

Research Article

Possibility Intuitionistic Fuzzy Soft Set

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Possibility intuitionistic fuzzy soft set and its operations are introduced, and a few of their properties are studied. An application of possibility intuitionistic fuzzy soft sets in decision making is investigated. A similarity measure of two possibility intuitionistic fuzzy soft sets has been discussed. An application of this similarity measure in medical diagnosis has been shown.

1. Introduction

In most real-life problems in social sciences, engineering, medical sciences, and economics the data involved are imprecise in nature. The solutions of such problems involve the use of mathematical principles based on uncertainty and imprecision. A number of theories have been proposed for dealing with uncertainties in an efficient way. Fuzzy set was introduced by Zadeh in [1] as a mathematical way to represent and deal with vagueness in everyday life. Then Atanassov [2] defined the concept of intuitionistic fuzzy set which is more general than fuzzy set. Molodtsov [3] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. Maji et al. [4, 5] have further studied the theory of soft sets and used this theory to solve some decision-making problems. Alkhazaleh et al. [6] introduced soft multiset as a generalisation of Molodtsov's soft set. Alkhazaleh and Salleh [7] defined the concept of soft expert set and they gave an application of this concept to decision making. Also Maji et al. [8] introduced the concept of fuzzy soft set and studied its properties and also Roy and Maji used this theory to solve some decision-making problems [9]. Majumdar and Samanta [10] defined and studied the generalised fuzzy soft sets, where the degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. In 2010 Baesho [11] introduced the concept of generalised intuitionistic fuzzy soft sets, where the degree is attached with the

parameterization of fuzzy sets while defining an intuitionistic fuzzy soft set (see also Baesho et al. [12]). Dinda et al. [13] introduced the same concept independently, and Agarwal et al. [14] introduced the same concept in 2011. Alkhazaleh et al. defined in [15] the concept of fuzzy parameterized interval-valued fuzzy soft set and gave its applications in decision making and medical diagnosis. Alkhazaleh et al. [16] defined the concept of possibility fuzzy soft set and gave its applications in decision making and medical diagnosis. Maji [17] defined the concept of intuitionistic fuzzy soft set, and Liang and Shi in [18] defined some new operations on intuitionistic fuzzy soft sets and studied some results relating to the properties of these operations. Salleh [19] gave a brief survey from soft set to intuitionistic fuzzy soft set. In this paper, we generalise the concept of possibility fuzzy soft set to the possibility intuitionistic fuzzy soft set. In our generalisation, a possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining an intuitionistic fuzzy soft set. We also give some applications of the possibility intuitionistic fuzzy soft set in decision-making problem and medical diagnosis.

2. Preliminaries

In this section we recall some definitions and properties regarding intuitionistic fuzzy soft set and a *possibility* fuzzy soft set required in this paper.

Let U be a universe set and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 (see [12]). Consider U and E as a universe set and a set of parameters, respectively. Let $P(U)$ denote the set of all intuitionistic fuzzy sets of U . Let $A \subseteq E$. A pair (F, E) is an intuitionistic fuzzy soft set over U , where F is mapping given by $F : A \rightarrow P(U)$.

Definition 2.2 (see [2]). An intuitionistic fuzzy set (IFS) A in a nonempty set U (a universe of discourse) is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in U \}$, where the functions $\mu_A(x) : U \rightarrow [0, 1]$, $\nu_A(x) : U \rightarrow [0, 1]$, denotes the degree of membership and degree of nonmembership of each element $x \in U$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in U$.

The following definitions and propositions are due to Alkhazaleh et al. [16].

Definition 2.3. Let $U = \{x_1, x_2, \dots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \dots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \rightarrow I^U$ and μ be a fuzzy subset of E , that is $\mu : E \rightarrow I^U$, where I^U is the collection of all fuzzy subsets of U . Let $F_\mu : E \rightarrow I^U \times I^U$ be a function defined as follows:

$$F_\mu(e) = (F(e)(x), \mu(e)(x)), \quad \forall x \in U. \quad (2.1)$$

Then F_μ is called a *possibility fuzzy soft set* (PFSS in short) over the soft universe (U, E) . For each parameter e_i , $F_\mu(e_i) = (F(e_i)(x), \mu(e_i)(x))$ indicates not only the degree of belongingness

of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $\mu(e_i)$. So one can write $F_\mu(e_i)$ as follows:

$$F_\mu(e_i) = \left\{ \left(\frac{x_1}{F(e_i)(x_1)}, \mu(e_i)(x_1) \right), \left(\frac{x_2}{F(e_i)(x_2)}, \mu(e_i)(x_2) \right), \dots, \left(\frac{x_n}{F(e_i)(x_n)}, \mu(e_i)(x_n) \right) \right\}. \quad (2.2)$$

Sometimes one writes F_μ as (F_μ, E) . If $A \subseteq E$ one can also have a PFSS (F_μ, A) .

Definition 2.4. Let F_μ and G_δ be two PFSSs over (U, E) . F_μ is said to be a *possibility fuzzy soft subset* (PFS subset) of G_δ and one writes $F_\mu \subseteq G_\delta$ if

- (i) $\mu(e)$ is a fuzzy subset of $\delta(e)$, for all $e \in E$,
- (ii) $F(e)$ is a fuzzy subset of $G(e)$, for all $e \in E$.

Definition 2.5. Let F_μ and G_δ be two PFSSs over (U, E) . Then F_μ and G_δ are said to be *equal* and we write $F_\mu = G_\delta$ if F_μ is a PFS subset of G_δ and G_δ is a PFS subset of F_μ . In other words, $F_\mu = G_\delta$ if the following conditions are satisfied:

- (i) $\mu(e)$ is equal to $\delta(e)$, for all $e \in E$,
- (ii) $F(e)$ is equal to $G(e)$, for all $e \in E$.

Definition 2.6. A PFSSs is said to be a *possibility null fuzzy soft set*, denoted by ϕ_0 , if $\phi_0 : E \rightarrow I^U \times I^U$ such that

$$\phi_0(e) = (F(e)(x), \mu(e)(x)), \quad \forall e \in E, \quad (2.3)$$

where $F(e) = 0$, and $\mu(e) = 0$, for all $e \in E$.

Definition 2.7. A PFSSs is said to be a *possibility absolute fuzzy soft set*, denoted by A_1 , if $A_1 : E \rightarrow I^U \times I^U$ such that

$$A_1(e) = (F(e)(x), \mu(e)(x)), \quad \forall e \in E, \quad (2.4)$$

where $F(e) = 1$ and $\mu(e) = 1$, for all $e \in E$.

Definition 2.8. Union of two PFSSs F_μ and G_δ , denoted by $F_\mu \tilde{\cup} G_\delta$, is a PFSSs $H_\nu : E \rightarrow I^U \times I^U$ defined by

$$H_\nu(e) = (H(e)(x), \nu(e)(x)), \quad \forall e \in E, \quad (2.5)$$

such that $H(e) = s(F(e), G(e))$, and $\nu(e) = s(\mu(e), \delta(e))$ where s is an s -norm.

Definition 2.9. Intersection of two PFSSs F_μ and G_δ , denoted by $F_\mu \tilde{\cap} G_\delta$, is a PFSSs $H_\nu : E \rightarrow I^U \times I^U$ defined by

$$H_\nu(e) = (H(e)(x), \nu(e)(x)), \quad \forall e \in E \quad (2.6)$$

such that $H(e) = t(F(e), G(e))$ and $\nu(e) = t(\mu(e), \delta(e))$ where t is a fuzzy t -norm.

Proposition 2.10. Let F_μ be a PFSSs over (U, E) . Then the following results hold:

- (i) $F_\mu \tilde{\cup} F_\mu = F_\mu$,
- (ii) $F_\mu \tilde{\cap} F_\mu = F_\mu$,
- (iii) $F_\mu \tilde{\cup} A_\mu = A_\mu$,
- (iv) $F_\mu \tilde{\cap} A_\mu = F_\mu$,
- (v) $F_\mu \tilde{\cup} \phi_\mu = F_\mu$,
- (vi) $F_\mu \tilde{\cap} \phi_\mu = \phi_\mu$.

Proposition 2.11. Let F_μ, G_δ , and H_ν be any three PFSSs over (U, E) , then the following results hold:

- (i) $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$,
- (ii) $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$,
- (iii) $F_\mu \tilde{\cup} (G_\delta \tilde{\cup} H_\nu) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cup} H_\nu$,
- (iv) $F_\mu \tilde{\cap} (G_\delta \tilde{\cap} H_\nu) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cap} H_\nu$.

Definition 2.12. Similarity between two GFSSs F_μ and G_δ , denoted by $S(F_\mu, G_\delta)$, is defined as follows:

$$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot m(\mu(e), \delta(e)) \quad (2.7)$$

such that, $M(F(e), G(e)) = \max_i M_i(F(e), G(e))$, where

$$M_i(F(e), G(e)) = 1 - \frac{\sum_{j=1}^n |F_{ij}(e) - G_{ij}(e)|}{\sum_{j=1}^n |F_{ij}(e) + G_{ij}(e)|}, \quad (2.8)$$

$$m(\mu(e), \delta(e)) = 1 - \frac{\sum |\mu(e) - \delta(e)|}{\sum |\mu(e) + \delta(e)|}.$$

Definition 2.13. Similarity between two PFSSs F_μ and G_δ , denoted by $S(F_\mu, G_\delta)$, is defined as follows:

$$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot M(\mu(e), \delta(e)), \quad (2.9)$$

such that, $M(F(e), G(e)) = \max_i M_i(F(e), G(e))$ and $M(\mu(e), \delta(e)) = \max_i M_i(\mu(e), \delta(e))$ where

$$M_i(F(e), G(e)) = 1 - \frac{\sum_{j=1}^n |F_{ij}(e) - G_{ij}(e)|}{\sum_{j=1}^n |F_{ij}(e) + G_{ij}(e)|}, \quad (2.10)$$

$$M_i(\mu(e), \delta(e)) = 1 - \frac{\sum_{j=1}^n |\mu_{ij}(e) - \delta_{ij}(e)|}{\sum_{j=1}^n |\mu_{ij}(e) + \delta_{ij}(e)|}. \quad (2.11)$$

Definition 2.14. If (F_μ, A) and (G_δ, B) are two PFSSs then “ (F_μ, A) AND (G_δ, B) ”, denoted by $(F_\mu, A) \wedge (G_\delta, B)$, is defined by

$$(F_\mu, A) \wedge (G_\delta, B) = (H_\lambda, A \times B), \quad (2.12)$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta)(x), \nu(\alpha, \beta)(x))$, for all $(\alpha, \beta) \in A \times B$, such that $H(\alpha, \beta) = t(F(\alpha), G(\beta))$ and $\nu(\alpha, \beta) = t(\mu(\alpha), \delta(\beta))$, for all $(\alpha, \beta) \in A \times B$, where t is a t -norm.

Definition 2.15. If (F_μ, A) and (G_δ, B) are two PFSSs then “ (F_μ, A) OR (G_δ, B) ”, denoted by $(F_\mu, A) \vee (G_\delta, B)$, is defined by

$$(F_\mu, A) \vee (G_\delta, B) = (H_\lambda, A \times B), \quad (2.13)$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta)(x), \nu(\alpha, \beta)(x))$, for all $(\alpha, \beta) \in A \times B$, such that $H(\alpha, \beta) = s(F(\alpha), G(\beta))$ and $\nu(\alpha, \beta) = s(\mu(\alpha), \delta(\beta))$, for all $(\alpha, \beta) \in A \times B$, where s is an s -norm.

Proposition 2.16. Let F_μ and G_δ be any two GFSSs over (U, E) . Then the following holds:

- (i) $S(F_\mu, G_\delta) = S(G_\delta, F_\mu)$,
- (ii) $0 \leq S(F_\mu, G_\delta) \leq 1$,
- (iii) $F_\mu = G_\delta \Rightarrow S(F_\mu, G_\delta) = 1$,
- (iv) $F_\mu \subseteq G_\delta \subseteq H_\lambda \Rightarrow S(F_\mu, H_\lambda) \leq S(G_\delta, H_\lambda)$,
- (v) $F_\mu \tilde{\cap} G_\delta = \phi \Leftrightarrow S(F_\mu, G_\delta) = 0$.

3. Possibility Intuitionistic Fuzzy Soft Sets

In this section we generalise the concept of possibility fuzzy soft sets as introduced by Alkhazaleh et al. [16]. In our generalisation of a possibility fuzzy soft set, a possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining an intuitionistic fuzzy soft set.

Definition 3.1. Let $U = \{x_1, x_2, \dots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \dots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \rightarrow (I \times I)^U \times I^U$ where $(I \times I)^U$ is the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all fuzzy subsets of U . Let p be a fuzzy subset of E , that is, $p : E \rightarrow I^U$ and let $F_p : E \rightarrow (I \times I)^U \times I^U$ be a function defined as follows:

$$F_p(e) = (F(e)(x), p(e)(x)), \quad \text{where } F(e)(x) = (\mu(x), \nu(x)) \quad \forall x \in U. \quad (3.1)$$

Then F_p is called a *possibility intuitionistic fuzzy soft set* (PIFSS in short) over the soft universe (U, E) . For each parameter e_i , $F_p(e_i) = (F(e_i)(x), p(e_i)(x))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $p(e_i)$. So we can write $F_p(e_i)$ as follows:

$$F_p(e_i) = \left\{ \left(\frac{x_1}{F(e_i)(x_1)}, p(e_i)(x_1) \right), \left(\frac{x_2}{F(e_i)(x_2)}, p(e_i)(x_2) \right), \dots, \left(\frac{x_n}{F(e_i)(x_n)}, p(e_i)(x_n) \right) \right\}. \quad (3.2)$$

Sometime we write F_p as (F_p, E) . If $A \subseteq E$ we can also have a PIFSS (F_p, A) .

Example 3.2. Let $U = \{x_1, x_2, x_3\}$ be a universe set. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters and let $p : E \rightarrow I^U$. We define a function $F_p : E \rightarrow (I \times I)^U \times I^U$ as follows:

$$\begin{aligned} F_p(e_1) &= \left\{ \left(\frac{x_1}{(0.4, 0.3)}, 0.7 \right), \left(\frac{x_2}{(0.7, 0.1)}, 0.5 \right), \left(\frac{x_3}{(0.5, 0.2)}, 0.6 \right) \right\}, \\ F_p(e_2) &= \left\{ \left(\frac{x_1}{(0.5, 0.1)}, 0.6 \right), \left(\frac{x_2}{(0.6, 0)}, 0.5 \right), \left(\frac{x_3}{(0.6, 0.3)}, 0.5 \right) \right\}, \\ F_p(e_3) &= \left\{ \left(\frac{x_1}{(0.7, 0)}, 0.5 \right), \left(\frac{x_2}{(0.6, 0.2)}, 0.5 \right), \left(\frac{x_3}{(0.5, 0.1)}, 0.7 \right) \right\}. \end{aligned} \quad (3.3)$$

Then F_p is a PIFSS over (U, E) . In matrix notation we write

$$F_p = \begin{pmatrix} (0.4, 0.3), 0.7 & (0.7, 0.1), 0.5 & (0.5, 0.2), 0.6 \\ (0.5, 0.1), 0.6 & (0.6, 0), 0.5 & (0.6, 0.3), 0.5 \\ (0.7, 0), 0.5 & (0.6, 0.2), 0.5 & (0.5, 0.1), 0.7 \end{pmatrix}. \quad (3.4)$$

Definition 3.3. Let F_p and G_q be two PIFSSs over (U, E) . F_p is said to be a *possibility intuitionistic fuzzy soft subset* (PIFS subset) of G_q and one writes $F_p \subseteq G_q$ if

- (i) $p(e)$ is a fuzzy subset of $q(e)$, for all $e \in E$,
- (ii) $F(e)$ is an intuitionistic fuzzy subset of $G(e)$, for all $e \in E$.

Example 3.4. Let $U = \{x_1, x_2, x_3\}$ be a universe set. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters and let $p : E \rightarrow I^U$. We define a function $F_p : E \rightarrow (I \times I)^U \times I^U$ as follows:

$$\begin{aligned} F_p(e_1) &= \left\{ \left(\frac{x_1}{(0.2, 0.1)}, 0.4 \right), \left(\frac{x_2}{(0.6, 0.3)}, 0.5 \right), \left(\frac{x_3}{(0.5, 0.4)}, 0.6 \right) \right\}, \\ F_p(e_2) &= \left\{ \left(\frac{x_1}{(0.7, 0.2)}, 0.5 \right), \left(\frac{x_2}{(0.6, 0.4)}, 0.6 \right), \left(\frac{x_3}{(0.8, 0.2)}, 0.6 \right) \right\}, \\ F_p(e_3) &= \left\{ \left(\frac{x_1}{(0, 0.7)}, 0.1 \right), \left(\frac{x_2}{(0.5, 0.1)}, 0.3 \right), \left(\frac{x_3}{(0.3, 0.3)}, 0.1 \right) \right\}. \end{aligned} \quad (3.5)$$

Let $G_q : E \rightarrow (I \times I)^U \times I^U$ be another PIFSS over (U, E) defined as follows:

$$\begin{aligned} G_q(e_1) &= \left\{ \left(\frac{x_1}{(0.3, 0)}, 0.6 \right), \left(\frac{x_2}{(0.7, 0.2)}, 0.6 \right), \left(\frac{x_3}{(0.6, 0.3)}, 0.7 \right) \right\}, \\ G_q(e_2) &= \left\{ \left(\frac{x_1}{(0.8, 0.1)}, 0.6 \right), \left(\frac{x_2}{(0.7, 0.3)}, 0.7 \right), \left(\frac{x_3}{(0.9, 0.1)}, 0.8 \right) \right\}, \\ G_q(e_3) &= \left\{ \left(\frac{x_1}{(0.1, 0.6)}, 0.2 \right), \left(\frac{x_2}{(0.6, 0)}, 0.5 \right), \left(\frac{x_3}{(0.5, 0.1)}, 0.2 \right) \right\}. \end{aligned} \quad (3.6)$$

It is clear that F_p is a PIFS subset of G_q .

Definition 3.5. Let F_p and G_q be two PIFSSs over (U, E) . Then F_p and G_q are said to be *equal* and one writes $F_p = G_q$ if F_p is a PIFS subset of G_q and G_q is a PIFS subset of F_p .

In other words, $F_p = G_q$ if the following conditions are satisfied:

- (i) $p(e)$ is equal to $q(e)$, for all $e \in E$,
- (ii) $F(e)$ is equal to $G(e)$, for all $e \in E$.

Definition 3.6. A PIFSS is said to be a *possibility null intuitionistic fuzzy soft set*, denoted by ϕ_0 , if $\phi_0 : E \rightarrow (I \times I)^U \times I^U$ such that

$$\phi_0(e) = (F(e)(x), p(e)(x)), \quad \forall e \in E, \quad (3.7)$$

where $F(e) = (0, v(e))$, and $p(e) = 0$, for all $e \in E$.

Definition 3.7. A PIFSS is said to be a *possibility absolute intuitionistic fuzzy soft set*, denoted by A_1 , if $A_1 : E \rightarrow (I \times I)^U \times I^U$ such that

$$A_1(e) = (F(e)(x), P(e)(x)), \quad \forall e \in E, \quad (3.8)$$

where $F(e) = (1, 0)$ and $P(e) = 1$, for all $e \in E$.

Example 3.8. Let $U = \{x_1, x_2, x_3\}$ be a universe set. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters and let $p : E \rightarrow I^U$. We define a function $F_p : E \rightarrow (I \times I)^U \times I^U$ as follows:

$$\begin{aligned} F_p(e_1) &= \left\{ \left(\frac{x_1}{(0,0.3)}, 0 \right), \left(\frac{x_2}{(0,0.5)}, 0 \right), \left(\frac{x_3}{(0,0.6)}, 0 \right) \right\}, \\ F_p(e_2) &= \left\{ \left(\frac{x_1}{(0,1)}, 0 \right), \left(\frac{x_2}{(0,0.2)}, 0 \right), \left(\frac{x_3}{(0,0.3)}, 0 \right) \right\}, \\ F_p(e_3) &= \left\{ \left(\frac{x_1}{(0,0.2)}, 0 \right), \left(\frac{x_2}{(0,0.6)}, 0 \right), \left(\frac{x_3}{(0,0.9)}, 0 \right) \right\}. \end{aligned} \quad (3.9)$$

Then F_p is a possibility null intuitionistic fuzzy soft set.

Let $p : E \rightarrow I^U$, and we define the function $F_p : E \rightarrow (I \times I)^U \times I^U$ which is a PIFSS over (U, E) as follows:

$$\begin{aligned} F_p(e_1) &= \left\{ \left(\frac{x_1}{(1,0)}, 1 \right), \left(\frac{x_2}{(1,0)}, 1 \right), \left(\frac{x_3}{(1,0)}, 1 \right) \right\}, \\ F_p(e_2) &= \left\{ \left(\frac{x_1}{(1,0)}, 1 \right), \left(\frac{x_2}{(1,0)}, 1 \right), \left(\frac{x_3}{(1,0)}, 1 \right) \right\}, \\ F_p(e_3) &= \left\{ \left(\frac{x_1}{(1,0)}, 1 \right), \left(\frac{x_2}{(1,0)}, 1 \right), \left(\frac{x_3}{(1,0)}, 1 \right) \right\}. \end{aligned} \quad (3.10)$$

Then F_p is a possibility absolute intuitionistic fuzzy soft set.

4. Union and Intersection of PIFSS

In this section we introduce the definitions of union and intersection of PIFSS, derive some properties and give some examples.

Definition 4.1. Union of two PIFSSs F_p and G_q , denoted by $F_p \tilde{\cup} G_q$, is a PIFSSs $H_r : E \rightarrow (I \times I)^U \times I^U$ defined by

$$H_r(e) = (H(e)(x), r(e)(x)), \quad \forall e \in E, \quad (4.1)$$

such that $H(e) = \bigcup_{\text{Atan}} (F(e), G(e))$ and $r(e) = s(p(e), q(e))$, where \bigcup_{Atan} is Atanassov union and s is an s -norm.

Example 4.2. Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$. Let F_μ be a PIFSS defined as follows:

$$\begin{aligned} F_p(e_1) &= \left\{ \left(\frac{x_1}{(0.3, 0.4)}, 0.1 \right), \left(\frac{x_2}{(0.7, 0.1)}, 0.4 \right), \left(\frac{x_3}{(0.2, 0.6)}, 0.6 \right) \right\}, \\ F_p(e_2) &= \left\{ \left(\frac{x_1}{(0.2, 0.6)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.5)}, 0.2 \right), \left(\frac{x_3}{(0.1, 0.3)}, 0.4 \right) \right\}, \\ F_p(e_3) &= \left\{ \left(\frac{x_1}{(0.7, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.5)}, 0 \right), \left(\frac{x_3}{(0.5, 0.3)}, 0.6 \right) \right\}. \end{aligned} \quad (4.2)$$

Let G_q be another PIFSS over (U, E) defined as follows:

$$\begin{aligned} G_q(e_1) &= \left\{ \left(\frac{x_1}{(0.1, 0.4)}, 0.3 \right), \left(\frac{x_2}{(0.3, 0.3)}, 0.6 \right), \left(\frac{x_3}{(0, 0.5)}, 0.2 \right) \right\}, \\ G_q(e_2) &= \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.6, 0.1)}, 0.6 \right), \left(\frac{x_3}{(0.7, 0)}, 0.6 \right) \right\}, \\ G_q(e_3) &= \left\{ \left(\frac{x_1}{(0, 0)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.1)}, 0.8 \right), \left(\frac{x_3}{(0.3, 0.1)}, 0.2 \right) \right\}. \end{aligned} \quad (4.3)$$

By using the Atanassov union which is the basic intuitionistic fuzzy union we have

$$\begin{aligned} H_r(e_1) &= \left\{ \left(\frac{x_1}{(\max(0.3, 0.1), \min(0.4, 0.4))}, \max(0.1, 0.3) \right), \right. \\ &\quad \left(\frac{x_1}{(\max(0.7, 0.3), \min(0.1, 0.3))}, \max(0.4, 0.6) \right), \\ &\quad \left. \left(\frac{x_1}{(\max(0.2, 0), \min(0.6, 0.5))}, \max(0.6, 0.2) \right) \right\}, \\ H_r(e_1) &= \left\{ \left(\frac{x_1}{(0.3, 0.4)}, 0.3 \right), \left(\frac{x_2}{(0.7, 0.1)}, 0.6 \right), \left(\frac{x_3}{(0.2, 0.5)}, 0.6 \right) \right\}. \end{aligned} \quad (4.4)$$

Similarly we get

$$\begin{aligned} H_r(e_2) &= \left\{ \left(\frac{x_1}{(0.2, 0.1)}, 0.3 \right), \left(\frac{x_2}{(0.6, 0.1)}, 0.6 \right), \left(\frac{x_3}{(0.7, 0)}, 0.6 \right) \right\}, \\ H_r(e_3) &= \left\{ \left(\frac{x_1}{(0.7, 0)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.1)}, 0.8 \right), \left(\frac{x_3}{(0.5, 0.1)}, 0.6 \right) \right\}. \end{aligned} \quad (4.5)$$

Remark 4.3. The Atanassov union can be replaced by any S-norm which is a general intuitionistic fuzzy union (see Fathi [20]).

Definition 4.4. Intersection of two PIFSSs F_p and G_q , denoted by $F_p \tilde{\cap} G_q$, is a PIFSSs $H_r : E \rightarrow (I \times I)^U \times I^U$ defined by

$$H_r(e) = (H(e)(x), r(e)(x)), \quad \forall e \in E \quad (4.6)$$

such that $H(e) = \underset{\text{Atan}}{\cap} (F(e), G(e))$ and $r(e) = t(p(e), q(e))$, where $\underset{\text{Atan}}{\cap}$ is Atanassov intersection and t is a t -norm.

Example 4.5. Consider Example 4.2 where F_p and G_q are PIFSSs defined as follows:

$$\begin{aligned} F_p(e_1) &= \left\{ \left(\frac{x_1}{(0.3, 0.2)}, 0.2 \right), \left(\frac{x_2}{(0.4, 0.1)}, 0.3 \right), \left(\frac{x_3}{(0, 0.1)}, 0.4 \right) \right\}, \\ F_p(e_2) &= \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.6)}, 0.4 \right), \left(\frac{x_3}{(0.3, 0)}, 0.2 \right) \right\}, \\ F_p(e_3) &= \left\{ \left(\frac{x_1}{(0.7, 0.1)}, 0.3 \right), \left(\frac{x_2}{(0.1, 0.2)}, 0.1 \right), \left(\frac{x_3}{(0.4, 0.1)}, 0.4 \right) \right\}, \\ G_q(e_1) &= \left\{ \left(\frac{x_1}{(0.7, 0.1)}, 0.3 \right), \left(\frac{x_2}{(0.1, 0.2)}, 0.1 \right), \left(\frac{x_3}{(0.4, 0.1)}, 0.4 \right) \right\}, \\ G_q(e_2) &= \left\{ \left(\frac{x_1}{(0.1, 0.6)}, 0.1 \right), \left(\frac{x_2}{(0.3, 0.2)}, 0.3 \right), \left(\frac{x_3}{(0.8, 0)}, 0.4 \right) \right\}, \\ G_q(e_3) &= \left\{ \left(\frac{x_1}{(0, 0.5)}, 0.4 \right), \left(\frac{x_2}{(0.1, 0.4)}, 0.1 \right), \left(\frac{x_3}{(0.9, 0)}, 0.3 \right) \right\}. \end{aligned} \quad (4.7)$$

By using the Atanassov intersection which is the basic intuitionistic fuzzy union we have

$$\begin{aligned} H_r(e_1) &= \left\{ \left(\frac{x_1}{(\min(0.3, 0.7), \max(0.2, 0.1))}, \min(0.2, 0.3) \right), \right. \\ &\quad \left(\frac{x_1}{(\min(0.4, 0.1), \max(0.1, 0.2))}, \min(0.3, 0.1) \right), \\ &\quad \left. \left(\frac{x_1}{(\min(0, 0.4), \max(0.1, 0.1))}, \min(0.4, 0.4) \right) \right\}, \\ &= \left\{ \left(\frac{x_1}{(0.3, 0.2)}, 0.2 \right), \left(\frac{x_2}{(0.1, 0.2)}, 0.1 \right), \left(\frac{x_3}{(0, 0.1)}, 0.4 \right) \right\}. \end{aligned} \quad (4.8)$$

Similarly we get

$$\begin{aligned} H_r(e_2) &= \left\{ \left(\frac{x_1}{(0, 0.6)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.6)}, 0.3 \right), \left(\frac{x_3}{(0.3, 0)}, 0.2 \right) \right\}, \\ H_r(e_3) &= \left\{ \left(\frac{x_1}{(0, 0.5)}, 0.3 \right), \left(\frac{x_2}{(0.1, 0.4)}, 0.1 \right), \left(\frac{x_3}{(0.4, 0.1)}, 0.3 \right) \right\}. \end{aligned} \quad (4.9)$$

Remark 4.6. The Atanassov intersection can be replaced by any T -norm which is a general intuitionistic fuzzy intersection (see Fathi [20]).

Proposition 4.7. *Let $F_p, G_q,$ and H_r be any three PIFSSs over (U, E) , then the following results hold:*

- (i) $F_p \tilde{\cup} G_q = G_p \tilde{\cup} F_q,$
- (ii) $F_p \tilde{\cap} G_q = G_p \tilde{\cap} F_q,$
- (iii) $F_p \tilde{\cup} (G_q \tilde{\cup} H_r) = (F_p \tilde{\cup} G_q) \tilde{\cup} H_r,$
- (iv) $F_p \tilde{\cap} (G_q \tilde{\cap} H_r) = (F_p \tilde{\cap} G_q) \tilde{\cap} H_r.$

Proof. The proof is straightforward by using the fact that the union and intersection of fuzzy sets and intuitionistic fuzzy sets are commutative and associative. \square

Proposition 4.8. *Let F_p be a PFSSs over (U, E) . Then the following results hold:*

- (i) $F_p \tilde{\cup} F_p = F_p,$
- (ii) $F_p \tilde{\cap} F_p = F_p,$
- (iii) $F_p \tilde{\cup} A_p = A_p,$
- (iv) $F_p \tilde{\cap} A_p = F_p.$

Proof. The proof is straightforward by using the definitions of union and intersection. \square

Proposition 4.9. *Let $F_p, G_q,$ and H_r be any three PIFSS over (U, E) . Then the following results hold:*

- (i) $F_p \tilde{\cup} (G_q \tilde{\cap} H_r) = (F_p \tilde{\cup} G_q) \tilde{\cap} (F_p \tilde{\cup} H_r),$
- (ii) $F_p \tilde{\cap} (G_q \tilde{\cup} H_r) = (F_p \tilde{\cap} G_q) \tilde{\cup} (F_p \tilde{\cap} H_r).$

Proof. (i) For all $x \in E,$

$$\begin{aligned}
 \lambda_{F(x)\tilde{\cup}(G(x)\tilde{\cap}H(x))}(x) &= \cup\{\lambda_{F(x)}(x), \lambda_{(G(x)\tilde{\cap}H(x))}(x)\} \\
 &= \cup\{\lambda_{F(x)}(x), \cap(\lambda_{G(x)}(x), \lambda_{H(x)}(x))\} \\
 &= \{\langle x, \max(\mu_{F(x)}(x), \min(\mu_{G(x)}(x), \mu_{H(x)}(x))), \\
 &\quad \min(\nu_{F(x)}, \max(\nu_{G(x)}(x), \nu_{H(x)}(x))) \rangle : x \in U\} \\
 &= \{\langle x, \min(\max(\mu_{F(x)}(x), \mu_{G(x)}(x)), \max(\mu_{F(x)}(x), \mu_{H(x)}(x))), \\
 &\quad \max(\min(\nu_{F(x)}(x), \nu_{G(x)}(x)), \min(\nu_{F(x)}(x), \nu_{H(x)}(x))) \rangle : x \in U\} \\
 &= \cap(\cup(\lambda_{F(x)\cup G(x)}(x)), \lambda_{F(x)\cup H(x)}(x)) \\
 &= \lambda_{(F(x)\cup G(x))\cap(F(x)\cup H(x))}(x),
 \end{aligned}$$

(4.10)

$$\begin{aligned}
\gamma_{\mu(x)\tilde{\cup}(\delta(x)\tilde{\cap}v(x))}(x) &= \max\{\gamma_{p(x)}(x), \gamma_{(q(x)\tilde{\cap}r(x))}(x)\} \\
&= \max\{\gamma_{p(x)}(x), \min(\gamma_{q(x)}(x), \gamma_{r(x)}(x))\} \\
&= \min\{\max(\gamma_{p(x)}(x), \gamma_{q(x)}(x)), \max(\gamma_{p(x)}(x), \gamma_{r(x)}(x))\} \\
&= \min\{\gamma_{(\mu(x)\cup\delta(x))}(x), \gamma_{(\mu(x)\cup v(x))}(x)\} \\
&= \gamma_{(\mu(x)\cup\delta(x))\cap(\mu(x)\cup v(x))}(x).
\end{aligned} \tag{4.11}$$

We can use the same method in (i). □

5. AND and OR Operations on PIFSS with Applications in Decision Making

In this section we introduce the definitions of AND and OR operations on possibility intuitionistic fuzzy soft sets. Applications of possibility fuzzy soft sets in decision-making problem are given.

Definition 5.1. If (F_μ, A) and (G_δ, B) , are two PIFSSs then “ (F_μ, A) AND (G_δ, B) ” denoted by $(F_\mu, A) \wedge (G_\delta, B)$ is defined by

$$(F_\mu, A) \wedge (G_\delta, B) = (H_\lambda, A \times B), \tag{5.1}$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta)(x), \wedge(\alpha, \beta)(x))$, for all $(\alpha, \beta) \in A \times B$, such that $H(\alpha, \beta) = T(F(\alpha), G(\beta))$ and $\wedge(\alpha, \beta) = t(\mu(\alpha), \delta(\beta))$, for all $(\alpha, \beta) \in A \times B$, where T is a T -norm and t is a t -norm.

Example 5.2. Suppose the universe consists of three machines x_1, x_2, x_3 that is, $U = \{x_1, x_2, x_3\}$ and there are three parameters $E = \{e_1, e_2, e_3\}$ which describe their performances according to certain specific task. Suppose a firm wants to buy one such machine depending on any two of the parameters only. Let there be two observations F_μ and G_δ by two experts defined as follows:

$$\begin{aligned}
F_\mu(e_1) &= \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.6)}, 0.4 \right), \left(\frac{x_3}{(0.3, 0)}, 0.2 \right) \right\}, \\
F_\mu(e_2) &= \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.2 \right), \left(\frac{x_2}{(0.1, 0.3)}, 0.6 \right), \left(\frac{x_3}{(0.8, 0)}, 0.1 \right) \right\}, \\
F_\mu(e_3) &= \left\{ \left(\frac{x_1}{(0.3, 0.1)}, 0.4 \right), \left(\frac{x_2}{(0, 0.5)}, 0.1 \right), \left(\frac{x_3}{(0.5, 0)}, 0.2 \right) \right\}, \\
G_\delta(e_1) &= \left\{ \left(\frac{x_1}{(0.1, 0.6)}, 0.1 \right), \left(\frac{x_2}{(0.3, 0.2)}, 0.3 \right), \left(\frac{x_3}{(0.8, 0)}, 0.4 \right) \right\}, \\
G_\delta(e_2) &= \left\{ \left(\frac{x_1}{(0, 0)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.4)}, 0.2 \right), \left(\frac{x_3}{(0.6, 0)}, 0.1 \right) \right\}, \\
G_\delta(e_3) &= \left\{ \left(\frac{x_1}{(0.6, 0)}, 0.7 \right), \left(\frac{x_2}{(0.3, 0.1)}, 0.4 \right), \left(\frac{x_3}{(0, 0)}, 0.4 \right) \right\}.
\end{aligned} \tag{5.2}$$

By using the Atanassov intersection which is the basic intuitionistic fuzzy union we have

$$H_v(e_1, e_1) = \left\{ \left(\frac{x_1}{(\min(0, 0.1), \max(0.1, 0.6))}, \min(0.1, 0.1) \right), \right. \\ \left(\frac{x_1}{(\min(0.2, 0.3), \max(0.6, 0.2))}, \min(0.4, 0.3) \right), \\ \left. \left(\frac{x_1}{(\min(0.3, 0.8), \max(0, 0))}, \min(0.2, 0.4) \right) \right\}. \tag{5.3}$$

$$H_\lambda(e_1, e_1) = \left\{ \left(\frac{x_1}{(0, 0.6)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.6)}, 0.3 \right), \left(\frac{x_3}{(0.3, 0)}, 0.2 \right) \right\}.$$

Similarly we get

$$H_\lambda(e_1, e_2) = \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.6)}, 0.2 \right), \left(\frac{x_3}{(0.3, 0)}, 0.1 \right) \right\}, \\ H_\lambda(e_1, e_3) = \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.7 \right), \left(\frac{x_2}{(0.2, 0.6)}, 0.4 \right), \left(\frac{x_3}{(0, 0)}, 0.2 \right) \right\}, \\ H_\lambda(e_2, e_1) = \left\{ \left(\frac{x_1}{(0, 0.6)}, 0.1 \right), \left(\frac{x_2}{(0.1, 0.3)}, 0.3 \right), \left(\frac{x_3}{(0.6, 0)}, 0.1 \right) \right\}, \\ H_\lambda(e_2, e_2) = \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.2 \right), \left(\frac{x_2}{(0.1, 0.4)}, 0.2 \right), \left(\frac{x_3}{(0.6, 0)}, 0.1 \right) \right\}, \\ H_\lambda(e_2, e_3) = \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.2 \right), \left(\frac{x_2}{(0.1, 0.3)}, 0.4 \right), \left(\frac{x_3}{(0, 0)}, 0.1 \right) \right\}, \\ H_\lambda(e_3, e_1) = \left\{ \left(\frac{x_1}{(0.1, 0.6)}, 0.1 \right), \left(\frac{x_2}{(0, 0.5)}, 0.1 \right), \left(\frac{x_3}{(0.5, 0)}, 0.2 \right) \right\}, \\ H_\lambda(e_3, e_2) = \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.3 \right), \left(\frac{x_2}{(0, 0.5)}, 0.1 \right), \left(\frac{x_3}{(0.5, 0)}, 0.1 \right) \right\}, \\ H_\lambda(e_3, e_3) = \left\{ \left(\frac{x_1}{(0.3, 0.1)}, 0.4 \right), \left(\frac{x_2}{(0, 0.5)}, 0.1 \right), \left(\frac{x_3}{(0, 0)}, 0.2 \right) \right\}. \tag{5.4}$$

In matrix notation we have

$$(F_\mu, A) \wedge (G_\delta, B) = \begin{pmatrix} ((0, 0.6), 0.1) & ((0.2, 0.6), 0.3) & ((0.3, 0), 0.2) \\ ((0, 0.1), 0.1) & ((0.2, 0.6), 0.2) & ((0.3, 0), 0.1) \\ ((0, 0.1), 0.7) & ((0.2, 0.6), 0.4) & ((0, 0), 0.2) \\ ((0, 0.6), 0.1) & ((0.1, 0.3), 0.3) & ((0.6, 0), 0.1) \\ ((0, 0.1), 0.2) & ((0.1, 0.4), 0.2) & ((0.6, 0), 0.1) \\ ((0, 0.1), 0.2) & ((0.1, 0.3), 0.4) & ((0, 0), 0.1) \\ ((0.1, 0.6), 0.1) & ((0, 0.5), 0.1) & ((0.5, 0), 0.2) \\ ((0, 0.1), 0.3) & ((0, 0.5), 0.1) & ((0.5, 0), 0.1) \\ ((0.3, 0.1), 0.4) & ((0, 0.5), 0.1) & ((0, 0), 0.2) \end{pmatrix}. \tag{5.5}$$

Now to determine the best machine we first calculate the difference between the membership and non-membership values and then we mark the highest numerical grade (indicated in parenthesis) in each row. Now the score of each of such machine is calculated by taking the sum of the products of these different numerical grades with the corresponding value of λ . The machine with the highest score is the desired machine. We do not consider the different numerical grades of the machine against the pairs (e_i, e_i) , $i = 1, 2, 3$, as both parameters are the same.

Matrix of different numerical grades is shown below:

$$\begin{pmatrix} (-0.6, 0.1) & (-0.4, 0.3) & (0.3, 0.2) \\ (-0.1, 0.1) & (-0.4, 0.2) & (0.3, 0.1) \\ (-0.1, 0.7) & (-0.4, 0.4) & (0, 0.2) \\ (-0.6, 0.1) & (-0.2, 0.3) & (0.6, 0.1) \\ (-0.1, 0.2) & (-0.3, 0.2) & (0.6, 0.1) \\ (-0.1, 0.2) & (-0.2, 0.4) & (0, 0.1) \\ (-0.5, 0.1) & (-0.5, 0.1) & (0.5, 0.2) \\ (-0.1, 0.3) & (-0.5, 0.1) & (0.5, 0.1) \\ (0.2, 0.4) & (-0.5, 0.1) & (0, 0.2) \end{pmatrix}. \tag{5.6}$$

Grade Table

H	x_i	highest numerical grade	λ_i
(e_1, e_1)	x_3	×	×
(e_1, e_2)	x_3	0.3	0.1
(e_1, e_3)	x_3	0	0.2
(e_2, e_1)	x_3	0.6	0.1
(e_2, e_2)	x_3	×	×
(e_2, e_3)	x_3	0	0.1
(e_3, e_1)	x_3	0.5	0.2
(e_3, e_2)	x_3	0.5	0.1
(e_3, e_3)	x_1	×	×

$$\tag{5.7}$$

Score $(x_1) = 0$,

Score $(x_2) = 0$,

Score $(x_3) = (0.3 \times 0.1) + (0 \times 0.2) + (0.6 \times 0.1) + (0 \times 0.1) + (0.5 \times 0.2) \times (0.5 \times 0.1) = 0.24$.

Then the firm will select the machine with the highest score. Hence, they will buy machine x_3 .

Definition 5.3. If (F_μ, A) and (G_δ, B) are two PIFSS then “ (F_μ, A) OR (G_δ, B) ”, denoted by $(F_\mu, A) \vee (G_\delta, B)$, is defined by

$$(F_\mu, A) \vee (G_\delta, B) = (H_\lambda, A \times B), \tag{5.8}$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta)(x), \vee(\alpha, \beta)(x))$ for all $(\alpha, \beta) \in A \times B$, such that $H(\alpha, \beta) = S(F(\alpha), G(\beta))$ and $\vee(\alpha, \beta) = s(\mu(\alpha), \delta(\beta))$, for all $(\alpha, \beta) \in A \times B$, where S is an S -norm and s is an s -norm.

Example 5.4. Let $U = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2, e_3\}$ consider F_μ and G_δ as in Example 5.2. Suppose now the firm wants to buy a machine depending on any one of two parameters. Then we have $(F_\mu, A) \vee (G_\delta, B) = (H_\lambda, A \times B)$ where

$$\begin{aligned} H(e_1, e_1) &= \left\{ \left(\frac{x_1}{(\max(0, 0.1), \min(0.1, 0.6))}, \max(0.1, 0.1) \right), \right. \\ &\quad \left(\frac{x_1}{(\max(0.2, 0.3), \min(0.6, 0.2))}, \max(0.4, 0.3) \right), \\ &\quad \left. \left(\frac{x_1}{(\max(0.3, 0.8), \min(0, 0))}, \max(0.2, 0.4) \right) \right\}, \\ H_\lambda(e_1, e_1) &= \left\{ \left(\frac{x_1}{(0.1, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.3, 0.2)}, 0.4 \right), \left(\frac{x_3}{(0.8, 0)}, 0.4 \right) \right\}. \end{aligned} \tag{5.9}$$

Similarly we get

$$\begin{aligned} H_\lambda(e_1, e_2) &= \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.4)}, 0.4 \right), \left(\frac{x_3}{(0.6, 0)}, 0.2 \right) \right\}, \\ H_\lambda(e_1, e_3) &= \left\{ \left(\frac{x_1}{(0.6, 0)}, 0.7 \right), \left(\frac{x_2}{(0.3, 0.1)}, 0.4 \right), \left(\frac{x_3}{(0.3, 0)}, 0.4 \right) \right\}, \\ H_\lambda(e_2, e_1) &= \left\{ \left(\frac{x_1}{(0.1, 0.1)}, 0.2 \right), \left(\frac{x_2}{(0.2, 0.3)}, 0.6 \right), \left(\frac{x_3}{(0.8, 0)}, 0.4 \right) \right\}, \\ H_\lambda(e_2, e_2) &= \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.3)}, 0.6 \right), \left(\frac{x_3}{(0.8, 0)}, 0.1 \right) \right\}, \\ H_\lambda(e_2, e_3) &= \left\{ \left(\frac{x_1}{(0.6, 0)}, 0.7 \right), \left(\frac{x_2}{(0.3, 0.1)}, 0.6 \right), \left(\frac{x_3}{(0.8, 0)}, 0.4 \right) \right\}, \\ H_\lambda(e_3, e_1) &= \left\{ \left(\frac{x_1}{(0.3, 0.1)}, 0.4 \right), \left(\frac{x_2}{(0.3, 0.2)}, 0.1 \right), \left(\frac{x_3}{(0.8, 0)}, 0.4 \right) \right\}, \\ H_\lambda(e_3, e_2) &= \left\{ \left(\frac{x_1}{(0.3, 0)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.4)}, 0.2 \right), \left(\frac{x_3}{(0.6, 0)}, 0.2 \right) \right\}, \\ H_\lambda(e_3, e_3) &= \left\{ \left(\frac{x_1}{(0.6, 0)}, 0.7 \right), \left(\frac{x_2}{(0.3, 0.1)}, 0.4 \right), \left(\frac{x_3}{(0.5, 0)}, 0.4 \right) \right\}. \end{aligned} \tag{5.10}$$

In matrix notation we have

$$(F_{\mu}, A) \vee (G_{\delta}, B) = \begin{pmatrix} ((0.1, 0.1), 0.1) & ((0.3, 0.2), 0.4) & ((0.8, 0), 0.4) \\ ((0, 0.1), 0.3) & ((0.2, 0.4), 0.4) & ((0.6, 0), 0.2) \\ ((0.6, 0), 0.7) & ((0.3, 0.1), 0.4) & ((0.3, 0), 0.4) \\ ((0.1, 0.1), 0.2) & ((0.2, 0.3), 0.6) & ((0.8, 0), 0.4) \\ ((0, 0.1), 0.3) & ((0.2, 0.3), 0.6) & ((0.8, 0), 0.1) \\ ((0.6, 0), 0.7) & ((0.3, 0.1), 0.6) & \underline{((0.8, 0), 0.4)} \\ ((0.3, 0.1), 0.4) & \underline{((0.3, 0.2), 0.1)} & ((0.8, 0), 0.4) \\ ((0.3, 0), 0.1) & ((0.2, 0.4), 0.2) & \underline{((0.6, 0), 0.2)} \\ ((0.6, 0), 0.7) & ((0.3, 0.1), 0.4) & ((0.5, 0), 0.4) \end{pmatrix}. \quad (5.11)$$

Now to determine the best machine we first calculate the difference between the membership and non-membership values and then we mark the highest numerical grade (indicated in parenthesis) in each row. Now the score of each of such machine is calculated by taking the sum of the products of these different numerical grades with the corresponding value of λ . The machine with the highest score is the desired machine. We do not consider the different numerical grades of the machine against the pairs (e_i, e_i) , $i = 1, 2, 3$, as both parameters are the same.

Matrix of different numerical grades is shown below:

$$\begin{pmatrix} (0, 0.1) & (0.1, 0.4) & (0.8, 0.4) \\ (-0.1, 0.3) & (-0.2, 0.4) & (0.6, 0.2) \\ (0.6, 0.7) & (0.2, 0.4) & (0.3, 0.4) \\ (0, 0.2) & (-0.1, 0.6) & (0.8, 0.4) \\ (-0.1, 0.3) & (-0.1, 0.6) & (0.8, 0.1) \\ (0.6, 0.7) & (0.2, 0.6) & (0.8, 0.4) \\ (0.2, 0.4) & (0.1, 0.1) & (0.8, 0.4) \\ (0.3, 0.1) & (-0.2, 0.2) & (0.6, 0.2) \\ (0.6, 0.7) & (0.2, 0.4) & (0.5, 0.4) \end{pmatrix}. \quad (5.12)$$

Grade Table

$$\begin{pmatrix}
 H & x_i & \text{highest numerical grade} & \lambda_i \\
 (e_1, e_1) & x_3 & \times & \times \\
 (e_1, e_2) & x_3 & 0.6 & 0.2 \\
 (e_1, e_3) & x_1 & 0.6 & 0.7 \\
 (e_2, e_1) & x_3 & 0.8 & 0.4 \\
 (e_2, e_2) & x_3 & \times & \times \\
 (e_2, e_3) & x_3 & 0.8 & 0.4 \\
 (e_3, e_1) & x_3 & 0.8 & 0.4 \\
 (e_3, e_2) & x_3 & 0.6 & 0.2 \\
 (e_3, e_3) & x_3 & \times & \times
 \end{pmatrix} \quad (5.13)$$

Score $(x_1) = (0.6 \times 0.7) = 0.42,$

Score $(x_2) = 0,$

Score $(x_3) = (0.6 \times 0.2) + (0.8 \times 0.4) + (0.8 \times 0.4) + (0.8 \times 0.4) + (0.6 \times 0.2) = 1.2.$

Then the firm will select the machine with the highest score. Hence, they will buy machine x_3 .

6. An Application of PIFSS in Decision Making

In this section we present an application of PIFSS in decision making problem. We shall use the algorithm introduced by Dinda et al. [13]. Suppose that there are three schools in universe $U = \{x_1, x_2, x_3\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, each, $e_i, 1 \leq i \leq 6$ indicates a specific criterion for the schools

- e_1 stands for "international".
- e_2 stands for "English".
- e_3 stands for "high efficiency".
- e_4 stands for "modern".
- e_5 stands for "full day".
- e_6 stands for "half day".

Suppose Madam X wants to pick a good school for her son on the basis of her wishing parameters among those listed above. Our aim is to find out the most appropriate school for her son.

Suppose the wishing parameters of Madam X is $A \subseteq E$ where $A = \{e_1, e_3, e_6\}$.

Let $p : E \rightarrow I^U$ be a fuzzy subset of E , defined by Madam X.

Consider the PIFSS defined as follows:

$$\begin{aligned} F_\mu(e_1) &= \left\{ \left(\frac{x_1}{(0.3, 0.5)}, 0.1 \right), \left(\frac{x_2}{(0.5, 0.3)}, 0.4 \right), \left(\frac{x_3}{(0.3, 0.4)}, 0.5 \right) \right\}, \\ F_\mu(e_2) &= \left\{ \left(\frac{x_1}{(0, 0.8)}, 0.1 \right), \left(\frac{x_2}{(1, 0)}, 0.3 \right), \left(\frac{x_3}{(0.9, 0.1)}, 0.2 \right) \right\}, \\ F_\mu(e_3) &= \left\{ \left(\frac{x_1}{(0.6, 0.3)}, 0.4 \right), \left(\frac{x_2}{(0.5, 0.4)}, 0.3 \right), \left(\frac{x_3}{(0.6, 0.3)}, 0.1 \right) \right\}. \end{aligned} \quad (6.1)$$

Now we introduce the following operations:

- (i) for membership function: $\alpha(e_i) = \mu_i + \gamma_i - \mu_i\gamma_i$,
- (ii) for non-membership function $\beta = \nu_i\gamma_i$, for $i = 1, 2, 3$.

Actually we have taken these two operations to ascend the membership value and descend the non-membership value of $F(e_i)$ on the basis of the degree of preference of Madam X. Then the PIFSS $F_\mu(e_i)$ reduced to an intuitionistic fuzzy soft set $\Psi(e_i)$ given as follows:

$$\begin{aligned} \Psi(e_1) &= \left\{ \frac{x_1}{(0.37, 0.05)', \frac{x_2}{(0.78, 0.12)', \frac{x_3}{(0.65, 0.2)' } \right\}, \\ \Psi(e_2) &= \left\{ \frac{x_1}{(0, 0.08)', \frac{x_2}{(1, 0)', \frac{x_3}{(0.92, 0.02)' } \right\}, \\ \Psi(e_3) &= \left\{ \frac{x_1}{(0.76, 0.12)', \frac{x_2}{(0.65, 0.12)', \frac{x_3}{(0.64, 0.03)' } \right\}. \end{aligned} \quad (6.2)$$

Definition 6.1 (see [13]). A comparison table is a square table in which number of rows and number of columns are equal and both are labeled by object name of the universe such as x_1, x_2, \dots, x_n and the entries are c_{ij} , where c_{ij} = the number of parameters for which the value of x_i exceeds or equal to the value of x_j .

Algorithm 6.2.

- (i) Input the set $A \subseteq E$ of choice of parameters of Madam X.
- (ii) Consider the reduced intuitionistic fuzzy soft set.
- (iii) Consider the tabular representation of membership function and non-membership function (see Table 1 and Table 4 respectively).
- (iv) Compute the comparison table membership function and non-membership function (see Table 2 and Table 5 respectively).
- (v) Compute the membership score and non-membership score (see Table 3 and Table 6 respectively).
- (vi) Compute the final score by subtracting non-membership score from membership score (see Table 7).
- (vii) Find the maximum score, if it occurs in i th row then Madam X will choose school x_i .

Table 1: Tabular representation of membership function.

U	e_1	e_3	e_6
x_1	0.37	0	0.76
x_2	0.78	1	0.65
x_3	0.65	0.92	0.64

Table 2: Comparison table of the above table.

	x_1	x_2	x_3
x_1	3	1	1
x_2	2	3	3
x_3	2	0	3

Table 3: Membership score table.

	Row sum (a)	Column sum (b)	Membership score ($a - b$)
x_1	5	7	-2
x_2	8	4	4
x_3	5	-7	-2

Table 4: Tabular representation of non-membership function.

	e_1	e_3	e_6
x_1	0.05	0.08	0.12
x_2	0.12	0	0.12
x_3	0.2	0.02	0.03

Decision:

Madam X will choose the school x_2 . In case, if she does not want to choose x_2 due to certain reasons, her second choice will be x_1 .

7. Similarity between Two Possibility Intuitionistic Fuzzy Soft Sets

Similarity measures have extensive application in several areas such as pattern recognition, image processing, region extraction, and coding theory and so forth. We are often interested to know whether two patterns or images are identical or approximately identical or at least to what degree they are identical.

Several researchers have studied the problem of similarity measurement between fuzzy sets, intuitionistic fuzzy sets and Liang and Shi [18] have studied the similarity measures on intuitionistic fuzzy sets. Shawkat et al. [16] have studied the similarity between two possibility fuzzy soft sets.

In this section we introduce a measure of similarity between two PIFSSs. The set theoretic approach has been taken in this regard because it is easier for calculation and is a very popular method too.

Table 5: Comparison table of the above table.

	x_1	x_2	x_3
x_1	3	2	2
x_2	2	3	1
x_3	1	2	3

Table 6: Non-membership score table.

	Row sum (c)	Column sum (d)	Non-membership score ($c - d$)
x_1	7	6	1
x_2	6	7	-1
x_3	6	6	0

Table 7

	Membership score (m)	Non-membership score (n)	Finale score ($m - n$)
x_1	-2	1	3
x_2	4	-1	5
x_3	-2	0	-2

Clearly the maximum score is 5 scored by the school x_2

Definition 7.1. Similarity between two PIFSSs F_μ and G_δ , denoted by $S(F_\mu, G_\delta)$, is defined as follows:

$$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot M(\mu(e), \delta(e)), \tag{7.1}$$

such that

$$M(F(e), G(e)) = \max_i M_i(F(e), G(e)), \quad M(\mu(e), \delta(e)) = \max_i M_i(\mu(e), \delta(e)), \tag{7.2}$$

where

$$M_i(F(e), G(e)) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^n (\phi_{F(e)}(i) - \phi_{G(e)}(i))^p} \quad 1 \leq p \leq \infty, \tag{7.3}$$

such that and

$$\begin{aligned} \phi_{F(e)}(i) &= \frac{\mu_{F(e)} + \nu_{F(e)}}{2}, & \phi_{G(e)}(i) &= \frac{\mu_{G(e)} + \nu_{G(e)}}{2}, \\ M_i(\mu(e), \delta(e)) &= 1 - \frac{\sum_{j=1}^n |\mu_{ij}(e) - \delta_{ij}(e)|}{\sum_{j=1}^n |\mu_{ij}(e) + \delta_{ij}(e)|}. \end{aligned} \tag{7.4}$$

Definition 7.2. Let F_μ and G_δ be two PIFSSs over (U, E) . We say that F_μ and G_δ are significantly similar if $S(F_\mu, G_\delta) \geq 1/2$.

Proposition 7.3. Let F_μ and G_δ be any two PIFSSs over (U, E) . Then the following holds:

- (i) $S(F_\mu, G_\delta) = S(G_\delta, F_\mu)$,
- (ii) $0 \leq S(F_\mu, G_\delta) \leq 1$,
- (iii) $F_\mu = G_\delta \Rightarrow S(F_\mu, G_\delta) = 1$,
- (iv) $F_\mu \subseteq G_\delta \subseteq H_\lambda \Rightarrow S(F_\mu, H_\lambda) \leq S(G_\delta, H_\lambda)$,
- (v) $F_\mu \tilde{\cap} G_\delta = \phi \Leftrightarrow S(F_\mu, G_\delta) = 0$.

Proof. The proof is straightforward and follows from Definition 6.1. □

Example 7.4. Consider Example 4.2 where F_μ and G_δ are defined as follows:

$$\begin{aligned}
 F_\mu(e_1) &= \left\{ \left(\frac{x_1}{(0.3, 0.4)}, 0.1 \right), \left(\frac{x_2}{(0.7, 0.1)}, 0.4 \right), \left(\frac{x_3}{(0.2, 0.6)}, 0.6 \right) \right\}, \\
 F_\mu(e_2) &= \left\{ \left(\frac{x_1}{(0.2, 0.6)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.5)}, 0.2 \right), \left(\frac{x_3}{(0.1, 0.3)}, 0.4 \right) \right\}, \\
 F_\mu(e_3) &= \left\{ \left(\frac{x_1}{(0.7, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.2, 0.5)}, 0 \right), \left(\frac{x_3}{(0.5, 0.3)}, 0.6 \right) \right\}, \\
 G_\delta(e_1) &= \left\{ \left(\frac{x_1}{(0.1, 0.4)}, 0.3 \right), \left(\frac{x_2}{(0.3, 0.3)}, 0.6 \right), \left(\frac{x_3}{(0, 0.5)}, 0.2 \right) \right\}, \\
 G_\delta(e_2) &= \left\{ \left(\frac{x_1}{(0, 0.1)}, 0.1 \right), \left(\frac{x_2}{(0.6, 0.1)}, 0.6 \right), \left(\frac{x_3}{(0.7, 0)}, 0.6 \right) \right\}, \\
 G_\delta(e_3) &= \left\{ \left(\frac{x_1}{(0, 0)}, 0.3 \right), \left(\frac{x_2}{(0.2, 0.1)}, 0.8 \right), \left(\frac{x_3}{(0.3, 0.1)}, 0.2 \right) \right\}.
 \end{aligned} \tag{7.5}$$

Here

$$\begin{aligned}
 M_1(\mu(e), \delta(e)) &= 1 - \frac{\sum_{j=1}^3 |\mu_{1j}(e) - \delta_{1j}(e)|}{\sum_{j=1}^3 |\mu_{1j}(e) + \delta_{1j}(e)|} \\
 &= 1 - \frac{|(0.1 - 0.3)| + |(0.4 - 0.6)| + |(0.6 - 0.2)|}{|(0.1 + 0.3)| + |(0.4 + 0.6)| + |(0.6 + 0.2)|} = 0.39.
 \end{aligned} \tag{7.6}$$

Similarly we get $M_2(\mu(e), \delta(e)) = 0.38$ and $M_3(\mu(e), \delta(e)) = 0.3$. Then

$$\begin{aligned}
 M(\mu(e), \delta(e)) &= \max(M_1(\mu(e), \delta(e)), M_2(\mu(e), \delta(e)), M_3(\mu(e), \delta(e))) = 0.39, \\
 M_1(F(e), G(e)) &= 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^n (\phi_{F(e)}(i) - \phi_{G(e)}(i))^p} \\
 &= 1 - \frac{1}{\sqrt{3}} \sqrt{((0.35 - 0.25) + (0.4 - 0.3) + (0.4 - 0.25))} = 0.66.
 \end{aligned} \tag{7.7}$$

Table 8: Model PIFSS for swamp fever.

F_μ	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
y	(1,0)	(0,1)	(0,1)	(1,0)	(1,0)	(1,0)	(1,0)	(0,1)	(1,0)	(1,0)
μ_y	1	1	1	1	1	1	1	1	1	1
n	(0,1)	(1,0)	(1,0)	(0,1)	(0,1)	(0,1)	(0,1)	(1,0)	(0,1)	(0,1)
μ_n	1	1	1	1	1	1	1	1	1	1

Table 9: PIFSS for the sick person.

G_α	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
y	(0.3,0)	(0,0.7)	(0.5,0.1)	(0.3,0)	(0.4,0)	(0.1,0)	(0,0)	(0.7,0.1)	(0,0)	(0.4,0.2)
μ_y	0.1	0.2	0.1	0	0.5	0.1	0.2	0.2	1	0.4
n	(0.6,0.2)	(0.1,0)	(0.4,0)	(0.6,0)	(0.4,0.2)	(0.6,0.3)	(0.8,0)	(0.1,0.3)	(0.8,0)	(0.5,0.3)
μ_n	0.5	0.3	0.2	0.1	0.3	0.2	0.4	0.1	0.5	0

Similarly we get $M_2(F(e), G(e)) = 0.63$ and $M_3(F(e), G(e)) = 0.48$. Then

$$M(F(e), G(e)) = \max(M_1(F(e), G(e)), M_2(F(e), G(e)), M_3(F(e), G(e))) = 0.66. \quad (7.8)$$

Hence the similarity between the two PIFSS F_μ and G_δ is given by

$$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot M(\mu(e), \delta(e)) = 0.66 \times 0.39 \cong 0.26. \quad (7.9)$$

8. Application of Similarity Measure in Medical Diagnosis

In the following example we will try to estimate the possibility that a sick person having certain visible symptoms is suffering from swamp fever. For this we first construct a model possibility intuitionistic fuzzy soft set for swamp fever and the possibility intuitionistic fuzzy soft set of symptoms for the sick person. Next we find the similarity measure of these two sets. If they are significantly similar then we conclude that the person is possibly suffering from swamp fever.

Let our universal set contain only two elements "yes" and "no," that is, $U = \{y, n\}$. Here the set of parameters E is the set of certain visible symptoms. Let $E = (e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10})$, where $e_1 =$ trembling, $e_2 =$ cough with chest congestion, $e_3 =$ muscles pain, $e_4 =$ nausea, $e_5 =$ headache, $e_6 =$ low heart rate (bradycardia), $e_7 =$ pain upon moving the eyes, $e_8 =$ fever, $e_9 =$ a flushing or pale pink rash comes over the face, $e_{10} =$ vomiting.

Our model possibility intuitionistic fuzzy soft set for swamp fever F_μ is given in Table 8 and this can be prepared with the help of a physician.

After talking to the sick person we can construct his PIFSS G_δ as in Table 9. Now we find the similarity measure of these two sets (as in Example 7.4), here $S(F_\mu, G_\delta) \cong 0.38 < 1/2$. Hence the two PIFSSs are not significantly similar. Therefore we conclude that the person is not suffering from swamp fever.

9. Conclusion

In this paper we have introduced the concept of possibility intuitionistic fuzzy soft set and studied some of its properties. Applications of this theory have been given to solve a decision-making problem. Similarity measure of two possibility intuitionistic fuzzy soft sets is discussed and an application of this to medical diagnosis has been shown.

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