International Scholarly Research Network
ISRN Geometry
Volume 2012, Article ID 317253, 10 pages
doi:10.5402/2012/317253

Research Article

On ϕ-Recurrent Para-Sasakian Manifold Admitting Quarter-Symmetric Metric Connection

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Received 3 November 2011; Accepted 6 December 2011

Academic Editors: T. Friedrich, M. Korkmaz, O. Mokhov, and R. Vázquez-Lorenzo

We obtained the relation between the Riemannian connection and the quarter-symmetric metric connection on a para-Sasakian manifold. Further, we study ϕ-recurrent and concircular ϕ-recurrent para-Sasakian manifolds with respect to quarter-symmetric metric connection.

1. Introduction

The idea of metric connection with torsion in a Riemannian manifold was introduced by Hayden [1]. Further, some properties of semisymmetric metric connection have been studied by Yano [2]. In [3], Golab defined and studied quarter-symmetric connection on a differentiable manifold with affine connection, which generalizes the idea of semisymmetric connection. Various properties of quarter-symmetric metric connection have been studied by many geometers like Rastogi [4, 5], Mishra and Pandey [6], Yano and Imai [7], De et al. [8, 9], Pradeep Kumar et al. [10], and many others.

The notion of local symmetry of a Riemannian manifold has been weakened by many authors in several ways to a different extent. As a weaker version of local symmetry, Takahashi [11] introduced the notion of local ϕ-symmetry on a Sasakian manifold. Generalizing the notion of ϕ-symmetry, the authors De et al. [12] introduced the notion of ϕ-recurrent Sasakian manifolds.

A linear connection $\tilde{\nabla}$ on an n-dimensional differentiable manifold is said to be a quarter-symmetric connection [3] if its torsion tensor T is of the form

$$T(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y] = \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.1}$$

where η is a 1-form and ϕ is a tensor of type $(1,1)$. In particular, if we replace ϕX by X and ϕY by Y, then the quarter-symmetric connection reduces to the semisymmetric connection [13]. Thus, the notion of quarter-symmetric connection generalizes the idea of the semisymmetric connection. And if quarter-symmetric linear connection $\widetilde{\nabla}$ satisfies the condition

$$\left(\widetilde{\nabla}_X g\right)(Y,Z) = 0, \tag{1.2}$$

for all $X, Y, Z \in \mathcal{X}(M)$, where $\mathcal{X}(M)$ is the Lie algebra of vector fields on the manifold M, then $\widetilde{\nabla}$ is said to be a quarter-symmetric metric connection.

2. Preliminaries

An n-dimensional differentiable manifold M is called an almost paracontact manifold if it admits an almost paracontact structure (ϕ, ξ, η) consisting of a $(1,1)$ tensor field ϕ, a vector field ξ, and a 1-form η satisfying

$$\phi^2 X = X - \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = 1, \qquad \phi \circ \xi = 0, \qquad \eta \circ \phi = 0. \tag{2.2}$$

If g is a compatible Riemannian metric with (ϕ, ξ, η), that is,

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(X,\xi) = \eta(X) \tag{2.3}$$

$$g(X, \phi Y) = g(\phi X, Y), \tag{2.4}$$

for all vector fields X and Y on M, then M becomes a almost paracontact Riemannian manifold equipped with an almost paracontact Riemannian structure (ϕ, ξ, η, g).

An almost paracontact Riemannian manifold is called a para-Sasakian manifold if it satisfies

$$(\nabla_X \phi)Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.5}$$

where ∇ denotes the operator of covariant differentiation. From the above equation it follows that

$$\nabla_X \xi = \phi X, \qquad (\nabla_X \eta)Y = g(X, \phi Y) = (\nabla_Y \eta)X. \tag{2.6}$$

In an n-dimensional para-Sasakian manifold M, the following relations hold [14, 15]:

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{2.7}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.8}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{2.9}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.10}$$

for any vector fields X, Y, and Z, where R and S are the Riemannian curvature tensor and the Ricci tensor of M, respectively.

A para-Sasakian manifold M is said to be η-Einstein if its Ricci tensor S is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{2.11}$$

for any vector fields X and Y, where a and b are some functions on M.

Definition 2.1. A para-Sasakian manifold is said to be locally ϕ-symmetric if

$$\phi^2((\nabla_W R)(X,Y)Z) = 0, \tag{2.12}$$

for all vector fields X, Y, Z, W orthogonal to ξ. This notion was introduced for Sasakian manifold by Takahashi [11].

Definition 2.2. A para-Sasakian manifold is said to be locally concircular ϕ-symmetric if

$$\phi^2\left(\left(\nabla_W \overline{C}\right)(X,Y)Z\right) = 0, \tag{2.13}$$

for all vector fields X, Y, Z, W orthogonal to ξ. Where the concircular curvature tensor $\overline{C}$ is given by [16]

$$\overline{C}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\left[g(Y,Z)X - g(X,Z)Y\right], \tag{2.14}$$

where R is the Riemannian curvature tensor and r is the scalar curvature.

Definition 2.3. A para-Sasakian manifold is said to be ϕ-recurrent if there exists a nonzero 1-form A such that

$$\phi^2((\nabla_W R)(X,Y)Z) = A(W)R(X,Y)Z, \tag{2.15}$$

where A is a 1-form and it is defined by

$$A(W) = g(W,\rho), \tag{2.16}$$

and ρ is a vector field associated with the 1-form A.

3. Quarter-Symmetric Metric Connection

Let $\tilde{\nabla}$ be a linear connection and ∇ a Riemannian connection of an almost contact metric manifold M such that

$$\tilde{\nabla}_X Y = \nabla_X Y + U(X, Y), \tag{3.1}$$

where U is a tensor of type $(1, 1)$. For $\tilde{\nabla}$ to be a quarter-symmetric metric connection in M, then we have [3]

$$U(X, Y) = \frac{1}{2}\left[T(X, Y) + T'(X, Y) + T'(Y, X)\right], \tag{3.2}$$

$$g\left(T'(X, Y), Z\right) = g(T(Z, X), Y). \tag{3.3}$$

From (1.1) and (3.3), we get

$$T'(X, Y) = \eta(X)\phi Y - g\left(\phi X, Y\right)\xi. \tag{3.4}$$

Using (1.1) and (3.4) in (3.2), we obtain

$$U(X, Y) = \eta(Y)\phi X - g\left(\phi X, Y\right)\xi. \tag{3.5}$$

Thus a quarter-symmetric metric connection $\tilde{\nabla}$ in a para-Sasakian manifold is given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g\left(\phi X, Y\right)\xi. \tag{3.6}$$

Hence (3.6) is the relation between Riemannian connection and the quarter-symmetric metric connection on a para-Sasakian manifold.

A relation between the curvature tensor of M with respect to the quarter-symmetric metric connection $\tilde{\nabla}$ and the Riemannian connection ∇ is given by

$$\begin{aligned}\tilde{R}(X, Y)Z = R(X, Y)Z &+ 3g\left(\phi X, Z\right)\phi Y - 3g\left(\phi Y, Z\right)\phi X + \eta(Z)\left[\eta(X)Y - \eta(Y)X\right] \\ &- \left[\eta(X)g(Y, Z) - \eta(Y)g(X, Z)\right]\xi,\end{aligned} \tag{3.7}$$

where $\tilde{R}$ and R denote the Riemannian curvatures of the connections $\tilde{\nabla}$ and ∇, respectively. From (3.7), it follows that

$$\tilde{S}(Y, Z) = S(Y, Z) + 2g(Y, Z) - (n + 1)\eta(Y)\eta(Z), \tag{3.8}$$

where $\tilde{S}$ and S are the Ricci tensors of the connections $\tilde{\nabla}$ and ∇, respectively.

Contracting (3.8), we get

$$\tilde{r} = r + (n - 1), \tag{3.9}$$

where $\tilde{r}$ and r are the scalar curvatures of the connections $\tilde{\nabla}$ and ∇, respectively.

4. ϕ-Recurrent Para-Sasakian Manifold with respect to Quarter-Symmetric Metric Connection

A para-Sasakian manifold is called ϕ-recurrent with respect to the quarter-symmetric metric connection if its curvature tensor $\tilde{R}$ satisfies the condition

$$\phi^2\left(\left(\tilde{\nabla}_W\tilde{R}\right)(X,Y)Z\right) = A(W)\tilde{R}(X,Y)Z. \tag{4.1}$$

By virtue of (2.1) and (4.1), we have

$$\left(\tilde{\nabla}_W\tilde{R}\right)(X,Y)Z - \eta\left(\left(\tilde{\nabla}_W\tilde{R}\right)(X,Y)Z\right)\xi = A(W)\tilde{R}(X,Y)Z. \tag{4.2}$$

From which, it follows that

$$g\left(\left(\tilde{\nabla}_W\tilde{R}\right)(X,Y)Z,U\right) - \eta\left(\left(\tilde{\nabla}_W\tilde{R}\right)(X,Y)Z\right)g(\xi,U) = A(W)g\left(\tilde{R}(X,Y)Z,U\right). \tag{4.3}$$

Let $\{e_i\}$, $i = 1,2,\ldots,n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (4.3) and taking summation over i, $1 \le i \le n$, we get

$$\left(\tilde{\nabla}_W\tilde{S}\right)(Y,Z) - \sum_{i=1}^{n}\eta\left(\left(\tilde{\nabla}_W\tilde{R}\right)(e_i,Y)Z\right)\eta(e_i) = A(W)\tilde{S}(Y,Z). \tag{4.4}$$

The second term of (4.4) by putting $Z = \xi$ takes the form

$$\begin{aligned} g\left(\left(\tilde{\nabla}_W\tilde{R}\right)(e_i,Y)\xi,\xi\right) = {} & g\left(\tilde{\nabla}_W\tilde{R}(e_i,Y)\xi,\xi\right) - g\left(\tilde{R}\left(\tilde{\nabla}_W e_i,Y\right)\xi,\xi\right) \\ & - g\left(\tilde{R}\left(e_i,\tilde{\nabla}_W Y\right)\xi,\xi\right) - g\left(\tilde{R}(e_i,Y)\tilde{\nabla}_W\xi,\xi\right). \end{aligned} \tag{4.5}$$

On simplification we obtain

$$g\left(\left(\tilde{\nabla}_W\tilde{R}\right)(e_i,Y)Z,\xi\right) = 0. \tag{4.6}$$

Therefore (4.4) can be written in the form

$$\left(\tilde{\nabla}_W\tilde{S}\right)(Y,Z) = A(W)\tilde{S}(Y,Z). \tag{4.7}$$

Replacing Z by ξ in the above relation, then using (3.8) and (2.9), we have

$$\left(\tilde{\nabla}_W\tilde{S}\right)(Y,\xi) = -2(n-1)A(W)\eta(Y). \tag{4.8}$$

We know that

$$\left(\tilde{\nabla}_W\tilde{S}\right)(Y,\xi)=\tilde{\nabla}_W\tilde{S}(Y,\xi)-\tilde{S}\left(\tilde{\nabla}_WY,\xi\right)-\tilde{S}\left(Y,\tilde{\nabla}_W\xi\right). \tag{4.9}$$

Using (3.8), (2.6) and (2.9) in the above relation, we get

$$\left(\tilde{\nabla}_W\tilde{S}\right)(Y,\xi)=-4(n-1)g(Y,\phi W)-2S(Y,\phi W)+4g(Y,\phi W). \tag{4.10}$$

In view of (4.8) and (4.10), we obtain

$$-4(n-1)g(Y,\phi W)-2S(Y,\phi W)+4g(Y,\phi W)=-2(n-1)A(W)\eta(Y). \tag{4.11}$$

Replacing Y by ϕY in (4.11) and then using (2.3) and (2.10), we have

$$S(Y,W)=-2(n-2)g(Y,W)+(n-3)\eta(Y)\eta(W). \tag{4.12}$$

Hence, we can state the following.

Theorem 4.1. *If para-Sasakian manifold is ϕ-recurrent with respect to quarter-symmetric metric connection then it is an η-Einstein manifold with respect to Riemannian connection.*

5. Concircular ϕ-Recurrent Para-Sasakian Manifold with respect to Quarter-Symmetric Metric Connection

A concircular ϕ-recurrent para-Sasakian manifold with respect to the quarter-symmetric metric connection is defined by

$$\phi^2\left(\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z\right)=A(W)\tilde{\tilde{C}}(X,Y)Z, \tag{5.1}$$

where $\tilde{\tilde{C}}$ is a concircular curvature tensor with respect to the quarter-symmetric metric connection given by

$$\tilde{\tilde{C}}(X,Y)Z=\tilde{R}(X,Y)Z-\frac{\tilde{r}}{n(n-1)}\left[g(Y,Z)X-g(X,Z)Y\right]. \tag{5.2}$$

By virtue of (2.1) and (5.1), we have

$$\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z-\eta\left(\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z\right)\xi=A(W)\tilde{\tilde{C}}(X,Y)Z, \tag{5.3}$$

from which it follows that

$$g\left(\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z,U\right)-\eta\left(\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z\right)g(\xi,U)=A(W)g\left(\tilde{\tilde{C}}(X,Y)Z,U\right), \tag{5.4}$$

where

$$\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z = ((\nabla_W R)(X,Y)Z) + 6\left[g(\phi Y,Z)g(W,X) - g(\phi X,Z)g(W,Y)\right]\xi \\ + 6\left[\eta(Y)g(W,Z) + \eta(Z)g(W,Y)\right]\phi X \\ - 6\left[\eta(X)g(W,Z) + \eta(Z)g(W,X)\right]\phi Y \\ + 2\left[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)\right]\phi W \\ + 6\left[\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi X,Z)\right]W \\ + 12\eta(W)\eta(Z)\left[\eta(X)\phi Y - \eta(Y)\phi X\right] + \eta(Z)\left[g(W,Y)X - g(W,X)Y\right] \\ + 2\eta(W)\eta(Z)\left[\eta(X)Y - \eta(Y)X\right] \\ + 12\eta(W)\left[\eta(Y)g(\phi X,Z) - \eta(X)g(\phi Y,Z)\right]\xi \\ + \eta(W)\left[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)\right]\xi + \eta(Z)\left[g(\phi W,X)Y - g(\phi W,Y)X\right] \\ + g(W,Z)\left[\eta(Y)X - \eta(X)Y\right] + \left[g(W,X)g(Y,Z) - g(W,Y)g(X,Z)\right]\xi \\ - \left[g(\phi W,X)g(Y,Z) - g(\phi W,Y)g(X,Z)\right]\xi + g(\phi W,Z)\left[\eta(X)Y - \eta(Y)X\right] \\ - \frac{\nabla_W r}{n(n-1)}\left[g(Y,Z)X - g(X,Z)Y\right]. \tag{5.5}$$

Let $\{e_i\}$, $i = 1,2,\ldots,n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (5.4) and taking summation over i, $1 \le i \le n$, we get

$$(\nabla_W S)(Y,Z) = \frac{\nabla_W r}{n}g(Y,Z) + (n+4)\eta(Z)g(\phi W,Y) + (n+3)\eta(Y)g(\phi W,Z) \\ + (2n-3)\eta(W)\eta(Y)\eta(Z) - (n-1)\eta(Y)g(W,Z) \\ - \frac{\nabla_W r}{n(n-1)}\left[g(Y,Z) - \eta(Y)\eta(Z)\right] + A(W)S(Y,Z) \\ - A(W)\left\{(n+1)\eta(Y)\eta(Z) + \frac{r-(n+1)}{n}g(Y,Z)\right\}. \tag{5.6}$$

Replacing Z by ξ in (5.6) and using (2.9), we have

$$(\nabla_W S)(Y,\xi) = \frac{\nabla_W r}{n}\eta(Y) + (n+4)g(\phi W,Y) + (n-2)\eta(W)\eta(Y) \\ - A(W)\eta(Y)\left[2n + \frac{r-(n+1)}{n}\right]. \tag{5.7}$$

We know that

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W\xi). \tag{5.8}$$

Using (2.6) and (2.9) in the above relation, it follows that

$$(\nabla_W S)(Y,\xi) = -(n-1)\left[g(\phi W,Y)\right] - S(Y,\phi W). \tag{5.9}$$

In view of (5.7) and (5.9), we obtain

$$\begin{aligned} S(Y,\phi W) = &-(n-1)g(\phi W,Y) - \frac{\nabla_W r}{n}\eta(Y) - (n+4)g(\phi W,Y) \\ &- (n-2)\eta(W)\eta(Y) + A(W)\eta(Y)\left[2n + \frac{r-(n+1)}{n}\right]. \end{aligned} \tag{5.10}$$

Replacing Y by ϕY in (5.10) and then using (2.3) and (2.10), we obtain

$$S(Y,W) = -(2n+3)g(W,Y) + (n+4)\eta(W)\eta(Y). \tag{5.11}$$

This leads to the following theorem.

Theorem 5.1. *If para-Sasakian manifold is concircular ϕ-recurrent with respect to quarter-symmetric metric connection then it is an η-Einstein manifold with respect to Riemannian connection.*

Now from (5.3), we have

$$\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z = \eta\left(\left(\tilde{\nabla}_W\tilde{\tilde{C}}\right)(X,Y)Z\right)\xi + A(W)\tilde{\tilde{C}}(X,Y)Z. \tag{5.12}$$

This gives

$$\begin{aligned} ((\nabla_W R)(X,Y)Z) = \eta((\nabla_W R)(X,Y)Z)\xi &+ 6[\eta(Y)g(W,Z) - \eta(Z)g(W,Y)]\phi X \\ &+ 6[\eta(X)g(W,Z)+\eta(Z)g(W,X)]\phi Y + 2[\eta(X)g(Y,Z)-\eta(Y)g(X,Z)]\phi W \\ &- 6[\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi X,Z)]W - 2\eta(W)\eta(Z)[\eta(X)Y - \eta(Y)X] \\ &+ 12\eta(W)\eta(Z)[\eta(Y)\phi X - \eta(X)\phi Y] - \eta(Z)[g(W,Y)X - g(W,X)Y] \\ &+ \eta(Z)[g(\phi W,Y)X - g(\phi W,X)Y] + \eta(Z)[\eta(X)g(W,Y) - \eta(Y)g(W,X)]\xi \\ &+ \eta(Z)[\eta(Y)g(\phi W,X) - \eta(X)g(\phi W,Y)]\xi - g(W,Z)[\eta(Y)X - \eta(X)Y] \\ &+ g(\phi W,Z)[\eta(Y)X - \eta(X)Y] + 6\eta(W)[\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi X,Z)]\xi \\ &+ \frac{\nabla_W r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y - \eta(X)g(Y,Z)\xi + \eta(Y)g(X,Z)\xi] \\ &+ A(W)R(X,Y)Z + 3A(W)[g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X] \\ &+ A(W)\eta(Z)[\eta(X)Y - \eta(Y)X] - A(W)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi \\ &- \frac{r+(n-1)}{n(n-1)}A(W)[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{5.13}$$

Now from (5.13) and Bianchi's second identity, we have

$$\begin{aligned}
&A(W)\eta(R(X,Y)Z) + A(X)\eta(R(Y,W)Z) + A(Y)\eta(R(W,X)Z)\\
&\quad= \frac{(n+1)(n-1)+r}{n(n-1)}A(W)\left[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\right]\\
&\qquad+ \frac{(n+1)(n-1)+r}{n(n-1)}A(X)\left[\eta(Y)g(W,Z) - \eta(W)g(Y,Z)\right]\\
&\qquad+ \frac{(n+1)(n-1)+r}{n(n-1)}A(Y)\left[\eta(W)g(X,Z) - \eta(X)g(W,Z)\right].
\end{aligned} \tag{5.14}$$

By virtue of (2.7), we obtain from (5.14) that

$$\begin{aligned}
&A(W)\left[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)\right]\\
&\qquad+ A(X)\left[g(Y,Z)\eta(W) - g(W,Z)\eta(Y)\right]\\
&\qquad+ A(Y)\left[g(W,Z)\eta(X) - g(X,Z)\eta(W)\right]\\
&\quad= \frac{(n+1)(n-1)+r}{n(n-1)}A(W)\left[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\right]\\
&\qquad+ \frac{(n+1)(n-1)+r}{n(n-1)}A(X)\left[\eta(Y)g(W,Z) - \eta(W)g(Y,Z)\right]\\
&\qquad+ \frac{(n+1)(n-1)+r}{n(n-1)}A(Y)\left[\eta(W)g(X,Z) - \eta(X)g(W,Z)\right].
\end{aligned} \tag{5.15}$$

Putting $Y = Z = e_i$ in (5.15) and taking summation over i, $1 \le i \le n$, we get

$$A(W)\eta(X) = A(X)\eta(W), \tag{5.16}$$

for all vector fields X, W. Replacing X by ξ in (5.16), we get

$$A(W) = \eta(W)\eta(\rho), \tag{5.17}$$

for any vector field W.

Hence from (5.16) and (5.17), we can state the following.

Theorem 5.2. *In a concircular ϕ-recurrent para-Sasakian manifold with respect to quarter-symmetric metric connection, the characteristic vector field ξ and the vector field ρ associated to the 1-form A are in codirectional and the 1-form A is given by* (5.17).

Acknowledgments

The authors express their thanks to DST (Department of Science and Technology), Government of India, for providing financial assistance under major research project (no. SR/S4/MS: 482/07). They are also thankful to the referees for valuable suggestions.

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