International Scholarly Research Network
ISRN Algebra
Volume 2011, Article ID 953124, 8 pages
doi:10.5402/2011/953124

Research Article

On Hyperideals in Left Almost Semihypergroups

Kostaq Hila and Jani Dine

Department of Mathematics & Computer Science, Faculty of Natural Sciences, University of Gjirokastra, Gjirokastra 6001, Albania

Correspondence should be addressed to Kostaq Hila, khila@uogj.edu.al

Received 7 June 2011; Accepted 11 July 2011

Academic Editors: A. V. Kelarev and A. Kiliçman

This paper deals with a class of algebraic hyperstructures called left almost semihypergroups (LA-semihypergroups), which are a generalization of LA-semigroups and semihypergroups. We introduce the notion of LA-semihypergroup, the related notions of hyperideal, bi-hyperideal, and some properties of them are investigated. It is a useful nonassociative algebraic hyperstructure, midway between a hypergroupoid and a commutative hypersemigroup, with wide applications in the theory of flocks, and so forth. We define the topological space and study the topological structure of LA-semihypergroups using hyperideal theory. The topological spaces formation guarantee for the preservation of finite intersection and arbitrary union between the set of hyperideals and the open subsets of resultant topologies.

1. Introduction and Preliminaries

The applications of mathematics in other disciplines, for example in informatics, play a key role, and they represent, in the last decades, one of the purposes of the study of the experts of hyperstructures theory all over the world. Hyperstructure theory was introduced in 1934 by a French mathematician Marty [1], at the 8th Congress of Scandinavian Mathematicians, where he defined hypergroups based on the notion of hyperoperation, began to analyze their properties, and applied them to groups. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics and computer science by many mathematicians. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Some principal notions about hyperstructures and semihypergroups theory can be found in [1–7].

The Theory of ideals, in its modern form, is a contemporary development of mathematical knowledge to which mathematicians of today may justly point with pride. Ideal theory is important not only for the intrinsic interest and purity of its logical structure but because it is a necessary tool in many branches of mathematics and its applications such as in informatics, physics, and others. As an example of applications of the concept of an ideal in informatics, let us mention that ideals of algebraic structures have been used recently to design efficient classification systems, see [8–12].

The study of LA-semigroup as a generalization of commutative semigroup was initiated in 1972 by Kazim and Naseeruddin [13]. They have introduced the concept of an LA-semigroup and have investigated some basic but important characteristics of this structure. They have generalized some useful results of semigroup theory. Since then, many papers on LA-semigroups appeared showing the importance of the concept and its applications [13–23]. In this paper, we generalize this notion introducing the notion of LA-semihypergroup which is a generalization of LA-semigroup and semihypergroup, proposing so a new kind of hyperstructure for further studying. It is a useful nonassociative algebraic hyperstructure, midway between a hypergroupoid and a commutative hypersemigroup, with wide applications in the theory of flocks etc. Although the hyperstructure is nonassociative and noncommutative, nevertheless, it possesses many interesting properties which we usually find in associative and commutative algebraic hyperstructures. A several properties of hyperideals of LA-semihypergroup are investigated. In this note, we define the topological space and study the topological structure of LA-semihypergroups using hyperideal theory. The topological spaces formation guarantee for the preservation of finite intersection and arbitrary union between the set of hyperideals and the open subsets of resultant topologies.

Recall first the basic terms and definitions from the hyperstructure theory.

Definition 1.1. A map $\circ : H \times H \to \wp^*(H)$ is called *hyperoperation* or *join operation* on the set H, where H is a nonempty set and $\wp^*(H) = \wp(H) \setminus \{\emptyset\}$ denotes the set of all nonempty subsets of H.

Definition 1.2. A *hyperstructure* is called the pair $(H, \circ)$, where $\circ$ is a hyperoperation on the set H.

Definition 1.3. A hyperstructure $(H, \circ)$ is called a *semihypergroup* if for all $x, y, z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v. \tag{1.1}$$

If $x \in H$ and A, B are nonempty subsets of H, then

$$A \circ B = \bigcup_{a \in A,\, b \in B} a \circ b, \qquad A \circ x = A \circ \{x\}, \qquad x \circ B = \{x\} \circ B. \tag{1.2}$$

Definition 1.4. A nonempty subset B of a semihypergroup H is called a *sub-semihypergroup* of H if $B \circ B \subseteq B$, and H is called in this case *super-semihypergroup* of B.

Definition 1.5. Let $(H, \circ)$ be a semihypergroup. Then H is called a *hypergroup* if it satisfies the reproduction axiom, for all $a \in H$, $a \circ H = H \circ a = H$.

Definition 1.6. A hypergrupoid $(H, \circ)$ is called an *LA-semihypergroup* if, for all $x, y, z \in H$,

$$(x \circ y) \circ z = (z \circ y) \circ x. \tag{1.3}$$

Every LA-semihypergroup $(H, \circ)$ satisfies the *medial law*, that is, for all $x, y, z, w \in H$,

$$(x \circ y) \circ (z \circ w) = (x \circ z) \circ (y \circ w). \tag{1.4}$$

In every LA-semihypergroup with left identity, the following law holds:

$$(x \circ y) \circ (z \circ w) = (w \circ y) \circ (z \circ x), \tag{1.5}$$

for all $x, y, z, w \in H$.

An element e in an *LA-semihypergroup* H is called *identity* if $x \circ e = e \circ x = \{x\}$, for all $x \in H$. An element 0 in a semihypergroup H is called *zero element* if $x \circ 0 = 0 \circ x = \{0\}$, for all $x \in H$. A subset I of an *LA-semihypergroup* H is called a *right* (*left*) *hyperideal* if $I \circ H \subseteq I(H \circ I \subseteq I)$ and is called a *hyperideal* if it is two-sided hyperideal, and if I is a left hyperideal of H, then $I \circ I = I^2$ becomes a hyperideal of H. By a *bi-hyperideal* of an *LA-semihypergroup* H, we mean a sub-LA-semihypergroup B of H such that $(B \circ H) \circ B \subseteq B$. It is easy to note that each right hyperideal is a bi-hyperideal. If H has a left identity, then it is not hard to show that B^2 is a bi-hyperideal of H and $B^2 \subseteq H \circ B^2 = B^2 \circ H$. If $E(B_H)$ denotes the set of all idempotents subsets of H with left identity e, then $E(B_H)$ forms a hypersemilattice structure, also if $C = C^2$, then $(C \circ H) \circ C \in E(B_H)$. The intersection of any set of bi-hyperideals of an *LA-semihypergroup* H is either empty or a bi-hyperideal of H. Also the intersection of prime bi-hyperideals of an *LA-semihypergroup* H is a semiprime bi-hyperideal of H.

2. Main Results

Proposition 2.1. *Let H be an LA-semihypergroup with left identity, T a left hyperideal, and B a bi-hyperideal of H. Then $B \circ T$ and $T^2 \circ B$ are bi-hyperideals of H.*

Proof. Using the medial law (1.4), we get

$$((B \circ T) \circ H) \circ (B \circ T) = ((B \circ T) \circ B) \circ (H \circ T) \subseteq ((B \circ H) \circ B) \circ T \subseteq B \circ T, \tag{2.1}$$

also

$$(B \circ T) \circ (B \circ T) = (B \circ B) \circ (T \circ T) \subseteq B \circ T. \tag{2.2}$$

Hence, $B \circ T$ is a bi-hyperideal of H. we obtain

$$\begin{aligned}\left(\left(T^2 \circ B\right) \circ H\right) \circ \left(T^2 \circ B\right) &= \left(\left(T^2 \circ H\right) \circ (B \circ H)\right) \circ \left(T^2 \circ B\right) \subseteq \left(T^2 \circ (B \circ H)\right) \circ \left(T^2 \circ B\right) \\ &= \left(T^2 \circ T^2\right) \circ ((B \circ H) \circ B) \subseteq T^2 \circ B,\end{aligned} \tag{2.3}$$

also

$$\left(T^2 \circ B\right) \circ \left(T^2 \circ B\right) = \left(T^2 \circ T^2\right) \circ (B \circ B) \subseteq T^2 \circ B. \tag{2.4}$$

Hence, $T^2 \circ B$ is a bi-hyperideal of H. □

Proposition 2.2. *Let H be an LA-semihypergroup with left identity and B_1, B_2 two bi-hyperideals of H. Then $B_1 \circ B_2$ is a bi-hyperideal of H.*

Proof. Using (1.4), we get

$$\begin{aligned}((B_1 \circ B_2) \circ H) \circ (B_1 \circ B_2) &= ((B_1 \circ B_2) \circ (H \circ H)) \circ (B_1 \circ B_2) \\ &= ((B_1 \circ H) \circ (B_2 \circ H)) \circ (B_1 \circ B_2) \\ &= ((B_1 \circ H) \circ B_1) \circ ((B_2 \circ H) \circ B_2) \subseteq B_1 \circ B_2.\end{aligned} \tag{2.5}$$

□

By the above, if B_1 and B_2 are nonempty, then $B_1 \circ B_2$ and $B_2 \circ B_1$ are connected bi-hyperideals. Proposition 2.1 leads us to an easy generalization, that is, if $B_1, B_2, B_3, \ldots, B_n$ are bi-hyperideals of an *LA-semihypergroup* H with left identity, then

$$(\cdots((B_1 \circ B_2) \circ B_3) \circ \cdots) \circ B_n, \qquad \left(\cdots\left(\left(B_1^2 \circ B_2^2\right) \circ B_3^2\right) \circ \cdots\right) \circ B_n^2 \tag{2.6}$$

are bi-hyperideals of H, consequently the set $\mathcal{C}(H_B)$ of bi-hyperideals forms an *LA-semihypergroup*.

If H is an *LA-semihypergroup* with left identity e, then $\langle a \rangle_L = H \circ a$, $\langle a \rangle_R = a \circ H$ and $\langle a \rangle_H = (H \circ a) \circ H$ are bi-hyperideals of H. It can be easily shown that $\langle a \circ b \rangle_L = \langle a \rangle_L \circ \langle b \rangle_L$, $\langle a \circ b \rangle_R = \langle a \rangle_R \circ \langle b \rangle_R$, and $\langle a \circ b \rangle_R = \langle b \rangle_L \circ \langle a \rangle_L$. Hence, this implies that $\langle a \rangle_R \circ \langle b \rangle_R = \langle b \rangle_L \circ \langle a \rangle_L$ and $\langle a \rangle_L \circ \langle b \rangle_L = \langle b \rangle_R \circ \langle a \rangle_R$. Also, $\langle a \rangle_L \circ \langle b \rangle_R = \langle b \rangle_L \circ \langle a \rangle_R$, $\langle a \circ a \rangle_L = \langle a \rangle_L^2$, $\langle a \circ a \rangle_R = \langle a \rangle_R^2$, $\langle a \circ a \rangle_L = \langle a \circ a \rangle_R$, and $\langle a \rangle_L = \langle a \rangle_R$ (if a is an idempotent), consequently $\langle a \circ a \rangle_L = \langle a \circ a \rangle_R$. It is easy to show that $\langle a \rangle_R \circ a^2 = a^2 \circ \langle a \rangle_L$.

Lemma 2.3. *Let H be an LA-semihypergroup with left identity, and let B be an idempotent bi-hyperideal of H. Then B is a hyperideal of H.*

Proof. By the definition of LA-semihypergroup (1.3), we have

$$B \circ H = (B \circ B) \circ H = (H \circ B) \circ B = \left(H \circ B^2\right) \circ B = \left(B^2 \circ H\right) \circ B = (B \circ H) \circ B, \tag{2.7}$$

and every right hyperideal in H with left identity is left. □

Lemma 2.4. *Let H be an LA-semihypergroup with left identity e, and let B be a proper bi-hyperideal of H. Then $e \notin B$.*

Proof. Let us suppose that $e \in B$. Since $h \circ b \subseteq (e \circ h) \circ b \subseteq B$, using (1.3), we have $h \in (e \circ e) \circ h = (h \circ e) \circ e \subseteq (S \circ B) \circ B \subseteq B$. It is impossible. So, $e \notin B$. □

It can be easily noted that $\{x \in H : (x \circ a) \circ x = e\} \nsubseteq B$.

Proposition 2.5. *Let H be an LA-semihypergroup with left identity, and let A, B be bi-hyperideals of H. Then the following statements are equivalent:*

(1) *every bi-hyperideal is idempotent,*

(2) $A \cap B = A \circ B$,

(3) *the hyperideals of H form a hypersemilattice $(L_H, \wedge)$, where $A \wedge B = A \circ B$.*

Proof. (1)⇒(2). Using Lemma 2.3, it is easy to note that $A \circ B \subseteq A \circ B$. Since $A \cap B \subseteq A, B$ implies $(A \cap B)^2 \subseteq A \circ B$, hence $A \cap B \subseteq A \circ B$.

(2)⇒(3). $A \wedge B = A \circ B = A \cap B = B \cap A = B \wedge A$ and $A \wedge A = A \circ A = A \cap A = A$. Similarly, associativity follows. Hence, $(L_H, \wedge)$ is a hypersemilattice.

(3)⇒(1). $A = A \wedge A = A \circ A$. □

A bi-hyperideal B of an *LA-semihypergroup* H is called a prime bi-hyperideal if $B_1 \circ B_2 \subseteq B$ implies either $B_1 \subseteq B$ or $B_2 \subseteq B$ for every bi-hyperideal B_1 and B_2 of H. The set of bi-hyperideals of H is totally ordered under the set inclusion if for all bi-hyperideals I, J either $I \subseteq J$ or $J \subseteq I$.

Theorem 2.6. *Let H be an LA-semihypergroup with left identity. Every bi-hyperideal of H is prime if and only if it is idempotent and the set of the bi-hyperideals of H is totally ordered under the set inclusion.*

Proof. Let us assume that every bi-hyperideal of H is prime. Since B^2 is a hyperideal and so is prime which implies that $B \subseteq B \circ B$, hence B is idempotent. Since $B_1 \cap B_2$ is a bi-hyperideal of H (where B_1 and B_2 are bi-hyperideals of H) and so is prime. Now by Lemma 2.3, either $B_1 \subseteq B_1 \cap \mathrm{B}_2$ or $B_2 \subseteq B_1 \cap B_2$ which further implies that either $B_1 \subseteq B_2$ or $B_2 \subseteq B_1$. Hence, the set of bi-hyperideals of H is totally ordered under set inclusion.

Conversely, let us assume that every bi-hyperideals of H is idempotent and the set of bi-hyperideals of H is totally ordered under set inclusion. Let B_1, B_2 and B be the bi-hyperideals of H with $B_1 \circ B_2 \subseteq B$ and without loss of generality assume that $B_1 \subseteq B_2$. Since B_1 is an idempotent, so $B_1 = B_1 \circ B_1 \subseteq B_1 \circ B_2 \subseteq B$ implies that $B_1 \subseteq B$, and, hence, every bi-hyperideal of H is prime. □

A bi-hyperideal B of an *LA-semihypergroup* H is called *strongly irreducible* bi-hyperideal if $B_1 \cap B_2 \subseteq B$ implies either $B_1 \subseteq B$ or $B_2 \subseteq B$ for every bi-hyperideal B_1 and B_2 of H.

Theorem 2.7. *Let H be an LA-semihypergroup with zero. Let D be the set of all bi-hyperideals of H, and Ω the set of all strongly irreducible proper bi-hyperideals of H, then $\Gamma(\Omega) = \{O_B : B \in D\}$ forms a topology on the set Ω, where $O_B = \{J \in \Omega; B \nsubseteq J\}$ and $\phi :$ Bi-hyperideal$(H) \rightarrow \Gamma(\Omega)$ preserves finite intersection and arbitrary union between the set of bi-hyperideals of H and open subsets of Ω.*

Proof. Since $\{0\}$ is a bi-hyperideal of H and 0 belongs to every bi-hyperideal of H, then $O_B = \{J \in \Omega, \{0\} \nsubseteq J\} = \{\}$, also $O_H = \{J \in \Omega, H \nsubseteq J\} = \Omega$ which is the first axiom for the topology. Let $\{O_{B_\alpha} : \alpha \in I\} \subseteq \Gamma(\Omega)$, then $\cup O_{B_\alpha} = \{J \in \Omega, B_\alpha \nsubseteq J, \text{ for some } \alpha \in I\} = \{J \in \Omega, \langle \cup B_\alpha \rangle \nsubseteq J\} = O_{\cup B_\alpha}$, where $\langle \cup B_\alpha \rangle$ is a bi-hyperideal of H generated by $\langle \cup B_\alpha \rangle$. Let O_{B_1} and $O_{B_2} \in \Gamma(\Omega)$, if $J \in O_{B_1} \cap O_{B_2}$, then $J \in \Omega$ and $B_1 \nsubseteq, B_2 \nsubseteq J$. Let us suppose $B_1 \cap B_2 \subseteq J$, this implies that either $B_1 \subseteq J$ or $B_2 \subseteq J$. It is impossible. Hence, $B_1 \cap B_2 \nsubseteq J$ which further implies that $J \in O_{B_1 \cap B_2}$. Thus $O_{B_1} \cap O_{B_2} \subseteq O_{B_1 \cap B_2}$. Now if $J \in O_{B_1 \cap B_2}$, then $J \in \Omega$ and $B_1 \cap B_2 \nsubseteq J$. Thus $J \in O_{B_1}$ and $J \in O_{B_2}$, therefore $J \in O_{B_1} \cap O_{B_2}$, which implies that $O_{B_1 \cap B_2} \subseteq O_{B_1} \cap O_{B_2}$. Hence $\Gamma(\Omega)$ is the topology on Ω. Define $\phi :$ Bi-hyperideal$(H) \rightarrow \Gamma(\Omega)$ by $\phi(B) = O_B$, then it is easy to note that ϕ preserves finite intersection and arbitrary union. □

A hyperideal P of an *LA-semihypergroup* H is called *prime* if $A \circ B \subseteq P$ implies that either $A \subseteq P$ or $B \subseteq P$ for all hyperideals A and B in H.

Let P_H denotes the set of proper prime hyperideals of an *LA-semihypergroup* H absorbing 0. For a hyperideal I of H, we define the sets $\Theta_I = \{J \in P_H : I \nsubseteq J\}$ and $\Gamma(P_H) = \{\Theta_I, I \text{ is a hyperideal of } H\}$.

Theorem 2.8. *Let H be an LA-semihypergroup with zero. The set $\Gamma(P_H)$ constitutes a topology on the set P_H.*

Proof. Let $\Theta_{I_1}, \Theta_{I_2} \in \Gamma(P_H)$, if $J \in \Theta_{I_1} \cap \Theta_{I_2}$, then $J \in P_H$ and $I_1 \nsubseteq J$ and $I_2 \nsubseteq J$. Let $I_1 \cap I_2 \subseteq J$ which implies that either $I_1 \subseteq J$ or $I_2 \subseteq J$, which is impossible. Hence, $J \in \Theta_{I_1 \cap I_2}$. Similarly $\Theta_{I_1 \cap I_2} \subseteq \Theta_{I_1} \cap \Theta_{I_2}$. The remaining proof follows from Theorem 2.7. □

The assignment $I \rightarrow \Theta_I$ preserves finite intersection and arbitrary union between the hyperideal(H) and their corresponding open subsets of Θ_I.

Let P be a left hyperideal of an *LA-semihypergroup* H. P is called *quasiprime* if for left hyperideals A, B of H such that $A \circ B \subseteq P$, we have $A \subseteq P$ or $B \subseteq P$.

Theorem 2.9. *Let H be an LA-semihypergroup with left identity e. Then a left hyperideal P of H is quasiprime if and only if $(H \circ a) \circ b \subseteq P$ implies that either $a \in P$ or $b \in P$.*

Proof. Let P be a left hyperideal of H. Let us assume that $(H \circ a) \circ b \subseteq P$, then

$$H \circ ((H \circ a) \circ b) \subseteq H \circ P \subseteq P, \tag{2.8}$$

that is,

$$H \circ ((H \circ a) \circ b) = (H \circ a) \circ (H \circ b). \tag{2.9}$$

Hence, either $a \in P$ or $b \in P$.

Conversely, let us assume that $A \circ B \subseteq P$, where A and B are left hyperideal of H such that $A \nsubseteq P$. Then there exists $x \in A$ such that $x \notin P$. Now, by the hypothesis, we have $(H \circ x) \circ y \subseteq (H \circ A) \circ B \subseteq A \circ B \subseteq P$ for all $y \in B$. Since $x \notin P$, so by hypothesis, $y \in P$ for all $y \in B$, we obtain $B \subseteq P$. This shows that P is quasiprime. □

An *LA-semihypergroup* H is called an *antirect*angular if $a \in (b \circ a) \circ b$, for all $a, b \in H$. It is easy to see that $H = H \circ H$. In the following results for an antirectangular LA-semihypergroup H, $e \notin H$.

Proposition 2.10. *Let H be an LA-semihypergroup. If A, B are hyperideals of H, then $A \circ B$ is a hyperideal.*

Proof. Using (1.4), we have

$$(A \circ B) \circ H = (A \circ B) \circ (H \circ H) = (A \circ H) \circ (B \circ H) \subseteq A \circ B, \tag{2.10}$$

also

$$H \circ (A \circ B) = (H \circ H) \circ (A \circ B) = (H \circ A) \circ (H \circ B) \subseteq A \circ B, \tag{2.11}$$

which shows that $A \circ B$ is a hyperideal. □

Consequently, if $I_1, I_2, \ldots, I_n$ are hyperideals of H, then

$$(\cdots \circ ((I_1 \circ I_2) \circ I_3) \circ \cdots \circ I_n), \qquad \left(\cdots \circ \left(\left(I_1^2 \circ I_2^2\right) \circ I_3^2\right) \circ \cdots \circ I_n^2\right) \tag{2.12}$$

are hyperideals of H and the set $\mathscr{H}(I)$ of hyperideals of H form an antirectangular LA-semihypergroup.

Lemma 2.11. *Let H be an antirectangular LA-semihypergroup. Any subset of H is left hyperideal if and only if it is right.*

Proof. Let I be a right hyperideal of H, then using (1.3), we get $h \circ i \subseteq ((k \circ h) \circ k) \circ i \subseteq (i \circ k) \circ (k \circ h) \subseteq I$.

Conversely, let us suppose that I is a left hyperideal of H, then using (1.3), we have $i \circ h \subseteq ((t \circ i) \circ t) \circ h \subseteq (h \circ t) \circ (t \circ i) \subseteq I$. □

It is fact that $H \circ I = I \circ H$. From the above lemma, we remark that every quasiprime hyperideal becomes prime in an antirectangular LA-semihypergroup.

Lemma 2.12. *Let H be an antirectangular LA-semihypergroup. If I is a hyperideal of H, then $S(a) = \{x \in H : a \in (x \circ a) \circ x,\ \text{for}\ a \in I\} \subseteq I$.*

Proof. Let $y \in S(a)$, then $y \in (y \circ a) \circ y \subseteq (H \circ I) \circ H \subseteq I$. Hence $H(a) \subseteq I$. Also, $S(a) = \{x \in H : x \in (x \circ a) \circ x,\ \text{for}\ a \in I\} \subseteq I$. □

An hyperideal I of an *LA-semihypergroup* H is called an *idempotent* if $I \circ I = I$. An *LA-semihypergroup* H is said to be *fully idempotent* if every hyperideal of H is idempotent.

Proposition 2.13. *Let H be an antirectangular LA-semihypergroup, and, A, B be hyperideals of H. Then the following statements are equivalent:*

(1) *H is fully idempotent,*

(2) *$A \cap B = A \circ B$,*

(3) *the hyperideals of H form a hypersemilattice $(L_H, \wedge)$ where $A \wedge B = A \circ B$.*

The proof follows from Proposition 2.5.

The set of hyperideals of H is totally ordered under set inclusion if for all hyperideals I, J either $I \subseteq J$ or $J \subseteq I$ and denoted by hyperideal(H).

Theorem 2.14. *Let H be an antirectangular LA-semihypergroup. Then every hyperideal of H is prime if and only if it is idempotent and hyperideal (H) is totally ordered under set inclusion.*

Proof. The proof follows from Theorem 2.6. □

In conclusion, let us mention that it would be interesting to investigate whether it is possible to apply hyperideals of hyperstructures to the construction of classification systems similar to those introduced in [8–12].

Acknowledgment

The authors are highly grateful to referees for their valuable comments and suggestions.

References

[1] F. Marty, "Sur une generalization de la notion de group, 8th Congres Math," *Scandinaves*, pp. 45–49, 1934.

[2] P. Corsini, *Prolegomena of Hypergroup Theory*, Aviani Editore, 2nd edition, 1993.

[3] P. Corsini and V. Leoreanu, *Applications of Hyperstructure Theory*, vol. 5 of *Advances in Mathematics*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.
[4] B. Davvaz and V. Leoreanu-Fotea, *Hyperring Theory and Applications*, International Academic Press, 2007.
[5] T. Vougiouklis, *Hyperstructures and Their Representations*, Hadronic Press, Palm Harbor, Fla, USA, 1994.
[6] K. Hila, B. Davvaz, and K. Naka, "On quasi-hyperideals insemihypergroups," *Communications in Algebra*. In press.
[7] K Hila, B. Davvaz, and J. Dine, "Study on the structure of Γ-semihypergroups," *Communications in Algebra*. In press.
[8] A. V. Kelarev, J. L. Yearwood, and M. A. Mammadov, "A formula for multiple classifiers in data mining based on Brandt semigroups," *Semigroup Forum*, vol. 78, no. 2, pp. 293–309, 2009.
[9] A. V. Kelarev, J. L. Yearwood, and P. W. Vamplew, "A polynomial ring construction for the classification of data," *Bulletin of the Australian Mathematical Society*, vol. 79, no. 2, pp. 213–225, 2009.
[10] A. V. Kelarev, J. L. Yearwood, and P. Watters, "Rees matrix constructions for clustering of data," *Journal of the Australian Mathematical Society*, vol. 87, no. 3, pp. 377–393, 2009.
[11] A. V. Kelarev, J. L. Yearwood, P. Watters, X. Wu, J. H. Abawajy, and L. Pan, "Internet security applications of the Munn rings," *Semigroup Forum*, vol. 81, no. 1, pp. 162–171, 2010.
[12] A. V. Kelarev, J. L. Yearwood, and P. A. Watters, "Optimization of classifiers for data mining based on combinatorial semigroups," *Semigroup Forum*, vol. 82, no. 2, pp. 242–251, 2011.
[13] M. A. Kazim and M. Naseeruddin, "On almost semigroups," *The Aligarh Bulletin of Mathematics*, vol. 2, pp. 1–7, 1972.
[14] Q. Mushtaq and S. M. Yusuf, "On LA-semigroups," *The Aligarh Bulletin of Mathematics*, vol. 8, pp. 65–70, 1978.
[15] Q. Mushtaq and S. M. Yusuf, "On locally associative LA-semigroups," *The Journal of Natural Sciences and Mathematics*, vol. 19, no. 1, pp. 57–62, 1979.
[16] Q. Mushtaq, "Abelian groups defined by LA-semigroups," *Studia Scientiarum Mathematicarum Hungarica*, vol. 18, no. 2–4, pp. 427–428, 1983.
[17] Q. Mushtaq and Q. Iqbal, "Decomposition of a locally associative LA-semigroup," *Semigroup Forum*, vol. 41, no. 2, pp. 155–164, 1990.
[18] Q. Mushtaq and M. S. Kamran, "On left almost groups," *Proceedings of the Pakistan Academy of Sciences*, vol. 33, no. 1-2, pp. 53–55, 1996.
[19] P. V. Protić and N. Stevanović, "On Abel-Grassmann's groupoids," in *Proceedings of the Mathematical Conference in Priština*, pp. 31–38.
[20] Q. Mushtaq and M. S. Kamran, "On LA-semigroup with weak associative law," *Scientiffic Khyber*, vol. 1, pp. 69–71, 1989.
[21] Q. Mushtaq and M. Khan, "M-systems in LA-semigroups," *Southeast Asian Bulletin of Mathematics*, vol. 33, no. 2, pp. 321–327, 2009.
[22] Q. Mushtaq and M. Khan, "Topological structures on Abel-Grassman's grupoids," arXiv:0904.1650v1, 2009.
[23] P. Holgate, "Groupoids satisfying a simple invertive law," *The Mathematics Student*, vol. 61, no. 1–4, pp. 101–106, 1992.

Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization