

## Research Article

# Typical Set Cognitive Sensing

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In cognitive radio (CR) cooperative sensing schemes, wireless sensor nodes deployed in the network sense the licensed spectrum and send their local sensing decisions to a fusion center (FC) that makes a global decision on whether to allow the unlicensed user transmit on the licensed spectrum, based on a decision (fusion) rule.  $k$ -out-of- $N$  is widely used in the literature owing to its practical simplicity. Regrettably, it exhibits a tradeoff between the achievable probabilities of false alarm and miss detection, which could have consequent effects on the performance of CR. In this paper, based on the notion of typical sequences, we propose a novel fusion rule in which the false alarm and miss detection probabilities can be *simultaneously* made as small as desired (asymptotically zero as the number of sensors goes to infinity).

## 1. Introduction

Cognitive Radio (CR) has recently emerged as a topic of interest in wireless research, following the findings of the United States' Federal Communications Commission's (FCC) Spectrum Policy Task Force Report [1] that most radio spectra go unused most of the time. CR has therefore been proposed as a tool to increase the spectrum usage efficiency. Unlicensed users (called secondary users or SU) are allowed to temporarily access licensed bands that are not utilized by their corresponding licensed users (called primary users or PU), provided that the formers do not cause significant interference to them.

Hence, an important task in CR is for an SU to efficiently monitor spectrum and sense any transmission from a PU on its licensed band. Traditional spectrum sensing techniques are energy detection [2, 3], matched filter detection [4, 5], and cyclostationary feature detection [6, 7]. Energy detection is the simplest method to detect the presence of a PU signal based on the amount of energy. As such it does not require any knowledge about the PU's or channel features, yet achieves less reliable detection than the other schemes. Matched filter is the optimal sensing method, though it requires a priori knowledge of the PU's channel parameters and signal. Finally, cyclostationary feature detection

outperforms energy detection by being able to distinguish white noise from signals, but fails to distinguish the PU's signal from other noisy signals such as very far signals or unintended transmissions from neighboring cells [8]. As each approach is limited, spectrum sensing appears to be one of the most challenging tasks in CR, particularly in the very low SNR case (where it is hard to distinguish signals from noise) or in a hidden terminal scenario (where primary users are shadowed or in a deep fade).

Very recently, cooperative spectrum sensing has been suggested to overcome the aforementioned issues. A network operator deploys a (large) centralized wireless network of sensor nodes. These periodically sense the spectrum in search of any PU transmission, then feed their sensing findings (local decisions) back to a fusion center (FC). Based on these reports, the FC makes an educated guess on whether to allow the secondary user(s) to transmit over the PU's channel. As it is unlikely that all sensors suffer from very low SNR or incur a hidden terminal problem, it is expected that cooperative sensing can overcome these issues that are usually difficult to solve in traditional noncooperative sensing. A crucial task, however, is the global decision rule (by the FC) also known as the *fusion rule*, as it will impact on the CR global performance. This issue shall be the focus of this paper.

Assume that all sensors' decisions are independent (uncorrelated). Of particular interest is the case where there is disagreement among the sensors (i.e., some sensors claim the PU is active while others claim the PU is silent). In such a case, the decision making process becomes less obvious. The aim of the FC is to *minimize* two probabilities of error:

- (i) *a probability of false alarm (PFA)*: probability that the FC wrongfully decides that PU is *active* (while it is, in fact, *silent*),
- (ii) *a probability of miss detection (PMD)*: probability that the FC wrongfully decides that PU is *silent* (while it is, in fact, *active*).

There has been a significant number of research works on deriving efficient fusion rules, see, for example, [9] and references therein for a survey on the main fusion rules in literature. In particular, two rules are more commonly used in CR. A first fusion rule, called *k-out-of-N* [9, 10], requires the FC to allow the SU to transmit if at least  $k$  sensors out of  $N$  assume PU is silent. Another rule is the Bayesian rule [10, 11], in which a likelihood (or log-likelihood) ratio is computed at the FC and compared to a given threshold in order to determine which hypothesis (PU is silent versus PU is active) is more likely. In [10], the authors showed that *both fusion rules are equivalent*.

Although such rules enjoy practical simplicity, unfortunately they cannot minimize both probabilities (PFA and PMD) at the same time:

- (i) if  $k$  is too small (close to 0), the FC will, most of the time, assume that PU is silent. Then, clearly  $\text{PMD} \approx 0$  and  $\text{PFA} \approx 1$  (this also can be seen in (14)-(15), by tending  $K$  to 0). Subsequently, the FC will allow the SU to transmit, most of the time. In such a case, the SU will enjoy a good data rate but the PU will suffer from substantial interference from the SU,
- (ii) contrarily, if  $k$  is too large (close to  $N$ ), the FC will, most of the time, assume that PU is active. Then, clearly  $\text{PFA} \approx 0$  but  $\text{PMD} \approx 1$  (this also can be seen in (14)-(15), by tending  $K$  to  $N$ ). Subsequently, the FC will forbid the SU from transmitting, most of the time. In such a case, the PU will enjoy an interference-free transmission but the SU will have a disrupted transmission (which makes CR less interesting).

In practice, usually CR systems are designed such that one probability is minimized while the other has a tolerable (but nonnegligible) value [10].

Contrarily, we show in this work that it may be possible to minimize both the PFA and a PMD, using the *asymptotic equipartition property* (AEP), a law first stated by Claude Shannon in his seminal work [12] and one that derives from the law of large numbers. Though it may have a rather evolved definition, the AEP has a simpler interpretation:

- (i) of all possible events, some events are more likely to occur than others (in other words, not all events are equally likely),

- (ii) most-likely events are fewer than unlikely events (in other words, of all possible events, only a fraction of them occur most of the time).

Precisely, let us assume that the  $N$  sensors report their binary local sensing decisions (1: PU is active, 0: PU is silent) to the FC. Then, the AEP simply states that, of all the  $2^N$  combinations (sequences) of possible reports, only a *fraction* of them are *almost surely*, (that is, with probability tending to 1 as  $N$  goes to infinity,) likely to be received, should PU be indeed, say, active. Such sequences are called *typical sequences* [13].

Therefore, we suggest in this work that the FC simply ignores *nontypical* sensor reports, and only believe *typical* ones, as they occur most of the time. By disregarding unlikely (nontypical) report sequences, we demonstrate in this work that the probabilities of false alarm and miss detection at the FC using can be *simultaneously* made as small as desired, and tend to zero as the number of sensors is increased.

## 2. Preliminaries

*2.1. Notations.* The following notations will be considered in this work. When  $x$  is a variable,  $p(x)$  denotes the probability of occurrence of a realization of  $x$  while  $p_x(x)$  denotes the distribution (probability density function, pdf) of the random variable  $x$ .  $H(x)$  denotes the Shannon entropy [13]:

$$H(x) \triangleq -\mathbb{E}\{\log_2(p_x)\}, \quad (1)$$

where  $\mathbb{E}\{\cdot\}$  denotes the mathematical expectation and  $\log_2$  denotes the base-2 logarithm. When  $\mathcal{A}$  is a finite set of possible realizations of a random variable  $x$ , let  $|\mathcal{A}|$  denote its cardinality, that is, the number of elements in  $\mathcal{A}$ . Finally,  $\Pr\{\mathcal{A}\}$  denotes the probability that a realization of the random variable  $x$  lies in the set  $\mathcal{A}$ , that is,  $\Pr\{\mathcal{A}\} \triangleq \Pr\{x \in \mathcal{A}\}$ .

### 2.2. System Model and Assumptions

*2.2.1. Network Model.* We consider a centralized wireless sensor network made of an FC and  $N$  sensors. We assume that sensors are deployed to periodically sense the spectrum, detect the activity of the PU, and make a local decision on whether the PU, is active or silent. Two hypotheses (sensing outcomes) are possible.

- (i) Hypothesis  $\mathcal{H}_1$ : a sensor decides that PU is active.
- (ii) Hypothesis  $\mathcal{H}_0$ : a sensor decides that PU is silent.

After making a local sensing decision on the activity of the PU, the sensors report their findings to the FC.

*2.3. Channel Model.* Each sensor is assumed to be equipped with a single antenna. For  $i = 1, \dots, N$ , sensor  $i$  senses at time  $k$ ,  $1 \leq k \leq K$  a signal sample  $r_i[k]$  corrupted by an additive white Gaussian noise  $z_i[k]$  with zero mean and variance

denoted as  $\sigma^2$ , through a Rayleigh flat-fading channel with equivalent baseband channel  $h_i$ , given by [14]:

$$h_i = \frac{1}{(d_i/d_0)^{\eta/2}} e^{j\phi_i} \alpha_i 10^{\beta_i/20}, \quad (2)$$

where (i)  $d_i$  denotes the distance between the PU and sensor  $i$ , (ii)  $d_0$  denotes a reference distance, (iii)  $\eta$  denotes a path loss exponent, (iv)  $\alpha_i$  denotes a random variable that accounts for Rayleigh fading, (v)  $\phi_i$  denotes a random variable that accounts for a phase shift uniformly distributed over  $[-\pi/2, \pi/2]$ , and (vi)  $\beta_i$  denotes a random variable that accounts for Log-normal shadowing.

#### 2.4. Cooperative Spectrum Sensing Model

**2.4.1. Locally Sensing the PU's Signal.** Each sensor filters its received signal  $r_i$  in the sensed spectrum then performs sampling. Let  $K$  denote the number of samples obtained from the received signals, which we assume common to all sensors and in agreement with Nyquist's rule (at least twice as large as the product of the sensed bandwidth and sensing time). Subsequently, energy detection is used for spectrum sensing [8, 9]:

$$y_i \triangleq \sum_{k=1}^K |r_i[k]|^2. \quad (3)$$

Therefore, the hypotheses  $\mathcal{H}_0$  and  $\mathcal{H}_1$  (mentioned earlier in Section 2.2.1) can be defined as follows:

$$\begin{aligned} \mathcal{H}_0 : r_i[k] &= z_i[k], \\ \mathcal{H}_1 : r_i[k] &= h_i s[k] + z_i[k]. \end{aligned} \quad (4)$$

In other words,  $\mathcal{H}_0$  represents the hypothesis that the sensed signal  $r_i$  is only the additive noise  $z_i$  (PU is silent) whereas  $\mathcal{H}_1$  represents the hypothesis that the sensed signal  $r_i$  is the sum of a noise  $z_i$  and a signal  $s$  transmitted by PU (PU is active) and received through the channel  $h_i$ . Using the energy detector, the local decision test is given by [8, 9]:

$$\begin{aligned} y_i > \gamma_0 &\implies \mathcal{H}_1, \\ y_i < \gamma_0 &\implies \mathcal{H}_0, \end{aligned} \quad (5)$$

where  $\gamma_0$  is a local decision threshold that we assume common to all sensors. The sensors' decisions are assumed *independent*. (In practice, independence can be achieved by sufficiently (a few times the wavelength of their antennas) spacing the wireless nodes. For instance, for GHz transmissions, it suffices that sensors be spaced a few centimeters apart from each other.) and *identically distributed*. (Such assumption may hold in many settings, particularly when sensors have similar sensing capabilities, use the same sensing method/threshold, and are uniformly distributed in the area of concern.) Under hypothesis  $\mathcal{H}_0$ , such distribution is known to be chi-squared with  $2K$  degrees of freedom, that is [8, 9],

$$p_{y_i}(y; \mathcal{H}_0) = \frac{1}{2^K \Gamma(K)} y_i^{K-1} e^{-y_i/\sigma^2}, \quad (6)$$

while the test distribution under hypothesis  $\mathcal{H}_1$  is noncentral chi-squared with  $2K$  degrees of freedom and noncentrality parameter  $\lambda = 2\gamma_0$ , that is, [8, 9],

$$p_{y_i}(y; \mathcal{H}_1) = \frac{1}{2} e^{-(y_i/2\gamma_0)} \left( \frac{y_i}{2\gamma_0} \right)^{(k-1)/2} I_{k-1}(\sqrt{2\gamma_0 y_i}), \quad (7)$$

where  $I_k(\cdot)$  denotes the Bessel function of the first kind.

**2.4.2. Sending the Local Reports to the FC.** Let  $x_i$  denote the local test decision of sensor  $i$ ,  $1 \leq i \leq N$ . Then, local decisions  $x_i$  can be modeled as binary i.i.d. random variables. Precisely, for all  $i$ , let:

$$x_i = \begin{cases} 1, & \text{if sensor } i \text{ favors } \mathcal{H}_1, \\ 0, & \text{if sensor } i \text{ favors } \mathcal{H}_0, \end{cases} \quad (8)$$

then, the set of all reports received by the FC can be modeled as a *sequence*  $\mathbf{x} = (x_1, \dots, x_N)$  of binary i.i.d. random variables, each taking 0 with a probability  $p_0$  and 1 with probability  $1 - p_0$ . Subsequently, the sequence  $\mathbf{x}$  of sensing reports  $(x_1, \dots, x_N)$  has the following probability of occurrence:

$$p(\mathbf{x}) = p_0^{n_0} (1 - p_0)^{N - n_0}, \quad (9)$$

where  $n_0$  denotes the number of zeros within the sequence  $\mathbf{x}$ . We also shall assume (for simplicity) that the sensing reports  $x_i$ ,  $1 \leq i \leq N$  are transmitted error-freely to the FC (an assumption that can be achieved through the use of a powerful (low-rate) error-correcting code and a transmit rate that is no larger than the capacity of the channel between the sensors and the FC). Finally, based on the received sequence  $\mathbf{x}$  of sensing reports, the FC attempts to make a decision on whether the PU is indeed silent or active (according to a *fusion rule*), while trying to *minimize* the PFA and the PMD.

### 3. The PFA-PMD Tradeoff

As explained in the introduction, so far in the literature it has been very difficult to simultaneously minimize both probabilities (PFA and PMD). We explain here why. In all that follows,  $p_{\text{MD}}$ ,  $p_{\text{FA}}$  respectively denote the (local) PMD and PFA at each sensor. (According to our model, all sensors have the same local probabilities of false alarm and miss-detection, see Section 2.4.1 for further details.), while  $Q_{\text{MD}}$ ,  $Q_{\text{FA}}$ , respectively, denote the (global) PMD and PFA at the FC.

**3.1. The Single-Sensor Case.** Let us start by considering the simplest scenario where only one sensor node makes up the sensor network. According to our local sensing model in Section 2.4.1, the probability of miss detection is given by

$$\begin{aligned} p_{\text{MD}} &\triangleq \Pr\{x_i = 0, \mathcal{H}_1\} \\ &= \Pr\{y_i < \gamma_0, \mathcal{H}_1\} \\ &= \int_0^{\gamma_0} p_y(y, \mathcal{H}_1) dy. \end{aligned} \quad (10)$$

Likewise, the probability of false alarm reads

$$\begin{aligned} p_{\text{FA}} &\triangleq \Pr\{x_i = 1, \mathcal{H}_0\} \\ &= \Pr\{y_i > \gamma_0, \mathcal{H}_0\} \\ &= \int_{\gamma_0}^{\infty} p_y(y, \mathcal{H}_0) dy. \end{aligned} \quad (11)$$

As we only have 1 sensor in the network, it is evident that the FC is wrong *iff* the sensor is wrong. Thus

$$\begin{aligned} Q_{\text{MD}} &= p_{\text{MD}} = \int_0^{\gamma_0} p_y(y, \mathcal{H}_1) dy, \\ Q_{\text{FA}} &= p_{\text{FA}} = \int_{\gamma_0}^{\infty} p_y(y, \mathcal{H}_0) dy. \end{aligned} \quad (12)$$

Now, we shall explain the tradeoff between the PFA and the PMD at the FC.

- (i) If we are to minimize  $Q_{\text{MD}}$ , we have to minimize  $\int_0^{\gamma_0} p_y(y, \mathcal{H}_1) dy$ . As  $p_y(y, \mathcal{H}_1) \geq 0$ , this integral is minimized *iff*  $\gamma_0$  is *minimized*.
- (ii) If we are to minimize  $Q_{\text{FA}}$ , we have to minimize  $\int_{\gamma_0}^{\infty} p_y(y, \mathcal{H}_0) dy$ . As  $p_y(y, \mathcal{H}_0) \geq 0$ , this integral is minimized *iff*  $\gamma_0$  is *maximized*.

Hence, it is not possible to *simultaneously* minimize both  $Q_{\text{MD}}$  and  $Q_{\text{FA}}$  if the network is made by merely one sensor. Figure 1 illustrates the tradeoff between minimizing the local PFA and minimizing the local PMD.

**3.2. The Multiple-Sensor Case.** Now, let us assume that the wireless sensor network is made up by  $N > 1$  sensors. The PMD and PFA probabilities ( $Q_{\text{MD}}$  and  $Q_{\text{FA}}$ ) at the FC depend upon the fusion rule.  $k$ -out-of- $N$  [9, 10, 14] is a widely used fusion rule owing to its simplicity. Therefore, we illustrate the tradeoff between PFA and PMD when using this fusion rule at the FC. Basically, a hypothesis, say  $\mathcal{H}_1$ , is favored if at least  $k$  sensors out of  $N$  confirm it [15]:

$$\begin{aligned} \sum_{i=1}^N x_i \geq k &\implies \mathcal{H}_1, \\ \sum_{i=1}^N x_i < k &\implies \mathcal{H}_0. \end{aligned} \quad (13)$$

Subsequently, the probability of false alarm at the FC is given by [15]

$$Q_{\text{FA}} = \sum_{j=k}^N \binom{N}{j} p_{\text{FA}}^j (1 - p_{\text{FA}})^{N-j}, \quad (14)$$

whereas the probability of miss detection at the FC is given by [15]:

$$Q_{\text{MD}} = 1 - \sum_{j=k}^N \binom{N}{j} p_{\text{MD}}^j (1 - p_{\text{MD}})^{N-j}. \quad (15)$$

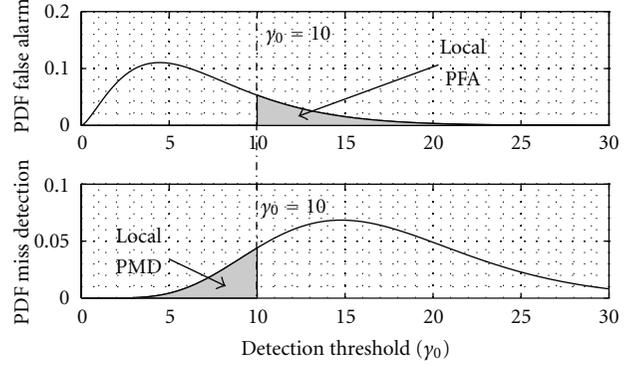


FIGURE 1: The tradeoff between minimizing local PMD and minimizing local PFA (within a single sensor). As depicted in this figure, for example, maximizing  $\gamma_0$  minimizes the local PFA (the gray area in the first subplot), yet inevitably increases the PMD (the gray area in the second subplot).

As we have,

$$\begin{aligned} &\underbrace{\sum_{j=N}^N \binom{N}{j} p_{\text{MD}}^j (1 - p_{\text{MD}})^{N-j}}_{\triangleq Q_{\text{FA}}(k=N)} \\ &= p_{\text{MD}}^N \\ &\leq \underbrace{p_{\text{MD}}^N + N p_{\text{MD}}^{N-1} (1 - p_{\text{MD}})}_{\triangleq Q_{\text{FA}}(k=N-1)} \\ &\vdots \\ &\leq \underbrace{p_{\text{MD}}^N + \dots + N p_{\text{MD}} (1 - p_{\text{MD}})^{N-1}}_{\triangleq Q_{\text{FA}}(k=1)}, \end{aligned} \quad (16)$$

then, we can see that

$$Q_{\text{FA}} \text{ is minimized} \iff k \text{ is maximized.} \quad (17)$$

Likewise, we can infer from (15) that

$$Q_{\text{MD}} \text{ is minimized} \iff k \text{ is minimized.} \quad (18)$$

Therefore, it is not possible to simultaneously minimize both PFA and PMD in the multiple-sensor case when using  $k$ -out-of- $N$  as a fusion rule.

## 4. Proposed Fusion Rule

**4.1. Problem Statement.** As explained earlier, even in the multiple-sensor case, it is not possible to simultaneously minimize the probabilities of false alarm and miss detection when using  $k$ -out-of- $N$ , the conventional fusion rule. Therefore, the aim of this work is to propose a fusion rule at the FC that achieves probabilities of miss detection and false alarm that can be made simultaneously asymptotically zero.

4.2. *How to Read This Section.* For convenience in exposition, we summarize the main content of this section.

- (i) First, we provide a paragraph where we familiarize the reader with the notion of typical sequences, through a summary of the main results as well as an intuitive interpretation.
- (ii) Then, we intuitively explain the motivation behind applying such notion to our problem (minimizing PFA and PMD).
- (iii) Then, we provide a detailed description of the proposed fusion rule (based on the notion of typical sequences).
- (iv) Finally, we state Theorem 3, the main contribution of this work.

#### 4.3. Typical Sequences: A Bird-Eye's View

*Definition 1* (Typical Set [13]). Let  $x_1, \dots, x_N$  denote  $N$  i.i.d. random variables,  $p(x)$  their (any) probability density function, and  $\epsilon$  a strictly positive real number. A sequence  $(x_1, \dots, x_N)$  of realizations of the aforesaid random variables is said to be  $\epsilon$ -typical if

$$2^{-N(H(x)+\epsilon)} \leq p(x_1, \dots, x_N) \leq 2^{-N(H(x)-\epsilon)}. \quad (19)$$

The set  $\mathcal{A}_\epsilon^{(N)}$  of  $\epsilon$ -typical sequences is called *typical set*.

Put into simpler terms, the typical set is the set of sequences whose probability of occurrence is roughly  $2^{-NH(x)}$ .

**Theorem 2** (properties of a typical set [13]). *The typical set has the following properties when  $N$  is sufficiently large:*

- (i)  $\Pr\{\mathcal{A}_\epsilon^{(N)}\} > 1 - \epsilon$ ,
- (ii)  $(1 - \epsilon)2^{N(H(x)-\epsilon)} \leq |\mathcal{A}_\epsilon^{(N)}| \leq 2^{N(H(x)+\epsilon)}$ .

In other words,

- (i) the typical set has a probability of occurrence that nears 1,
- (ii) the number of sequences in the typical set is nearly  $2^{NH(x)}$ .

These properties of typical sequences follow from the law of large numbers. Simply put, Theorem 2 states that if we draw a sequence of  $(x_1, \dots, x_N)$  i.i.d. variables, then this sequence will *almost surely* (i.e., with probability  $1 - \epsilon$ , with  $\epsilon$  tending to zero when  $N$  goes to infinity) lie in the typical set  $\mathcal{A}_\epsilon^{(N)}$ . In what follows, based on the notion of typical sequences, we provide a hard fusion rule at the FC that achieves arbitrarily small probabilities of (global) miss detection and false alarm. First we outline the proposed scheme, then we justify why it achieves such unprecedented performance.

4.4. *Motivation behind Applying the Notion of Typical Sequences to Cooperative Spectrum Sensing in CR.* In a network of  $N$ -sensors, FC receives a report  $(x_1, \dots, x_N)$

made up by  $N$  binary random variables. Therefore, there are at most  $2^N$  possible reports that can be received by the FC. However, according to Theorem 2, these  $2^N$  do not necessarily have the same probability of occurrence. As a matter of fact, Theorem 2 tells us that of all these  $2^N$  possible sequences, *only* a fraction of them (roughly  $2^{NH(x)}$ ) *occur most of the time when a certain hypothesis holds*. Without loss of generality, let us consider the set of likely (typical) events given hypothesis. (This choice is arbitrary. We could have defined the typicality with respect to hypothesis  $\mathcal{H}_0$ . Regardless of this choice, we shall show later that we can simultaneously minimize both the PFA and the PMD)  $\mathcal{H}_1$ . Motivated by the result in Theorem 2, we raise the following question: rather than believing all report sequences that are received (as suggested by conventional fusion rules), why not trust only report sequences that are *likely* to be received should hypothesis  $\mathcal{H}_1$  be true? Indeed, according to Theorem 2, if we ignore unlikely (nontypical) sequences, we will be right most (precisely,  $1 - \epsilon$ ) of the time, and wrong on much rarer occasions ( $\epsilon$  of the time). Therefore, even if we disregard such unlikely reports and they turn out to be true, they will have very little impact on the average probability of error.

4.4.1. *Outline of the Proposed Sequence-Typicality-Based Fusion Rule.* Motivated by the previous reflections, we propose the following fusion rule.

- (1) Set initial parameters: the local sensing threshold  $\gamma_0$ , the local test statistics with respect to hypotheses  $\mathcal{H}_0$ ,  $\mathcal{H}_1$ . The probabilities  $p_0$  and  $p_1$  can therefore be computed.
- (2) Make a list  $\mathcal{A}_\epsilon^{(N)}$  of all typical report sequences with respect to the hypothesis  $\mathcal{H}_1$ , given a sufficiently large number of nodes  $N$  and an arbitrary real strictly positive number  $\epsilon$ : *what are the sequences that are likely to be received when hypothesis  $\mathcal{H}_1$  is indeed true?*
- (3) Whenever a sequence of reports  $p(x_1, \dots, x_N)$  is received, the FC performs the following computationally simple test.
  - (a) If the sequence  $p(x_1, \dots, x_N)$  is in the list  $\mathcal{A}_\epsilon^{(N)}$ , FC goes in favor of hypothesis  $\mathcal{H}_1$  (i.e., assumes that PU is active).
  - (b) If the sequence  $p(x_1, \dots, x_N)$  is not in the list  $\mathcal{A}_\epsilon^{(N)}$ , FC goes in favor of hypothesis  $\mathcal{H}_0$  (i.e., assume that PU is silent).

4.4.2. *Generation of the Typical Set  $\mathcal{A}_\epsilon^{(N)}$ .* According to (1), a sequence  $(x_1, \dots, x_N)$  of sensor nodes' decisions is  $\epsilon$ -typical if

$$2^{-N(H(x)+\epsilon)} \leq p(x_1, \dots, x_N) \leq 2^{-N(H(x)-\epsilon)}. \quad (20)$$

Therefore, in order to determine such sequences, we need to determine the following

- (i) The probability of occurrence of any possible sequence  $p(x_1, \dots, x_N)$ , according to (9).

- (ii) The Shannon entropy  $H(x)$  of the i.i.d. random variables  $x_1, \dots, x_N$ . The reports  $x_i$  are i.i.d. discrete (binary) random variables that follow a Bernoulli distribution. (A one is received with probability  $p_0$  and a zero is received with probability  $(1 - p_0)$ .) From the definition (1), the entropy  $H(x)$  reads

$$H(x) = -p_0 \log_2(p_0) - (1 - p_0) \log_2(1 - p_0). \quad (21)$$

4.4.3. *Determining Which Sequences Are Typical.* Now that FC has computed both the probability of occurrence  $p(x_1, \dots, x_N)$  of any sequence  $(x_1, \dots, x_N)$  and the Shannon entropy  $H(x)$ , it can determine which sequences are in the typical set, as follows.

- (i) For every sequence  $(x_1, \dots, x_N)$  (out of the  $2^N$  possible sequences)
- if  $2^{-N(H(x)+\epsilon)} \leq p(x_1, \dots, x_N) \leq 2^{-N(H(x)-\epsilon)}$ , then the sequence is *in* the typical set. If such sequence is received, FC will decide in favor of hypothesis  $\mathcal{H}_1$ ;
  - otherwise (if  $p(x_1, \dots, x_N) < 2^{-N(H(x)+\epsilon)}$  or  $p(x_1, \dots, x_N) > 2^{-N(H(x)-\epsilon)}$ ), then the sequence is *not in* the typical set. If such sequence is received, FC will decide in favor of hypothesis  $\mathcal{H}_0$ . If such sequence is received, FC will decide in favor of hypothesis  $\mathcal{H}_0$ .

For clarity, a flowchart in Figure 2 illustrates the various steps in the implementation of the proposed fusion rule, as discussed in this paragraph.

#### 4.5. Analysis of the Probability of Error (False Alarm or Miss Detection)

4.5.1. *Main Result.* Theorem 3 constitutes the main contribution in this work.

**Theorem 3.** *Let  $Q_{PF}$  and  $Q_{MD}$  respectively denote the probabilities of (global) false alarm and miss detection of the FC's sensing decision. Then, irrespective of the local decision threshold  $\gamma_0$ ,  $Q_{PF}$  and  $Q_{MD}$  are asymptotically zero when using the proposed fusion rule, that is,*

$$\lim_{N \rightarrow \infty} Q_{MD} = \lim_{N \rightarrow \infty} Q_{PF} = 0. \quad (22)$$

*Proof.* The proof that we provide is inspired by the proof of Theorem 15.3.1. in [13, pages 530–532] on a different topic (the achievability of the capacity region for the multiple-access channel).

Using the proposed fusion rule, only 4 situations are possible, as summarized in Table 1. Of these 4 possible scenarios, only two represent a global decision error at the FC. Precisely, using the proposed fusion rule, FC is wrong if either of the following situations occur.

- (i) Situation  $A_0$ : when the PU is active but  $(x_1, \dots, x_N)$  is *not in* the typical set: this is a *miss detection*, because under the proposed rule the FC would assume the PU silent.

TABLE 1: Summary of all the sensing outcomes for every hypothesis. In this table, O represents a correct global decision (i.e., FC's decision = real event), while  $\times$  represents an erroneous global decision (i.e., FC's decision  $\neq$  real event).

Real event	$(x_1, \dots, x_N) \in \mathcal{A}_\epsilon^N$	$(x_1, \dots, x_N) \notin \mathcal{A}_\epsilon^N$
PU is active	O	$\times$ (miss detection)
PU is silent	$\times$ (false alarm)	O

- (ii) Situation  $A_1$ : when the PU is silent but  $(x_1, \dots, x_N)$  is *in* the typical set: this is a *false alarm*, because under the proposed rule the FC would assume the PU active.

Let  $Q_{MD}$  denote the probability that situation  $A_0$  occurs,

$$\begin{aligned} Q_{MD} &\triangleq \Pr\{(x_1, \dots, x_N) \notin \mathcal{A}_\epsilon^N, \mathcal{H}_1\} \\ &= 1 - \Pr\{(x_1, \dots, x_N) \in \mathcal{A}_\epsilon^N, \mathcal{H}_1\}. \end{aligned} \quad (23)$$

Owing to Theorem 2, we have

$$\Pr\{(x_1, \dots, x_N) \in \mathcal{A}_\epsilon^N, \mathcal{H}_1\} > 1 - \epsilon. \quad (24)$$

Therefore, it follows that

$$Q_{MD} < 1 - (1 - \epsilon) = \epsilon. \quad (25)$$

Hence, using the proposed fusion rule, the probability of miss detection goes to zero when  $N$  goes to infinity. We have by now proved the first part of Theorem 3.

Now, let us consider the second part (i.e., the probability of false alarm). The probability of occurrence of event  $A_1$  is

$$Q_{PF} \triangleq \Pr\{(x_1, \dots, x_N) \in \mathcal{A}_\epsilon^N, \mathcal{H}_0\}. \quad (26)$$

Owing to Theorem 3, we know that the typical set  $\mathcal{A}_\epsilon^N$  has at most  $2^{N(H(x)+\epsilon)}$  elements. Let us denote these elements as  $\alpha_1, \dots, \alpha_{2^{N(H(x)+\epsilon)}}$ . Subsequently, we get

$$Q_{PF} = \sum_{k=1}^{2^{N(H(x)+\epsilon)}} \Pr\{(x_1, \dots, x_N) = \alpha_k, \mathcal{H}_0\}. \quad (27)$$

On the other hand, the law of total probabilities tells us that, for all  $k$ :

$$\begin{aligned} \Pr\{(x_1, \dots, x_N) = \alpha_k, \mathcal{H}_0\} &= \Pr\{(x_1, \dots, x_N) = \alpha_k\} \\ &\quad - \Pr\{(x_1, \dots, x_N) = \alpha_k, \mathcal{H}_1\}. \end{aligned} \quad (28)$$

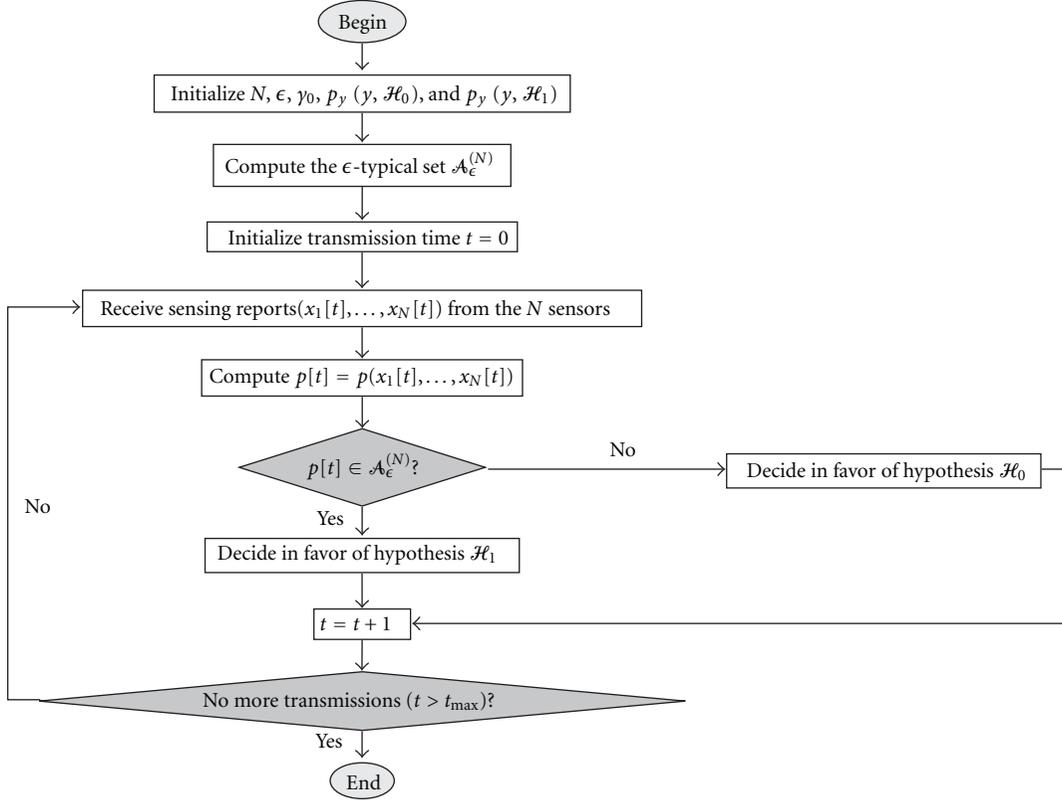


FIGURE 2: Practical implementation of the proposed fusion rule for cooperative sensing in wireless sensor networks. In this chart,  $t_{\max}$  denotes the total observation (sensing) time.

Thus, we get:

$$\begin{aligned}
 Q_{\text{PF}} &= \sum_{k=1}^{2^{N(H(x)+\epsilon)}} \Pr\{(x_1, \dots, x_N) = \alpha_k\} \\
 &\quad - \sum_{k=1}^{2^{N(H(x)+\epsilon)}} \Pr\{(x_1, \dots, x_N) = \alpha_k, \mathcal{H}_1\} \\
 &\leq \underbrace{\sum_{k=1}^{2^{N(H(x)+\epsilon)}} 2^{N(H(x)+\epsilon)}}_{=1} - \underbrace{\sum_{k=1}^{2^{N(H(x)+\epsilon)}} \Pr\{(x_1, \dots, x_N) = \alpha_k, \mathcal{H}_1\}}_{>1-\epsilon} \\
 &< 1 - (1 - \epsilon) = \epsilon \rightarrow 0.
 \end{aligned} \tag{29}$$

Hence,  $Q_{\text{PF}}$  also converges to zero when  $N$  goes to infinity when using the proposed fusion rule.  $\square$

**4.5.2. Commentary.** We explain in this paragraph the consequences of Theorem 3. By applying the formal definition of the limit of a function in mathematics, (22) can be rewritten as follows:

$$\begin{aligned}
 \forall \epsilon > 0, \quad \exists n_1 \in \mathbb{N}, \quad \forall n \in \mathbb{N} : n > n_1 \implies Q_{\text{MD}} < \epsilon, \\
 \forall \epsilon > 0, \quad \exists n_2 \in \mathbb{N}, \quad \forall n \in \mathbb{N} : n > n_2 \implies Q_{\text{PF}} < \epsilon.
 \end{aligned} \tag{30}$$

By taking  $n_0 = \max(n_1, n_2)$ , we get

$$\forall \epsilon > 0, \quad \exists n_0 \in \mathbb{N}, \quad \forall n \in \mathbb{N} : n > n_0 \implies \begin{cases} Q_{\text{MD}} < \epsilon \\ Q_{\text{PF}} < \epsilon. \end{cases} \tag{31}$$

Therefore, it appears that for any real number  $\epsilon$ , there exists a certain number of sensor nodes,  $n_0$ , such that the proposed scheme can achieve  $Q_{\text{FA}}$  and  $Q_{\text{MD}}$  probabilities that are *both* below  $\epsilon$ .

**4.5.3. On the Tightest Bound  $\epsilon_{\min}$  for a Given Sequence  $(x_1, \dots, x_N)$ .** Let us fix an arbitrary bound  $\epsilon > 0$ . Let  $\mathbf{x} = (x_1, \dots, x_N)$  be an  $\epsilon$ -typical sequence. Then, by definition, the sequence  $\mathbf{x}$  has a probability of occurrence,  $p(x_1, \dots, x_N)$ , such that

$$2^{-N(H(x)+\epsilon)} \leq p(x_1, \dots, x_N) \leq 2^{-N(H(x)-\epsilon)}. \tag{32}$$

Further, if the FC assumes such sequence of reports correct (as suggested in our fusion rule), then PFA and QFA will be lower than  $\epsilon$ . However,  $2^{-N(H(x)\pm\epsilon)}$  is *not necessarily* the tightest bound on  $p(x_1, \dots, x_N)$ .

Indeed, for some sequences, there may be an  $\epsilon_1 < \epsilon$  such that

$$\begin{aligned}
 2^{-N(H(x)+\epsilon)} &< 2^{-N(H(x)+\epsilon_1)} \leq p(x_1, \dots, x_N) \\
 &\leq 2^{-N(H(x)-\epsilon_1)} < 2^{-N(H(x)-\epsilon)}.
 \end{aligned} \tag{33}$$

Therefore, one conclusion we may draw is that *even within the typical set, some sequences are more reliable than others*. Precisely, the closer the probability of occurrence,  $p(x_1, \dots, x_N)$ , of a sequence  $(x_1, \dots, x_N)$  to the true ( $\epsilon = 0$ ) typical probability  $2^{-NH(x)}$ , the more reliable the sequence is (in terms of PFA and PMD).

Quantitatively, let  $\Delta C$  denote the gap between  $p(x_1, \dots, x_N)$  and  $2^{-NH(x)}$

$$\Delta C = \left| 2^{-NH(x)} - p(x_1, \dots, x_N) \right|. \quad (34)$$

Let  $\epsilon_{\min}$  denote the smallest upper bound on PFA and QMD of  $(x_1, \dots, x_N)$ , that is,

$$\begin{aligned} \epsilon_{\min} &\triangleq \min \left\{ \epsilon > 0 : 2^{-N(H(x)+\epsilon)} \leq p(x_1, \dots, x_N) \right. \\ &\quad \left. \leq 2^{-N(H(x)-\epsilon)} \right\}. \end{aligned} \quad (35)$$

Then, one may write:

$$\begin{aligned} 2^{-N(H(x)-\epsilon_{\min})} &= \Delta C \\ &= \left| 2^{-NH(x)} - p(x_1, \dots, x_N) \right|. \end{aligned} \quad (36)$$

It follows that

$$\epsilon_{\min} = \log_2 \left| 1 - 2^{NH(x)} p(x_1, \dots, x_N) \right|. \quad (37)$$

## 5. Numerical Examples

In this section, we report numerical examples obtained through computer simulations. The goals behind such examples are twofold.

- (1) First, we provide an illustrative example to give insight on how the proposed fusion rule can be applied in practice. Particularly, we explain how the list  $\mathcal{A}_\epsilon^{(N)}$  can be generated, given a number of active sensors  $N$  in the network and an *arbitrary* threshold  $\epsilon$  on *both probabilities* (PFA and PMD).
- (2) Second, we provide a numerical evaluation of the bounds on PFA and PMD for the proposed fusion rule compared with the conventional rule ( $k$ -out-of- $N$ ).

Toward this end, we consider a wireless sensor network of  $N = 10$  sensor nodes that sense the spectrum using an energy detector by taking  $K$  samples, where  $K$  has been varied from 5 to 20. A node's spectrum sensing is subject to an additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2 = 10$ . Finally, local decision tests are with respect to a threshold  $\gamma_0$  that has been varied from 0 to 30.

**5.1. A Toy Example.** We start by explaining, through an illustrative example, how the proposed scheme can be implemented in practice. This toy example is inspired by Problem 3.13 provided in [13, pages 68–69], to illustrate the general notion of typical sequences.

We consider  $N = 10$  sensor nodes in the network. (While  $N = 10$  is not really a large number, 1024 (the number

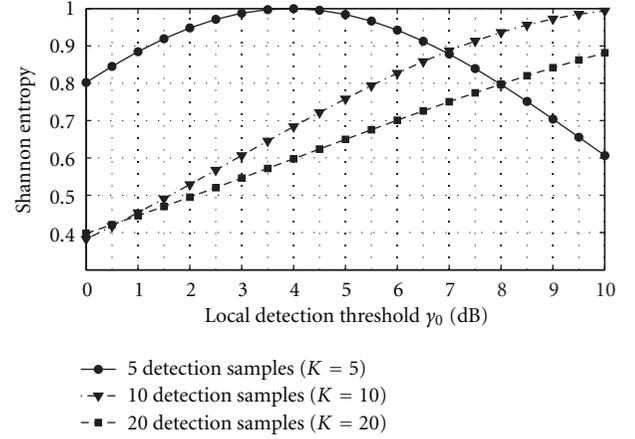


FIGURE 3: The Shannon entropy of the local test decisions for different values of  $\gamma_0$  and  $K$ .

of possible sequences) is a sufficiently large number for the targeted PFA and PMD threshold,  $\epsilon$ . If much smaller probabilities are targeted, then there may be a need for a larger number of sensor nodes.) Thus, the number of possible sequences  $(x_1, \dots, x_N)$  is  $2^N = 1024$ .

Let us *arbitrarily* assume that the network operator wants *both* probabilities (PFA and PMD) to be less than, say, 10%.

According to Theorem 3,  $\epsilon$  is an upper bound on PFA and PMD. Therefore, a *sufficient* condition for such requirement would be to fix  $\epsilon = 0.1$ . Now, we shall determine the set of all possible  $\epsilon$ -typical sequences, that is, those whose probability is bounded as

$$2^{-N(H(x)+\epsilon)} \leq p(x_1, \dots, x_N) \leq 2^{-N(H(x)-\epsilon)}. \quad (38)$$

**5.1.1. Computing the Shannon Entropy.** A first step towards this goal is to compute  $H(x)$ . Figure 3 illustrates the Shannon entropy  $H(x)$  of any given report  $x_i$  relative to a sensor node  $i$ ,  $1 \leq i \leq N$ , for different local detection thresholds  $\gamma_0$  and detection samples  $K$ .  $H(x)$  has been computed according to (21). From these results, it appears that the entropy-maximizing threshold  $\gamma_0$  is increasing with the detection sample. As the number of typical sequences is nearly  $2^{NH(x)}$ , our simulation results give insights to the network operator on how the threshold  $\gamma_0$  should be set to manage the size of the typical set:

- (i) if the network operator wants to reduce the number of possible  $\epsilon$ -typical sequences,  $\gamma_0$  should be set to a relatively low value;
- (ii) if the network operator wants to increase the number of possible  $\epsilon$ -typical sequences,  $\gamma_0$  should be set to a high value.

**5.1.2. Determining the  $\epsilon$ -Typical Sequences.** After computing  $H(x)$  according to (21), we compute the probability of occurrence of every sequence (1024 in total), according to the formula in (9), for  $K = 5$  and  $\gamma_0 = 2$  (thereby yielding an entropy  $H(x) \approx 0.9$ ), as reported in Figure 4. On the same figure, we plot three thresholds.

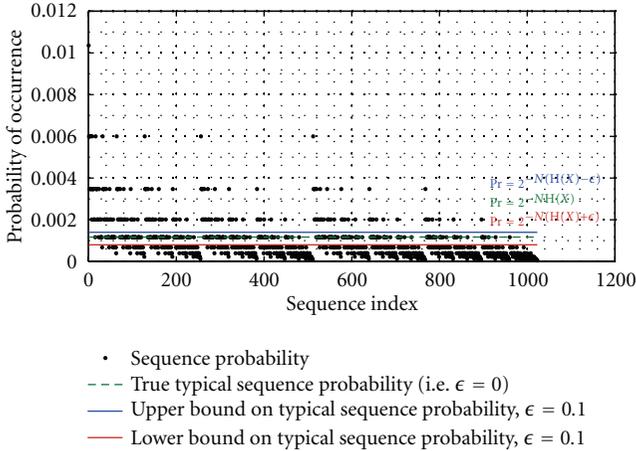


FIGURE 4: Probability of occurrence of all the possible sequences, when  $\epsilon = 0.1$ ,  $K = 5$ , and  $\gamma_0 = 2$ .  $\epsilon$ -typical sequences are the sequences between the two far-most lines.

- (i) The probability that a (true, i.e., when  $\epsilon \rightarrow 0$ ) typical sequence occurs. This probability is  $p_{\text{true}} = 1.17 \times 10^{-3}$ . It can be verified that  $2^{-N(H(x))} = 2^{-10 \times 0.3} = 2 \times 10^{-3} \approx p_{\text{true}}$ .
- (ii) The outer bound on the probability of an  $\epsilon$ -typical sequence, with  $\epsilon = 0.1$ . The outer bound is  $p_{\text{out}} = 2^{-N(H(x)-\epsilon)} = 1.4 \times 10^{-3}$ .
- (iii) The inner bound on the probability of an  $\epsilon$ -typical sequence, with  $\epsilon = 0.1$ . The inner bound is  $p_{\text{in}} = 2^{-N(H(x)+\epsilon)} = 0.7 \times 10^{-3}$ .

The sequences whose probabilities of occurrence are between the two far-most lines (i.e., when  $0.7 \times 10^{-3} \leq p(x_1, \dots, x_N) \leq 1.4 \times 10^{-3}$ ) are the  $\epsilon$ -typical sequences while the sequences that lie outside the two far-most lines are the non- $\epsilon$ -typical sequences. As claimed in Theorem 2 and depicted in Figure 4, we can see that nontypical sequences constitute the majority of the possible sequences.

**5.2. Performance Evaluation.** Figures 5 and 6 respectively illustrate the global PFA and PMD ( $Q_{\text{FA}}$  and  $Q_{\text{MD}}$  (14) and (15)) of the conventional fusion rule ( $K$ -out-of- $N$ ) versus the number of sensors in the network, for different values of  $K$ . On the same figures, the *upper bound*,  $\epsilon$ , on the global PFA and PMD for the proposed fusion rule is plotted.

First, by observing the performance of the conventional fusion rule ( $K$ -out-of- $N$ ), we confirm the tradeoff that we discussed earlier in Section 3.2. For instance, while settings where  $K$  is large ( $K = N - 1$ ,  $N - 2$ , the red curves) are ideal for cooperative spectrum sensing from the perspective of minimizing PFA (Figure 5), they are unattractive from the perspective of PMD (Figure 6), as the latter is *increasing* with the number of sensors (in other words, cooperation hurts). Similarly, decreasing  $K$  (the blue curves) yields a global PMD that is decreasing with the number of sensors but also a PFA that is increasing with the number of sensors.

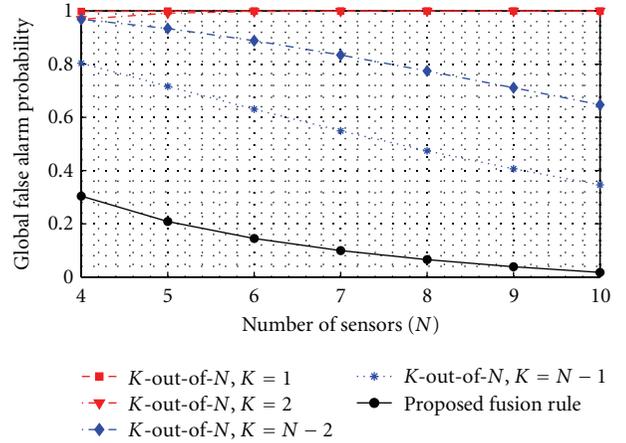


FIGURE 5: Upper bounds on the probabilities of false alarm at the FC when using the conventional ( $K$ -out-of- $N$ ) and the proposed fusion rules.

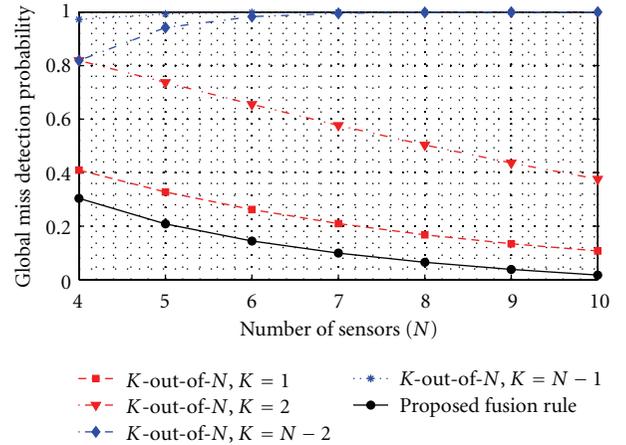


FIGURE 6: Upper bounds on the probabilities of miss detection at the FC when using the conventional ( $K$ -out-of- $N$ ) and the proposed fusion rules.

Contrarily, the proposed fusion rule achieves global PFA and PMD that are *both* decreasing with the number of sensors in the network. This confirms the theoretical result in Theorem 3 that the proposed fusion rule achieves asymptotically zero PFA and PMD probabilities when  $N$  goes to infinity. In other words, the proposed fusion rule can simultaneously achieve arbitrarily small PFA and PMD for a sufficient number of sensors.

Further, we observe that with the same number of sensor nodes in the network, the proposed fusion rule achieves the lowest PFA and PMD, which also suggests that the proposed fusion rule is more reliable than the conventional fusion rule, for a given number of sensor nodes.

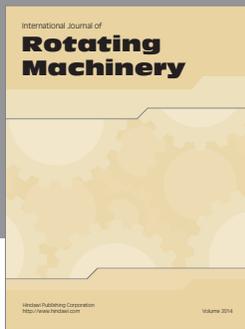
## 6. Conclusions

In CR cooperative sensing schemes, wireless sensor nodes deployed in the network sense the licensed spectrum and

send their local sensing decisions to a fusion center (FC) that makes a global decision on whether or not to allow the secondary user use the spectrum, based on a decision fusion rule.  $k$ -out-of- $N$  rule is widely used in the literature owing to its practical simplicity. Regrettably, it cannot minimize both probabilities of false alarm and miss detection, which could have consequent effects on the performance of CR. In this work, based on the notion of typical sequences, we propose a novel fusion rule in which the false alarm and miss detection probabilities can be *simultaneously* made as small as desired (asymptotically zero as the number of sensors goes to infinity).

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