

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