

Research Article

A Model for Conjoint Shape Memory and Pseudo-Elastic Effects during Martensitic Transformation

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Shape memory alloys (SMA) are metals which can restore their initial shape after having been subjected to a deformation. They exhibit in general both nonlinear shape memory and pseudoelastic effects. In this paper, shape memory alloy (SMA) and its constitutive model with an empirical kinetics equation are investigated. A new formulation to the martensite fraction-dependent Young modulus has been adopted and the plastic deformation was taken into account. To simulate the variations, a one-dimensional constitutive model was constructed based on the uniaxial tension features.

1. Introduction

Recently, smart metals and alloys have been extensively used in several metallurgical applications, due to their great potential in updated structures and design [1–10]. Among these materials, shape memory alloys (SMA) have attracted more attention, due to their ability to develop extremely large recoverable strains and great forces in the field of biomedical, metallurgy, aerospace, and civil structures [5–10]. In SMA matrices, pseudoplastic effect creates different stress strain behavior resulting in a stress strain curve which lies on the curve produced by the initial linear elastic response during loading. Consecutive and continued unloading may produce linear elastic behavior that eventually returns the structure to the zero stress strain state.

In the present work, an attempt is made to model typical martensitic transformations occurring in shape memory alloys, taking into account pseudoplasticity patterns. In this martensitic transformation, austenite undergoes transformation to form different variants of martensite under a controlled mechanical loading. The formation of martensite in the material is monitored through the coexistence of the initial austenite phases and martensite inside periodic units. Solutions for the implemented governing equations are obtained numerically via explicit numerical protocols and compared to some records presented in the recent related literature [11–18].

2. Model Patterns

2.1. Governing Equations. The studied system is a mono-dimensional rod subjected to axial solicitation (Figure 1). The phase transformations in this considered structure occur by nucleation and growth of platelet inclusions perpendicular to x -axis (Figure 1). In this configuration, elastic modulus local expression can be obtained considering the medium as a succession of austenite-martensite periodic units (Figure 1).

The main assumptions of the present model consists of setting one scalar internal variable η which represents the martensite fraction, along with linear kinetic rules in terms of the uniaxial stress T .

Young modulus E relative to the periodic unit can be computed via the elongation (x) of the periodic cell and the local strain ε :

$$\begin{aligned} x &= \varepsilon w = x_A + x_M = \varepsilon_A w_A + \varepsilon_M w_M, \\ \varepsilon &= \varepsilon_A \frac{w_A}{w} + \varepsilon_M \frac{w_M}{w}, \end{aligned} \quad (1)$$

where w , w_A , and w_M represent the total length of the periodic cell, the length of the austenite, and martensite fractions, respectively, (Figure 1). Subscripts A and M indicate austenite and martensite.

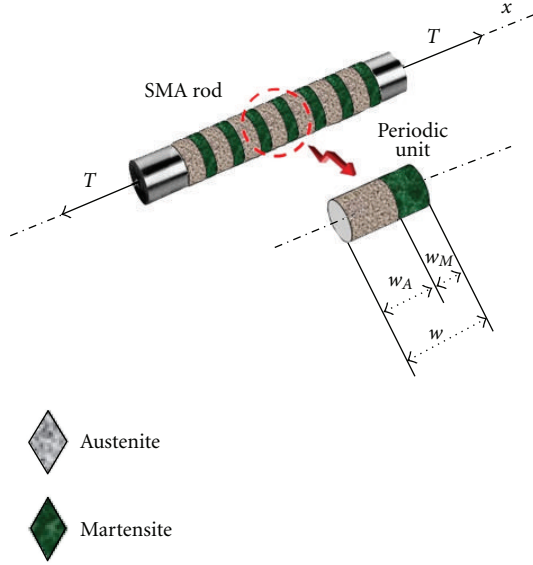


FIGURE 1: The studied model.

Since the martensite fraction η is defined as

$$\eta = \frac{w_M}{w}, \quad (2)$$

it gives

$$\varepsilon = \varepsilon_A(1 - \eta) + \varepsilon_M\eta. \quad (3)$$

The η -dependent boundary condition concerning Young modulus $E(\eta)$ are:

$$\begin{aligned} E(\eta)|_{\eta=0} &= E_A, \\ E(\eta)|_{\eta=1} &= E_M. \end{aligned} \quad (4)$$

The expression of $E(\eta)$ has been discussed by several authors. Tanaka-Mori [19, 20] scheme proposed the expression:

$$\begin{aligned} E(\eta) &= \frac{E_A E_M}{2} \left[\frac{(1 - \eta) + \eta \Pi^{MA}}{E_M(1 - \eta) + E_A \eta \Pi^{MA}} \right. \\ &\quad \left. + \frac{(1 - \eta) \Pi^{AM} + \eta}{E_M(1 - \eta) \Pi^{AM} + E_A \eta} \right], \\ \Pi^{MA} &= \frac{2E_M}{E_A + E_M}, \\ \Pi^{AM} &= \frac{2E_A}{E_A + E_M}, \end{aligned} \quad (5)$$

whereas Voigt [21–25] proposed a simple linear model:

$$E(\eta) = E_A(1 - \eta) + E_M(\eta). \quad (6)$$

Both expressions verify the imposed condition for Young modulus, but present the disadvantage of lack of control on the first derivatives of $E(\eta)$, which are either sophisticated (Tanaka-Mori scheme [19, 20]) or constant (Voigt [21–25]).

As per the model of derived by Tanaka and Nagaki [26], the main equation deduced from the first and second laws of thermodynamics can be formulated as

$$T(\varepsilon, \eta) = \rho \frac{\partial \varphi(\varepsilon, \eta)}{\partial \varepsilon}, \quad (7)$$

with

T : Second Piola-Kirchhoff stress,

ρ : Density,

φ : Helmholtz free energy,

ε : Green strain,

η : Martensite fraction.

In the case of pure mechanical constraint, and taking into accounts the presumptions of Chen et al. [27], the main equation becomes

$$T(\varepsilon, \eta) - T_0 = \rho \frac{\partial^2 \varphi}{\partial \varepsilon^2} (\varepsilon - \varepsilon_0) + \rho \frac{\partial^2 \varphi}{\partial \varepsilon \partial \eta} (\eta - \eta_0), \quad (8)$$

$$E(\eta) = \rho \frac{\partial^2 \varphi}{\partial \varepsilon^2},$$

where $E(\eta)$ is Young modulus and $T_0 \varepsilon_0$ and η_0 refer to initial condition for stress, strain and martensite fraction, respectively.

In the actual model, the transformation tensor $\rho(\partial^2 \varphi / \partial \varepsilon \partial \eta)$ is considered as constant, while the Young modulus $E(\eta)$ is expressed as

$$E(\eta) = E_M + \frac{E_M - E_A}{N_0} \sum_{k=1}^{N_0} \xi_k \times B_{4k}(\eta \times r_k), \quad (9)$$

where B_{4k} are the $4k$ -order Boubaker polynomials [28–44], r_k are B_{4k} minimal positive roots, N_0 is a prefixed integer, and $\xi_k |_{k=1, \dots, N_0}$ are unknown pondering real coefficients.

This expression refers to the Boubaker polynomials expansion scheme (BPES) [29–31]. This scheme is a resolution protocol which has been successfully applied to several applied-physics and mathematics problems. The BPES protocol ensures the validity of the related boundary conditions regardless of main equation features. The BPES is mainly based on Boubaker polynomials first derivatives properties:

$$\begin{aligned} \sum_{q=1}^N B_{4q}(x) \Big|_{x=0} &= -2N \neq 0, \\ \sum_{q=1}^N B_{4q}(x) \Big|_{x=r_q} &= 0, \end{aligned}$$

$$\sum_{q=1}^N \left. \frac{dB_{4q}(x)}{dx} \right|_{x=0} = 0,$$

$$\sum_{q=1}^N \left. \frac{dB_{4q}(x)}{dx} \right|_{x=r_q} = \sum_{q=1}^N H_q,$$

$$\text{with } H_n = B'_{4n}(r_n) = \left(\frac{4r_n[2 - r_n^2] \times \sum_{q=1}^n B_{4q}^2(r_n)}{B_{4(n+1)}(r_n)} + 4r_n^3 \right). \quad (10)$$

Several solutions have been proposed through the BPES in many fields such as numerical analysis [28–31], theoretical physics [31–34], mathematical algorithms [35], heat transfer [36], homodynamic [37, 38], material characterization [39], fuzzy systems modeling [40–42], and biology [43, 44].

The main advantage of this formulation (9) is the verification of boundary conditions, expressed in (4), in advance to resolution process, along with affording controllable and easy-to-access first derivatives of $E(\eta)$. In fact, thanks to the properties expressed in (10), these conditions are inherently verified.

$$E(\eta)|_{\eta=0} = E_M + \frac{E_M - E_A}{N_0} \sum_{k=1}^{N_0} \xi_k \times (-2) = E_A, \quad (11)$$

$$E(\eta)|_{\eta=1} = E_M + \frac{E_M - E_A}{N_0} \sum_{k=1}^{N_0} \xi_k \times (0) = E_M.$$

Plots of the η -dependent Young modulus $E(\eta)$ are gathered in Figure 2, for both actual and referred models [19–25].

2.2. Numerical Results and Discussion. Numerical simulation was achieved for the values of parameters gathered in Table 1. The system was taken as insulated thermally and pinned-end mechanically. A variable mechanical load T , expressed as follows, has been considered [45]

$$T = \begin{cases} 3500t, & t \in \left[0, \frac{\theta}{6}\right], \\ 3500(4 - t), & t \in \left[\frac{\theta}{6}, \frac{\theta}{3}\right], \\ 0, & t \in \left[\frac{\theta}{3}, \frac{\theta}{2}\right], \\ 3500(6 - t), & t \in \left[\frac{\theta}{2}, \frac{2\theta}{3}\right], \\ 3500(t - 10), & t \in \left[\frac{2\theta}{3}, \frac{5\theta}{6}\right], \\ 0, & t \in \left[\frac{5\theta}{6}, \theta\right]. \end{cases} \quad (12)$$

$\theta = 12.$

Consequently, the dynamics were simulated using (7)–(8) with the given load (12) and the BPES-related expression (9). Finite element simulations in simple uniaxial solicitation

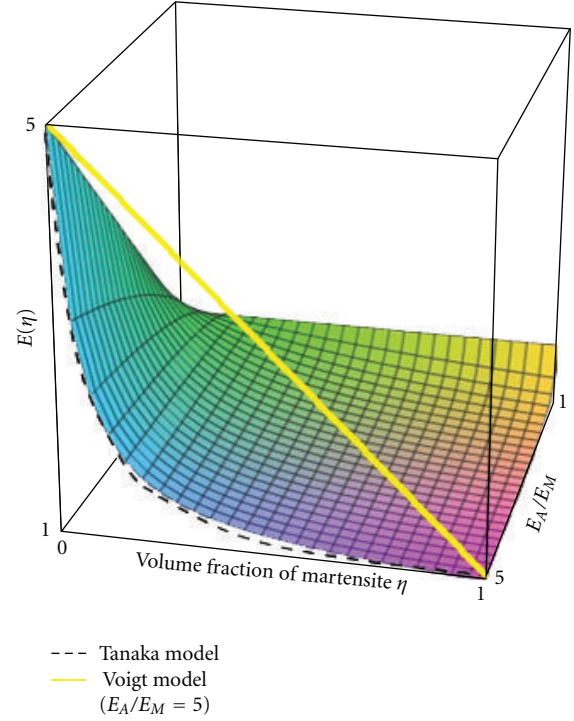


FIGURE 2: Young modulus $E(\eta)$ versus martensite fraction for different values of E_A/E_M .

TABLE 1: Values of d_{hkl} of relevant parameters.

Parameters	Value	Unit
E_A	$6.0 \cdot 10^5$	MPa
E_M	$7.5 \cdot 10^3$	MPa
Rod diameter	1.49	mm

mode were carried out with the preset load using load gradient and the preset η -dependent Young modulus expression as input. The resulting stress-strain response was plotted in Figure 3, along with some records from the related literature [46, 47].

3. Discussion and Perspectives

Numerical stress-strain plots obtained from the actual model confirm that the patterns of hysteresis loop generated in the positive quadrant (Figure 3) are in good agreement with the profiles presented by Auricchio [45], Motahari and Ghassemieh [46], and Sayyadi et al. [47].

Strain span shows also a good agreement with the model performed by Motahari and Ghassemieh [46]. The unique divergence lies in the upper zone ($\varepsilon > 5\%$) and may be attributed to the linear approximation of the η -dependent Young modulus $E(\eta)$ (Voigt-type) instead of the polynomial form of evolution equations assumed in our study. In this context, it can be stated that Tanaka-Mori [19, 20] and Voigt [21–25] models, cannot be efficiently used for predicting the shape memory effect behavior of SMA. This is due to the fact that in the constitutive equations used in those

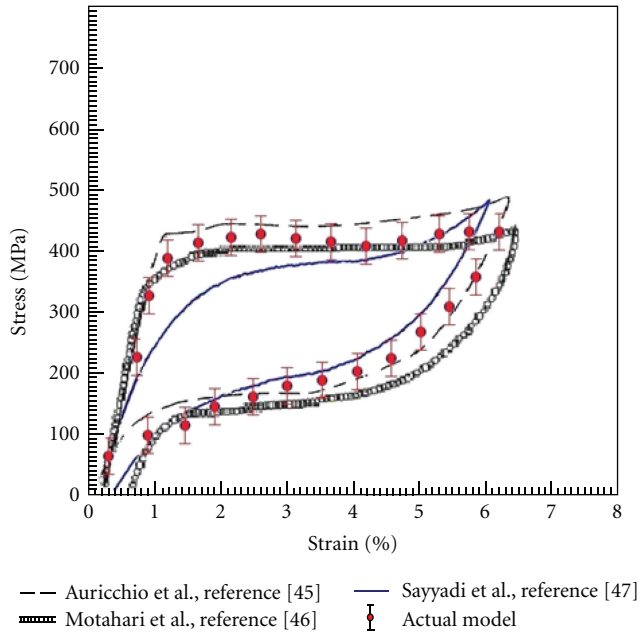


FIGURE 3: Solution plots represented in the Strain-Stress plane.

models, the transformational component is proportional to both martensite volume fraction and its derivatives. Indeed, these models, oppositely to the actual one, introduce a pre-fixed and uncontrollable derivative-dependence.

4. Conclusion

In summary, we have implemented constitutive model for shape-memory alloys capable of undergoing austenite to martensite phase transformation using fundamental thermodynamic laws and the principle of martensite fraction-dependence. The stress-strain plots obtained from uniaxial monitored load were predicted using the finite-element simulations. A key parameter of the performed model consists of avoiding singularity of the main equilibrium equation during the transition since the derivative of the preset η -dependent Young modulus expression are controllable. η -dependent boundary conditions were also inherently taken into account in the model.

Although applied to a particular geometry, the model should be suitable to study other configurations since it was based on a single scalar internal variable: the martensite fraction. This model may be extended to 2D and 3D, while other possible future developments are the inclusion of permanent inelastic effects, the prediction of coupled thermo-mechanical behavior, and the nonlinear hardening mechanisms.

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