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Research Article

Onset of Magnetic Monopole-Antimonopole Condensation

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We determine the critical strength of the effective electric coupling for the onset of Bose condensation of stable magnetic monopoles and antimonopoles in SU(2) Yang-Mills thermodynamics. Two scenarios are considered: infinitely fast and infinitely slow downward approaches of the critical temperature. Our results support the claim that the first lepton family and the weak interactions emerge from pure SU(2) gauge dynamics of scale ~0.5 MeV.

1. Introduction

A genuine understanding of fundamental physics requires well-controlled conditions. As for four-dimensional Quantum Yang-Mills theory unadulterated thermodynamics seems to be such a setting [1–3]. Because the term "understanding" not only refers to the ability to derive quantitative results but also refers to one's position to actually interpret them in deterministic terms a connection between effective (results) and fundamental (interpretation) fluctuations in the according formulations of the same partition function needs to be made. This connection also serves as the pointer to how a controlled deformation of the original thermodynamical setting can be carried out.

In this note we are concerned with the transition between deconfining and preconfining SU(2) Yang-Mills thermodynamics. In approaching this transition from above in a specified way, we ask the question what the critical value for the effective electric coupling e is at which the thermal ground state of the deconfining phase drastically rearranges such as to attribute mass to a formerly massless gauge mode (condensation of magnetic (anti)monopoles). Technically speaking, the answer to this question is obtained in a surprisingly simple way if the above-mentioned connection between effective and fundamental fluctuations is made. Starting from a useful a priori estimate of the deconfining thermal

ground state the computation of thermodynamical quantities is organized into a rapidly converging loop expansion carried by effective gauge-field fluctuations of trivial topology.

At high temperatures this expansion indicates the presence of stable and isolated (anti)monopoles of a given number density. For temperatures not far above the critical temperature T_c the collective dynamics of these defects are exhaustedly described by two-loop corrections to the pressure [4, 5]. They induce, depending on momentum, screening, or antiscreening of the tree-level massless, effective gauge mode [6, 7].

2. Some Remarks on Deconfining SU(2) Yang-Mills Thermodynamics

On the one-loop level the Yang-Mills system is approximated by a gas of noninteracting thermal quasiparticles fluctuating above a thermal ground state. The latter is described by an effective inert, adjoint scalar field, and a pure-gauge configuration, and the mass spectrum of thermal quasiparticles is made explicit in admissible unitary gauge [1–3]. On the fundamental level, this situation locally is induced by interacting (anti)calorons of topological charge-modulus unity whose small holonomy [8–12], temporarily created by the absorption of soft and fundamental propagating gauge fields, is insufficient for the permanent release of their magnetic monopole-antimonopole constituents [13]. This approximation captures the total pressure up to an error smaller than one percent. Effective radiative corrections describe a departure from this situation in so far as the thermal ground state is contributed to by domainized configurations of the adjoint scalar field with the vertices of sufficiently many domain boundaries [14] representing magnetic (anti)monopoles. At high temperature, an average over such configurations is performed implicitly by a particular two-loop diagram for the pressure [4, 5] whose ratio to the one-loop approximation with increasing temperature rapidly approaches a small negative constant. This constant represents the existence of an average density of highly nonrelativistic and screened magnetic (anti)monopoles which are released by the rare and irreversible dissociation of (anti)calorons [7, 13]. The irreversibility of (anti)caloron dissociation together with the fact that overall magnetic charge is nil due to pairwise monopole-antimonopole creation implies that the chemical potential associated with monopole-antimonopole pairs vanishes in the infinite-volume limit.

As temperature decreases, stable (anti)monopoles behave like comoving raisins, immersed in an expanding, infinitely extended dough, with their average distance set by the inverse temperature. In the hypothetic limit of isolation, see [7], the liability of a caloron or an anticaloron to dissociate into a pair of an isolated monopole of mass m_M and its antimonopole of mass m_A is determined solely by its holonomy [10–13]. For the realistic case of densely packed (anti)calorons the description of a single caloron by semiclassical methods fails [7]. An *average* over monopole-antimonopole creation processes, however, leads to a number density n_{M+A} of pairs which is determined by the following holonomy-independent sum of masses [10–12]:

$$m_{M+A} = m_M + m_A = \frac{8\pi^2 T}{e(\lambda)} + \text{non-BPS}, \tag{2.1}$$

where T and $\lambda \equiv 2\pi T/\Lambda$ are the dimensionful and the dimensionless versions of the temperature, respectively, and Λ denotes the Yang-Mills scale. The λ dependence of e is a consequence of the renormalization-group invariance of the a priori estimate of the Yang-Mills partition function under the spatial coarse-graining applied to derive the effective

theory [1–3]. This running of e with temperature describes the screening effects due to instable magnetic dipoles arising from (anti)calorons whose holonomy is only mildly deformed away from trivial. Notice that e approaches a plateau $e \equiv \sqrt{8}\pi$ very rapidly with increasing $\lambda > \lambda_c = 13.87$. For low temperatures, $\lambda_c \leq \lambda \leq 15.0$, we have,

$$e(\lambda) = -4.59 \log(\lambda - \lambda_c) + 18.42. \tag{2.2}$$

(The author would like to thank Markus Schwarz for performing this fit.) In (2.1) the term "non-BPS" refers to the effects which are induced by the presence of all other isolated, stable, and screened magnetic monopole-antimonopole pairs which, in contrast to the situation of screening by instable dipoles, introduce a mass scale into the decay properties of the magnetic potential of a given stable (anti)monopole. The correction to the BPS mass should be comparable to the dual-gauge mode's magnetic screening mass m_m which due to weak coupling is calculable in perturbation theory. (This yields an order-of-magnitude result which matches well with the exact result extracted from a two-loop correction of the pressure at high temperature [7].) To the lowest order in the magnetic coupling $g = 4\pi/e$, we have $m_m = 4\pi T/\sqrt{3}\, e$ [15]. Compared to the BPS term in (2.1) this is a correction of less than 10%.

3. Condensation of Monopole-Antimonopole Pairs

Based on the discussion presented by Huang [16] of thermalized, noninteracting Bose particles with mass m a relation was formulated in [17] between the total number density n and the density n_0 of particles residing in the condensate. For statistical weight unity (only one species of monopole-antimonopole pairs occurs in an SU(2) Yang-Mills theory) one has

$$\begin{aligned} n_0 &= n - n_c \\ &\equiv n - \frac{T^3}{2\pi^2}\mu^2 \sum_{l=1}^{\infty} e^{l\mu} \frac{K_2(l\mu)}{l}, \end{aligned} \tag{3.1}$$

where $\mu \equiv m/T$, and $K_2(x)$ is the modified Bessel function of the second kind. At the onset of Bose condensation, where n_0 is yet zero, the total number density n is given by the number density n_{fr} of freely fluctuating particles

$$n = n_{\mathrm{fr}} \equiv \frac{T^3}{2\pi^2} \int_0^{\infty} dx \frac{x^2}{e^{\sqrt{x^2+\mu^2}} - 1}. \tag{3.2}$$

So in the fully thermalized system, which takes place if T is slowly lowered towards T_c, we have at the onset of Bose condensation

$$\int_0^{\infty} dx \, \frac{x^2}{e^{\sqrt{x^2+\mu_C^2}} - 1} = n_c = \mu_c^2 \sum_{k=1}^{\infty} e^{k\mu_c} \, \frac{K_2(k\mu_c)}{k}. \tag{3.3}$$

Equation (3.3) determines the critical *ratio* μ_c between mass and temperature at which Bose condensation starts to occur. For conventional condensed-matter systems the mass m of a given species of bosonic particles is a predetermined quantity which does not depend on

temperature in the absence of interactions. The solution μ_c to (3.3) thus determines the critical temperature T_c for Bose condensation as $T_c = m/\mu_c$. In deconfining Yang-Mills thermodynamics, however, a pair of an isolated and screened magnetic monopole and its antimonopole owes its very existence to the presence of a heat bath of given temperature. The according relation between mass and temperature, see (2.1), together with μ_c determines the critical temperature for condensation to be the solution to the following equation:

$$T_c = \frac{m(T_c)}{\mu_c}. \tag{3.4}$$

Notice that the limit $\mu_c \to 0$ of (3.3) yields the identity $2\zeta(3) = 2\zeta(3)$, where $\zeta(z)$ is Riemann's zeta function. Since the left-hand side of (3.3) is monotonic decreasing and the right-hand side is monotonic increasing in μ_c, it follows that $\mu_c = 0$ is the only solution. Thus In deconfining SU(2) Yang-Mills thermodynamics (adiabatically slow approach of T_c from above), only massless monopoles and antimonopoles condense into a new ground state at the critical temperature T_c corresponding to the logarithmic pole in e described by (2.2).

Alternatively, one may ask the question of what happens in the limit, where T_c is rapidly approached from above. As we will see, such an adiabatic (sudden) approximation fully takes into account the static screening effects imposed by the system at high temperatures but neglects the influence of propagating dual-gauge modes not too far above T_c. Since the pole of the coupling $e(\lambda)$, see (2.2), is logarithmic (reflecting the fact that the Yang-Mills scale Λ nonperturbatively interferes with the dynamics of fundamental propagating gauge modes only shortly above λ_c [18]), we may consider the limit of large temperatures for the dependence on temperature of the density $n_{M+A,\mathrm{as}}$ of *interacting* but statically equilibrated monopole-antimonopole pairs. This situation is relevant for particle collisions at sufficiently high center-of-mass energy where locally a hot spot of deconfining phase is generated whose temperature quickly drops due to cooling and shrinking by evaporation [19].

Recall that $n_{M+A,\mathrm{as}}$ is extracted from a particular two-loop correction to the quasiparticle pressure as calculated in the effective theory [7]. One has

$$n_{M+A,\mathrm{as}} = (21.691)^{-3}T^3 \sim 9.8 \times 10^{-5}T^3. \tag{3.5}$$

At high temperatures isolated, stable, and screened (anti)monopoles are nonrelativistic [7]. If temperature is lowered towards T_c in a sufficiently rapid way then the generation of almost massless stable monopoles and antimonopoles by strong screening occurs quickly enough to not affect their nonrelativistic nature endowed by high-temperature physics. (To catch up in velocity (anti)monopoles must interact via the exchange of dual-gauge modes that are close to their mass shell and therefore propagate at a speed close to the velocity of light. In contrast to the portion of magnetic screening induced by an increased activity of instable monopole-antimonopole pairs and described by the effective-theory a priori estimate for the thermal ground state this part of the thermalization of (anti)monopoles—a loop correction in the effective theory—thus requires a finite amount of time.) The condensation condition (3.3) thus modifies as

$$\frac{n_{M+A,\mathrm{as}}}{T^3} = (21.691)^{-3} = \zeta\left(\frac{3}{2}\right)\left(\frac{\mu_{c,M+A}}{2\pi}\right)^{3/2}, \tag{3.6}$$

where the expression to the far right is obtained by considering $\mu_{c,M+A} \gg 1$ of n_c or, equivalently, of $1/2\pi^2$ times the right-hand side of (3.3). To summarize, (3.6) determines $\mu_{c,M+A}$ in a situation where highly nonrelativistic, stable, and isolated (anti)monopoles are adiabatically fast deprived of their mass by cooling (enhanced instantaneous screening by instable dipoles) so that no time is available for them to start moving.

The solution to (3.6) is $\mu_{c,M+A} = 7.04 \times 10^{-3}$. Note the amusing fact that the nonrelativistic nature of monopole-antimonopole pairs is assured by the large-mass limit of n_c while the solution to (3.6) actually corresponds to a small mass on the scale of temperature. The resolution of this apparent puzzle is grounded in the fact that the sudden approximation employed does not admit a thermodynamical interpretation: the expression for $n_{M+A,\mathrm{as}}$ at $T \gg T_c$ is analytically continued down to T_c.

Using

$$\mu_{M+A} = \left(8\pi^2 + \frac{4\pi}{\sqrt{3}}\right)e^{-1}, \tag{3.7}$$

compare with (2.1) and paragraph below (2.2), we obtain

$$e_c = 1.225 \times 10^4. \tag{3.8}$$

The result in (3.8) acts as a lower bound for the values of e_c occurring for finite-velocity approaches $\lambda \searrow \lambda_c$. That is, for this nonadiabatic situation $\mu_{c,M+A}$ must take values in between the extremes obtained at zero and infinite velocity:

$$0 \leq \mu_{c,M+A} \leq 7.04 \times 10^{-3} \text{ or } \infty \geq e_c \geq 1.225 \times 10^4. \tag{3.9}$$

4. Mass of Charged Vector Bosons in the Standard Model

The ratio of the charged-vector-boson-mass m_W to the electron mass m_e, as experimentally measured, is given as

$$\frac{m_W}{m_e} = 1.6 \times 10^5. \tag{4.1}$$

If we postulate that a pure SU(2) Yang-Mills theory of Yang-Mills scale $\Lambda_e \sim m_e$ is responsible for the emergence of the electron and its neutrino in its confining phase [19–22] and for the mediation of the weak force by its decoupling, dynamically massive gauge bosons [1, 2] ($W^{\pm}$ at the deconfining-preconfining transition and Z^0 at the preconfining-confining transition) then the value of e as calculated from the ratio in (4.1) should be contained in the range specified by (3.9).

Let us check whether this indeed is the case. Since in such a theory one would have

$$m_W = 2e_c|\phi|(T_c) = 2e_c\Lambda_e\lambda_c^{-1/2} = e_c\Lambda_e\sqrt{\frac{4}{13.87}}, \tag{4.2}$$

and since $\Lambda_e \sim m_e$ the value of e_c should relate to the experimentally determined ratio in (4.1) as follows

$$e_c \sim \frac{m_W}{m_e}\sqrt{\frac{13.87}{4}} = 2.98 \times 10^5. \tag{4.3}$$

Obviously, this value for e_c lies in the range given by (3.9).

5. Summary

In this paper we have considered two scenarios for the onset of magnetic monopole-antimonopole condensation at the deconfining-preconfining transition in SU(2) Yang-Mills thermodynamics: infinitely slow and infinitely fast downward approach of T_c. In the former situation, we have shown that pairs of stable monopoles and antimonopoles do only condense when they are massless, that is, at the pole position for the effective electric coupling e. This is a consequence of the fact that due to the irreversibility of the (anti)monopole creation process (dissociation of large-holonomy (anti)calorons) and due to overall charge neutrality (pairwise creation in an infinite spatial volume) the chemical potential associated with pairs is nil. Concerning the case of an infinitely fast approach of T_c, we obtain a lower bound on the value e_c of the critical coupling. Our results are compatible with the claim that a pure SU(2) Yang-Mills theory of scale $\Lambda_e \sim m_e \sim 0.5\,\text{MeV}$ is responsible for the emergence of the first lepton family and the weak interactions of the Standard Model of Particle Physics. Due to the experimental fact of a universal electric coupling of the photon—likely to emerge from an SU(2) Yang-Mills theory of scale $\Lambda_{\text{CMB}} \sim 10^{-4}$ eV [5, 6, 23]—to all charged leptons it is clear that Nature's SU(2) Yang-Mills theories of the same electric-magnetic parity mix maximally.

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