Research Article

Dynamic Analysis of Wind Turbine Blades Using Radial Basis Functions

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Wind turbine blades play important roles in wind energy generation. The dynamic problems associated with wind turbine blades are formulated using radial basis functions. The radial basis function procedure is used to transform partial differential equations, which represent the dynamic behavior of wind turbine blades, into a discrete eigenvalue problem. Numerical results demonstrate that rotational speed significantly impacts the first frequency of a wind turbine blade. Moreover, the pitch angle does not markedly affect wind turbine blade frequencies. This work examines the radial basis functions for dynamic problems of wind turbine blade.

1. Introduction

Wind energy will likely play an important role as a future energy source in countries worldwide. Energy is essential to economic growth. In response to environmental concerns, the amount of energy generated using renewable resources is increasing [1–3]. Herbert et al. [2] reviewed assessment models for wind resources, site selection models, and aerodynamic models, including the wake effect model. Varol et al. [3] demonstrated the positive effect of aerofoils surrounding wind blades. Their notion of surrounding aerofoils resembles that of steering blades in a water turbine. Spee et al. [4] created a novel control strategy for a brushless doubly fed machine applied to variable-speed wind-power generation systems. Vitale and Rossi [5] designed low-power, horizontal-axis wind turbine blades, using an iterative algorithm. With their software, one can determine the optimum blade shape for a wind turbine to satisfy the energy requirements of an electrical system with optimum rotor efficiency. Song and Dhinakaran [6] developed a nonlinear wind turbine control method. Storti and Aboelnaga [7] analyzed transverse deflections of a straight tapered symmetrical beam attached to a rotating hub as a model for bending vibration of blades in turbomachinery. The natural frequencies of a single tapered and pretwisted turboblade were calculated by Rao [8, 9], Abrate [10], Hodges et al. [11], and Dawson and Carneige [12, 13] using the Rayleigh-Ritz scheme. Gupta and Rao [14]

applied the finite element method to identify the frequencies of natural vibration of doubly tapered and twisted beams. Swaminathan and Rao [15] described the vibrations of a rotating pretwisted and tapered blade. Subrahmanyam et al. [16, 17] derived the natural frequencies and mode shapes of a uniform pretwisted cantilever blade using the Reissner approach. Chen and Keer [18] examined the transverse vibrations of a rotating pretwisted Timoshenko beam under axial loading. Storti and Aboelnaga [19] examined transverse deflections of a straight tapered symmetrical beam attached to a rotating hub as a model for the bending vibration of blades. Wagner [20] derived the forced vibration response of subsystems with different natural frequencies and damping attached to a foundation with a finite stiffness and mass. Griffin and Sinha [21–23] determined the dynamic responses of frictionally damped turbine blades. Determining the dynamic features of wind turbine blades is of significant importance. Radial basis functions are important elements of approaches generally called meshless methods. In this study, dynamic problems associated with wind turbine blades are formulated using radial basis functions.

2. Radial Basis Function

A radial basis function is a real-value function whose value depends on distance from an origin. Kansa [24, 25]

investigated a given function or partial derivatives of a function with respect to a coordinate direction expressed as a linear weighted sum of all functional values at all mesh points along the direction initiated based on the concept of the radial basis function. In their algorithm, node distribution was entirely unstructured. Wang and Liu [26] developed a point interpolation meshless method based on radial basis functions that incorporated the Galerkin weak form for solving partial differential equations. Elfelsoufi [27] investigated buckling, flutter, and vibration of beams using radial basis functions. Hon et al. [28] utilized radial basis functions for function fitting and solving partial differential equations using global nodes and collocation procedures. Liu et al. [29] constructed shape functions with the delta function property using radial and polynomial basis functions. Devi and Pepper [30] generated a simplified radial basis function approach for calculating coupled heat transfer of a fluid flow using a local pressure correction scheme. In their study, shape functions were constructed using radial basis functions. Vrankar et al. [31] applied a relatively new approach for modeling radionuclide migration through the geosphere using a radial basis function scheme. Qiao and Ernst [32] applied a nonlinear approach for constructing color conversions based on radial basis functions. Their experimental results demonstrated that color conversion can be achieved effectively using the radial basis function approach when constructing nonlinear data maps. A radial basis function can be expressed as follows [33, 34]:

$$B_i(r) = \sqrt{(r - r_i)^2 + c^2}, \quad (1)$$

where c is a shape parameter. The radial basis function is generally utilized to develop functional approximations in the following forms [33, 34]:

$$u_k(r, t) = \sum_{i=1}^N a_{ki}(t) B_i(r), \quad \text{for } k = 1, 2, 3, \quad (2)$$

$$v_k(r, t) = \sum_{i=1}^N \bar{a}_{ki}(t) B_i(r), \quad \text{for } k = 1, 2, 3, \quad (3)$$

$$U_k(r) = \sum_{i=1}^N b_{ki} B_i(r), \quad \text{for } k = 1, 2, 3, \quad (4)$$

$$V_k(r) = \sum_{i=1}^N \bar{b}_{ki} B_i(r), \quad \text{for } k = 1, 2, 3, \quad (5)$$

where a_{ki} , $\bar{a}_{ki}$, b_{ki} , and $\bar{b}_{ki}$ are coefficients to be determined. Blade deflection $u_k(r, t)$ is the sum of N radial basis functions, each associated with a different center r_i . Blade deflection $v_k(r, t)$ denotes the sum of N radial basis functions, each associated with a different center r_i . Blade deflection $U_k(r)$ is the sum of N radial basis functions, each associated with a different center r_i . Blade deflection $V_k(r)$ denotes the sum of N radial basis functions, each associated with a different center r_i . The domain contains N collocation points.

FIGURE 1: The first four frequencies of a wind turbine blade at different rotational speeds.

FIGURE 2: The first four frequencies of a wind turbine blade at different pitch angles.

3. Dynamic Analysis of Wind Turbine Blades

The kinetic energy of rotating wind turbine blades can be derived as

$$T^e = \sum_{k=1}^3 \frac{1}{2} \int_0^L \rho A \left(\left(\frac{\partial u_k}{\partial t} \right)^2 + \left(\frac{\partial v_k}{\partial t} \right)^2 + (\Omega v_k \cos \theta)^2 + (\Omega u_k \sin \theta)^2 \right) dr, \quad (6)$$

where Ω is hub rotational speed, A is cross-sectional area, θ is pitch angle, ρ is blade density, and L is blade length. The corresponding strain energy of rotating blades is

$$U^e = \sum_{k=1}^3 \left(\frac{1}{2} \int_0^L E \left(I_{yy} \left(\frac{\partial u_k}{\partial r} \right)^2 + 2I_{xy} \left(\frac{\partial u_k}{\partial r} \right) \left(\frac{\partial v_k}{\partial r} \right) + I_{xx} \left(\frac{\partial v_k}{\partial r} \right)^2 \right) dr + \frac{1}{2} \int_0^L \int_r^L \rho A \Omega^2 (r + \bar{r}_0) dr \times \left(\left(\frac{\partial u_k}{\partial r} \right)^2 + \left(\frac{\partial v_k}{\partial r} \right)^2 \right) dr \right), \quad (7)$$

where $\bar{r}_0$ is hub radius, $u_1(r, t)$ is the displacement of the first blade on the x -axis, $v_1(r, t)$ is the displacement of the first blade on the y -axis, $u_2(r, t)$ is the displacement of the second blade on the x -axis, $v_2(r, t)$ is the displacement of the second blade on the y -axis, $u_3(r, t)$ is the displacement of the third blade on the x -axis, $v_3(r, t)$ is the displacement of the third blade on the y -axis, and E is Young's modulus. By examining the internal and external damping effects of blades, virtual work δW^e in wind turbine blades can be derived as

$$\begin{aligned} \delta W^e &= \sum_{k=1}^3 \left(- \int_0^L C_0 \frac{\partial u_k}{\partial t} \delta u_k dr - \int_0^L C_1 \frac{\partial^2 EI_{yy}}{\partial r^2} \frac{\partial^3 u_k}{\partial t \partial r^2} \delta u_k dr \right. \\ &\quad - 2 \int_0^L C_1 \frac{\partial EI_{yy}}{\partial r} \frac{\partial^4 u_k}{\partial t \partial r^3} \delta u_k dr \\ &\quad - \int_0^L C_1 EI_{yy} \frac{\partial^5 u_k}{\partial t \partial r^4} \delta u_k dr \\ &\quad - \int_0^L C_0 \frac{\partial v_k}{\partial t} \delta v_k dr - \int_0^L C_1 \frac{\partial^2 EI_{xx}}{\partial r^2} \frac{\partial^3 v_k}{\partial t \partial r^2} \delta v_k dr \\ &\quad - 2 \int_0^L C_1 \frac{\partial EI_{xx}}{\partial r} \frac{\partial^4 v_k}{\partial t \partial r^3} \delta v_k dr \\ &\quad \left. - \int_0^L C_1 EI_{xx} \frac{\partial^5 v_k}{\partial t \partial r^4} \delta v_k dr \right), \quad (8) \end{aligned}$$

where C_0 and C_1 are external and internal damping coefficients of blades, respectively. Equations (6)–(8) are integrated into the Hamilton equation as follows:

$$\int_{t_1}^{t_2} (\delta T^e - \delta U^e + \delta W^e) dt = 0. \quad (9)$$

This leads to the following equations for the motion of the blades:

$$\begin{aligned} &\frac{\partial^2 EI_{yy}}{\partial r^2} \frac{\partial^2 u_k}{\partial r^2} + 2 \frac{\partial EI_{yy}}{\partial r} \frac{\partial^3 u_k}{\partial r^3} + EI_{yy} \frac{\partial^4 u_k}{\partial r^4} + \frac{\partial^2 EI_{xy}}{\partial r^2} \frac{\partial^2 v_k}{\partial r^2} \\ &\quad + 2 \frac{\partial EI_{xy}}{\partial r} \frac{\partial^3 v_k}{\partial r^3} + EI_{xy} \frac{\partial^4 v_k}{\partial r^4} - \rho A \Omega^2 u_k \sin^2 \theta \\ &\quad - \rho \Omega^2 \frac{\partial}{\partial r} \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{\partial u_k}{\partial r} \\ &\quad - \rho \Omega^2 \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{\partial^2 u_k}{\partial r^2} \\ &\quad + C_0 \frac{\partial u_k}{\partial t} + C_1 \frac{\partial^2 EI_{yy}}{\partial r^2} \frac{\partial^3 u_k}{\partial r^2 \partial t} + 2C_1 \frac{\partial EI_{yy}}{\partial r} \frac{\partial^4 u_k}{\partial r^3 \partial t} \\ &\quad + C_1 EI_{yy} \frac{\partial^5 u_k}{\partial r^4 \partial t} + C_1 \frac{\partial^2 EI_{xy}}{\partial r^2} \frac{\partial^3 v_k}{\partial r^2 \partial t} + 2C_1 \frac{\partial EI_{xy}}{\partial r} \frac{\partial^4 v_k}{\partial r^3 \partial t} \\ &\quad + C_1 EI_{xy} \frac{\partial^5 v_k}{\partial r^4 \partial t} + \rho A \frac{\partial^2 u_k}{\partial t^2} = 0 \quad \text{for } k = 1, 2, 3, \\ &\frac{\partial^2 EI_{xx}}{\partial r^2} \frac{\partial^2 v_k}{\partial r^2} + 2 \frac{\partial EI_{xx}}{\partial r} \frac{\partial^3 v_k}{\partial r^3} + EI_{xx} \frac{\partial^4 v_k}{\partial r^4} + \frac{\partial^2 EI_{xy}}{\partial r^2} \frac{\partial^2 u_k}{\partial r^2} \\ &\quad + 2 \frac{\partial EI_{xy}}{\partial r} \frac{\partial^3 u_k}{\partial r^3} + EI_{xy} \frac{\partial^4 u_k}{\partial r^4} - \rho A \Omega^2 v_k \cos^2 \theta \\ &\quad - \rho \Omega^2 \frac{\partial}{\partial r} \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{\partial v_k}{\partial r} \\ &\quad - \rho \Omega^2 \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{\partial^2 v_k}{\partial r^2} \\ &\quad + C_0 \frac{\partial v_k}{\partial t} + C_1 \frac{\partial^2 EI_{xx}}{\partial r^2} \frac{\partial^3 v_k}{\partial r^2 \partial t} + 2C_1 \frac{\partial EI_{xx}}{\partial r} \frac{\partial^4 v_k}{\partial r^3 \partial t} \\ &\quad + C_1 EI_{xx} \frac{\partial^5 v_k}{\partial r^4 \partial t} + C_1 \frac{\partial^2 EI_{xy}}{\partial r^2} \frac{\partial^3 u_k}{\partial r^2 \partial t} + 2C_1 \frac{\partial EI_{xy}}{\partial r} \frac{\partial^4 u_k}{\partial r^3 \partial t} \\ &\quad + C_1 EI_{xy} \frac{\partial^5 u_k}{\partial r^4 \partial t} + \rho A \frac{\partial^2 v_k}{\partial t^2} = 0 \quad \text{for } k = 1, 2, 3. \end{aligned} \quad (10)$$

The following equations are the corresponding boundary conditions:

$$\begin{aligned} u_k(0, t) &= 0 \quad \text{for } k = 1, 2, 3, \\ \frac{\partial u_k(0, t)}{\partial r} &= 0 \quad \text{for } k = 1, 2, 3, \\ EI_{yy} \frac{\partial^2 u_k(L, t)}{\partial r^2} &= 0 \quad \text{for } k = 1, 2, 3, \\ \frac{\partial}{\partial r} \left(EI_{yy} \frac{\partial^2 u_k(L, t)}{\partial r^2} \right) &= 0 \quad \text{for } k = 1, 2, 3, \\ v_k(0, t) &= 0 \quad \text{for } k = 1, 2, 3, \end{aligned}$$

$$\begin{aligned}
\frac{\partial v_k(0, t)}{\partial r} &= 0 \quad \text{for } k = 1, 2, 3, \\
EI_{xx} \frac{\partial^2 v_k(L, t)}{\partial r^2} &= 0 \quad \text{for } k = 1, 2, 3, \\
\frac{\partial}{\partial r} \left(EI_{xx} \frac{\partial^2 v_k(L, t)}{\partial r^2} \right) &= 0 \quad \text{for } k = 1, 2, 3.
\end{aligned} \tag{11}$$

A solution is thus assumed as

$$\begin{aligned}
u_k &= U_k(r) e^{\lambda t} \quad \text{for } k = 1, 2, 3, \\
v_k &= V_k(r) e^{\lambda t} \quad \text{for } k = 1, 2, 3.
\end{aligned} \tag{12}$$

This yields the following equations for the motion of blades:

$$\begin{aligned}
&\frac{d^2 EI_{yy}}{dr^2} \frac{d^2 U_k}{dr^2} + 2 \frac{d EI_{yy}}{dr} \frac{d^3 U_k}{dr^3} + EI_{yy} \frac{d^4 U_k}{dr^4} + \frac{d^2 EI_{xy}}{dr^2} \frac{d^2 V_k}{dr^2} \\
&+ 2 \frac{d EI_{xy}}{dr} \frac{d^3 V_k}{dr^3} + EI_{xy} \frac{d^4 V_k}{dr^4} - \rho A \Omega^2 U_k \sin^2 \theta \\
&- \rho \Omega^2 \frac{d}{dr} \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{d U_k}{dr} \\
&- \rho \Omega^2 \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{d^2 U_k}{dr^2} \\
&+ \lambda C_0 U_k + \lambda C_1 \frac{d^2 EI_{yy}}{dr^2} \frac{d^2 U_k}{dr^2} + 2 \lambda C_1 \frac{d EI_{yy}}{dr} \frac{d^3 U_k}{dr^3} \\
&+ \lambda C_1 EI_{yy} \frac{d^4 U_k}{dr^4} + \lambda C_1 \frac{d^2 EI_{xy}}{dr^2} \frac{d^2 V_k}{dr^2} \\
&+ 2 \lambda C_1 \frac{d EI_{xy}}{dr} \frac{d^3 V_k}{dr^3} + \lambda C_1 EI_{xy} \frac{d^4 V_k}{dr^4} \\
&+ \lambda^2 \rho A U_k = 0 \quad \text{for } k = 1, 2, 3, \\
&\frac{d^2 EI_{xx}}{dr^2} \frac{d^2 V_k}{dr^2} + 2 \frac{d EI_{xx}}{dr} \frac{d^3 V_k}{dr^3} + EI_{xx} \frac{d^4 V_k}{dr^4} + \frac{d^2 EI_{xy}}{dr^2} \frac{d^2 U_k}{dr^2} \\
&+ 2 \frac{d EI_{xy}}{dr} \frac{d^3 U_k}{dr^3} + EI_{xy} \frac{d^4 U_k}{dr^4} - \rho A \Omega^2 V_k \cos^2 \theta
\end{aligned}$$

$$\begin{aligned}
&- \rho \Omega^2 \frac{d}{dr} \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{d V_k}{dr} \\
&- \rho \Omega^2 \left(\int_r^L A(r + \bar{r}_0) dr \right) \frac{d^2 V_k}{dr^2} \\
&+ \lambda C_0 \frac{d V_k}{dt} + \lambda C_1 \frac{d^2 EI_{xx}}{dr^2} \frac{d^2 V_k}{dr^2} + 2 \lambda C_1 \frac{d EI_{xx}}{dr} \frac{d^3 V_k}{dr^3} \\
&+ \lambda C_1 EI_{xx} \frac{d^4 V_k}{dr^4} + \lambda C_1 \frac{d^2 EI_{xy}}{dr^2} \frac{d^2 U_k}{dr^2} + 2 \lambda C_1 \frac{d EI_{xy}}{dr} \frac{d^3 U_k}{dr^3} \\
&+ \lambda C_1 EI_{xy} \frac{d^4 U_k}{dr^4} + \lambda^2 \rho A V_k = 0 \quad \text{for } k = 1, 2, 3.
\end{aligned} \tag{13}$$

The corresponding boundary conditions are as follows:

$$\begin{aligned}
U_k(0) &= 0 \quad \text{for } k = 1, 2, 3, \\
\frac{d U_k(0)}{dr} &= 0 \quad \text{for } k = 1, 2, 3, \\
EI_{yy} \frac{d^2 U_k(L)}{dr^2} &= 0 \quad \text{for } k = 1, 2, 3, \\
\frac{d}{dr} \left(EI_{yy} \frac{d^2 U_k(L)}{dr^2} \right) &= 0 \quad \text{for } k = 1, 2, 3, \\
V_k(0) &= 0 \quad \text{for } k = 1, 2, 3, \\
\frac{d V_k(0)}{dr} &= 0 \quad \text{for } k = 1, 2, 3, \\
EI_{xx} \frac{d^2 V_k(L)}{dr^2} &= 0 \quad \text{for } k = 1, 2, 3, \\
\frac{d}{dr} \left(EI_{xx} \frac{d^2 V_k(L)}{dr^2} \right) &= 0 \quad \text{for } k = 1, 2, 3.
\end{aligned} \tag{14}$$

By employing the radial basis function approach, (2) and (3) can be substituted into (10). The equations of motion of wind turbine blades can be rearranged in the following matrix form:

$$\begin{aligned}
&[\rho A(r_i) B_1(r_i) \quad \rho A(r_i) B_2(r_i) \quad \cdots \quad \rho A(r_i) B_N(r_i)] \left[\frac{\partial^2 a_{k1}}{\partial t^2} \quad \frac{\partial^2 a_{k2}}{\partial t^2} \quad \cdots \quad \frac{\partial^2 a_{kN}}{\partial t^2} \right]^T \\
&+ [C_0 B_1(r_i) \quad C_0 B_2(r_i) \quad \cdots \quad C_0 B_N(r_i)] \left[\frac{\partial a_{k1}}{\partial t} \quad \frac{\partial a_{k2}}{\partial t} \quad \cdots \quad \frac{\partial a_{kN}}{\partial t} \right]^T \\
&+ \left[C_1 \frac{\partial^2 EI_{yy}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad C_1 \frac{\partial^2 EI_{yy}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \cdots \quad C_1 \frac{\partial^2 EI_{yy}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] \left[\frac{\partial a_{k1}}{\partial t} \quad \frac{\partial a_{k2}}{\partial t} \quad \cdots \quad \frac{\partial a_{kN}}{\partial t} \right]^T \\
&+ \left[2 C_1 \frac{\partial EI_{yy}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2 C_1 \frac{\partial EI_{yy}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \cdots \quad 2 C_1 \frac{\partial EI_{yy}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] \left[\frac{\partial a_{k1}}{\partial t} \quad \frac{\partial a_{k2}}{\partial t} \quad \cdots \quad \frac{\partial a_{kN}}{\partial t} \right]^T \\
&+ \left[C_1 EI_{yy}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad C_1 EI_{yy}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \cdots \quad C_1 EI_{yy}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] \left[\frac{\partial a_{k1}}{\partial t} \quad \frac{\partial a_{k2}}{\partial t} \quad \cdots \quad \frac{\partial a_{kN}}{\partial t} \right]^T
\end{aligned}$$

$$\begin{aligned}
& + \left[C_1 \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad C_1 \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad C_1 \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] \left[\frac{\partial \bar{a}_{k1}}{\partial t} \quad \frac{\partial \bar{a}_{k2}}{\partial t} \quad \dots \quad \frac{\partial \bar{a}_{kN}}{\partial t} \right]^T \\
& + \left[2C_1 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2C_1 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \dots \quad 2C_1 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] \left[\frac{\partial \bar{a}_{k1}}{\partial t} \quad \frac{\partial \bar{a}_{k2}}{\partial t} \quad \dots \quad \frac{\partial \bar{a}_{kN}}{\partial t} \right]^T \\
& + \left[C_1 EI_{xy}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad C_1 EI_{xy}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \dots \quad C_1 EI_{xy}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] \left[\frac{\partial \bar{a}_{k1}}{\partial t} \quad \frac{\partial \bar{a}_{k2}}{\partial t} \quad \dots \quad \frac{\partial \bar{a}_{kN}}{\partial t} \right]^T \\
& + \left[\frac{\partial^2 EI_{yy}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad \frac{\partial^2 EI_{yy}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad \frac{\partial^2 EI_{yy}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& + \left[2 \frac{\partial EI_{yy}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2 \frac{\partial EI_{yy}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \dots \quad 2 \frac{\partial EI_{yy}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& + \left[EI_{yy}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad EI_{yy}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \dots \quad EI_{yy}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& + \left[\frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& + \left[2 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \dots \quad 2 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& + \left[EI_{xy}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad EI_{xy}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \dots \quad EI_{xy}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& - \left[\rho A(r) \Omega^2 (\sin \theta)^2 B_1(r_i) \quad \rho A(r) \Omega^2 (\sin \theta)^2 B_2(r_i) \quad \dots \quad \rho A(r) \Omega^2 (\sin \theta)^2 B_N(r_i) \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& - \left[\rho \Omega^2 \frac{\partial}{\partial r} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{\partial B_1(r_i)}{\partial r} \quad \rho \Omega^2 \frac{\partial}{\partial r} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{\partial B_2(r_i)}{\partial r} \quad \dots \quad \rho \Omega^2 \frac{\partial}{\partial r} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{\partial B_N(r_i)}{\partial r} \right] \\
& \times [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& - \left[\rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] \\
& \times [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T = [0], \quad \text{for } i = 1, 2, \dots, N, \quad k = 1, 2, 3, \\
& [\rho A(r_i) B_1(r_i) \quad \rho A(r_i) B_2(r_i) \quad \dots \quad \rho A(r_i) B_N(r_i)] \left[\frac{\partial^2 \bar{a}_{k1}}{\partial t^2} \quad \frac{\partial^2 \bar{a}_{k2}}{\partial t^2} \quad \dots \quad \frac{\partial^2 \bar{a}_{kN}}{\partial t^2} \right]^T \\
& + [C_0 B_1(r_i) \quad C_0 B_2(r_i) \quad \dots \quad C_0 B_N(r_i)] \left[\frac{\partial \bar{a}_{k1}}{\partial t} \quad \frac{\partial \bar{a}_{k2}}{\partial t} \quad \dots \quad \frac{\partial \bar{a}_{kN}}{\partial t} \right]^T \\
& + \left[C_1 \frac{\partial^2 EI_{xx}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad C_1 \frac{\partial^2 EI_{xx}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad C_1 \frac{\partial^2 EI_{xx}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] \left[\frac{\partial \bar{a}_{k1}}{\partial t} \quad \frac{\partial \bar{a}_{k2}}{\partial t} \quad \dots \quad \frac{\partial \bar{a}_{kN}}{\partial t} \right]^T \\
& + \left[2C_1 \frac{\partial EI_{xx}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2C_1 \frac{\partial EI_{xx}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \dots \quad 2C_1 \frac{\partial EI_{xx}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] \left[\frac{\partial \bar{a}_{k1}}{\partial t} \quad \frac{\partial \bar{a}_{k2}}{\partial t} \quad \dots \quad \frac{\partial \bar{a}_{kN}}{\partial t} \right]^T \\
& + \left[C_1 EI_{xx}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad C_1 EI_{xx}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \dots \quad C_1 EI_{xx}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] \left[\frac{\partial \bar{a}_{k1}}{\partial t} \quad \frac{\partial \bar{a}_{k2}}{\partial t} \quad \dots \quad \frac{\partial \bar{a}_{kN}}{\partial t} \right]^T \\
& + \left[C_1 \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad C_1 \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad C_1 \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] \left[\frac{\partial a_{k1}}{\partial t} \quad \frac{\partial a_{k2}}{\partial t} \quad \dots \quad \frac{\partial a_{kN}}{\partial t} \right]^T
\end{aligned}$$

$$\begin{aligned}
& + \left[2C_1 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2C_1 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \dots \quad 2C_1 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] \left[\frac{\partial a_{k1}}{\partial t} \quad \frac{\partial a_{k2}}{\partial t} \quad \dots \quad \frac{\partial a_{kN}}{\partial t} \right]^T \\
& + \left[C_1 EI_{xy}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad C_1 EI_{xy}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \dots \quad C_1 EI_{xy}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] \left[\frac{\partial a_{k1}}{\partial t} \quad \frac{\partial a_{k2}}{\partial t} \quad \dots \quad \frac{\partial a_{kN}}{\partial t} \right]^T \\
& + \left[\frac{\partial^2 EI_{xx}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad \frac{\partial^2 EI_{xx}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad \frac{\partial^2 EI_{xx}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& + \left[2 \frac{\partial EI_{xx}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2 \frac{\partial EI_{xx}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \dots \quad 2 \frac{\partial EI_{xx}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& + \left[EI_{xx}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad EI_{xx}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \dots \quad EI_{xx}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& + \left[\frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad \frac{\partial^2 EI_{xy}(r_i)}{\partial r^2} \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& + \left[2 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_1(r_i)}{\partial r^3} \quad 2 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_2(r_i)}{\partial r^3} \quad \dots \quad 2 \frac{\partial EI_{xy}(r_i)}{\partial r} \frac{\partial^3 B_N(r_i)}{\partial r^3} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& + \left[EI_{xy}(r_i) \frac{\partial^4 B_1(r_i)}{\partial r^4} \quad EI_{xy}(r_i) \frac{\partial^4 B_2(r_i)}{\partial r^4} \quad \dots \quad EI_{xy}(r_i) \frac{\partial^4 B_N(r_i)}{\partial r^4} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& - \left[\rho A(r) \Omega^2 (\cos \theta)^2 B_1(r_i) \quad \rho A(r) \Omega^2 (\cos \theta)^2 B_2(r_i) \quad \dots \quad \rho A(r) \Omega^2 (\cos \theta)^2 B_N(r_i) \right] [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& - \left[\rho \Omega^2 \frac{\partial}{\partial r} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{\partial B_1(r_i)}{\partial r} \quad \rho \Omega^2 \frac{\partial}{\partial r} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{\partial B_2(r_i)}{\partial r} \quad \dots \quad \rho \Omega^2 \frac{\partial}{\partial r} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{\partial B_N(r_i)}{\partial r} \right] \\
& \times [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T \\
& - \left[\rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{\partial^2 B_1(r_i)}{\partial r^2} \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{\partial^2 B_2(r_i)}{\partial r^2} \quad \dots \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{\partial^2 B_N(r_i)}{\partial r^2} \right] \\
& \times [\bar{a}_{k1} \quad \bar{a}_{k2} \quad \dots \quad \bar{a}_{kN}]^T = [0] \quad \text{for } i = 1, 2, \dots, N, \quad k = 1, 2, 3,
\end{aligned} \tag{15}$$

where $\mathfrak{A}$ denotes $(r + \bar{r}_0)$.

Based on the radial basis function technique, (11) take the following discrete forms:

$$\begin{aligned}
& [B_1(r_1) \quad B_2(r_1) \quad \dots \quad B_N(r_1)] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{\partial B_1(r_1)}{\partial r} \quad \frac{\partial B_2(r_1)}{\partial r} \quad \dots \quad \frac{\partial B_N(r_1)}{\partial r} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[EI_{yy}(r_N) \frac{\partial^2 B_1(r_N)}{\partial r^2} \quad EI_{yy}(r_N) \frac{\partial^2 B_2(r_N)}{\partial r^2} \quad \dots \quad EI_{yy}(r_N) \frac{\partial^2 B_N(r_N)}{\partial r^2} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{\partial EI_{yy}(r_N)}{\partial r} \frac{\partial^2 B_1(r_N)}{\partial r^2} \quad \frac{\partial EI_{yy}(r_N)}{\partial r} \frac{\partial^2 B_2(r_N)}{\partial r^2} \quad \dots \quad \frac{\partial EI_{yy}(r_N)}{\partial r} \frac{\partial^2 B_N(r_N)}{\partial r^2} \right] [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T \\
& + \left[EI_{yy}(r_N) \frac{\partial^3 B_1(r_N)}{\partial r^3} \quad EI_{yy}(r_N) \frac{\partial^3 B_2(r_N)}{\partial r^3} \quad \dots \quad EI_{yy}(r_N) \frac{\partial^3 B_N(r_N)}{\partial r^3} \right] \\
& \times [a_{k1} \quad a_{k2} \quad \dots \quad a_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3,
\end{aligned}$$

$$\begin{aligned}
& [B_1(r_1) \ B_2(r_1) \ \cdots \ B_N(r_1)][\bar{a}_{k1} \ \bar{a}_{k2} \ \cdots \ \bar{a}_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{\partial B_1(r_1)}{\partial r} \ \frac{\partial B_2(r_1)}{\partial r} \ \cdots \ \frac{\partial B_N(r_1)}{\partial r} \right][\bar{a}_{k1} \ \bar{a}_{k2} \ \cdots \ \bar{a}_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[EI_{xx}(r_N) \frac{\partial^2 B_1(r_N)}{\partial r^2} \ EI_{xx}(r_N) \frac{\partial^2 B_2(r_N)}{\partial r^2} \ \cdots \ EI_{xx}(r_N) \frac{\partial^2 B_N(r_N)}{\partial r^2} \right][\bar{a}_{k1} \ \bar{a}_{k2} \ \cdots \ \bar{a}_{kN}]^T = [0], \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{\partial EI_{xx}(r_N)}{\partial r} \frac{\partial^2 B_1(r_N)}{\partial r^2} \ \frac{\partial EI_{xx}(r_N)}{\partial r} \frac{\partial^2 B_2(r_N)}{\partial r^2} \ \cdots \ \frac{\partial EI_{xx}(r_N)}{\partial r} \frac{\partial^2 B_N(r_N)}{\partial r^2} \right][\bar{a}_{k1} \ \bar{a}_{k2} \ \cdots \ \bar{a}_{kN}]^T \\
& + \left[EI_{xx}(r_N) \frac{\partial^3 B_1(r_N)}{\partial r^3} \ EI_{xx}(r_N) \frac{\partial^3 B_2(r_N)}{\partial r^3} \ \cdots \ EI_{xx}(r_N) \frac{\partial^3 B_N(r_N)}{\partial r^3} \right] \\
& \times [\bar{a}_{k1} \ \bar{a}_{k2} \ \cdots \ \bar{a}_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3.
\end{aligned} \tag{16}$$

By applying the radial basis function approach, (4) and (5) are substituted into (13). The following equations are then yielded

$$\begin{aligned}
& [\lambda^2 \rho A(r_i) B_1(r_i) \ \lambda^2 \rho A(r_i) B_2(r_i) \ \cdots \ \lambda^2 \rho A(r_i) B_N(r_i)][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + [\lambda C_0 B_1(r_i) \ \lambda C_0 B_2(r_i) \ \cdots \ \lambda C_0 B_N(r_i)][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + \left[\lambda C_1 \frac{d^2 EI_{yy}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \ \lambda C_1 \frac{d^2 EI_{yy}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \ \cdots \ \lambda C_1 \frac{d^2 EI_{yy}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + \left[2\lambda C_1 \frac{d EI_{yy}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \ 2\lambda C_1 \frac{d EI_{yy}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \ \cdots \ 2\lambda C_1 \frac{d EI_{yy}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + \left[\lambda C_1 EI_{yy}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \ \lambda C_1 EI_{yy}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \ \cdots \ \lambda C_1 EI_{yy}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + \left[\lambda C_1 \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \ \lambda C_1 \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \ \cdots \ \lambda C_1 \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right][\bar{b}_{k1} \ \bar{b}_{k2} \ \cdots \ \bar{b}_{kN}]^T \\
& + \left[2\lambda C_1 \frac{d EI_{xy}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \ 2\lambda C_1 \frac{d EI_{xy}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \ \cdots \ 2\lambda C_1 \frac{d EI_{xy}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right][\bar{b}_{k1} \ \bar{b}_{k2} \ \cdots \ \bar{b}_{kN}]^T \\
& + \left[\lambda C_1 EI_{xy}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \ \lambda C_1 EI_{xy}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \ \cdots \ \lambda C_1 EI_{xy}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right][\bar{b}_{k1} \ \bar{b}_{k2} \ \cdots \ \bar{b}_{kN}]^T \\
& + \left[\frac{d^2 EI_{yy}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \ \frac{d^2 EI_{yy}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \ \cdots \ \frac{d^2 EI_{yy}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + \left[2 \frac{d EI_{yy}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \ 2 \frac{d EI_{yy}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \ \cdots \ 2 \frac{d EI_{yy}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + \left[EI_{yy}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \ EI_{yy}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \ \cdots \ EI_{yy}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right][b_{k1} \ b_{k2} \ \cdots \ b_{kN}]^T \\
& + \left[\frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \ \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \ \cdots \ \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right][\bar{b}_{k1} \ \bar{b}_{k2} \ \cdots \ \bar{b}_{kN}]^T \\
& + \left[2 \frac{d EI_{xy}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \ 2 \frac{d EI_{xy}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \ \cdots \ 2 \frac{d EI_{xy}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right][\bar{b}_{k1} \ \bar{b}_{k2} \ \cdots \ \bar{b}_{kN}]^T
\end{aligned}$$

$$\begin{aligned}
& + \left[EI_{xy}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \quad EI_{xy}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \quad \dots \quad EI_{xy}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& - \left[\rho A(r_i) \Omega^2 (\sin \theta)^2 B_1(r_i) \quad \rho A(r_i) \Omega^2 (\sin \theta)^2 B_2(r_i) \quad \dots \quad \rho A(r_i) \Omega^2 (\sin \theta)^2 B_N(r_i) \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& - \left[\rho \Omega^2 \frac{d}{dr} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{dB_1(r_i)}{dr} \quad \rho \Omega^2 \frac{d}{dr} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{dB_2(r_i)}{dr} \quad \dots \quad \rho \Omega^2 \frac{d}{dr} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{dB_N(r_i)}{dr} \right] \\
& \times [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& - \left[\rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{d^2 B_1(r_i)}{dr^2} \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{d^2 B_2(r_i)}{dr^2} \quad \dots \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{d^2 B_N(r_i)}{dr^2} \right] \\
& \times [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T = [0] \quad \text{for } i = 1, 2, \dots, N, \quad k = 1, 2, 3, \\
& [\lambda^2 \rho A(r_i) B_1(r_i) \quad \lambda^2 \rho A(r_i) B_2(r_i) \quad \dots \quad \lambda^2 \rho A(r_i) B_N(r_i)] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + [\lambda C_0 B_1(r_i) \quad \lambda C_0 B_2(r_i) \quad \dots \quad \lambda C_0 B_N(r_i)] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[\lambda C_1 \frac{d^2 EI_{xx}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \quad \lambda C_1 \frac{d^2 EI_{xx}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \quad \dots \quad \lambda C_1 \frac{d^2 EI_{xx}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[2\lambda C_1 \frac{dEI_{xx}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \quad 2\lambda C_1 \frac{dEI_{xx}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \quad \dots \quad 2\lambda C_1 \frac{dEI_{xx}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[\lambda C_1 EI_{xx}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \quad \lambda C_1 EI_{xx}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \quad \dots \quad \lambda C_1 EI_{xx}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[\lambda C_1 \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \quad \lambda C_1 \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \quad \dots \quad \lambda C_1 \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& + \left[2\lambda C_1 \frac{dEI_{xy}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \quad 2\lambda C_1 \frac{dEI_{xy}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \quad \dots \quad 2\lambda C_1 \frac{dEI_{xy}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& + \left[\lambda C_1 EI_{xy}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \quad \lambda C_1 EI_{xy}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \quad \dots \quad \lambda C_1 EI_{xy}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& + \left[\frac{d^2 EI_{xx}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \quad \frac{d^2 EI_{xx}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \quad \dots \quad \frac{d^2 EI_{xx}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[2 \frac{dEI_{xx}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \quad 2 \frac{dEI_{xx}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \quad \dots \quad 2 \frac{dEI_{xx}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[EI_{xx}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \quad EI_{xx}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \quad \dots \quad EI_{xx}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[\frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_1(r_i)}{dr^2} \quad \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_2(r_i)}{dr^2} \quad \dots \quad \frac{d^2 EI_{xy}(r_i)}{dr^2} \frac{d^2 B_N(r_i)}{dr^2} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& + \left[2 \frac{dEI_{xy}(r_i)}{dr} \frac{d^3 B_1(r_i)}{dr^3} \quad 2 \frac{dEI_{xy}(r_i)}{dr} \frac{d^3 B_2(r_i)}{dr^3} \quad \dots \quad 2 \frac{dEI_{xy}(r_i)}{dr} \frac{d^3 B_N(r_i)}{dr^3} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& + \left[EI_{xy}(r_i) \frac{d^4 B_1(r_i)}{dr^4} \quad EI_{xy}(r_i) \frac{d^4 B_2(r_i)}{dr^4} \quad \dots \quad EI_{xy}(r_i) \frac{d^4 B_N(r_i)}{dr^4} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& - \left[\rho A(r_i) \Omega^2 (\cos \theta)^2 B_1(r_i) \quad \rho A(r_i) \Omega^2 (\cos \theta)^2 B_2(r_i) \quad \dots \quad \rho A(r_i) \Omega^2 (\cos \theta)^2 B_N(r_i) \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T
\end{aligned}$$

$$\begin{aligned}
& - \left[\rho \Omega^2 \frac{d}{dr} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{dB_1(r_i)}{dr} \quad \rho \Omega^2 \frac{d}{dr} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{dB_2(r_i)}{dr} \quad \dots \quad \rho \Omega^2 \frac{d}{dr} \left(\int_{r_i}^L A(r) \mathfrak{A} dr \right) \frac{dB_N(r_i)}{dr} \right] \\
& \times [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& - \left[\rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{d^2 B_1(r_i)}{dr^2} \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{d^2 B_2(r_i)}{dr^2} \quad \dots \quad \rho \Omega^2 \int_{r_i}^L A(r) \mathfrak{A} dr \frac{d^2 B_N(r_i)}{dr^2} \right] \\
& \times [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T = [0] \quad \text{for } i = 1, 2, \dots, N, \quad k = 1, 2, 3.
\end{aligned} \tag{17}$$

where $\mathfrak{A}$ denotes $(r + \bar{r}_0)$.

According to the radial basis function approach, the boundary conditions in (14) have the following discrete forms:

$$\begin{aligned}
& [B_1(r_1) \quad B_2(r_1) \quad \dots \quad B_N(r_1)] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{dB_1(r_1)}{dr} \quad \frac{dB_2(r_1)}{dr} \quad \dots \quad \frac{dB_N(r_1)}{dr} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[EI_{yy}(r_N) \frac{d^2 B_1(r_N)}{dr^2} \quad EI_{yy}(r_N) \frac{d^2 B_2(r_N)}{dr^2} \quad \dots \quad EI_{yy}(r_N) \frac{d^2 B_N(r_N)}{dr^2} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{dEI_{yy}(r_N)}{dr} \frac{d^2 B_1(r_N)}{dr^2} \quad \frac{dEI_{yy}(r_N)}{dr} \frac{d^2 B_2(r_N)}{dr^2} \quad \dots \quad \frac{dEI_{yy}(r_N)}{dr} \frac{d^2 B_N(r_N)}{dr^2} \right] [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T \\
& + \left[EI_{yy}(r_N) \frac{d^3 B_1(r_N)}{dr^3} \quad EI_{yy}(r_N) \frac{d^3 B_2(r_N)}{dr^3} \quad \dots \quad EI_{yy}(r_N) \frac{d^3 B_N(r_N)}{dr^3} \right] \\
& \times [b_{k1} \quad b_{k2} \quad \dots \quad b_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& [B_1(r_1) \quad B_2(r_1) \quad \dots \quad B_N(r_1)] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{dB_1(r_1)}{dr} \quad \frac{dB_2(r_1)}{dr} \quad \dots \quad \frac{dB_N(r_1)}{dr} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[EI_{xx}(r_N) \frac{d^2 B_1(r_N)}{dr^2} \quad EI_{xx}(r_N) \frac{d^2 B_2(r_N)}{dr^2} \quad \dots \quad EI_{xx}(r_N) \frac{d^2 B_N(r_N)}{dr^2} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3, \\
& \left[\frac{dEI_{xx}(r_N)}{dr} \frac{d^2 B_1(r_N)}{dr^2} \quad \frac{dEI_{xx}(r_N)}{dr} \frac{d^2 B_2(r_N)}{dr^2} \quad \dots \quad \frac{dEI_{xx}(r_N)}{dr} \frac{d^2 B_N(r_N)}{dr^2} \right] [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T \\
& + \left[EI_{xx}(r_N) \frac{d^3 B_1(r_N)}{dr^3} \quad EI_{xx}(r_N) \frac{d^3 B_2(r_N)}{dr^3} \quad \dots \quad EI_{xx}(r_N) \frac{d^3 B_N(r_N)}{dr^3} \right] \\
& \times [\bar{b}_{k1} \quad \bar{b}_{k2} \quad \dots \quad \bar{b}_{kN}]^T = [0] \quad \text{for } k = 1, 2, 3.
\end{aligned} \tag{18}$$

4. Results

Figure 1 illustrates the effects of various rotational speeds on calculated frequencies for a wind turbine blade. Computational results suggest that the first frequency of a wind turbine blade is strongly dependent on blade shaft speed. The first frequency of a wind turbine blade increases as

rotational speed increases. Numerical results obtained by this study suggest that rotational speed does not significantly affect the second, third, and fourth frequencies of a wind blade. Figure 2 lists the frequencies of a wind turbine blade at different pitch angles. Numerical results demonstrate that pitch angle does not significantly affect the first, second, third, and fourth frequencies of a wind turbine blade.

5. Concluding Remarks

The radial basis function scheme has been extensively adopted to solve various problems in different scientific and engineering fields. This study investigated how pitch angles and rotational speeds affect frequencies of a wind generator blade. The simplicity of this formulation makes it a good candidate for very complex applications.

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