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Research Article

Improved Expression for Estimation of Leakage Inductance in *E* Core Transformer Using Energy Method

**Sivananda Reddy Thondapu,[1] Mangesh B. Borage,[2]
Yashwant D. Wanmode,[1] and Purushottam Shrivastava[1]**

[1] *Pulse High Power Microwave Section, Raja Ramanna Centre for Advanced Technology, Indore 452013, India*
[2] *Power Supplies and Industrial Accelerator Division, Raja Ramanna Centre for Advanced Technology, Indore 452013, India*

Correspondence should be addressed to Sivananda Reddy Thondapu, sivananda@rrcat.gov.in

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This paper proposes a simpler and more accurate expression for estimation of leakage inductance in E core transformer, which is the most widely used transformer structure. The derived expression for leakage inductance accounts for the flux extending into air. The finite element method (FEM) analysis is made on the secondary shorted transformer to observe the H-field pattern. The results obtained from FEM analysis are used for approximating the field that is extending into air to derive an expression for leakage inductance. This expression is experimentally validated on prototype transformers of different core dimensions.

1. Introduction

Transformer is one of the basic building blocks of many power converters. The following are some of the cases where accurate estimation of leakage inductance is required.

(i) Different resonant converter topologies, discussed in [1–5], use parasitics of transformer as a part of resonant tank network. For designing power converter with such topologies, one requires accurate estimation of leakage inductance.

(ii) In hard switched converters, in every cycle the energy stored in the parasitics appears as loss in converter. In estimation of efficiency of such converters, one needs to estimate leakage inductance before hand.

(iii) For designing snubber circuits to limit device voltage during turn-off transients [6–8], one needs to estimate leakage inductance. These turn-off transients mainly occur due to energy stored in the leakage inductance of the transformer.

Methods that are usually employed for estimation of leakage inductance are (i) energy method [8–13] and (ii) method of mutual fluxes.

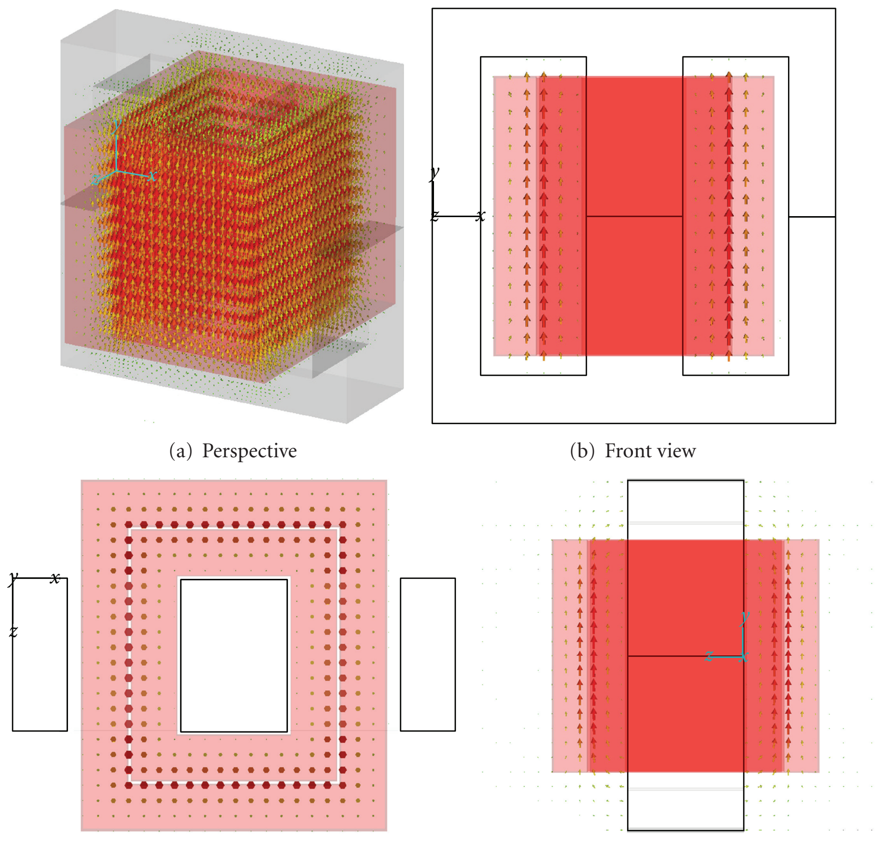
In energy method, the energy stored in magnetic field of the secondary shorted transformer is calculated and equated to $(1/2)L_{\text{leak}}I_p^2$ where L_{leak} is the leakage inductance of the transformer when referred to primary, and I_p is current flowing through primary.

The H-profile inside the coil is calculated using Ampere's law. The energy stored in magnetic field is calculated by evaluating the volume integral in (1):

$$E_{\text{stored}} = \frac{\mu}{2}\iiint H^2\,dv = \frac{1}{2}L_{\text{leak}}I_p^2. \quad (1)$$

The expression derived for leakage inductance using energy method is independent of frequency. Hence, it does not consider any frequency-dependent effects on leakage inductance. The energy method is used for comparing leakage inductance, in different winding configurations.

On the other hand, method of mutual fluxes uses Maxwell's equations to predict the leakage inductance more accurately at high frequencies. As this method accounts for frequency-dependent effects like eddy current losses and altered flux pattern due to eddy currents, it gives more accurate results, particularly at high frequencies. In [14], a frequency-dependent formula is presented to find leakage inductance in a toroidal core transformers.



(a) Perspective (b) Front view (c) Top view (d) Side view

Figure 1: Simulation result of FEM analysis.

In this paper, an expression for leakage inductance is derived using energy method. Expression derived accounts for the flux that extends into air in a secondary shorted transformer, which is not considered earlier [8, 10, 11]. Therefore, the estimated leakage inductance using this expression has better accuracy.

2. Estimation of Leakage Inductance on *E* Core Transformer

FEM analysis on secondary shorted transformer is made using CST Ver 2008.5 (with magneto-static solver). The simulation result of FEM analysis is shown in Figure 1. The simulation result is the H-field pattern of secondary shorted E core transformer. This gives an idea of profile of H-field from the surface of the core to the outer surface of the whole winding. The profile of H-field of a E core transformer and its winding configuration is shown in Figures 2 and 3, respectively.

Energy stored in the magnetic field in the secondary shorted transformer is equated to energy stored in leakage inductor, which is given in (1).

The volume represented with purple color in Figure 4 is considered for volume integration. This constitutes (i) the volume occupied by the coils and (ii) the volume formed by extrusion of the coils along the centre limb of the core, excluding the volume of the core. The flux extends into air shown in Figure 5. The volume shown in Figure 4 is divided into four parts, whose top view is shown in Figure 3. The profile of H-field is calculated using Ampere's law, the same profile is observed in FEM analysis. The H-profile is expressed mathematically in (2):

$$H(z) = \begin{cases} H_m \dfrac{z}{h_1} & 0 < z < h_1, \\ H_m & h_1 < z < h_1 + t, \\ H_m \left(\dfrac{h_1 + h_2 + t - z}{h_2} \right) & h_1 + t < z < h_1 + h_2 + t. \end{cases} \tag{2}$$

Using H-profile given in (2) to evaluate volume integral given in (1), we get expression of the leakage inductance:

$$L_{\text{leak}} = \frac{1}{3} \frac{\mu_o N_1^2}{F^2} (h + 2t) [FC + B(E + 2h)], \tag{3}$$

where $h = h_1 + h_2 + t$ is total thickness of the winding measured from the surface of the core to the outer surface of the outer winding, t is total thickness of the insulation used between the layers. B, C, E, F is are the dimensions of the E core shown in Figure 6.

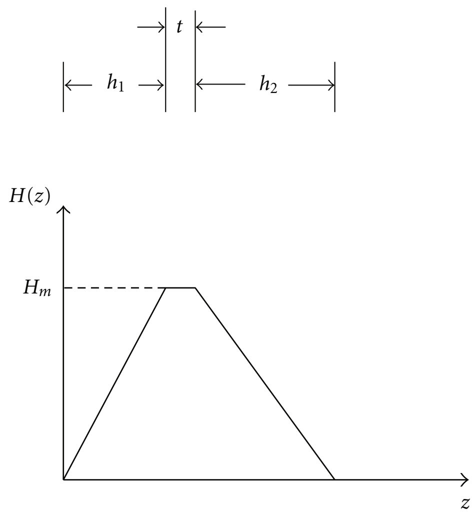


Figure 2: Profile of $H(z)$ from the surface of the core.

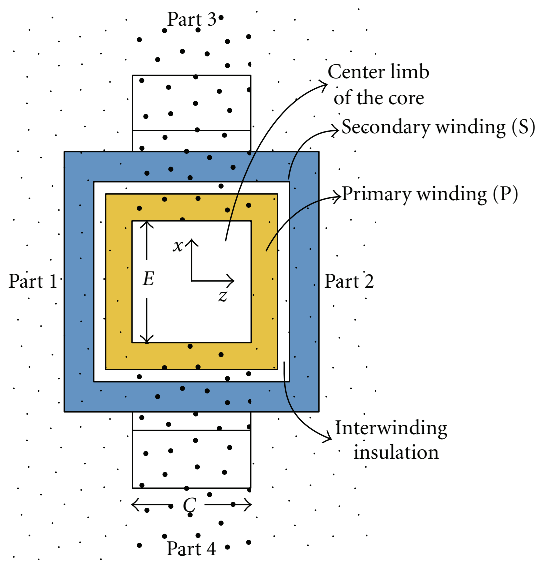


Figure 3: Top view of E core transformer with windings.

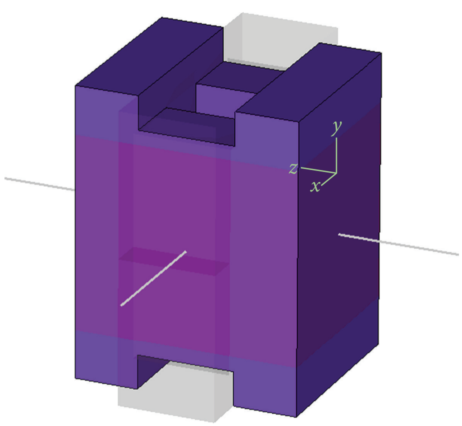


Figure 4: Volume around the core considered for integration.

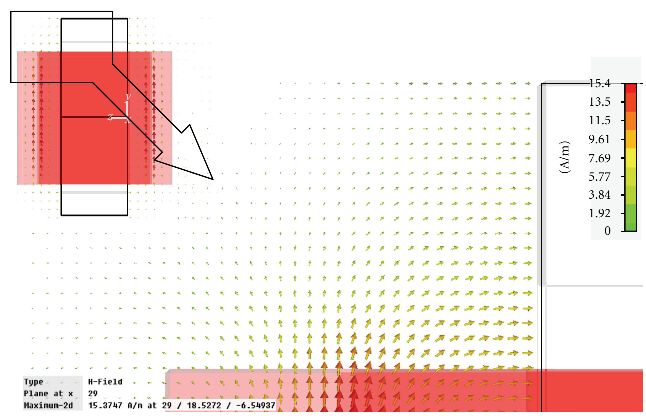
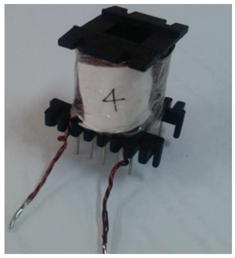
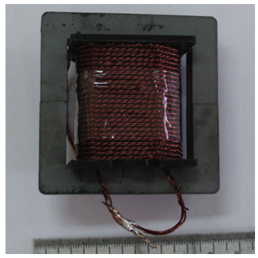
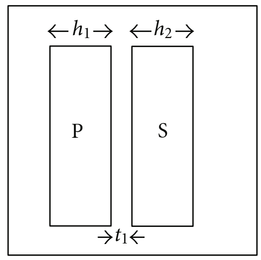


Figure 5: Scaled side view to show the flux that extends into air.

Table 1: EE core transformer with interwinding insulation.

Sample no.	1	2	
Core type	EE42/21/15	EE65/38/13	
F	14.45 mm	22.65 mm	
C	15.20 mm	13.45 mm	
B	21.10 mm	32.59 mm	
E	12.05 mm	19.77 mm	
h_1 thickness of primary	3.20 mm	3.81 mm	
h_2 thickness of secondary	1.90 mm	1.55 mm	
t thickness of insulation	1.27 mm	2.00 mm	
N_1	34	48	
N_2	17	24	Winding configuration
Photograph			
Leakage inductance $L_{leak,earlier}$ from (5)	11.91 μH	22.52 μH	
Leakage inductance L_{leak} from (3)	15.32 μH	28.21 μH	
Leakage inductance L_{exp} at 10 kHz measured	14.13 μH	26.76 μH	
% deviation of L_{exp} from calculated $L_{leak,earlier}$	15.71%	15.84%	
% deviation of L_{exp} from calculated L_{leak}	8.38%	5.12%	

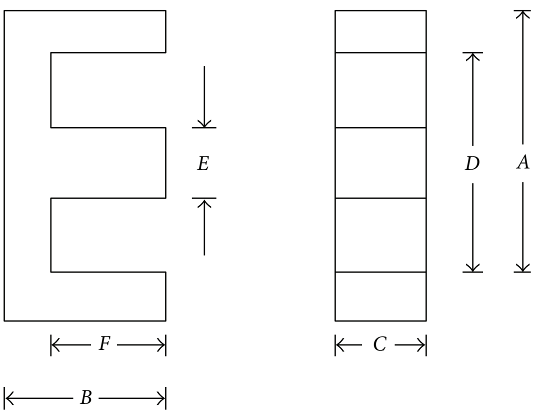


Figure 6: E core dimensions given in data sheets.

It is seen in [8, 11–13], by sandwiching the windings, leakage inductance reduces by p^2 times, where p is the number of interfaces between primary and secondary [8]:

$$L_{leak} = \frac{1}{3p^2}\frac{\mu_o N_1^2}{F^2}(h+2t)[FC+B(E+2h)]. \qquad (4)$$

3. Experimental Results

Three different samples are made to validate the derived expression. The results are compared with the expressions available in literature. The expression given in [8, 10, 11] is replaced with the dimensions of the core (shown in Figure 6) and the dimensions of windings will give (5):

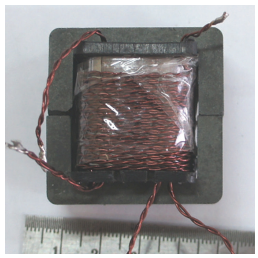
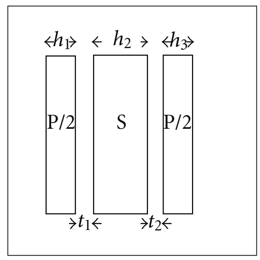
$$L_{leak,earlier} = \frac{1}{3p^2}\frac{\mu_o N_1^2}{F^2}(h+2t)F(C+E+2h). \qquad (5)$$

Sample no. 1 is made with E core (EE42/21/15) wound with stranded conductor made with three conductors of 24 SWG. With interwinding insulation of thickness 1.27 mm is used. Standard winding configuration as shown in Table 1 is used to wind primary and secondary.

Sample no. 2 is made with E core (EE65/38/13) wound with stranded conductor made with three conductors of 24 SWG. With interwinding insulation of thickness 2.00 mm. Standard winding configuration as shown in Table 1 is used to wind primary and secondary.

Sample no. 3 is made with E core (EE42/21/15) wound with stranded conductor made with two conductors of 24 SWG. With total interwinding insulation of thickness 0.46 mm + 0.26 mm = 0.72 mm is used. Sandwiched winding

Table 2: EE core transformer with sandwiched winding (P/2, S, P/2).

Sample no.	3	
Core type	EE42/21/15	
F	14.45 mm	
C	15.20 mm	
B	21.10 mm	
E	12.05 mm	
h_1 thickness of primary half	1.41 mm	
h_2 thickness of secondary	2.93 mm	
h_3 thickness of primary half	1.52 mm	
t_1 thickness of insulation	0.46 mm	
t_2 thickness of insulation	0.26 mm	
N_1	46	
N_2	44	
Photograph		Winding configuration 
Leakage inductance $L_{\text{leak,earlier}}$ from (5)	4.94 μH	
Leakage inductance L_{leak} from (4)	6.37 μH	
Leakage inductance L_{exp} at 10 kHz measured	5.91 μH	
% deviation of L_{exp} from calculated $L_{\text{leak,earlier}}$	16.41%	
% deviation of L_{exp} from calculated L_{leak}	7.82%	

configuration as shown in Table 2 is used to wind primary and secondary.

All the measurements for leakage inductance are made by shorting the secondary. The measurements are made with LCR meter (make: Gw instek Model: LCR-8101) at 10 kHz.

Tables 1 and 2 tabulate experimentally measured values, estimated values using the expression derived in this paper and expression derived earlier for leakage inductance and their corresponding errors when compared with the experimentally measured values.

4. Conclusion

In this paper an improved expression for leakage inductance has been derived, using energy method. To observe the H-field pattern, FEM analysis has been carried out on a secondary shorted transformer, with primary excited. Experimental results show that the formula derived in this paper has better accuracy. It is observed that the leakage inductance estimated using improved expression L_{leak} has less deviation from L_{exp} when compared with leakage inductance estimated from earlier expression $L_{\text{leak,earlier}}$. The accuracy of the estimated value is improved due to consideration of flux that is extending into air.

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