International Scholarly Research Network
ISRN Applied Mathematics
Volume 2012, Article ID 598539, 14 pages
doi:10.5402/2012/598539

Research Article

The Study of a Class of Pest Control Pollution Model with Stage-Structure and Time Delay

Jianwen Jia and Bo Wu

School of Mathematics and Computer Science, Shaanxi Normal University, Shanxi, Linfen 041004, China

Correspondence should be addressed to Jianwen Jia, jiajw.2008@163.com

Received 1 March 2012; Accepted 26 March 2012

Academic Editors: A. Bellouquid and C. Lu

We construct a pest control pollution model with stage-structure for pests and with epidemic in the predator by spraying pesticides and releasing susceptible predators together. We assume that only the pests and infective predators are affected by pesticide. We show that there exists a globally attractive pest-extinction periodic solution and we get the condition of global attractiveness of the pest-extinction periodic solution by applying comparison theorem of impulsive differential equation. Further, the condition for the permanence of the system is also given.

1. Introduction

With the rapid development of modern technology, industry, and agriculture, it is of great interest to consider the effects of toxicant on ecological communities from both an environmental and conservational point of view. Qualitatively estimating the effect of a toxicant on a population by mathematical models is a relatively new field that began only in the early 1980s [1–3]. Population toxicant coupling has been applied in several contexts including Lotka-Volterra and chemostat-like environments, resulting in ordinary, integrodifferential, and impulsive differential equation and stochastic models. So in this paper, we consider the above effects and introduce the pollution model to model the process of pest control problems and study its dynamics, and this is different from the previous pest control model which assumed that pests were reduced proportionally by spraying pesticides [4–6].

In the natural world, many species have a life history that takes their individual members through two stages: immature and mature; the authors of [7] studied an ecological model with stage-structure for predator. A general functional response was considered, and the authors analyzed the stability and the permanence of the system. The authors [8, 9] analyzed prey-predator models with age structure and impulsive control. The authors [10] investigated the dynamics of a pest control model with age structure for pests by introducing a constant periodic pesticide input and releasing natural enemies at different fixed moments. they analyzed the conditions for the global attractivity of the pest-extinction periodic solution and the permanence of the system.

Modeling studies on disease-dominated ecological populations have addressed issues like disease-related mortality, reduction in reproduction, change in population sizes, and disease-induced oscillation of population states. Chattopadhyay and Arino [11] formulated a prey-predator model with prey infection and observed destabilization due to infection. Venturino analyzed prey-predator models with disease in the prey [12] and the predator [13].

Motivated by the above, in this paper, we construct a pest control model with epidemic in the predator by spraying pesticides and releasing susceptible predators at the same time. The pest is stage-structured, and the effects of spraying pesticides into the environment and into the organism are considered. So the pollution model provides a natural description of such a system and should be introduced to our model.

The organization of this paper is as follows: in Section 2, we introduce a pest control pollution model with stage-structure for pest and with epidemic in the predator by introducing a constant periodic pesticide input and releasing susceptible predators together. In Section 3, we will introduce some definitions and lemmas which will be used in the paper. In Section 4, sufficient conditions are obtained for the global attractiveness of pest-extinction periodic solution. In Section 5, sufficient conditions are obtained for the permanence of the system. We give a brief conclusion of our results in the last section.

2. Model Formulation

In this paper, we suppose that pesticides hardly have influence on the susceptible predators, and the susceptible predators only feed on mature pests. Now we consider the following impulsive differential equation:

$$\begin{aligned}
&\left.\begin{aligned}
\dot{x}_1(t) &= \alpha x_2(t) - d_1 x_1(t) - \alpha e^{-d_1\tau} x_2(t-\tau) - r_1 c_o(t) x_1(t)\\
\dot{x}_2(t) &= \alpha e^{-d_1\tau} x_2(t-\tau) - f x_2^2(t) - \beta_1 x_2(t) S(t) - r_2 c_o(t) x_2(t)\\
\dot{S}(t) &= \frac{\beta_1 c x_2(t) S(t)}{1 + h\beta_1 x_2(t)} - \frac{\beta_2 S(t) I(t)}{1 + kI(t)} - d_2 S(t)\\
\dot{I}(t) &= \frac{\beta_2 S(t) I(t)}{1 + kI(t)} - d_3 I(t) - r_4 c_o(t) I(t)\\
\dot{c}_o(t) &= k c_e(t) - g c_o(t) - m c_o(t)\\
\dot{c}_e(t) &= -h c_e(t),
\end{aligned}\right\} \quad t \neq nT,\ n \in N,\\
&\left.\begin{aligned}
x_1(t^+) &= x_1(t), x_2(t^+) = x_2(t), S(t^+) = S(t) + p\\
I(t^+) &= I(t), c_o(t^+) = c_o(t), c_e(t^+) = c_e(t) + q,
\end{aligned}\right\} \quad t = nT,\ n \in N.
\end{aligned} \tag{2.1}$$

Here $x_1 = x_1(t)$ and $x_2 = x_2(t)$ represent the density of the immature and mature pest (the prey) at time t, respectively; $S = S(t)$ and $I = I(t)$ represent the density of susceptible predator and infective predator at time t, respectively; $c_e(t)$ represents the concentration of pesticide in the environment at time t; $c_o(t)$ represents the concentration of pesticide in the organism at time t; we use a special functional response, that is, when the number of the prey captured is less, the digestive capacity of the predator will increase with the density of the prey. Here α is the birth rate of the immature pest; d_1 is the death rate of the mature pest; d_2 and d_3 are the death rates of the susceptible predator and the infective predator, respectively; τ is the mean length of the juvenile period; β_1 represents the capturing rate of the susceptible predator; h represents digestive time of the susceptible predator; c is the transformation rate of the susceptible predator; f represents the intraspecific competition coefficient of mature pest; $\beta_2 S(t)I(t)/(1+kI(t))$ represents saturation incidence rate; T is the period of the impulsive effect; p is the releasing amount of the susceptible predator at $t=nT$; q is the amount of the pesticides spraying at every impulsive period nT; $r_1, r_2,$ and r_4 represent the decreasing rate of the intrinsic growth rate associated with the uptake of the pesticide in the organism for the immature pest, mature pest, susceptible predator, and infective predator, respectively; $kc_e(t)$ represents an organism's net uptake of toxin from the environment; $gc_o(t)$ and $mc_o(t)$ represent the digestion and depuration rates of pesticide in an organism, respectively; $hc_e(t)$ represents the loss of pesticide in the environment due to natural degradation. All the coefficients are positive constants.

The initial conditions of system (2.1) are

$$\begin{gathered}(x_1(t), x_2(t), S(t), I(t), c_o(t), c_e(t)) \in C\left([-\tau,0], R_+^6\right), \\ x_1(0)>0, \quad x_2(0)>0, \quad S(0)>0, \quad I(0)>0, \quad c_o(0)>0, \quad c_e(0)>0,\end{gathered} \tag{2.2}$$

where $R_+^6 = \{(x_1, x_2, S, I, c_o, c_e) : x_1 \geq 0,\ x_2 \geq 0,\ S \geq 0,\ I \geq 0,\ c_o \geq 0,\ c_e \geq 0\}$. To assure the continuity of the initial values, we assume that $x_1(0) = \int_{-\tau}^{0} \alpha e^{\gamma\theta} x_2(\theta)d\theta$. This suggests that if we know the properties of $x_2(t)$, then the properties of $x_1(t)$ can be obtained.

Note that the variable $x_1(t)$ does not appear in the second, third, forth, fifth, and sixth equations of system (2.1), hence we only need to consider the subsystem of (2.1) as follows:

$$\begin{aligned}
&\left.\begin{aligned}
\dot{x}_2(t) &= \alpha e^{-d_1\tau} x_2(t-\tau) - f x_2^2(t) - \beta_1 x_2(t)S(t) - r_2 c_o(t)x_2(t)\\
\dot{S}(t) &= \frac{\beta_1 c x_2(t)S(t)}{1+h\beta_1 x_2(t)} - \frac{\beta_2 S(t)I(t)}{1+kI(t)} - d_2 S(t)\\
\dot{I}(t) &= \frac{\beta_2 S(t)I(t)}{1+kI(t)} - d_3 I(t) - r_4 c_o(t)I(t)\\
\dot{c}_o(t) &= kc_e(t) - gc_o(t) - mc_o(t)\\
\dot{c}_e(t) &= -hc_e(t),
\end{aligned}\right\} \quad t \neq nT,\ n \in N,\\
&\left.\begin{aligned}
x_2(t^+) &= x_2(t), S(t^+) = S(t)+p, I(t^+) = I(t)\\
c_o(t^+) &= c_o(t), c_e(t^+) = c_e(t)+q,
\end{aligned}\right\} \quad t = nT,\ n \in N.
\end{aligned} \tag{2.3}$$

The initial conditions for system (2.3) are

$$\begin{gathered}(x_2(t), S(t), I(t), c_o(t), c_e(t)) \in C\left([-\tau,0], R_+^5\right), \\ x_2(0)>0, \quad S(0)>0, \quad I(0)>0, \quad c_o(0)>0, \quad c_e(0)>0,\end{gathered} \tag{2.4}$$

where $R_+^5 = \{(x_2,\ S,\ I, c_o,\ c_e) : x_2 \geq 0,\ S \geq 0,\ I \geq 0,\ c_o \geq 0,\ c_e \geq 0\}$.

3. Definitions and Lemmas

Let $R_+ = [0,\infty], R_+^5 = \{X = (x_2(t), S(t), I(t), c_o(t), c_e(t)) : x_2 \geq 0, S \geq 0, I \geq 0, c_o \geq 0, c_e \geq 0\}$, and N be the set of all nonnegative integers, let $f = (f_1, f_2, f_3, f_4, f_5)^T$ the map defined by the right of system (2.3). The solution of system (2.3), denoted by $X = (x_2(t),, S(t), I(t), c_o(t), c_e(t)) : R_+ \to R_+^5$, is continuously differentiable on $(nT, (n+1)T)$. Let $V : R_+ \times R_+^5 \to R_+$, then V is said to belong to class V_0 if;

(1) V is continuous on $(nT, (n+1)T] \times R_+^5$, and for each $X(t) \in R_+^5,\ n \in N$ and $\lim_{(t,\, y) \to (nT^+,\, x)} V(t, y) = V(nT^+, x)$ exist;

(2) V is locally Lipschitzian in X.

Definition 3.1. Let $V \in V_0$, then for $(t, X) \in (nT, (n+1)T] \times R_+^5$, the upper right derivative of $V(t, X)$ with respect to impulsive differential system (2.3) is defined as

$$D^+V(t,X) = \lim_{h \to 0^+} \sup \frac{1}{h}\left[V(t+h, X + hf(t,X)) - V(t,X)\right]. \tag{3.1}$$

Definition 3.2. System (2.3) is said to be permanent if there are constants m, $M > 0$ (independent of initial value) and a finite time T_0 such that for every positive solution $(x_2(t), S(t), I(t), c_o(t), c_e(t)) \in R_+^5$ with initial conditions of system (2.3) satisfies $m \leq x_2(t) \leq M$, $m \leq S(t) \leq M$, $m \leq I(t) \leq M$, $m \leq c_o(t) \leq M$, $m \leq c_e(t) \leq M$ for all $t \geq T_0$, here T_0 may depend on the initial condition of system (2.3).

Lemma 3.3 (the comparison theorem of impulsive differential equation [14]). *Let $V \in V_0$. Assume that*

$$\begin{aligned} &D^+V(t,x) \leq g(t, V(t,x)), \quad t \neq nT, \\ &V(t, X(t^+)) \leq \varphi_n(V(t,x)), \quad t = nT, \end{aligned} \tag{3.2}$$

where $g : R_+ \times R_+ \to R$ is continuous in $(nT, (n+1)T] \times R_+$, and for each $u \in R_+$, $n \in N$, $\lim_{(t,\, v) \to (nT^+,\, u)} g(t,v) = g(nT^+,\ u)$ exists and is finite; $\varphi_n : R_+ \to R_+$ is nondecreasing.

Let $r(t)$ be the maximal solution of the scalar impulsive differential equation defined on $[0, +\infty)$, then

$$\begin{aligned} \dot{u}(t) &= g(t, u(t)), \quad t \neq nT, \\ u(t^+) &= \varphi_n(u(t)), \quad t = nT, \\ u(0^+) &= u_0. \end{aligned} \tag{3.3}$$

So $V(0^+, X_0) \leq u_0$ implies that

$$V(t, X(t)) \leq r(t), \quad t \geq 0, \tag{3.4}$$

where $X(t)$ is any solution of system (2.3).

Lemma 3.4 (see [15]). *Consider the following equation:*

$$\frac{dx}{dt} = ax(t-\tau) - bx(t), \tag{3.5}$$

where $a, b, \tau > 0$, $x(t) > 0$ *for* $t \in [-\tau, 0]$, *one has*

(i) *if* $a < b$, *then* $\lim_{t\to\infty} x(t) = 0$,

(ii) *if* $a > b$, *then* $\lim_{t\to\infty} x(t) = +\infty$.

Remark 3.5 (see [16]). $c_o(t), c_e(t)$ are the concentration of toxicant. To assure $0 \le c_o(t) \le 1$, $0 \le c_e(t) \le 1$, it is necessary that $g \le k \le g+m$.

Remark 3.6 (see [16]). From the point of the biological meaning, we assume that $k < h$.

Lemma 3.7. *Consider the following subsystem of system* (2.3)

$$\begin{aligned} \left.\begin{aligned} \dot{c}_o(t) &= kc_e(t) - gc_o(t) - mc_o(t) \\ \dot{c}_e(t) &= -hc_e(t), \end{aligned}\right\} \quad & t \neq nT,\ n \in N, \\ \left.\begin{aligned} c_o(t^+) &= c_o(t) \\ c_e(t^+) &= c_e(t) + q, \end{aligned}\right\} \quad & t = nT,\ n \in N. \end{aligned} \tag{3.6}$$

Then, system (3.6) *has a unique positive T-periodic solution* $(c_o^*(t), c_e^*(t))$ *and for each solution* $(c_o(t), c_e(t))$ *of system* (3.6), $c_o(t) \to c_o^*(t)$ *and* $c_e(t) \to c_e^*(t)$ *as* $t \to +\infty$. *Where*

$$\begin{aligned} c_o^*(t) &= c_o^*(0)e^{-(g+m)(t-nT)} + \frac{kq\left(e^{-(g+m)(t-nT)} - e^{-h(t-nT)}\right)}{(h-g-m)\left(1-e^{-hT}\right)}, \\ c_e^*(t) &= \frac{qe^{-h(t-nT)}}{1-e^{-hT}}, \\ c_o^*(0) &= \frac{kq\left(e^{-(g+m)T} - e^{-hT}\right)}{(h-g-m)\left(1-e^{-hT}\right)\left(1-e^{-(g+m)T}\right)}, \\ c_e^*(0) &= \frac{q}{1-e^{-hT}}, \end{aligned} \tag{3.7}$$

for $t \in (nT, (n+1)T]$ *and* $n \in N$.

Lemma 3.8. *There exists a constant* $M > 0$ *such that* $x_1(t) \le M$, $x_2(t) \le M$, $S(t) \le M$, $I(t) \le M$, $c_o(t) \le M$, $c_e(t) \le M$ *for each solution* $X(t) = (x_1(t), x_2(t), S(t), I(t), c_o(t), c_e(t))$ *of system* (2.1) *with all t large enough.*

Proof. Define $V(t) = c(x_1 + x_2) + S + I + c_o + c_e$. Choose $0 < l < \min\{d_1, d_2, d_3, g + m, h - k\}$, we have $V \in V_0$ and

$$\begin{aligned} D^+V(t) + lV(t) &\le -cfx_2^2 + c(l+\alpha)x_2 + (l-d_2)S + (l-d_3)I \\ &\quad + \left(l-g-m\right)\, c_o + (l+k-h)\, c_e + c(l-d_1)x_1, \quad t \ne nT,\ n \in N, \\ &\le -cfx_2^2 + c(l+\alpha)x_2, \\ V(nT^+) &= V(nT) + p + q, \quad t = nT,\ n \in N. \end{aligned} \tag{3.8}$$

Hence there exists a positive constant K such that

$$\begin{aligned} D^+V(t) &\le K - lV(t), \quad t \ne nT,\ n \in N, \\ V(nT^+) &= V(nT) + p + q, \quad t = nT,\ n \in N, \end{aligned} \tag{3.9}$$

by Lemma 3.3, for $t \ge 0$, we have

$$V(t) \le \left(V(0^+) - \frac{K}{l}\right)e^{-lt} + \frac{(p+q)\left(1-e^{-nlT}\right)e^{-l(t-nT)}}{1-e^{-lT}} + \frac{K}{l}, \quad t \in (nT, (n+1)T],\ n \in N. \tag{3.10}$$

Then $\lim_{t\to\infty} V(t) \le (K/l) + (p+q)e^{lT}/(e^{lT}-1)$.

So $V(t)$ is uniformly ultimately bounded. By the definition of $V(t)$, there exists a constant $M > 0$ such that $x_1(t) \le M,\ x_2(t) \le M,\ S(t) \le M,\ I(t) \le M,\ c_o(t) \le M,\ c_e(t) \le M$ for t large enough. The proof is complete. □

4. The Global Attractivity of Periodic Solution

In this section, the sufficient conditions are obtained for the global attractivity of the pest-extinction periodic solution.

We first demonstrate the expression of the pest-extinction solution of system (2.3), in which the pest individual and infective predator individual are entirely absent from the model, that is, $x_2(t) = 0, I(t) = 0$ for all $t \ge 0$.

When $x_2(t) = 0$ and $I(t) = 0$, $S(t)$ satisfies the following system:

$$\begin{aligned} \dot{S}(t) &= -d_2S, \quad t \ne nT, \\ S(t^+) &= S(t) + p, \quad t = nT, \end{aligned} \tag{4.1}$$

$S(0^+) = S_0$. Clearly, we can obtain the unique positive periodic solution with the form

$$S^*(t) = \frac{pe^{-d_2(t-nT)}}{1-e^{-d_2T}}, \quad t \in (nT, (n+1)T],\ n \in N. \tag{4.2}$$

Therefore, $S(t) = (S(0^+) - p/1 - e^{-d_2T})e^{-d_2t} + S^*(t),\ t \in (nT, (n+1)T],\ n \in N$ is the solution of system (4.1) with initial value S_0.

Denote $\eta_1 = e^{-(\beta_2/k+d_2)T}/(1-e^{-(\beta_2/k+d_2)T})$, $\eta_2 = k(e^{-(g+m)T}-e^{-hT})e^{-(g+m)T}/(h-g-m)(1-e^{-(g+m)T})(1-e^{-hT})$.

Theorem 4.1. *Let* $X(t) = (x_2(t), S(t), I(t)c_o(t), c_e(t))$ *be any solution of system* (2.3) *with positive initial values, if*

$$\frac{\alpha e^{-d_1\tau}}{\beta_1\eta_1 p + r_2 q\eta_2} < 1, \qquad \frac{\beta_2 p e^{-d_2T}}{d_3(1-e^{-d_2T})} < 1, \tag{4.3}$$

then $(0, S^*(t), 0, c_o^*(t), c_e^*(t))$ *is globally attractive.*

Proof. From system (2.3), we have

$$\begin{aligned} \dot{S}(t) &\geq -\left(d_2 + \frac{\beta_2}{k}\right)S, \quad t \neq nT, \\ S(t^+) &= S(t) + p, \quad t = nT. \end{aligned} \tag{4.4}$$

Consider the following comparison system

$$\begin{aligned} \dot{u}(t) &= -\left(d_2 + \frac{\beta_2}{k}\right)u, \quad t \neq nT, \\ u(t^+) &= u(t) + p, \quad t = nT, \\ u(0^+) &= S(0^+). \end{aligned} \tag{4.5}$$

Obviously, system (4.5) has a positive periodic solution

$$u^*(t) = \frac{pe^{(-d_2-\beta_2/k)(t-nT)}}{1-e^{(-d_2-\beta_2/k)T}}, \quad t \in (nT, (n+1)T], \tag{4.6}$$

which is globally asymptotically stable. By Lemma 3.3, we conclude that for an arbitrary positive constant ε_1 small enough, there exists an $N_1 \in Z$ such that

$$S(t) \geq Z(t) > Z^*(t) - \varepsilon_1, \quad t \in (N_1T, (N_1+1)T]. \tag{4.7}$$

From which, we get

$$S(t) > \frac{pe^{(-d_2-\beta_2/k)T}}{1-e^{(-d_2-\beta_2/k)T}} - \varepsilon_1, \quad t \in (N_1T, (N_1+1)T]. \tag{4.8}$$

That is,

$$S(t) > p\eta_1 - \varepsilon_1, \quad t \in (N_1T, (N_1+1)T]. \tag{4.9}$$

By Lemma 3.7, we conclude that for a sufficiently small $\varepsilon_2 > 0$, there exists an $N_2 \in Z$ such that

$$c_0(t) > c_0^*(t) - \varepsilon_2, \quad t \in (N_2T, (N_2+1)T]. \tag{4.10}$$

that is,

$$c_0(t) > q\eta_2 - \varepsilon_2, \quad t \in (N_2T, (N_2+1)T]. \tag{4.11}$$

Let $\overline{T} = \max\{N_1T, N_2T\}$, from the first equation of systems (2.3), (4.9), and (4.11). We have

$$\dot{x}_2(t) < \alpha e^{-d_1\tau} x_2(t-\tau) - f x_2^2(t) - \beta_1 x_2(t)(p\eta_1 - \varepsilon_1) - r_2 x_2(t)(q\eta_2 - \varepsilon_2), \quad t > \overline{T} + \tau. \tag{4.12}$$

Now consider the following comparison equation:

$$\dot{P}(t) = \alpha e^{-d_1\tau} P(t-\tau) - f P^2(t) - [\beta_1(p\eta_1 - \varepsilon_1) + r_2(q\eta_2 - \varepsilon_2)] P(t). \tag{4.13}$$

Since the first condition of the theorem holds, we can choose the above $\varepsilon_1, \varepsilon_2$ small enough such that

$$\alpha e^{-d_1\tau} < \beta_1(p\eta_1 - \varepsilon_1) + r_2(q\eta_2 - \varepsilon_2). \tag{4.14}$$

By (4.14) and Lemma 3.4, we have $\lim_{t\to\infty} P(t) = 0$.

By Lemma 3.3, we get

$$\lim_{t\to\infty} x_2(t) \leq \lim_{t\to\infty} P(t) = 0. \tag{4.15}$$

Incorporating the positivity of $x_2(t)$, we get $\lim_{t\to\infty} x_2(t) = 0$.

Then for a sufficiently small $\varepsilon_3 \in (0, d_2)$ and t large enough, we have $0 < x_2(t) < \varepsilon_3/c\beta_1$, without loss of generality, we may assume $0 < x_2(t) < \varepsilon_3/c\beta_1$ as $t \geq 0$.

From the second equation of system (2.3), we have

$$\begin{aligned} \dot{S}(t) &\leq (\varepsilon_3 - d_2)S, \quad t \neq nT, \\ S(t^+) &= S(t) + p, \quad t = nT. \end{aligned} \tag{4.16}$$

Consider the following system:

$$\begin{aligned} \dot{G}(t) &= (\varepsilon_3 - d_2)G(t), \quad t \neq nT, \\ G(t^+) &= G(t) + p, \quad t = nT, \\ G(0^+) &= S(0^+). \end{aligned} \tag{4.17}$$

Obviously, system (4.17) has a positive periodic solution

$$G^*(t) = \frac{pe^{(-d_2+\varepsilon_3)(t-nT)}}{1-e^{(-d_2+\varepsilon_3)T}}, \quad nT < t \le (n+1)T, \tag{4.18}$$

which is globally asymptotically stable. Thus, for a sufficiently small $\varepsilon_4 > 0$, when t is large enough, we have

$$S(t) \le G(t) < V^*(t) + \varepsilon_4. \tag{4.19}$$

By the second condition of the theorem, when t is large enough, we have

$$\beta_2\left(\frac{pe^{-d_2T}}{1-e^{-d_2T}} + \varepsilon_4\right) - d_3 < 0. \tag{4.20}$$

By (4.19), when t is large enough,

$$S(t) < \frac{pe^{-d_2T}}{1-e^{-d_2T}} + \varepsilon_4. \tag{4.21}$$

Combining the third equation of system (2.3) with (4.21), we obtain

$$\dot{I}(t) \le \left[\beta_2\left(\frac{pe^{-d_2T}}{1-e^{-d_2T}} + \varepsilon_4\right) - d_3\right]I(t) . \tag{4.22}$$

By (4.20) and $I \ge 0$, we have $I(t) \to 0$ as $t \to \infty$.

Further, since $I(t) \to 0$, for an arbitrary positive constant ε_5 small enough, we have $I \le \varepsilon_5$ as t is large enough. Then

$$\begin{gathered} \dot{S}(t) \ge -(d_2 + \beta_2\varepsilon_5)S, \quad t \ne nT, \\ S(t^+) = S(t) + p, \quad t = nT. \end{gathered} \tag{4.23}$$

Consider the following system:

$$\begin{gathered} \dot{U}(t) = -(d_2 + \beta_2\varepsilon_5)U, \quad t \ne nT, \\ U(t^+) = U(t) + p, \quad t = nT, \\ U(0^+) = S(0^+). \end{gathered} \tag{4.24}$$

So system (4.24) has a positive periodic solution

$$U^*(t) = \frac{pe^{-(d_2+\beta_2\varepsilon_5)(t-nT)}}{1-e^{-(d_2+\beta_2\varepsilon_5)T}}, \quad nT < t \le (n+1)T, \tag{4.25}$$

which is globally asymptotically stable. Therefore, for an arbitrary positive constant ε_6 small enough, when t is large enough, we have

$$S(t) \geq U(t) > U^*(t) - \varepsilon_6. \tag{4.26}$$

Combining (4.26) with (4.19), we obtain $U^*(t) - \varepsilon_6 < S(t) < G^*(t) + \varepsilon_4$, since $\varepsilon_4, \varepsilon_5, \varepsilon_6$ are all sufficient small constants, we know

$$\lim_{t\to\infty} S(t) = S^*(t). \tag{4.27}$$

By Lemma 3.7, we get

$$\lim_{t\to\infty} c_0(t) = c_0^*(t), \qquad \lim_{t\to\infty} c_e(t) = c_e^*(t). \tag{4.28}$$

The proof is complete. □

Remark 4.2. Obviously, we know that the global attractiveness of pest-eradication periodic solution $(0, 0, S^*(t), 0, c_o^*(t), c_e^*(t))$ of system (2.2) is equivalent to the global attractiveness of mature pest-eradication periodic solution $(0, S^*(t), 0, c_o^*(t), c_e^*(t))$ of system (2.3).

5. Permanence

Theorem 5.1. *System* (2.3) *is permanent provided that*

$$\alpha e^{-d_1\tau} - \frac{\beta_1 p e^{-d_2 T}}{1 - e^{-d_2 T}} - \frac{r_2 k q \left(e^{-(g+m)T} - e^{-hT}\right)}{(h - g - m)\left(1 - e^{-hT}\right)\left(1 - e^{-(g+m)T}\right)} > 0, \qquad \frac{\beta_2 p e^{-d_2 T}}{\left(1 - e^{-d_2 T}\right) d_3} > 1. \tag{5.1}$$

Proof. By Lemma 3.8, we know that there exists an $M > 0$, and $M > (\alpha e^{-d_1\tau})/f$ such that $x_2(t) \leq M$, $S(t) \leq M$, $I(t) \leq M$, $c_o(t) \leq M$, $c_e(t) \leq M$ for all $t > 0$, we will prove the theorem through the following five steps.

Step 1. From (4.9), we know that there exists an $m_2 = p\eta_1 - \varepsilon_1$ such that $S(t) \geq m_2$ for t large enough.

Step 2. From (4.11), we know that there exists an $m_3 = q\eta_2 - \varepsilon_2$ such that $c_0(t) \geq m_3$ for t large enough.

Step 3. From Lemma 3.7, for an arbitrary positive constant ε_0 small enough, when t is large enough, $c_e(t) \geq q e^{-hT}/(1 - e^{-hT}) - \varepsilon_0 \triangleq m_4$.

Step 4. We will prove that there exists an $m_1 > 0$ such that $x_2(t) \geq m_1$ for t large enough, we will do it in the following two steps.

(i) By the condition of the theorem, we can select positive constants ε_7 and m_5 small enough such that

$$\begin{aligned} m_5 &< \frac{\alpha e^{-d_1\tau}}{f}, \qquad \delta = \beta_1 c m_5 < d_2, \\ \alpha e^{-d_1\tau} - f m_5 - &\frac{\beta_1 p e^{(-d_2+\delta)T}}{1-e^{(-d_2+\delta)T}} - \beta_1\varepsilon_7 - r_2\eta - r_2\varepsilon_2 > 0. \end{aligned} \tag{5.2}$$

Now we will prove that $x_2(t) < m_5$ cannot hold for all $t \geq 0$, otherwise,

$$\dot{S}(t) \leq (-d_2+\delta)S. \tag{5.3}$$

Let

$$\begin{aligned} \dot{R}(t) &= (-d_2+\delta)R, \quad t \neq nT, \\ R(t^+) &= R(t) + p, \quad t = nT, \\ R(0^+) &= S(0^+). \end{aligned} \tag{5.4}$$

By Lemma 3.3, $S(t) \leq R(t)$ and $R(t) \to R^*(t),\ t \to \infty$, where $R^*(t) = pe^{(-d_2+\delta)(t-nT)}/(1-e^{(-d_2+\delta)T}),\ t \in (nT,(n+1)T]$. So there exists a $T_1 > 0$ such that

$$S(t) \leq R(t) < R^*(t) + \varepsilon_7, \tag{5.5}$$

for $t > T_1$. From Lemma 3.7, we know that when t is large enough,

$$c_0(t) < c_o^*(0)e^{-(g+m)T} + \frac{kq\left(e^{-(g+m)T} - e^{-hT}\right)}{(h-g-m)\left(1-e^{-hT}\right)} + \varepsilon_2 \triangleq \eta + \varepsilon_2, \tag{5.6}$$

for a sufficiently small $\varepsilon_2 > 0$, then

$$\dot{x}_2(t) \geq \alpha e^{-d_1\tau} x_2(t-\tau) - \left[f m_5 + \beta_1\left(\frac{pe^{(-d_2+\delta)T}}{1-e^{(-d_2+\delta)T}} + \varepsilon_7\right) + r_2(\eta+\varepsilon_2)\right] x_2(t), \tag{5.7}$$

by Lemma 3.4, $x_2(t) \to \infty$, as $t \to \infty$. This is a contradiction to the boundedness of $x_2(t)$. So there exists a $t_1 > 0$ such that $x_2(t_1) \geq m_5$.

(ii) If $x_2(t) \geq m_5$ for all $t \geq t_1$, our aim is obtained. Otherwise, we consider that $x_2(t)$ is oscillating about m_5.

Let $t_2 = \inf_{t \geq t_1}\{x_2(t) < m_5\}$, then $x_2(t) \geq m_5$ for $t \in [t_1,t_2)$, since $x_2(t)$ is continuous, $x_2(t_2) = m_5$; and because $x_2(t)$ is oscillating about m_5, we know that there exists a $t_3 = \inf_{t \geq t_2}\{x_2(t) > m_5\}$, then $x_2(t) \leq m_5$ for $t \in [t_2,t_3)$, from the continuity of $x_2(t)$, we get $x_2(t_3) = m_5$; and continue we can obtain the time sequence $t_1 \leq t_2 < t_3 < \cdots < t_{2k} < t_{2k+1} < \cdots$, which satisfies

(a) when $i = 2,3,4,\ldots,\ x_2(t_i) = m_5$,

(b) when $t \in (t_{2k}, t_{2k+1}), k = 1,2,\ldots,\ x_2(t_i) < m_5$,

(c) when $t \in (t_{2k+1}, t_{2k+2}), k = 1,2,\ldots,\ x_2(t_i) > m_5$.

We claim that there exists a $T_0 = \sup\{t_{2k+1} - t_{2k},\ k \in N\}$, otherwise, there exists a subsequence $\{T_j \mid T_j = t_{2k_j+1} - t_{2k_j},\ j \in N\}$ such that $\lim_{j\to\infty} T_j = +\infty$; obviously, (5.7) holds when $t \in (t_{2k_j}, t_{2k_j+1})$.

By Lemma 3.4, we get $\lim_{j\to\infty} x_2(t_{2k_j+1}) = +\infty$; this is a contradiction to $x_2(t_{2k_j+1}) = m_5$.

By the boundedness of the system, we have

$$S(t) < M, \qquad c_0(t) < M, \tag{5.8}$$

for $t \geq t_1$. It is clear that

$$\dot{x}_2(t) \geq -(f m_5 + \beta_1 M + r_2 M) x_2(t), \qquad x_2(t_{2k}) = m_5, \tag{5.9}$$

for $t \in (t_{2k}, t_{2k+1})$, $k = 1,2,\ldots$ Let $m_1 = m_5 e^{-(f m_5 + \beta_1 M + r_2 M)T_0}$, then we have $x_2(t) \geq m_1$ for all $t \geq t_1$.

Step 5. We will prove that there exists an $m_6 > 0$ such that $I(t) \geq m_6$ for t large enough, we will do it in the following two steps.

(i) From the condition of the theorem, let $m_7 > 0$ and $\varepsilon_8 > 0$ be small enough such that

$$\frac{\beta_2}{1 + k m_7}\left(\frac{p e^{-(\beta_2 m_7 + d_2)T}}{1 - e^{-(\beta_2 m_7 + d_2)T}} - \varepsilon_8\right) - d_3 - r_4 M > 0. \tag{5.10}$$

We will prove $I(t) < m_7$ cannot hold for all $t \geq 0$. Otherwise, we have

$$\dot{S}(t) \geq -(\beta_2 m_7 + d_2) S(t). \tag{5.11}$$

Consider the following system:

$$\begin{gathered}
\dot{Q}(t) = -(d_2 + \beta_2 m_7) Q, \qquad t \neq nT, \\
Q(t^+) = Q(t) + p, \qquad t = nT, \\
Q(0^+) = S(0^+).
\end{gathered} \tag{5.12}$$

By Lemma 3.3, we have $S(t) \geq Q(t)$ and $Q(t) \to Q^*(t)$, as $t \to \infty$, where $Q^*(t) = p e^{-(d_2 + \beta_2 m_7)(t - nT)} / (1 - e^{-(d_2 + \beta_2 m_7)T})$, $t \in (nT, (n+1)T]$.

So there exists a $T_2 > 0$, when $t > T_2 S(t) \geq Q(t) > Q^*(t) - \varepsilon_8$.

Hence,

$$\dot{I}(t) \geq \left[\frac{\beta_2}{1 + k m_7}\left(\frac{p e^{-(\beta_2 m_7 + d_2)T}}{1 - e^{-(\beta_2 m_7 + d_2)T}} - \varepsilon_8\right) - d_3 - r_4 M\right] I(t), \tag{5.13}$$

from (5.10), we get $I(t) \to \infty$, as $t \to \infty$, this is a contradiction to the boundedness of $I(t)$.So there exists a $t_0 > 0$ such that $I(t_0) \geq m_7$.

(ii) Similar to the method of step (ii) of Step 4, we can find an $m_6 = m_7 e^{-d_3 T_0'}$ such that $I(t) \geq m_6$ for all $t \geq t_0$.

Therefore, $m \leq x_1(t), x_2(t), S(t), I(t), c_o(t), c_e(t) \leq M$ for t large enough, where $m = \min\{m_1, m_2, m_3, m_4, m_6\}$. The proof is complete.

□

6. Conclusion

In this paper, we propose and analyze a pest control model with age structure for pest and pulse spraying pesticides and pulse releasing infective predators. By Lemma 3.8, we know that any solution of system (2.1) is bounded for t large enough and get the specific form of the upper boundedness. From Theorem 4.1, we get the sufficient condition of global attractiveness of the pest-extinction periodic solution:

$$\frac{\alpha e^{-d_1\tau}}{\beta_1\eta_1 p + r_2 q\eta_2} < 1, \qquad \frac{\beta_2 p e^{-d_2 T}}{d_3\left(1 - e^{-d_2 T}\right)} < 1. \tag{6.1}$$

By Theorem 5.1, we get the sufficient condition for the permanence of the system:

$$\alpha e^{-d_1\tau} - \frac{\beta_1 p e^{-d_2 T}}{1 - e^{-d_2 T}} - \frac{r_2 k q\left(e^{-(g+m)T} - e^{-hT}\right)}{(h - g - m)\left(1 - e^{-hT}\right)\left(1 - e^{-(g+m)T}\right)} > 0, \qquad \frac{\beta_2 p e^{-d_2 T}}{\left(1 - e^{-d_2 T}\right) d_3} > 1. \tag{6.2}$$

References

[1] Z. E. Ma, B. J. Song, and T. G. Hallam, "The threshold of survival for systems in a fluctuating environment," *Bulletin of Mathematical Biology*, vol. 51, no. 3, pp. 311–323, 1989.

[2] Z. E. Ma and T. G. Hallam, "Effects of parameter fluctuations on community survival," *Mathematical Biosciences*, vol. 86, no. 1, pp. 35–49, 1987.

[3] H. P. Liu and Z. E. Ma, "The threshold of survival for system of two species in a polluted environment," *Journal of Mathematical Biology*, vol. 30, no. 1, pp. 49–61, 1991.

[4] B. Liu, Z. Teng, and L. Chen, "The effect of impulsive spraying pesticide on stage-structured population models with birth pulse," *Journal of Biological Systems*, vol. 13, no. 1, pp. 31–44, 2005.

[5] Z. Lu, X. Chi, and L. Chen, "Impulsive control strategies in biological control of pesticide," *Theoretical Population Biology*, vol. 64, no. 1, pp. 39–47, 2003.

[6] B. Liu, L. S. Chen, and Y. J. Zhang, "The dynamics of a prey-dependent consumption model concerning impulsive control strategy," *Applied Mathematics and Computation*, vol. 169, no. 1, pp. 305–320, 2005.

[7] X. Meng, J. Jiao, and L. Chen, "The dynamics of an age structured predator-prey model with disturbing pulse and time delays," *Nonlinear Analysis*, vol. 9, no. 2, pp. 547–561, 2008.

[8] K. Y. Liu, X. Z. Meng, and L. S. Chen, "A new stage structured predator-prey Gomportz model with time delay and impulsive perturbations on the prey," *Applied Mathematics and Computation*, vol. 196, no. 2, pp. 705–719, 2008.

[9] G. P. Pang, F. Y. Wang, and L. S. Chen, "Extinction and permanence in delayed stage-structure predator-prey system with impulsive effects," *Chaos, Solitons and Fractals*, vol. 39, no. 5, pp. 2216–2224, 2009.

[10] B. Liu, Q. Zhang, and Y. H. Gao, "The dynamics of pest control pollution model with age structure and time delay," *Applied Mathematics and Computation*, vol. 216, no. 10, pp. 2814–2823, 2010.
[11] J. Chattopadhyay and O. Arino, "A predator-prey model with disease in the prey," *Nonlinear Analysis B*, vol. 36, pp. 747–766, 1999.
[12] E. Venturino, "Epidemics in predator-prey models: diseases in the prey," in *Mathematical Population Dynamics: Analysis of Heterogeneity*, O. Arino, D. Axelrod, M. Kimmel, and M. Langlais, Eds., vol. 1 of *Theory of Epidemics*, pp. 381–393, Wuerz, Winnipeg, Canada, 1995.
[13] E. Venturino, "Epidemics in predator-prey models: disease in the predators," *IMA Journal of Mathematics Applied in Medicine and Biology*, vol. 19, no. 3, pp. 185–205, 2002.
[14] V. Lakshmikantham, D. D. Baĭnov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore, 1989.
[15] Y. Kuang, *Delay Differential Equations with Applications in Population Dynamics*, Academic Press, San Diego, Calif, USA, 1993.
[16] B. Liu, L. S. Chen, and Y. J. Zhang, "The effects of impulsive toxicant input on a population in a polluted environment," *Journal of Biological Systems*, vol. 11, no. 3, pp. 265–274, 2003.