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Research Article

GPC with Structured Perturbations: The Influence of Prefiltering and Terminal Equality Constraints

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It is widely accepted that in order to improve the robust stability of generalized predictive control (GPC), the use of a prefilter T and terminal equality constraints plays a fundamental role. In this work it is shown with straightforward counterexamples how, in the presence of structured uncertainties, neither the prefilter nor the equality constraints guarantee that the robust stability is improved. In fact, it can even worsen compared with "conventional" GPC.

1. Introduction

All predictive controllers share a common methodology: at each "present" instant t, future process outputs $y(t+k \mid t)$ are predicted for a certain time window, $k = 1, 2, \ldots, N_2$, using a model of the process. The optimal control law is obtained by minimizing a cost function as follows:

$$J(\Delta u, t) = E\left\{ \sum_{j=N_1}^{N_2} \gamma(j)\left[r(t+j \mid t) - y(t+j \mid t)\right]^2 + \sum_{j=1}^{N_u} \lambda(j)\left[\Delta u(t+j-1 \mid t)\right]^2 \right\}, \tag{1.1}$$

where $E\{\cdot\}$ is the expectation operator, N_1 is the minimum costing horizon, N_2 is the maximum costing horizon, N_u is the control horizon, γ is a future errors weighting sequence, and λ is a control weighting [1, 2]. (In this work only the unconstrained case is considered.)

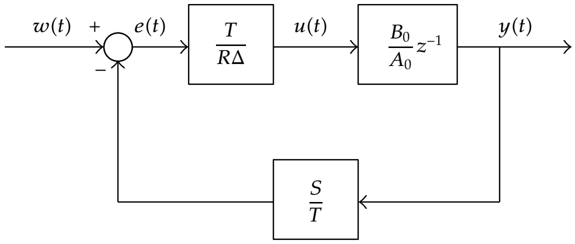


Figure 1: GPC structure.

Generalized predictive control (GPC) [3–5] is one of the most representative predictive controllers. GPC assumes a CARIMA model (a transfer function plus a colored and integrated white noise) to describe the following system dynamics:

$$A\left(z^{-1}\right)y(t) = B\left(z^{-1}\right)u(t) + \frac{T\left(z^{-1}\right)}{\Delta}\xi(t), \tag{1.2}$$

where Δ is the increment operator and $\xi(t)$ represents uncorrelated zero-mean white noise. $T(z^{-1})$ is a polynomial which implements a prefilter.

In practice, T is not considered a model parameter but a controller parameter, as its value is chosen to improve the closed-loop robustness. In fact, prefiltering is one of the most popular approaches to robust design in GPC.

The effects of prefiltering on robustness were initially considered by Robinson and Clarke [6], who gave some guidelines for selecting T. However, these results are applied only for mean-level or dead-beat GPC. Soeterboek [7] pointed out that robustness would be enhanced by choosing $T = A(1 - \alpha z^{-1})$, as α increases. Megías et al. [8] showed that α cannot be increased unlimitedly to improve robustness because performance would deteriorate. Yoon and Clarke [9] extended these works, suggesting that a more general guideline, $T = A(1 - \alpha z^{-1})^{N_1}$, improves the robust stability because it ensures the presence of a low-pass filter in the control loop that rejects the high frequency unmodeled responses. This recommendation is widely accepted in the literature and deserves some further explanations.

GPC control structure can be expressed in a classical LTI form (Figure 1). From it, it is easy to derive the expression of the closed-loop characteristic equation as follows:

$$\delta_0\left(z^{-1}\right) = RA_0\Delta + B_0 S z^{-1}, \tag{1.3}$$

where A_0 and B_0 represent the transfer function of the actual plant (generally different to A and B, the model transfer function used to design the GPC controller), R and S are the following polynomials:

$$R = \frac{T + \sum_{i=N_1}^{N_2} k_{1i} H_i}{\sum_{i=N_1}^{N_2} k_{1i} q^{-N_2+i}}, \qquad S = \frac{\sum_{i=N_1}^{N_2} k_{1i} F_i}{\sum_{i=N_1}^{N_2} k_{1i} q^{-N_2+i}}, \tag{1.4}$$

and H_i and F_i are polynomials that can be derived from some Diophantine equations [3, 4].

Assuming unstructured uncertainties and using the Small Gain Theorem, Yoon and Clarke [9] concluded that it is required that S/T be a low-pass filter and therefore $T \neq 1$ should be used, otherwise $S/T(= S)$ would be a high-pass filter. According to this, they proposed a "natural" choice for T as follows:

$$T = A\left(1 - \alpha z^{-1}\right)^{N_1}, \tag{1.5}$$

where α lies in the neighborhood of the dominant root of A.

The main handicap of this approach is that it has been shown that (1.5) is fairly simplistic. For this reason, deep analysis for each given case is needed. Sensitivity analysis (to multiplicative uncertainty, to disturbances, and to noise) has been proposed by Rossiter [10]. In practice, this analysis shows that the effect of T is a trade-off between disturbances or noise rejection and robustness to model parameter uncertainty at different frequency bands, in a way that is not always beneficial. If the chosen T-filter does not achieve the desired sensitivities, it may not be obvious how to redesign it to improve matters.

Following this line of argument, in this paper we will study the influence of T on robustness when structured perturbations are considered using tools applicable to polytopes of polynomials. Previous analysis [11] suggested that (1.5) might improve the robust stability under structured perturbations. Now, we will show how, in general, (1.5) can lead to poorer robustness when these perturbations are present and how this fact can be depicted in geometrical terms.

The use of equality constraints forcing the controlled output to match the reference signal at the end of the cost horizon (CRHPC, SIORHC) [12] is another important method to ensure the stability.

For completeness purposes, in this paper we will show that in presence of structured perturbations the close loop stability of CRHPC can be studied using polytopes theory and that CRHPC does not necessarily improve the stability compared with GPC [11].

In this work, the perturbations will be considered directly established on the plant coefficients, that is, structured perturbations. Robust stability analysis will focus on how prefiltering, T, and terminal equality constraints, two keys of predictive control, are affected when the knowledge about the process parameters is not exact.

This paper has been structured as follows. In Section 2 the GPC control of plants with structured uncertainties is introduced and it is shown by examples how the general guideline for choosing T is not always adequate. In Section 3 the CRHPC control of plants with structured uncertainties is introduced and it is shown that this controller does not improve the stability in presence of uncertainties compared with a common GPC. Finally, Section 4 draws the main conclusions of this paper.

2. GPC with Structured Uncertainties and the Study of Prefiltering

Structured perturbations mean that the uncertainties are in the coefficients, that is, the numerator and the denominator of the actual plant are given by uncertain polynomials. Affine linear uncertainty structures will be considered. Thus, given a set of real parameters q_i, $i = 0, \dots, l$, which can vary between a maximum and a minimum value, $q_i^- \leq q_i \leq q_i^+$, $i = 0, \dots, l$, the coefficients of the numerator and denominator polynomials are affine linear functions of the uncertainty parameter vector $\mathbf{q} = (q_0, \dots, q_l) \in \Re^l$, that is, $a_i(\mathbf{q}) = \alpha_i^T \mathbf{q} + \beta_i$, $i =$

$0, \dots, m$, $b_i(\mathbf{q}) = \gamma_i^T \mathbf{q} + \rho_i$, $i = 0, \dots, n$ where α_i and γ_i are $1 \times l$ vectors and β_i and ρ_i are scalars. Then, the actual plant is defined by the following family of plants:

$$G_0\left(\mathbf{q}, z^{-1}\right) = \frac{B_0(\mathbf{q}, z^{-1})}{A_0(\mathbf{q}, z^{-1})} = \frac{\sum_{i=0}^{n} b_i^0(\mathbf{q}) z^{-i}}{1 + \sum_{i=1}^{m} a_i^0(\mathbf{q}) z^{-i}}, \tag{2.1}$$

where $(m \geq n \geq 0)$. With this structure of uncertainties, $A_0(\mathbf{q}, z^{-1})$ and $B_0(\mathbf{q}, z^{-1})$ *are polytopes of polynomials in* z^{-1}, and $G_0(\mathbf{q}, z^{-1})$ *is a polytope of plants in* z^{-1}.

It has been shown that the family of characteristic polynomials $\delta_0(z^{-1})$ of the closed-loop system constituted by a GPC controller (1.4) and the family of plants (2.1) is a polytope of polynomials [11, 13, 14] as follows:

$$\delta_0\left(z^{-1}\right) = R\left(z^{-1}\right) A_0\left(\mathbf{q}, z^{-1}\right) \Delta + S\left(z^{-1}\right) B_0\left(\mathbf{q}, z^{-1}\right) z^{-1}. \tag{2.2}$$

This fact allows the use of a very mature theory from the point of view of the robust stability analysis. Nowadays it can be said that there are powerful results to analyze the stability and the robust performance of families of polynomials formed by interval polynomials or by polytopes of polynomials. The main tools for the analysis of polynomial families are Kharitonov Theorem [15] for interval polynomials and the Edge Theorem [16] and Rantzer Theorem [17] for polytopes of polynomials.

In order to determine which T leads to the best robustness when structured uncertainties are present, we propose the analysis of the stability region in the parameter space derived from the closed loop characteristic equation [18]. The method of Ackermann [19] will be used to draw this region because it has low computational cost and no conservatism. The stability hypersphere around the nominal process will be also analyzed.

The following two examples will illustrate how guideline (1.5) influences robustness in the presence of structured uncertainties.

Example 2.1. Let us revisit an example proposed by Yoon and Clarke [9] as follows:

$$G = \frac{B}{A} = \frac{0.2}{1 - 0.8z^{-1}}, \tag{2.3}$$

and let us assume that the plant is actually represented by the following family (interval) of plants:

$$G_0 = \frac{0.2}{1 + (-0.8 + \alpha_1)z^{-1} + \alpha_2 z^{-2}}, \tag{2.4}$$

with α_1 and α_2 the uncertainty parameters. The GPC controller is tuned with the following predictive control settings: $N_1 = 1$, $N_u = 2$, $N_2 = 5$, and $\lambda = 0.01$.

Case 1 ($T = 1$). In the absence of prefiltering (i.e., $T = 1$) the polynomials R and S (1.4) are the following:

$$
\begin{aligned}
R\left(z^{-1}\right) &= 0.2624, \\
S\left(z^{-1}\right) &= 1.9231 - 0.9231z^{-1},
\end{aligned}
\tag{2.5}
$$

and the closed-loop characteristic equation is

$$
\begin{aligned}
\delta_0(z, \alpha_1, \alpha_2) = {} & 0.2624z^3 + (-0.0877 + 0.2624\alpha_1)z^2 \\
& + (0.0253 - 0.2624\alpha_1 + 0.2624\alpha_2)z \\
& - 0.2624\alpha_2.
\end{aligned}
\tag{2.6}
$$

Case 2 ($T = A(1 - 0.8z^{-1})$). Now we will follow the standard guideline (1.5): $T = A(1 - \alpha z^{-1})^{N_1}$, with $\alpha \in (0, 1)$. It is extensively accepted that the robustness will be better when α lies in the neighborhood of the dominant root of A. Therefore, we will take $\alpha = 0.8$ as it was proposed in [10]. Thus R and S are

$$
\begin{aligned}
R\left(z^{-1}\right) &= 0.2624 - 0.0752z^{-1} + 0.0202z^{-2}, \\
S\left(z^{-1}\right) &= 0.2 - 0.16z^{-1},
\end{aligned}
\tag{2.7}
$$

$$
\begin{aligned}
\delta_0(z, \alpha_1, \alpha_2) = {} & 0.2624z^5 + (-0.5075 + 0.2624\alpha_1)z^4 \\
& + (0.3335 - 0.3376\alpha_1 + 0.2624\alpha_2)z^3 \\
& + (-0.09652 + 0.0954\alpha_1 - 0.3376\alpha_2)z^2 \\
& + (0.01616 - 0.0202\alpha_1 + 0.0954\alpha_2)z - 0.0202\alpha_2.
\end{aligned}
\tag{2.8}
$$

The families of polynomials (2.6) and (2.8) are polytopes as it was stated above (2.2). Figure 2 shows their stability regions.

The area of the stability region for $T = 1$ is smaller than the one that follows the recommendation (1.5). Therefore, it could be concluded that the robust stability, in terms of the associated stability areas, has been improved with prefiltering.

However, the radius of the stability hypersphere (in this case just a circle) around the nominal process is smaller with prefiltering. Even though the stability region is bigger, from this point of view, prefiltering following (1.5) deteriorates the robustness of the closed-loop system.

Example 2.2. Now one of the examples proposed by Rossiter [10] is considered as follows

$$
G = \frac{1}{1 - 1.4z^{-1} + 0.45z^{-2}},
\tag{2.9}
$$

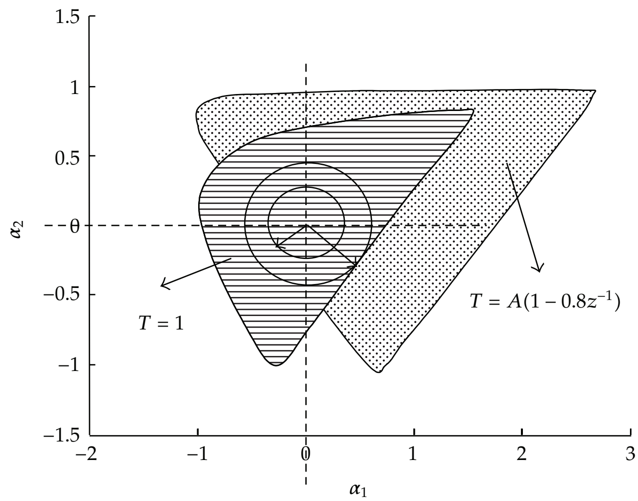


Figure 2: Example 2.1. Stability regions for $T = 1$ and $T = A(1 - 0.8z^{-1})$.

assuming that the actual perturbations are structured and therefore the actual plant is given by a family (interval) of plants as

$$G_0 = \frac{1}{1 + (-1.40 + \alpha_1)z^{-1} + (0.45 + \alpha_2)z^{-2}}, \tag{2.10}$$

with α_1 and α_2 the uncertainty parameters. The GPC controller has been tuned with $N_1 = 1$, $N_u = 1$, $N_2 = 10$, and $\lambda = 0.01$.

This example illustrates the situations considered in [10]. No prefiltering (i.e., $T = 1$, the situation rejected by Yoon and Clarke [9]) and T following the general recommendation $(1 - 0.8z^{-1})^n$, $n = 1$ or 2. (This choice of α is intuitive in that if sampling at about 1/10 of the rise time, a common guideline for predictive control, then a typical dominant process pole would be around 0.8. Hence this is a sensible pole for a low-pass filter on output measurements.)

For the nominal system, the sensitivity analysis performed in [10] concludes that the inclusion of a T-filter has given good reductions in every sensitive function over the high frequency: the sensitivity function to multiplicative uncertainty is actually better over the whole frequency range, the output sensitivity is worse at mid and low frequencies and better at high frequencies, and the input sensitivity is better over the whole frequency range.

Now let us consider the effect of T-filter on structured uncertainties .

Case 1 ($T = 1$). The controller is given by the following expressions:

$$\begin{gathered} R\left(z^{-1}\right) = 8.9245, \\ S\left(z^{-1}\right) = 9.9977 - 13.0137z^{-1} + 4.0159z^{-2}, \end{gathered} \tag{2.11}$$

and the closed-loop characteristic equation is

$$\begin{aligned}\delta_0(z,\alpha_1,\alpha_2) = {} & 8.9245z^3 + (-11.421 + 8.9245\alpha_1)z^2 \\ & + (3.4963 - 8.9245\alpha_1 + 8.9245\alpha_2)z \\ & - 8.9246\alpha_2 - 0.000125.\end{aligned} \tag{2.12}$$

Case 2 ($T = (1 - 0.8z^{-1})$). Now we obtain

$$\begin{aligned}R\left(z^{-1}\right) &= 8.9245 - 0.0001z^{-1}, \\ S\left(z^{-1}\right) &= 2.8583 - 3.8772z^{-1} + 1.2189z^{-2},\end{aligned} \tag{2.13}$$

$$\begin{aligned}\delta_0(z,\alpha_1,\alpha_2) = {} & 8.9245z^4 + (-18.561 + 8.9245\alpha_1)z^3 \\ & + (12.633 - 8.9246\alpha_1 + 8.9245\alpha_2)z^2 \\ & + (-2.7973 - 0.0001\alpha_1 - 8.9246\alpha_2)z \\ & - 0.0001\alpha_2 + 0.000045.\end{aligned} \tag{2.14}$$

Case 3 ($T = (1 - 0.8z^{-1})^2$). In this case, we obtain

$$\begin{aligned}R\left(z^{-1}\right) &= 8.9245 - 4.9728z^{-1} + 0.0001z^{-2}, \\ S\left(z^{-1}\right) &= 0.6914 - 0.9632z^{-1} + 0.3118z^{-2},\end{aligned} \tag{2.15}$$

$$\begin{aligned}\delta_0(z,\alpha_1,\alpha_2) = {} & 8.9245z^5 + (-25.7 + 8.9245\alpha_1)z^4 \\ & + (27.482 + 13.897\alpha_1 + 8.9245\alpha_2)z^3 \\ & + (-12.904 + 4.9729\alpha_1 - 0.9632\alpha_2)z^2 \\ & + (2.2379 - 0.0001\alpha_1 + 4.9729\alpha_2)z \\ & - 0.0001\alpha_2 - 0.000045.\end{aligned} \tag{2.16}$$

Figure 3 shows the stability regions of the polytopes (2.12), (2.14), and (2.16). The stability region for $T = 1$ is smaller than the ones that follow the recommendation (1.5). However, as in the previous example the stability hypersphere around $(0,0)$—absence of uncertainties—is bigger when there is no prefiltering ($T = 1$). Therefore, in this sense robustness is not improved by the guideline.

3. CRHPC with Structured Uncertainties

This controller was developed under the necessity of guaranteeing stability. The special feature of this controller is the use of equality constraints forcing the controlled output to

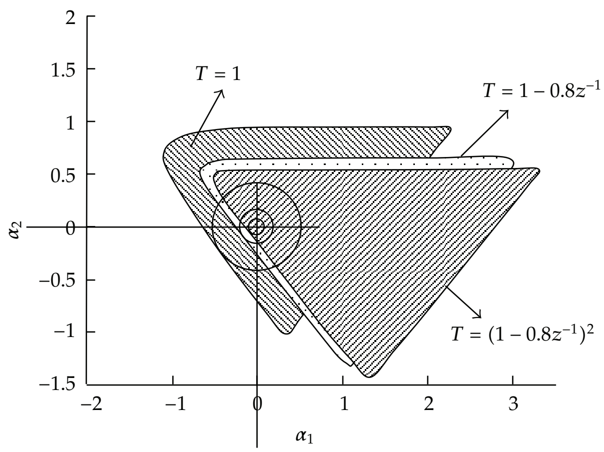


Figure 3: Example 2.2. Stability regions for $T = 1$, $T = A(1 - 0.8z^{-1})$, and $T = A(1 - 0.8z^{-1})^2$.

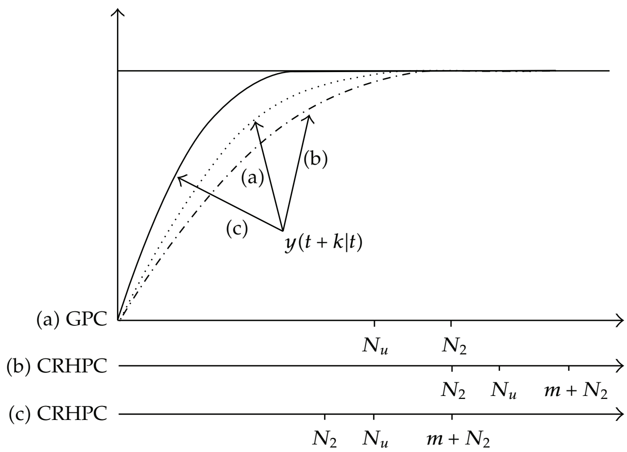


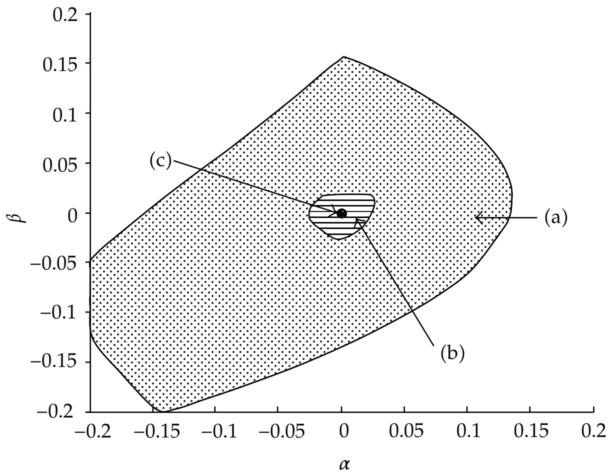
Figure 4: Horizons for the different controllers.

match the reference signal over a terminal constraint window m at the end of the costing horizon. The controller horizons must be chosen according to the following rules to guarantee the stability: $N_1 = 1$, $N_u = N_2 + 2 - d$, where d is the delay, $\lambda(j) > 0$ and m must be $0 \leq m \leq N_u$ and equal to n, the order of the incremental system, and $n = \max[\deg(A), \deg(B)] + 1$.

The objective is to analyze the stability of this controller under the assumption that the model is different to the process. To carry out this, it is necessary to calculate the analytic control law of this controller and to obtain the expression of the characteristic equation.

The solution of the constrained minimization problem associated with CRHPC leads to a control law with R and S as in (3.1), where Π_1, defined following [20], is a vector that plays the same role as $\mathbf{k}_1$ in GPC Consider the following:

$$R = \frac{T + z^{-1} \sum_{i=N_1}^{N+m} \Pi_{1i} H_i}{\sum_{i=N_1}^{N+m} \Pi_{1i}}, \qquad S = \frac{\sum_{i=N_1}^{N+m} \Pi_{1i} F_i}{\sum_{i=N_1}^{N+m} \Pi_{1i}}. \tag{3.1}$$



(a) GPC
(b) CRHPC (equality constraint after the predicted output)
(c) CRHPC (equality constraint over the predicted output)

Figure 5: Stability regions.

From the analysis of the characteristic equation we obtain that the family of characteristic polynomials $\delta(z^{-1})$ of the closed-loop system constituted by a CRHPC predictive controller (3.1) and the family of plants (2.1) is a polytope of polynomials [20]. Again, this result permits us to apply the theory based on polytopes in order to study the robust stability of CRHPC.

When the model is equal to the process the real output matches the reference signal due to the equality constraints and the stability properties of the control law may be stated. On the other hand, when the model is different to the process, multiple possible outputs exist, one for each plant of the family, and obviously it is impossible to force every real outputs to unique value given by the equality constraint. The only thing we can do in order to implement this kind of controller when the process is represented by a family of plants is to apply the equality constraint on the predicted output. Obviously, as the process is different to the model, the real output is different to the predicted output, and the real output does not necessary match with the reference signal. Even so, it seems reasonable in the following conjecture: "CRHPC improves the stability compared with GPC in the presence of uncertainties." However, it will be shown in the following counterexample that this conjecture is not true.

3.1. Example: CRHPC with Uncertain Parameters

The model is described by the following transfer function:

$$\begin{gathered} B = 0.65 - 0.1950z^{-1} - 0.26z^{-2}, \\ A = 1 - 0.9z^{-1} - 0.01z^{-2} + 0.1050z^{-3}. \end{gathered} \tag{3.2}$$

Let us assume the process with uncertainties in the following coefficients:

$$B_0 = 0.65 - 0.1950z^{-1} - 0.26z^{-2},$$
$$A_0 = 1 - 0.9z^{-1} + (-0.01 + \alpha)z^{-2} + (0.1050 + \beta)z^{-3}. \tag{3.3}$$

Firstly, a GPC is applied with the parameters tuned to $N_1 = 1$, $N_2 = 7$, $N_u = 4$, $\lambda = 0.1$, and $m = 0$, (without equality constraints).

The characteristic equation is represented by this polytope as follows:

$$\begin{aligned}\delta = {} & 0.8578z^6 + 0.5668z^5 + (-0.1123 + 0.8578\alpha)z^4 \\ & + (0.02857 + 0.8578\beta - 1.1433\alpha)z^3 + (-0.01224 - 0.02318\alpha - 1.1433\beta)z^2 \\ & + (-0.0232\beta + 0.3087\alpha)z + 0.3087\beta.\end{aligned} \tag{3.4}$$

Secondly, the study is repeated using CRHPC. We consider the following two cases (see Figure 4):

(i) $N_1 = 1$, $N_2 = 7$, $N_u = 8$, $\lambda = 0.1$, and $m = 4$ leads to:

$$\begin{aligned}\delta = {} & 0.6935z^6 - 0.4252z^5 + (-0.0861 + 0.6935\alpha)z^4 \\ & + (0.02245 - 2.4041\alpha + 0.6935\beta)z^3 + (-0.0093 + 0.7199\alpha - 2.4041\beta)z^2 \\ & + (0.9909\alpha + 0.7199\beta)z + 0.9909\beta;\end{aligned} \tag{3.5}$$

(ii) $N_1 = 1$, $N_2 = 3$, $N_u = 4$, $\lambda = 0.1$, and $m = 4$ leads to:

$$\begin{aligned}\delta = {} & 0.195z^6 + 0.195\alpha z^4 + (-6.2178\alpha + 0.195\beta)z^3 \\ & + (2.9711\alpha - 6.2178\beta)z^2 + (3.0517\alpha + 2.9711\beta)z + 3.0517\beta.\end{aligned} \tag{3.6}$$

Figure 5 shows the stability regions for these three controllers. It can be concluded that the stability of CRHPC does not improve. The stability regions are smaller when the equality constraints have been considered than when they were not included in the optimisation and the computational cost has been higher.

Summarising, the fact of using a CRHPC instead of GPC under the assumption that the process presents uncertainties does not improve necessarily the robust stability. It can be found that a GPC controller works better than a CRHPC controller with a lower computational cost.

4. Conclusions

This paper has focused on the study of the influence of prefilter T and equality constraints on the robustness of GPC against structured uncertainties. Given that the closed loop is a polytope of polynomials, it is possible to analyze the robust stability with tools based on polytopes with ease and no conservatism.

Some examples have shown that the widely accepted guideline for choosing T does not guarantee better robustness. In fact, this has been shown with simple geometrical measurements, such as the area of the stability region or the radius of the stability hypersphere in the uncertainties space.

Examples have shown that there exist "directions" in the uncertainties space where the robust stability margins are better and "directions" where they are worse. This situation is somehow similar to the sensitivity analysis when unstructured disturbances are considered, where there could be frequency bands where sensitivity is improved and bands where it is worsened.

In relation to the influence of equality constraints (CRHPC), it has been shown that they do not necessary improve the stability region compared with a conventional GPC.

For these reasons, no matter the type of disturbances that are present, a deep analysis of robustness (sensitivity functions, stability regions, etc.) is needed.

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