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Research Article

Some More Results on IF Soft Rough Approximation Space

Sharmistha Bhattacharya (Halder)[1] and Bijan Davvaz[2]

[1] *Department of Mathematics, Tripura University, Suryamaninagar, Tripura 799130, India*
[2] *Department of Mathematics, Yazd University, Yazd 89195-741, Iran*

Correspondence should be addressed to Bijan Davvaz, davvaz@yazduni.ac.ir

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Fuzzy sets, rough sets, and later on IF sets became useful mathematical tools for solving various decision making problems and data mining problems. Molodtsov introduced another concept soft set theory as a general frame work for reasoning about vague concepts. Since most of the data collected are either linguistic variable or consist of vague concepts so IF set and soft set help a lot in data mining problem. The aim of this paper is to introduce the concept of IF soft lower rough approximation and IF upper rough set approximation. Also, some properties of this set are studied, and also some problems of decision making are cited where this concept may help. Further research will be needed to apply this concept fully in the decision making and data mining problems.

1. Introduction

Data mining is a technique of extracting meaningful information from large and mostly unorganized data banks. Data mining is one of the areas in which rough set is widely used. Data mining is the process of automatically searching large volumes of data for patterns using tools such as classifications, association, rule mining, and clustering. The rough set theory is a well understood format framework for building data mining models in the form of logic rules on the bases of which it is possible to issue predictions that allow classifying new cases.

In general whenever data are collected they are linguistic variables. Not only this, the answers are not always in Yes/No form. So, in this case to deal with such type of data IF set is a very important tool.

Data are in most of the cases a relation between object and attribute. Soft set is an important tool to deal with such types of data.

So, throughout this paper a combined approach of soft set, IF set, rough set is studied. Further study is required to find the application of this concept in the field of data mining.

Zadeh in 1965 [1] introduced the concept of fuzzy set. This set contains only a membership function lying between 0 and 1. But while collecting data many cases may be there where data are missing so IF sets are reqd which consists of both membership value and nonmembership value. Atanassov [2] introduced the concept of IF set. Atanassov named it intuitionistic fuzzy set. But nowadays a problem arose due to the already introduced concept of intuitionistic logic. Hence, instead of intuitionistic fuzzy set, throughout this paper we are using the nomenclature IF set.

Rough sets introduced by Pawlak [3] are also a very useful tool for data mining problems where vagueness is the key factor. Molodtsov [4] introduced the concept of soft set, and in 2009 Feng et al. [5] introduced a combined notion of fuzzy set, rough set, and soft set to deal with complex data which arises in the most social science problems.

In this paper, our aim is to introduce the concepts of IF soft lower and IF soft upper rough approximations which help a lot for sorting the vague data and tending towards decision.

2. Basic Definitions

In this section, some of the important required concepts necessary to go further through this paper are shown.

Let X be a nonempty set, and let I be the unit interval $[0,1]$. According to [2], an *intuitionistic fuzzy set* (IFS for short) U is an object having the form

$$U = \{\langle x, \mu_u(x), \gamma_u(x)\rangle : x \in X\}, \tag{2.1}$$

where the functions $\mu_u : X \to [0,1]$ and $\gamma_u : X \to [0,1]$ denote, respectively, the degree of membership and the degree of nonmembership of each element $x \in X$ to the set U, and $0 \le \mu_u(x) + \gamma_u(x) \le 1$ for each $x \in X$. An *intuitionistic fuzzy topology* (IFT for short) on a nonempty set X is a family τ of IFS's in X containing $0_\sim, 1_\sim$ and closed under arbitrary infimum and finite supremum [6]. In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS for short) and each IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS for short). The compliment U^c of an IFOS is called an *intuitionistic fuzzy closed set* (IFCS for short).

Let X be a nonempty set and let IFS's U and V be in the following forms:

$$U = \{\langle x, \mu_u(x), \gamma_u(x)\rangle : x \in X\}, \qquad V = \{\langle x, \mu_v(x), \gamma_v(x)\rangle : x \in X\}. \tag{2.2}$$

Then,

(1) $U^c = \{\langle x, \gamma_u(x), \mu_u(x)\rangle : x \in X\}$,

(2) $U \cap V = \{\langle x, \mu_u(x) \wedge \mu_v(x),\ \gamma_u(x) \vee \gamma_v(x)\rangle \ : x \in X\}$,

(3) $U \cup V = \{\langle x, \mu_u(x) \vee \mu_v(x),\ \gamma_u(x) \wedge \gamma_v(x)\rangle \ : x \in X\}$,

(4) $0_\sim = \{\langle x, 0, 1\rangle : x \in X\},\ 1_\sim = \{\langle x, 1, 0\rangle \ : x \in X\}$,

(5) $(U^c)^c = U,\ 0_\sim^c = 1_\sim,\ 1_\sim^c = 0_\sim$.

Let U be a finite nonempty set, called *universe* and R an equivalence relation on U, called *indiscernibility relation*. The pair (U, R) is called an *approximation space*. By $R(x)$ we mean that the set of all y such that xRy, that is, $R(x) = [x]_R$ is containing the element x. Let X be a subset of U. We want to characterize the set X with respect to R. According to Pawlak's paper [3], the lower approximation of a set X with respect to R is the set of all objects, which surely belong to X, that is, $R_*(X) = \{x : R(x) \subseteq X\}$, and the upper approximation of X with respect to R is the set of all objects, which are partially belonging to X, that is, $R^*(X) = \{x : R(x) \cap X \neq \phi\}$. For an approximation space (U, θ), by a rough approximation in (U, θ) we mean a mapping $\mathrm{Apr} : \wp(U) \to \wp(U) \times \wp(U)$ defined by for every $X \in \wp(U)$,

$$\mathrm{Apr}(X) = (R_*(X), R^*(X)). \tag{2.3}$$

Given an approximation space (U, R), a pair $(A, B) \in \wp(U) \times \wp(U)$ is called a *rough set* in (U, R) if $(A, B) = \mathrm{Apr}(X)$ for some $X \in \wp(U)$.

Fuzzy set is defined by employing the fuzzy membership function, whereas rough set is defined by approximations. The difference of the upper and the lower approximation is a boundary region. Any rough set has a nonempty boundary region whereas any crisp set has an empty boundary region. The lower approximation is called *interior,* and the upper approximation is called *closure* of the set. By using these concepts, we can make a topological space.

A set T is said to be a *topological space* if with every $X \subset T$ there is an associated set $IX \subset T$ such that the following conditions are satisfied: for any $X, Y \subset T$, $I(X \cap Y) = IX \cap IY$, $IX \subset X$, $IIX = IX$, and $IT = T$. The operation I is called an *interior operation*. This topological space is written by (T, I).

Let U be a universal set and let E be a set of parameters. According to [4], a pair (F, A) is called a *soft set* over U, where $A \subseteq E$ and $F : A \to P(U)$, the power set of U, is a set-valued mapping.

Let (U, R) be a Pawlak approximation space. For a fuzzy set $\mu \in F(U)$, the lower and upper rough approximations of μ in (U, R) are denoted by $\underline{R}(\mu)$ and $\overline{R}(\mu)$, respectively, which are fuzzy sets defined by

$$\begin{aligned} \underline{R}(\mu)(x) &= \bigwedge\{\mu(y) : y \in [x]_R\}, \\ \overline{R}(\mu)(x) &= \bigvee\{\mu(y) : y \in [x]_R\}, \end{aligned} \tag{2.4}$$

for all $x \in U$. The operators $\underline{R}$ and $\overline{R}$ are called the *lower and upper rough approximation operators on fuzzy sets*. If $\underline{R} = \overline{R}$ the fuzzy set μ is said to be *definable*, otherwise μ is called a *rough fuzzy set*.

A soft set (F, A) over U is called a *full soft set* if $\bigcup_{a \in A} F(a) = U$. Let (F, A) be a full soft set over U, and let $\mathcal{S} = (U, E)$ be a soft approximation space. For a fuzzy set $\mu \in F(U)$ the lower and upper soft rough approximations of μ with respect to S are denoted by $\underline{\mathrm{sap}}_S(\mu)$ and $\overline{\mathrm{sap}}_S(\mu)$, respectively, which are fuzzy sets in U given by

$$\begin{aligned} \underline{\mathrm{sap}}_S(\mu)(x) &= \bigwedge\{\mu(y) : \exists a \in A,\ \{x, y\} \subseteq F(a)\}, \\ \overline{\mathrm{sap}}_S(\mu)(x) &= \bigvee\{\mu(y) : \exists a \in A,\ \{x, y\} \subseteq F(a)\}, \end{aligned} \tag{2.5}$$

Table 1

	d_1	d_2	d_3	d_4	d_5	d_6	d_7
t_1	1	1	0	0	0	0	0
t_2	0	0	1	1	1	0	0
t_3	0	0	0	0	0	1	1
t_4	1	1	0	0	0	0	0
t_5	0	0	0	0	1	1	0
t_6	0	0	1	1	0	0	0

for all $x \in U$. The operators $\underline{\mathrm{sap}}_S(\mu)$ and $\overline{\mathrm{sap}}_S(\mu)$ are called the *lower and upper soft rough approximation operators on fuzzy sets*. If both the operators are the same then μ is said to be *soft definable*, otherwise μ is said to be *soft rough fuzzy set*.

3. On IF Soft Rough Approximations

In this section, we introduce the concept of IF soft rough approximation. Some of its properties are studied and examples are presented. The main focus of this paper is to show the scope of this newly introduced concept in the field of data mining and decision making.

Definition 3.1. Let $\mathcal{E} = (F, A)$be a full soft set over U and $\mathcal{S} = (U, E)$ a soft approximation space. For an IF set $\langle \mu, \gamma \rangle$, the IF soft lower rough approximation and IF soft upper rough approximation with respect to the soft approximation space S are denoted by $\underline{\mathrm{sap}}^*_S(\langle \mu, \gamma \rangle)$ and $\overline{\mathrm{sap}}^*_S(\langle \mu, \gamma \rangle)$ and are defined as follows:

$$\begin{aligned}
\underline{\mathrm{sap}}^*_S(\langle \mu, \gamma \rangle)(x) &= \sup_{\gamma} \inf_{\mu} \{ \langle \mu(y), \gamma(y) \rangle : \exists a \in A,\ \{x, y\} \subseteq F(a) \}, \\
\overline{\mathrm{sap}}^*_S(\langle \mu, \gamma \rangle)(x) &= \inf_{\mu} \sup_{\gamma} \{ \langle \mu(y), \gamma(y) \rangle : \exists a \in A,\ \{x, y\} \subseteq F(a) \},
\end{aligned} \tag{3.1}$$

for all $x \in U$.

Example 3.2. Suppose that $U = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$ is the universe of the days of a week and the set of parameters are given by $E = \{t_1, t_2, t_3, t_4, t_5, t_6\}$, where t_i $(i = 1, \ldots, 6)$ stands for hot, medium, cold, heavy rain, medium rainy, and not raining. Let us consider a soft set (F, E) describing the weather. Let us represent Table 1.

Then, $F(t_1) = \{d_1, d_2\}$, $F(t_2) = \{d_3, d_4, d_5\}$, $F(t_3) = \{d_6, d_7\}$, $F(t_4) = \{d_1, d_2\}$, $F(t_5) = \{d_5, d_6\}$, $F(t_6) = \{d_3, d_4\}$.

Suppose that

$$\langle \mu, \gamma \rangle = \left\{ \frac{\langle 0.6, 0.3 \rangle}{d_1}, \frac{\langle 0.6, 0.4 \rangle}{d_2}, \frac{\langle 0.6, 0.2 \rangle}{d_3}, \frac{\langle 0.5, 0.2 \rangle}{d_4}, \frac{\langle 0.5, 0.5 \rangle}{d_5}, \frac{\langle 0.3, 0.7 \rangle}{d_6}, \frac{\langle 0.3, 0.2 \rangle}{d_7} \right\}. \tag{3.2}$$

Then,

$$\underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle) = \left\{\frac{\langle 0.6,0.4\rangle}{d_1},\frac{\langle 0.6,0.4\rangle}{d_2},\frac{\langle 0.5,0.5\rangle}{d_3},\frac{\langle 0.5,0.5\rangle}{d_4},\frac{\langle 0.3,0.7\rangle}{d_5},\frac{\langle 0.3,0.7\rangle}{d_6},\frac{\langle 0.3,0.7\rangle}{d_7}\right\},$$
$$\overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle) = \left\{\frac{\langle 0.6,0.4\rangle}{d_1},\frac{\langle 0.6,0.4\rangle}{d_2},\frac{\langle 0.5,0.5\rangle}{d_3},\frac{\langle 0.5,0.5\rangle}{d_4},\frac{\langle 0.3,0.7\rangle}{d_5},\frac{\langle 0.3,0.7\rangle}{d_6},\frac{\langle 0.3,0.7\rangle}{d_7}\right\}. \tag{3.3}$$

Remark 3.3. (1) $\underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle) \nleq (\langle\mu,\gamma\rangle)$ which follows from the above example. But it completely is a part of the same object.

(2) If any object is of the form $\langle 0,1\rangle$, then $\underline{\mathrm{sap}}^*_S\langle 0,1\rangle = \overline{\mathrm{sap}}^*_S\langle 0,1\rangle = \langle 0,1\rangle$, since 0 is the infimum of all members and 1 is the supremum of all non members.

(3) If any object is of the form $\langle 1,0\rangle$, then $\underline{\mathrm{sap}}^*_S\langle 1,0\rangle$ need not be $\langle 1,0\rangle$, since there may exist many other elements whose membership value is less than 1, but if $\underline{\mathrm{sap}}^*_S\langle 1,0\rangle = \langle 1,0\rangle$, then no other object is in the same mapping F. Similarly, $\overline{\mathrm{sap}}^*_S\langle 1,0\rangle \neq \langle 1,0\rangle$.

(4) Let any object d be of the form $\langle 0,0\rangle$. Now, if $\underline{\mathrm{sap}}^*_S\langle 0,0\rangle = \langle 0,0\rangle$, then also there does not exist any object in the same mapping with membership 0 but if other object exists with membership nonzero its nonmembership must be 0.

Remark 3.4. (1) If $\underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x) = \overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x)$, then the IF soft rough approximation is said to be simply IF soft approximation.

(2) If for some of the object $\overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x) = \underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x)$, then the IF soft rough approximation is said to be simply IF soft oscillating approximation.

(3) If for none of the object $\overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x) = \underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x)$, then the IF soft rough approximation is said to be completely IF soft rough approximation. For this case we may consider two more definitions which are known as IF soft stable lower rough approximation and IF soft stable upper rough approximations and are denoted by $\underline{\mathrm{sap}}_S(\langle\mu,\gamma\rangle)$ and $\overline{\mathrm{sap}}_S(\langle\mu,\gamma\rangle)(x)$.

Definition 3.5. The positive difference between $\underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)$ and $\overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x)$ is denoted by O_S and is said to oscillate in the approximation space, that is,

$$O_S = \left|\overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle) - \underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)\right|, \tag{3.4}$$

where "$|\cdot|$" is required since otherwise the membership value of the difference may be negative.

Example 3.6. Consider the Example 3.2. Then, we have

$$O_S = \left\{\frac{\langle 0,0\rangle}{d_1},\frac{\langle 0,0\rangle}{d_2},\frac{\langle 0,0\rangle}{d_3},\frac{\langle 0,0\rangle}{d_4},\frac{\langle 0,0\rangle}{d_5},\frac{\langle 0,0\rangle}{d_6},\frac{\langle 0,0\rangle}{d_7}\right\} = \underline{O}. \tag{3.5}$$

Now, let us consider another case of Example 3.2. Suppose that

$$\langle \mu, \gamma \rangle = \left\{ \frac{\langle 0.3, 0.6 \rangle}{d_1}, \frac{\langle 0.2, 0.5 \rangle}{d_2}, \frac{\langle 0.4, 0.5 \rangle}{d_3}, \frac{\langle 0.3, 0.5 \rangle}{d_4}, \frac{\langle 0.5, 0.5 \rangle}{d_5}, \frac{\langle 0.2, 0.7 \rangle}{d_6}, \frac{\langle 0.4, 0.3 \rangle}{d_7} \right\}. \tag{3.6}$$

Then,

$$\begin{aligned}
\underline{\mathrm{sap}}^*_S(\langle \mu, \gamma \rangle)(x) &= \left\{ \frac{\langle 0.2, 0.5 \rangle}{d_1}, \frac{\langle 0.2, 0.5 \rangle}{d_2}, \frac{\langle 0.3, 0.5 \rangle}{d_3}, \frac{\langle 0.3, 0.5 \rangle}{d_4}, \frac{\langle 0.2, 0.7 \rangle}{d_5}, \frac{\langle 0.2, 0.7 \rangle}{d_6}, \frac{\langle 0.2, 0.7 \rangle}{d_7} \right\}, \\
\overline{\mathrm{sap}}^*_S(\langle \mu, \gamma \rangle)(x) &= \left\{ \frac{\langle 0.3, 0.6 \rangle}{d_1}, \frac{\langle 0.3, 0.6 \rangle}{d_2}, \frac{\langle 0.3, 0.5 \rangle}{d_3}, \frac{\langle 0.3, 0.5 \rangle}{d_4}, \frac{\langle 0.2, 0.7 \rangle}{d_5}, \frac{\langle 0.2, 0.7 \rangle}{d_6}, \frac{\langle 0.2, 0.7 \rangle}{d_7} \right\}, \\
O_S &= \left\{ \frac{\langle 0.1, 0.1 \rangle}{d_1}, \frac{\langle 0.1, 0.1 \rangle}{d_2}, \frac{\langle 0, 0 \rangle}{d_3}, \frac{\langle 0, 0 \rangle}{d_4}, \frac{\langle 0, 0 \rangle}{d_5}, \frac{\langle 0, 0 \rangle}{d_6}, \frac{\langle 0, 0 \rangle}{d_7} \right\} \neq \underline{O}.
\end{aligned} \tag{3.7}$$

Theorem 3.7. *If $O_S = \langle 0, 0 \rangle$ then we obtain object IF soft approximation space.*

Proof. If $O_S = \langle 0, 0 \rangle$, then the following two cases may arise:

(1) all the object has the same nonzero value for $\underline{\mathrm{sap}}^*_S$ and $\overline{\mathrm{sap}}^*_S$. Hence, from Remark 3.4, we obtain an IF soft approximation space.

(2) If $\underline{\mathrm{sap}}^*_S = \langle 0, 0 \rangle = \overline{\mathrm{sap}}^*_S$, then the conclusions may be drawn from Remark 3.3. □

Theorem 3.8. (1) *O_S can never be $\langle 1, 0 \rangle$ for any object.*

(2) *O_S can never be $\langle 0, 1 \rangle$ for any object.*

Proof. (1) Suppose that $O_S = \langle 1, 0 \rangle$, then from the definition we have $|\underline{\mathrm{sap}}^*_S \langle \mu, \gamma \rangle - \overline{\mathrm{sap}}^*_S \langle \mu, \gamma \rangle| = \langle 1, 0 \rangle$, that is,

$$\begin{aligned}
&\sup_{\gamma} \inf_{\mu} \{ \langle \mu(y), \gamma(y) \rangle : \exists a \in A, \ \{x, y\} \subseteq F(a) \} \\
&\quad - \sup_{\mu} \inf_{\gamma} \{ \langle \mu(y), \gamma(y) \rangle : \exists a \in A, \{x, y\} \subseteq F(a) \} = \langle 1, 0 \rangle.
\end{aligned} \tag{3.8}$$

Let $\langle a, b \rangle - \langle c, d \rangle = \langle 1, 0 \rangle$, that is, $a - c = 1$, $d - b = 0$, that is, $d = b$. Since $a \not> 1$, so $a - c = 1$ gives $a = 1$ and $c = 0$. Since $a = 1$, $b = 0$ gives $d = 0$, that is,

$$\underline{\mathrm{sap}}^*_S \langle \mu, \gamma \rangle = \langle 1, 0 \rangle, \qquad \overline{\mathrm{sap}}^*_S \langle \mu, \gamma \rangle = \langle 0, 0 \rangle, \tag{3.9}$$

but if $\langle 1, 0 \rangle$ and $\langle 0, 0 \rangle$ are members of the same mapping F, then $\underline{\mathrm{sap}}^*_S \langle \mu, \gamma \rangle = \langle 0, 0 \rangle$, which is a contradiction. Hence, $O_S \neq \langle 1, 0 \rangle$.

(2) can be proved similarly. □

Remark 3.9. (1) If $O_S = \langle 0,0\rangle$ for all object then the approximation space is IF soft approximation space for all object.

(2) If $O_S = \langle 0,0\rangle$ for some object then the approximation space is If soft oscillating space.

(3) If $O_S \neq \langle 0,0\rangle$ for all objects then the approximation space is IF soft rough approximation space.

In such cases we need to define an IFSLR set approximation space which is stable; else decisions cannot be drawn for any particular object.

Definition 3.10. An IF soft stable lower rough approximation (IFSSLRA) of $\langle \mu,\gamma\rangle$ with respect to S is denoted by

$$\underline{\mathrm{sap}}_S\langle \mu\cdot\gamma\rangle = \underline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle \cap \overline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle \tag{3.10}$$

and an IF soft stable upper rough approximation by

$$\overline{\mathrm{sap}}_S\langle \mu\cdot\gamma\rangle = \underline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle \cup \overline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle. \tag{3.11}$$

Example 3.11. Let us consider Example 3.2. Then, $\underline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle = \overline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle$, that is,

$$\underline{\mathrm{sap}}_S\langle \mu\cdot\gamma\rangle = \underline{\mathrm{sap}}^*_S\langle \mu\cdot\gamma\rangle = \overline{\mathrm{sap}}^*_S\langle \mu\cdot\gamma\rangle = \overline{\mathrm{sap}}_S\langle \mu\cdot\gamma\rangle. \tag{3.12}$$

Now, if we consider Example 3.6 then $\underline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle \neq \overline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle$. Therefore,

$$\underline{\mathrm{sap}}_S\langle \mu,\gamma\rangle = \left\{ \frac{\langle 0.2,0.6\rangle}{d_1}, \frac{\langle 0.2,0.6\rangle}{d_2}, \frac{\langle 0.3,0.5\rangle}{d_3}, \frac{\langle 0.3,0.5\rangle}{d_4}, \frac{\langle 0.2,0.7\rangle}{d_5}, \frac{\langle 0.2,0.7\rangle}{d_6}, \frac{\langle 0.2,0.7\rangle}{d_7} \right\}. \tag{3.13}$$

Theorem 3.12. *Let $\mathcal{E} = (F,A)$ be a full soft set over $\mathcal{U}$, and let $\mathcal{S} = (\mathcal{U},E)$ be a soft approximation space. Then, we have:*

(1) $\underline{\mathrm{sap}}_S\langle \mu,\gamma\rangle = inf\{\langle \mu(y),\gamma(y)\rangle : \exists a \in A,\ \{x,y\} \subseteq F(a)\}$ *and* $\overline{\mathrm{sap}}_S\langle \mu,\gamma\rangle = sup\{<\mu(y),\gamma(y)>: \exists a \in A,\ \{x,y\} \subseteq F(a)\}$,

(2) $\underline{\mathrm{sap}}_S\langle \mu,\gamma\rangle \leq \langle \mu,\gamma\rangle \leq \overline{\mathrm{sap}}_S\langle \mu,\gamma\rangle$, $\underline{\mathrm{sap}}_S\langle \mu,\gamma\rangle \leq \underline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle \leq \overline{\mathrm{sap}}_S\langle \mu,\gamma\rangle$, $\underline{\mathrm{sap}}_S\langle \mu,\gamma\rangle \leq \overline{\mathrm{sap}}^*_S\langle \mu,\gamma\rangle \leq \overline{\mathrm{sap}}_S\langle \mu,\gamma\rangle$,

(3) $\underline{\mathrm{sap}}_S\langle 0,1\rangle = \langle 0,1\rangle$ *for any object* $d\langle 0,1\rangle$ *in* $\langle \mu,\gamma\rangle$,

(4) $\underline{\mathrm{sap}}_S\langle 0,0\rangle \neq \langle 0,1\rangle$ *for any object* $d\langle 0,0\rangle$ *in* $\langle \mu,\gamma\rangle$,

(5) $\underline{\mathrm{sap}}_S\phi = \phi = \overline{\mathrm{sap}}_S\phi$,

(6) $\underline{\mathrm{sap}}_S\mathcal{U} = \overline{\mathrm{sap}}_S\mathcal{U}$,

(7) $\underline{\mathrm{sap}}(\langle \mu,\gamma\rangle \cap \langle \nu,\beta\rangle) = \underline{\mathrm{sap}}\langle \mu,\gamma\rangle \cap \underline{\mathrm{sap}}\langle \nu,\beta\rangle$,

(8) $\langle \mu,\gamma\rangle \subseteq \langle \nu,\beta\rangle$, $\underline{\mathrm{sap}}\langle \mu,\gamma\rangle \subseteq \underline{\mathrm{sap}}\langle \nu,\beta\rangle$,

(9) $\underline{\mathrm{sap}}(\langle \mu,\gamma\rangle \cup \langle \nu,\beta\rangle) \subseteq \underline{\mathrm{sap}}\langle \mu,\gamma\rangle \cup \underline{\mathrm{sap}}\langle \nu,\beta\rangle$.

Table 2

	1	2	3	4	5	6	7	8
ht_s	1	1	0	0	0	0	0	1
ht_t	0	0	1	1	1	1	1	0
h_b	1	1	0	0	0	1	0	1
h_r	0	0	1	0	0	0	0	0
h_d	0	0	0	1	1	0	1	0
e_b	1	0	1	1	0	1	0	0
e_{br}	0	1	0	0	1	0	1	1

Table 3

	1	2	3	4	5	6
H	0	1	1	0	1	0
M	1	0	1	1	0	1
T	1	1	1	0	1	1

Proof. It is straightforward. □

Remark 3.13. If $\underline{\text{sap}}_S\langle\mu,\gamma\rangle = \langle\mu,\gamma\rangle$, then $\langle\mu,\gamma\rangle$ is an IF soft open set. In Example 3.11, $\{d_4, d_6\}$ are soft open objects, and their memberships are soft open members. Also, if $\overline{\text{sap}}_S\langle\mu,\gamma\rangle = \langle\mu,\gamma\rangle$, then $\langle\mu,\gamma\rangle$ is a closed set.

Remark 3.14. (1) Here $B_S = \overline{\text{sap}}_S\langle\mu,\gamma\rangle - \underline{\text{sap}}_S\langle\mu,\gamma\rangle$ is the IFSR boundary region.

(a) If $B_S = \langle 0,0\rangle$ then the data is IF soft set.

If $B_S \neq \langle 0,1\rangle$, then the data are IF soft rough set. In Example 3.11, the first $\langle\mu,\gamma\rangle$ is IF soft set and is not IF soft rough set but the second one is IFSR set.

(b) $B_S = \langle 0,0\rangle$ if and only if $O_S = \langle 0,0\rangle$.

(2) Here $N_S = I - \overline{\text{sap}}_S\langle\mu,\gamma\rangle$, where I denotes that the value $\langle 1,0\rangle$ for every object is the IFSR negative region.

(a) If $N_S = 0$, then $\overline{\text{sap}}_S\langle\mu,\gamma\rangle = I$ if and only if $\langle\mu,\gamma\rangle = I$, by Remark 3.3.

(b) $N_S = I$ if $\overline{\text{sap}}_S\langle\mu,\gamma\rangle = 0$ if and only if $\langle\mu,\gamma\rangle = O$, where O is the value $< 0,0 >$ for every object.

Now, let us take an example from [7].

Example 3.15. Suppose that $U = \{1,2,3,4,5,6,7,8\}$ is the universe consisting of eight persons and the set of parameters are given by $E = \{ht_s, ht_t, h_b, h_r, h_d, e_b e_{br}\}$, where ht_s implies short height, ht_t implies tall height, h_b implies blond hair, h_r implies red hair, h_d implies dark hair, e_b implies blue eyes, and e_{br} implies brown eyes. Let us consider a soft set (F,E) describing the "*attractive person*". Let us represent Table 2.

Let $F(ht_s) = \{1,2,8\}$, $F(ht_t) = \{3,4,5,6,7\}$, $F(h_b) = \{1,2,8\}$, $F(h_r) = \{3\}$, $F(h_d) = \{4,5,7\}$, and $F(e_b) = \{1,3,4,5,6\}$, $F(e_{br}) = \{2,7,8\}$.

Let us now consider the IF set of an attractive person as per our choice as

$$\langle\mu,\gamma\rangle = \left\{ \frac{\langle 0.8,0.1\rangle}{1}, \frac{\langle 0.3,0.7\rangle}{2}, \frac{\langle 0.6,0.3\rangle}{3}, \frac{\langle 0.3,0.5\rangle}{4}, \frac{\langle 0.4,0.5\rangle}{5}, \frac{\langle 0.7,0.2\rangle}{6}, \frac{\langle 0.2,0.8\rangle}{7}, \frac{\langle 0.3,0.5\rangle}{8} \right\},$$

$$\begin{aligned}
\underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x) &= \left\{\frac{\langle 0.3,0.7\rangle}{1},\frac{\langle 0.2,0.8\rangle}{2},\frac{\langle 0.2,0.8\rangle}{3},\frac{\langle 0.2,0.8\rangle}{4},\frac{\langle 0.2,0.8\rangle}{5},\frac{\langle 0.2,0.8\rangle}{6},\right.\\
&\qquad\left.\frac{\langle 0.2,0.8\rangle}{7},\frac{\langle 0.2,0.8\rangle}{8}\right\},\\
\overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x) &= \left\{\frac{\langle 0.3,0.7\rangle}{1},\frac{\langle 0.3,0.7\rangle}{2},\frac{\langle 0.2,0.8\rangle}{3},\frac{\langle 0.2,0.8\rangle}{4},\frac{\langle 0.2,0.8\rangle}{5},\frac{\langle 0.2,0.8\rangle}{6},\right.\\
&\qquad\left.\frac{\langle 0.2,0.8\rangle}{7},\frac{\langle 0.2,0.8\rangle}{8}\right\},\\
O_S &= \left\{\frac{\langle 0,0\rangle}{1},\frac{\langle 0.1,0.1\rangle}{2},\frac{\langle 0,0\rangle}{3},\frac{\langle 0,0\rangle}{4},\frac{\langle 0,0\rangle}{5},\frac{\langle 0,0\rangle}{6},\frac{\langle 0,0\rangle}{7},\frac{\langle 0,0\rangle}{8}\right\},\\
\underline{\mathrm{sap}}_S(\langle\mu,\gamma\rangle)(x) &= \left\{\frac{\langle 0.3,0.7\rangle}{1},\frac{\langle 0.2,0.8\rangle}{2},\frac{\langle 0.2,0.8\rangle}{3},\frac{\langle 0.2,0.8\rangle}{4},\frac{\langle 0.2,0.8\rangle}{5},\frac{\langle 0.2,0.8\rangle}{6},\right.\\
&\qquad\left.\frac{\langle 0.2,0.8\rangle}{7},\frac{\langle 0.2,0.8\rangle}{8}\right\},\\
\overline{\mathrm{sap}}_S(\langle\mu,\gamma\rangle)(x) &= \left\{\frac{\langle 0.3,0.7\rangle}{1},\frac{\langle 0.3,0.7\rangle}{2},\frac{\langle 0.2,0.8\rangle}{3},\frac{\langle 0.2,0.8\rangle}{4},\frac{\langle 0.2,0.8\rangle}{5},\frac{\langle 0.2,0.8\rangle}{6},\right.\\
&\qquad\left.\frac{\langle 0.2,0.8\rangle}{7},\frac{\langle 0.2,0.8\rangle}{8}\right\},\\
B_S &= O_S,\\
N_S &= \left\{\frac{\langle 0.7,0.3\rangle}{1},\frac{\langle 0.7,0.3\rangle}{2},\frac{\langle 0.8,0.2\rangle}{3},\frac{\langle 0.8,0.2\rangle}{4},\frac{\langle 0.8,0.2\rangle}{5},\frac{\langle 0.8,0.2\rangle}{6},\right.\\
&\qquad\left.\frac{\langle 0.8,0.2\rangle}{7},\frac{\langle 0.8,0.2\rangle}{8}\right\}.
\end{aligned} \tag{3.14}$$

Here $B_S = O_S$ which implies that the approximation space is stable and the IF set taken for the persons is correct and of less error.

Finally, we consider another example from [3].

Example 3.16. Suppose that $U = \{1,2,3,4,5,6\}$ is the universe consisting of six persons and the set of parameters are given by $E = \{H,M,T\}$, where H implies headache, M implies musclepain,and T implies temperature. Let us consider a soft set (F,E) describing the "flu infected person". Let us represent Table 3.

Let $F(H) = \{2,3,5\}$, $F(M) = \{3,4,6\}$, and $F(T) = \{1,2,3,5,6\}$. Let us now consider the IF set of a flu infected person as per our choice as

$$\begin{aligned}
\langle\mu,\gamma\rangle &= \left\{\frac{\langle 0.8,0.1\rangle}{1},\frac{\langle 0.5,0.4\rangle}{2},\frac{\langle 0.9,0.1\rangle}{3},\frac{\langle 0.3,0.4\rangle}{4},\frac{\langle 0.6,0.3\rangle}{5},\frac{\langle 0.7,0.2\rangle}{6}\right\},\\
\underline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x) &= \left\{\frac{\langle 0.5,0.4\rangle}{1},\frac{\langle 0.5,0.4\rangle}{2},\frac{\langle 0.3,0.4\rangle}{3},\frac{\langle 0.3,0.4\rangle}{4},\frac{\langle 0.5,0.4\rangle}{5},\frac{\langle 0.5,0.4\rangle}{6}\right\},\\
\overline{\mathrm{sap}}^*_S(\langle\mu,\gamma\rangle)(x) &= \left\{\frac{\langle 0.9,0.1\rangle}{1},\frac{\langle 0.9,0.1\rangle}{2},\frac{\langle 0.9,0.1\rangle}{3},\frac{\langle 0.9,0.1\rangle}{4},\frac{\langle 0.9,0.1\rangle}{5},\frac{\langle 0.9,0.1\rangle}{6}\right\},
\end{aligned}$$

$$O_S = \left\{ \frac{\langle 0.4, 0.3 \rangle}{1}, \frac{\langle 0.4, 0.3 \rangle}{2}, \frac{\langle 0.6, 0.3 \rangle}{3}, \frac{\langle 0.6, 0.3 \rangle}{4}, \frac{\langle 0.4, 0.3 \rangle}{5}, \frac{\langle 0.4, 0.3 \rangle}{6} \right\},$$

$$\underline{\mathrm{sap}}_S(\langle \mu, \gamma \rangle)(x) = \left\{ \frac{\langle 0.3, 0.4 \rangle}{1}, \frac{\langle 0.3, 0.4 \rangle}{2}, \frac{\langle 0.3, 0.4 \rangle}{3}, \frac{\langle 0.3, 0.4 \rangle}{4}, \frac{\langle 0.3, 0.4 \rangle}{5}, \frac{\langle 0.3, 0.4 \rangle}{6} \right\},$$

$$\overline{\mathrm{sap}}_S(\langle \mu, \gamma \rangle)(x) = \left\{ \frac{\langle 0.9, 0.1 \rangle}{1}, \frac{\langle 0.9, 0.1 \rangle}{2}, \frac{\langle 0.9, 0.1 \rangle}{3}, \frac{\langle 0.9, 0.1 \rangle}{4}, \frac{\langle 0.9, 0.1 \rangle}{5}, \frac{\langle 0.9, 0.1 \rangle}{6} \right\},$$

$$B_S = \left\{ \frac{\langle 0.6, 0.3 \rangle}{1}, \frac{\langle 0.6, 0.3 \rangle}{2}, \frac{\langle 0.6, 0.3 \rangle}{2}, \frac{\langle 0.6, 0.3 \rangle}{4}, \frac{\langle 0.6, 0.3 \rangle}{5}, \frac{\langle 0.6, 0.3 \rangle}{6} \right\}. \tag{3.15}$$

Here $B_S \neq O_S$ which implies that the approximation space may not be stable and the IF set taken for the persons is not perfectly correct.

4. Conclusion

The concepts of IF lower soft rough approximation and IF upper soft rough approximation space are introduced. In the most of the cases, IFSLRA is not stable for that we had introduced a new concept of IFSSLRA space. In some sense almost all concepts we are meeting in every day life are vague rather than precise. This gap between real world and traditional mathematics becomes smaller in recent year. In order to remove this gap, rough set, IF set, a soft set help a lot. The data mining and decision making processes may cross a new milestone after introduction of this new hybridized model. Further study will be needed to establish the utilities of the notions indicated in this paper.

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