Research Article

Nonnormal Edge-Transitive Cubic Cayley Graphs of Dihedral Groups

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A Cayley graph of a finite group G is called *normal edge transitive* if its automorphism group has a subgroup which both normalizes G and acts transitively on edges. In this paper we determine all cubic, connected, and undirected edge-transitive Cayley graphs of dihedral groups, which are not normal edge transitive. This is a partial answer to the question of Praeger (1999).

1. Introduction

Let G be a finite group, and let S be a subset of G such that $1_G \notin S$. The *Cayley graph* $X = \text{Cay}(G, S)$ of G on S is defined as the graph with a vertex set $V(X) = G$ and edge set $E(X) = \{\{g, sg\} \mid g \in G, s \in S\}$. Immediately from the definition there are three obvious facts (1) $\text{Aut}(X)$, the automorphism group of X , contains the right regular representation $R(G)$ of G ; (2) X is connected if and only if $G = \langle S \rangle$; (3) X is undirected if and only if $S = S^{-1}$.

A part of $\text{Aut}(X)$ may be described in terms of automorphisms of G , that is, the normalizer $N_{\text{Aut}(X)}(G) = G \rtimes \text{Aut}(G, S)$, a semidirect product of G by $\text{Aut}(G, S)$, where $\text{Aut}(G, S) = \{\sigma \in \text{Aut}(G) \mid S^\sigma = S\}$.

We simply use $A(X)$ to denote the arc set of X . A Cayley graph $X = \text{Cay}(G, S)$ is said to be *vertex transitive*, *edge transitive*, and *arc transitive* if its automorphism group $\text{Aut}(X)$ is transitive on the vertex set $V(X)$, edge set $E(X)$, and arc set $A(X)$, respectively. For $s \geq 1$, an s -arc in a graph X is an ordered $(s+1)$ -tuple $(v_0, v_1, \dots, v_s)$ of vertices of X such that v_{i-1} is adjacent to v_i for $1 \leq i \leq s$ and $v_{i-1} \neq v_{i+1}$ for $1 \leq i < s$ in other words, a directed walk of length s which never includes a backtracking. A graph X is said to be *s arc transitive* if $\text{Aut}(X)$ is transitive on the set of s -arcs in X . In particular, 0 arc transitive means vertex transitive, and

1-arc transitive means arc transitive or *symmetric*. A subgroup of the automorphism group of a graph X is said to be *s-regular* if it acts regularly on the set of s -arcs of X .

It is difficult to find the full automorphism group of a graph in general, and so this makes it difficult to decide whether it is edge-transitive, even for a Cayley graph. As an accessible kind of edge transitive graphs, Praeger [1] focuses attention on those graphs for which $N_{\text{Aut}(X)}(G)$ is transitive on edges, and such a graph is said to be *normal edge transitive*. By the definition, every normal edge-transitive Cayley graph is edge-transitive, but not every edge-transitive Cayley graph is normal edge-transitive.

Independently for our investigation, and as another attempt to study the structure of finite Cayley graphs, Xu [2] defined a Cayley graph $X = \text{Cay}(G, S)$ to be *normal* if $R(G)$ is normal subgroup of the full automorphism group $\text{Aut}(X)$. Xu's concept of normality for a Cayley graph is a very strong condition. For example, K_n is normal if and only if $n < 4$. However any edge-transitive Cayley graph which is normal, in the sense of Xu's definition, is automatically normal edge transitive.

Praeger posed the following question in [1]: what can be said about the structure of Cayley graphs which are edge transitive but not normal edge transitive? In [3], Alaeiyan et al. have given partial answer to this question for abelian groups of valency at most 5, and also Sim and Kim [4] determined normal edge-transitive circulant graphs. In the next theorem we will identify all cubic edge transitive Cayley graphs of dihedral group which are not normal edge-transitive. This is a partial answer to Question 5 of [1]. Throughout of this paper, we suppose that $D_{2n} = \langle a, b \mid a^n = b^2 = 1, bab^{-1} = a^{-1} \rangle$, and $X = \text{Cay}(D_{2n}, S)$ is connected and undirected cubic Cayley graph. The main result of this paper is the following theorem.

Theorem 1.1. *Let $G = D_{2n}$ be a dihedral group, and let $X = \text{Cay}(G, S)$ be a connected cubic Cayley graph. If X is an edge-transitive Cayley graph but is not normal edge transitive, then X, G satisfy one of the following:*

- (1) $n = 4, S = \{b, ab, a^2b\}, X \cong K_{4,4} - 4K_2$;
- (2) $n = 8, S = \{b, ab, a^3b\}, X \cong P(8, 3)$, the generalized Peterson graph.

2. Basic Facts

In this section we give some facts on Cayley graphs, which will be useful for our purpose. First we make some comments about the normalizer $N_{\text{Aut}(X)}(G)$ of the regular subgroup G . As before, the normalizer of the regular subgroup G in the symmetric group $\text{Sym}(G)$ is the holomorph of G , that is, the semidirect product $G \rtimes \text{Aut}(G)$. Thus,

$$N_{(\text{Aut}(X))}(G) = (G \rtimes \text{Aut}(G)) \cap \text{Aut}(X) = G \rtimes (\text{Aut}(G) \cap \text{Aut}(X)) = G \rtimes \text{Aut}(G, S). \quad (2.1)$$

The following lemmas are basic for our purpose. Now we have the first lemma from [1].

Lemma 2.1 (see [1, Proposition 1]). *Let $X = \text{Cay}(G, S)$ be a Cayley graph for a finite group G . Then X is normal edge transitive if and only if $\text{Aut}(G, S)$ is either transitive on S or has two orbits in S which are inverse of each other.*

Lemma 2.2 (see [2, Proposition 1.5]). *Let $X = \text{Cay}(G, S)$, and $A = \text{Aut}(X)$. Then X is normal if and only if $A_1 = \text{Aut}(G, S)$, where A_1 is the stabilizer of 1 in A .*

Lemma 2.3 (see [5, Lemma 4.4]). *All 1-regular cubic Cayley graphs on the dihedral group D_{2n} are normal.*

Lemma 2.4 (see [6, Lemma 3.2]). *Let Γ be a connected cubic graph on dihedral group D_{2n} , and let B_1 and B_2 be two orbits of $C = \langle a \rangle$. Also let G^* be the subgroup of G fixing setwise B_1 and B_2 , respectively. If G^* acts unfaithfully on one of B_1 and B_2 , then $\Gamma \cong K_{3,3}$.*

Let C_G be the core of $C = \langle a \rangle$ in $\text{Aut}(X)$. By assuming the hypothesis in the above lemma, we have the following results

Lemma 2.5 (see [6, Lemma 3.5]). *If C_G is a proper subgroup of C , then X is isomorphic to $\text{Cay}(D_{14}, \{b, ab, a^3b\})$ or $\text{Cay}(D_{16}, \{b, ab, a^3b\})$.*

Lemma 2.6 (see [6, Lemma 3.6]). *If $C_G = C$, then X is isomorphic to $\text{Cay}(D_{2n}, \{b, ab, a^kb\})$, where $k^2 - k + 1 = 0 \pmod{n}$, and $n \geq 13$.*

Let $G = D_{2n}$. Then the elements of G are a^i and a^ib , where $i = 0, 1, \dots, n-1$. All a^ib are involutions, and a^i is an involution if and only if n is even and $i = n/2$. Finally in this section we obtain a preliminary result restricting S for cubic Cayley graphs of $\text{Cay}(D_{2n}, S)$. We can easily prove the following lemma

Lemma 2.7. *Let $\Gamma = \text{Cay}(G, S)$ be Cayley graphs of $G = D_{2n}$. Then Γ is cubic, connected and, undirected if and only if one of the following conditions holds*

(1) *When n is odd, one has*

$$\begin{aligned} S_{o_1} &= \{a^ib, a^jb, a^kb\}, \quad 0 \leq i < j < k < n, \\ S_{o_2} &= \{a^i, a^{-i}, a^jb\}, \quad 0 < i < \frac{n}{2}, \quad 0 \leq j < n, \end{aligned} \tag{2.2}$$

(2) *When n is even, one has*

$$\begin{aligned} S_{e_1} &= \{a^ib, a^jb, a^kb\}, \quad 0 \leq i < j < k < n, \\ S_{e_2} &= \{a^i, a^{-i}, a^jb\}, \quad 0 < i < \frac{n}{2}, \quad 0 \leq j < n, \\ S_{e_3} &= \left\{ a^{\frac{n}{2}}, a^i, a^{-i} \right\}, \quad 0 < i < \frac{n}{2}, \\ S_{e_4} &= \{a^{n/2}, a^ib, a^jb\}, \quad 0 \leq i < j < n. \end{aligned} \tag{2.3}$$

Let X and Y be two graphs. The *direct product* $X \times Y$ is defined as the graph with vertex set $V(X \times Y) = V(X) \times V(Y)$ such that for any two vertices $u = [x_1, y_1]$ and $v = [x_2, y_2]$ in $V(X \times Y)$, $[u, v]$ is an edge in $X \times Y$ whenever $x_1 = x_2$ and $[y_1, y_2] \in E(Y)$ or $y_1 = y_2$ and

$[x_1, x_2] \in E(X)$. Two graphs are called relatively prime if they have no nontrivial common direct factor. The *lexicographic product* $X[Y]$ is defined as the graph with vertex set $V(X[Y]) = V(X) \times V(Y)$ such that for any two vertices $u = [x_1, y_1]$ and $v = [x_2, y_2]$ in $V(X[Y])$, $[u, v]$ is an edge in $X[Y]$ whenever $[x_1, x_2] \in E(X)$ or $x_1 = x_2$ and $[y_1, y_2] \in E(Y)$. Let $V(Y) = \{y_1, y_2, \dots, y_n\}$. Then there is a natural embedding nX in $X[Y]$, where for $1 \leq i \leq n$, the i th copy of X is the subgraph induced on the vertex subset $\{(x, y_i) \mid x \in V(X)\}$ in $X[Y]$. The *deleted lexicographic product* $X[Y] - nX$ is the graph obtained by deleting all the edges (natural embedding) of nX from $X[Y]$.

3. Proof of Theorem 1.1

As we have seen in Section 1, each edge transitive Cayley graph which is normal is automatically normal edge transitive. Hence for the proof of Theorem 1.1, we must determine all nonnormal connected undirected cubic Cayley graphs for dihedral group D_{2n} . If $n = 2$, then dihedral group D_4 is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$, and so it is easy to show that the cubic Cayley graph $\text{Cay}(D_4, S)$ is normal. So from now we assume that $n \geq 3$. Also, since $\text{Cay}(D_{2n}, S)$ when $S = S_{e_3}$ is disconnected, thus we do not consider this case for the proof of the main theorem. First we prove the following lemma.

Lemma 3.1. *Let G be the dihedral group D_{2n} with $n \geq 3$, and let $\Gamma = \text{Cay}(G, S)$ be a cubic Cayley graph. Then*

- (a) *if S is S_{e_4} and Γ is connected, then $S \cap (S^2 - \{1\}) = \emptyset$ holds;*
- (b) *if S is S_{o_2} or S_{e_2} and Γ is connected, then $S \cap (S^2 - \{1\}) = \emptyset$ if $n > 3$. For $n = 3$, and S is S_{o_2} or S_{e_2} , one has; $S \cap (S^2 - \{1\}) \neq \emptyset$ and $\Gamma = \text{Cay}(D_6, S)$ is connected and normal;*
- (c) *if S is S_{o_1} or S_{e_1} , then $S \cap (S^2 - \{1\}) = \emptyset$ always holds.*

Proof. (a) Suppose first that $S = S_{e_4} = \{a^{n/2}, a^i b, a^j b\}$. Then

$$S^2 - \{1\} = \{a^{n/2+i}b, a^{n/2+j}b, a^{i-j}, a^{j-i}\}. \quad (3.1)$$

We show that $S \cap (S^2 - \{1\}) = \emptyset$. Suppose to the contrary that $S \cap (S^2 - \{1\}) \neq \emptyset$. We may suppose that $a^{i-j} = a^{j-i} = a^{n/2}$. Now we have $\Gamma = nK_1[Y]$, where $Y = K_4$. Hence Γ is not connected, which is a contradiction.

(b) Now suppose that $S = S_{o_2}$ or $S = S_{e_2}$, that is, $S = \{a^i, a^{-i}, a^j b\}$. For $n > 3$, we have $S^2 - \{1\} = \{a^{2i}, a^{-2i}, a^{i+j}b, a^{j-i}b\}$. We claim that $S \cap (S^2 - \{1\}) = \emptyset$. Suppose to the contrary that $S \cap (S^2 - \{1\}) \neq \emptyset$. We may suppose that $a^{2i} = a^{-i}$. Then $\Gamma = mK_1[Y]$, where $Y = \text{Cay}(S, \langle S \rangle)$ and $|D_{2n} : \langle S \rangle| = m$. So Γ is not connected, which is a contradiction. Now let $n = 3$. Then $S = \{a, a^{-1}, b\}$, $\{a, a^{-1}, ab\}$, or $\{a, a^{-1}, a^2 b\}$, respectively. Therefore $S^2 - 1 = \{a^2, ab, a^2 b, a\}$, $\{a^2, a^2 b, a, b\}$, or $\{a^2, b, a, ab\}$, respectively. Obviously Γ is connected, and $G \cong D_6$. Also we have $S \cap (S^2 - \{1\}) \neq \emptyset$, and $\text{Cay}(D_6, \{a, a^{-1}, b\}) \cong \text{Cay}(D_6, \{a, a^{-1}, ab\}) \cong \text{Cay}(D_6, \{a, a^{-1}, a^2 b\})$. Let σ be an automorphism of $\Gamma = \text{Cay}(D_6, \{a, a^{-1}, b\})$, which fixes 1 and all elements of S . Since $a^\sigma = a$, and $(a^2)^\sigma = a^2$, we have $\{1, a^2, a^2 b\}^\sigma = \{1, a^2, a^2 b\}$ and $\{1, a, ab\}^\sigma = \{1, a, ab\}$. Therefore $(ab)^\sigma = ab$, and $(a^2 b)^\sigma = a^2 b$, and hence σ fixes all elements of S^2 . Thus $\sigma = 1$, and A_1 acts faithfully on S . So we may view A_1 as a permutation group on S . Now let α be an arbitrary element of A_1 . Since $1^\alpha = 1$, we have $\{a, a^2, b\}^\alpha = \{a, a^2, b\}$. If $b^\alpha = a$ or $b^\alpha = a^2$, then $\{1, ab, a^2 b\}^\alpha = \{1, a^2 b, a^2\}$ or $\{1, ab, a^2 b\}^\alpha = \{1, ab, a\}$, which is a contradiction.

Thus $b^a = b$, and A_1 is generated by the permutation (a, a^2) . So $|A_1| = 2$. On the other hand, $\beta : a^i b^j \rightarrow a^{2i} b^j$ is an element of $\text{Aut}(G, S)$. Therefore $|A_1| = |\text{Aut}(G, S)| = 2$, and hence by Lemma 2.2, Γ is normal.

(c) Finally, suppose that $S = S_{o_1}$ or $S = S_{e_1}$, that is, $S = \{a^i b, a^j b, a^k b\}$. Then $S^2 - \{1\} = \{a^{i-j}, a^{j-i}, a^{i-k}, a^{k-i}, a^{j-k}, a^{k-j}\}$. Clearly $S \cap (S^2 - \{1\}) = \emptyset$. The results now follow.

By considering this lemma, we prove the following proposition. This result will be used in the proof of Theorem 1.1. $\square$

Proposition 3.2. *Let G be the dihedral group D_{2n} ($n \geq 3$), and let $X = \text{Cay}(G, S)$ be a connected and undirected cubic Cayley graph. Then X is normal except y one of the following cases happens:*

- (1) $n = 4$, $S = \{b, ab, a^2 b\}$, $X \cong K_{4,4} - 4K_2$;
- (2) $n = 8$, $S = \{b, ab, a^3 b\}$, $X \cong P(8, 3)$, (the generalized Peterson graph);
- (3) $n = 3$, $S = \{b, ab, a^2 b\}$, $X \cong K_{3,3}$;
- (4) $n = 7$, $S = \{b, ab, a^3 b\}$, $X \cong S(7)$, (Heawood's graph).

Proof. First assume that $S = S_{e_4}$. Since Γ is connected, by Lemma 3.1(a), $S \cap (S^2 - \{1\}) = \emptyset$. Now consider the graph $\Gamma_2(1)$, and let σ be an automorphism of $\Gamma = \text{Cay}(D_{2n}, \{a^{n/2}, a^i b, a^j b\})$, which fixes 1 and all elements of S . Since $(a^{n/2})^\sigma = a^{n/2}$, $(a^i b)^\sigma = a^i b$, and $(a^j b)^\sigma = a^j b$, we have $\{1, a^{n/2+i} b, a^{n/2+j} b\}^\sigma = \{1, a^{n/2+i} b, a^{n/2+j} b\}$, $\{1, a^{n/2+i} b, a^{j-i}\}^\sigma = \{1, a^{n/2+i} b, a^{j-i}\}$, and $\{1, a^{n/2+j} b, a^{i-j}\}^\sigma = \{1, a^{n/2+j} b, a^{i-j}\}$, respectively. Therefore $(a^{n/2+i} b)^\sigma = a^{n/2+i} b$, $(a^{n/2+j} b)^\sigma = a^{n/2+j} b$, $(a^{j-i})^\sigma = a^{j-i}$, and $(a^{i-j})^\sigma = a^{i-j}$, and hence σ fixes all elements of S^2 . Because of the connectivity of Γ , this automorphism is the identity in $\text{Aut}(\Gamma)$. Therefore A_1 acts faithfully on S . So we may view A_1 as a permutation group on S . Now let α be an arbitrary element of A_1 . Since $1^\alpha = 1$, we have $\{a^{n/2}, a^i b, a^j b\}^\alpha = \{a^{n/2}, a^i b, a^j b\}$. If $(a^{n/2})^\alpha = a^i b$, or $(a^{n/2})^\alpha = a^j b$, then $\{1, a^{n/2+i} b, a^{n/2+j} b\}^\alpha = \{1, a^{n/2+i} b, a^{j-i}\}$, or $\{1, a^{n/2+i} b, a^{n/2+j} b\}^\alpha = \{1, a^{n/2+j} b, a^{i-j}\}$, respectively. Now again we consider $\Gamma_2(1)$. In this subgraph, $a^{n/2+i} b$ and $a^{n/2+j} b$ have valency 2, and a^{i-j}, a^{j-i} have valency 1. This implies a contradiction. Thus $(a^{n/2})^\alpha = a^{n/2}$, and A_1 is generated by the permutation $(a^i b, a^j b)$. So $|A_1| = 2$. On the other hand, $\beta : a^i b^j \rightarrow a^{-i} (a^{2j} b)^j$ is an element of $\text{Aut}(G, S)$. Therefore $|A_1| = |\text{Aut}(G, S)| = 2$, and hence by Lemma 2.2, Γ is normal.

Now assume that $S = S_{e_2} = \{a^i, a^{-i}, a^j b\}$, or $S = S_{o_2} = \{a^i, a^{-i}, a^j b\}$. If $n = 3$, then by Lemma 3.1 (b), $\Gamma = \text{Cay}(D_6, S)$, and Γ is normal. Now if $n > 3$, then again by Lemma 3.1(b), $S \cap (S^2 - \{1\}) = \emptyset$. Considering the graph $\Gamma_2(1)$, with the same reason as before if an automorphism of Γ fixes 1 and all elements of S , then it also fixes all elements of S^2 . Because of the connectivity of Γ , this automorphism is the identity in $\text{Aut}(\Gamma)$. Therefore A_1 acts faithfully on S . So we may view A_1 as a permutation group on S . We can easily see that A_1 is generated by the permutation (a^i, a^{-i}) . So $|A_1| = 2$. On the other hand, $\sigma : a^i b^j \rightarrow a^{-i} (a^{2j} b)^j$ is an element of $\text{Aut}(G, S)$. Therefore $|A_1| = |\text{Aut}(G, S)| = 2$, and hence by Lemma 2.2, Γ is normal.

Finally assume that $S = S_{e_1} = \{a^i b, a^j b, a^k b\}$, or $S = S_{o_1} = \{a^i b, a^j b, a^k b\}$. Up to graph isomorphism, $S = \{b, a^j b, a^k b\}$, where $j, k \geq Z_n^*$. In this case, Γ is a bipartite graph with the partition $B = B_1 \cup B_2$, where B_1 and B_2 are just two orbits of $C = \langle a \rangle$, and we assume the block B_1 contains 1. Let G^* be the subgroup of G fixing setwise B_1 and B_2 , respectively. If G^* acts unfaithfully on one of B_1 and B_2 , then by Lemma 2.4, $\Gamma \cong K_{3,3}$, and $\sigma = (b, ab)$ is not in $\text{Aut}(G, S)$ but in A_1 , and so Γ is not normal. Let G^* act faithfully on B_1 and B_2 . Then $n \neq 3$. If $n = 4$, then Γ is isomorphic to $K_{4,4} - 4K_2$, and $\sigma = (b, ab)(a^2, a^3)$ is not in $\text{Aut}(G, S)$ but in A_1 , and so Γ is not normal. From now on we assume $n \geq 5$. Now suppose that C_G , the core of C in G , is a proper subgroup of C .

Then by Lemma 2.5, $\Gamma \cong \text{Cay}(D_{14}, \{b, ab, a^3b\})$ or $\Gamma \cong \text{Cay}(D_{16}, \{b, ab, a^3b\})$. For the first case, $\sigma = (a, a^2, a^3, a^6)(a^4, a^5)(a^2b, a^6b, a^5b, a^4b)(ab, b)$ is not in $\text{Aut}(G, S)$ but in A_1 , and so Γ is not normal. For the second case, $\sigma = (a, a^7, a^6)(a^2, a^5, a^3)(b, ab, a^3b)(a^4b, a^5b, a^7b)$ is not in $\text{Aut}(G, S)$ but in A_1 , and so Γ is not normal. Finally we suppose that $C_G = C$. Then by Lemma 2.6, Γ is isomorphic to the $\text{Cay}(D_{2n}, \{b, ab, a^kb\})$, where $k^2 - k + 1 \equiv 0 \pmod{n}$ and $n \geq 13$. The Cayley graph Γ is 1-regular, and by Lemma 2.3, Γ is normal. The result now follows.

Now we complete the proof of Theorem 1.1. We remind that any edge transitive Cayley graph which is normal, in the sense of Xu's definition, is also normal edge transitive. Thus this implies that we must consider nonnormal Cayley graphs which were obtained in Proposition 3.2. So we consider four cases in Proposition 3.2. For case (1), we claim that there is no automorphism of G such that b maps to ab . Suppose to the contrary that there is an automorphism σ such that b maps to ab . Then a must be mapped to a^i , where $(i, 4) = 1$, and so with the simple check it is easy to see that this is a contradiction. Also in case (2), with the same reason as above there is a contradiction. Hence $\text{Aut}(G, S)$ does not act transitively on S also does not have two orbits in S which are inverse of each other. Now by using Lemma 2.1 these graphs are not normal edge transitive. For the last two cases it is easy to show that $\text{Aut}(G, S)$ acts transitively on S , and hence by Lemma 2.1, these graphs are normal edge transitive. Now the proof is complete as claimed. $\square$

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