Hindawi Publishing Corporation
Physics Research International
Volume 2012, Article ID 879537, 5 pages
doi:10.1155/2012/879537

Research Article

Heat Transfer to MHD Oscillatory Viscoelastic Flow in a Channel Filled with Porous Medium

Rita Choudhury[1] and Utpal Jyoti Das[2]

[1] *Department of Mathematics, Gauhati University, Assam, Guwahati 781014, India*
[2] *Department of Mathematics, Rajiv Gandhi University, Rono Hills, Arunachal Pradesh, Itanagar 791112, India*

Correspondence should be addressed to Utpal Jyoti Das, utpaljyotidas@yahoo.co.in

Received 2 April 2011; Revised 28 July 2011; Accepted 16 October 2011

Academic Editor: Baruch Meerson

The combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin viscoelastic fluid through a channel filled with saturated porous medium and nonuniform walls temperature has been discussed. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Closed-form analytical solutions are constructed for the problem. The effects of the radiation and the magnetic field parameters on velocity profile and shear stress for different values of the viscoelastic parameter with the combination of the other flow parameters are illustrated graphically, and physical aspects of the problem are discussed.

1. Introduction

The flow of an electrically conducting fluid has important applications in many branches of engineering science such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactor, geothermal energy extraction, electromagnetic propulsion, and the boundary layer control in the field of aerodynamics. In the light of these applications, MHD flow in a channel has been studied by many authors; some of them are Nigam and Singh [1], Soundalgekar and Bhat [2], Vajravelu [3], and Attia and Kotb [4]. A survey of MHD studies in the technological fields can be found in Moreau [5]. The flow of fluids through porous media is an important topic because of the recovery of crude oil from the pores of the reservoir rocks; in this case, Darcy's law represents the gross effect. Raptis et al. [6] have analysed the hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [7] have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde and Mhone [8] have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium.

In this study, an attempt has been made to extend the problem studied by Makinde and Mhone [8] to the case of viscoelastic fluid characterised by second-order fluid.

The constitutive equation for the incompressible second-order fluid is of the form

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2, \tag{1}$$

where σ is the stress tensor, p is the hydrostatic pressure, I is the unit tensor, A_n $(n = 1, 2)$ are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2, and μ_3 are the material coefficients describing viscosity, elasticity, and cross-viscosity, respectively. The material coefficients μ_1, μ_2, and μ_3 have taken constants with μ_1 and μ_3 as positive and μ_2 as negative (Markovitz and Coleman [9]). Equation (1) was derived by Coleman and Noll [10] from that of the simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

2. Mathematical Formulation of the Problem

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Figure 1. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The x-axis

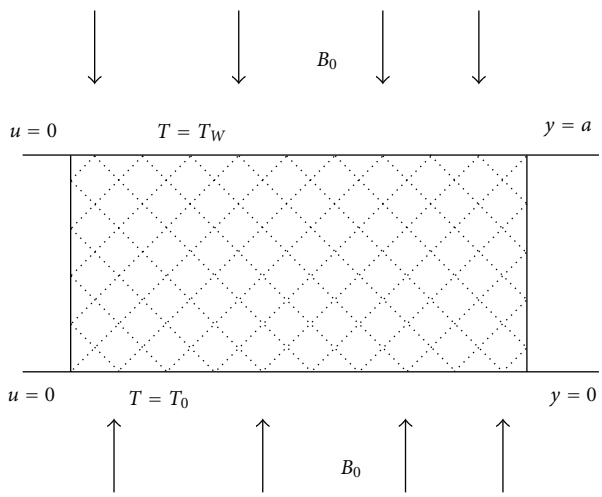

Figure 1: Geometry of the problem.

is taken along the centre of the channel, and the y-axis is taken normal to it. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v_1\frac{\partial^2 u}{\partial y^2} + v_2\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{v_1 u}{k} - \frac{\sigma_e {B_0}^2 u}{\rho} + g\beta(T - T_0), \tag{2}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y}, \tag{3}$$

subject to boundary conditions

$$\begin{aligned} u = 0, \quad T = T_w \quad \text{on } y = 1, \\ u = 0, \quad T = T_0 \quad \text{on } y = 0, \end{aligned} \tag{4}$$

where u is the axial velocity, t is the time, T is the fluid temperature, P is the pressure, g is the gravitational force, q is the radiative heat flux, β is the co-efficient of volume expansion due to temperature, C_p is the specific heat at constant pressure, k is the thermal conductivity, K is the porous medium permeability co-efficient, B_0 $(= \mu_e H_0)$ is the electromagnetic induction, μ_e is the magnetic permeability, H_0 is the intensity of the magnetic field, σ_e is the conductivity of the fluid, ρ is the fluid density, and $v_i = \mu_i/\rho$, $(i = 1, 2)$. It is assumed that both walls of temperature T_0, T_w are high enough to induce radiative heat transfer. Following Cogley et al. [11], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2(T_0 - T), \tag{5}$$

where α is the mean radiation absorption co-efficient.

The following nondimensional quantities are introduced:

$$\begin{gathered} \mathrm{Re} = \frac{Ua}{v_1}, \qquad \bar{x} = \frac{x}{a}, \qquad \bar{y} = \frac{y}{a}, \qquad \bar{u} = \frac{u}{U}, \\ \theta = \frac{T - T_0}{T_w - T_0}, \qquad H^2 = \frac{a^2 \sigma_e {B_0}^2}{\rho v_1}, \\ \bar{t} = \frac{tU}{a}, \qquad \bar{P} = \frac{aP}{\rho v_1 U}, \\ \mathrm{Da} = \frac{K}{a^2}, \qquad \mathrm{Gr} = \frac{g\beta(T_w - T_0)a^2}{v_1 U}, \\ \mathrm{Pe} = \frac{Ua\rho C_p}{k}, \qquad N^2 = \frac{4\alpha^2 a^2}{k}, \end{gathered} \tag{6}$$

where U is the flow mean velocity.

The dimensionless governing equations together with the appropriate boundary conditions (neglecting the bars for clarity) can be written as

$$\mathrm{Re}\ \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2 + H^2)u + \mathrm{Gr}\ T + \gamma\frac{\partial^3 u}{\partial y^2 \partial t}, \tag{7}$$

$$\mathrm{Pe}\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta, \tag{8}$$

with

$$\begin{aligned} u = 0, \quad \theta = 1 \quad \text{on } y = 1, \\ u = 0, \quad \theta = 0 \quad \text{on } y = 0, \end{aligned} \tag{9}$$

where Gr, H, N, Pe, Re, Da, $S(= 1/\mathrm{Da})$, and $\gamma = (v_2\,\mathrm{Re})/a^2$ are Grashoff number, Hartmann number, Radiation parameter, Péclet number, Reynolds number, Darcy number, porous medium shape factor parameter, and viscoelastic parameter, respectively.

3. Method of Solution

In order to solve (7) and (8) for purely oscillatory flow, let

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, \qquad u(y,t) = u_0(y)e^{i\omega t}, \qquad \theta(y,t) = \theta_0(y)e^{i\omega t}, \tag{10}$$

where λ is a constant and ω is the frequency of oscillation.

Substituting the above expressions into (7) and (8) and using (9), we get

$$(1 + i\gamma\omega)\frac{d^2 u_0}{dy^2} - {m_2}^2 u_0 = -\lambda - \mathrm{Gr}\theta_0, \tag{11}$$

$$\frac{d^2\theta_0}{dy^2} + {m_1}^2\theta_0 = 0, \tag{12}$$

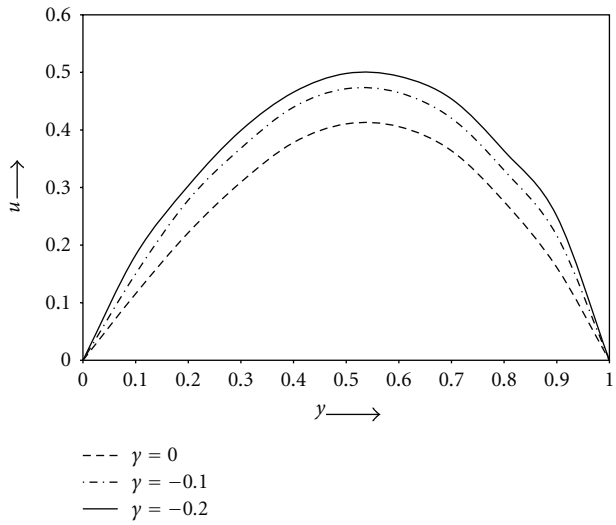

Figure 2: Variation of u against y when $H = 0.5$, $N = 1.5$.

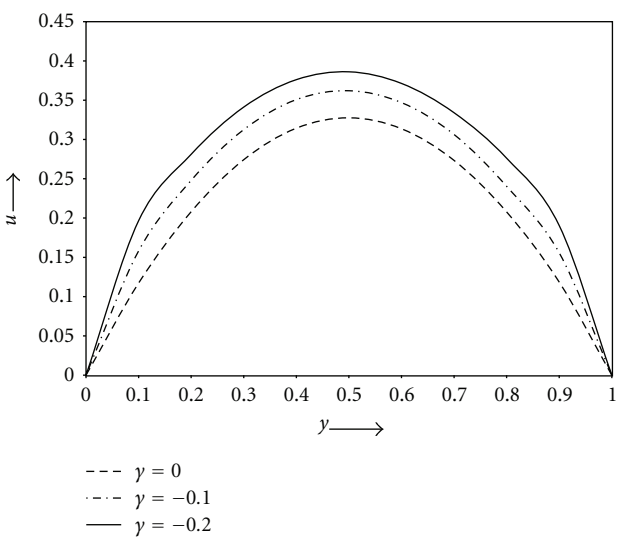

Figure 3: Variation of u against y when $H = 0.5$, $N = 2.5$.

subject to boundary conditions

$$\begin{aligned} u_0 &= 0, \quad \theta_0 = 1 \quad \text{on } y = 1, \\ u_0 &= 0, \quad \theta_0 = 0 \quad \text{on } y = 0, \end{aligned} \tag{13}$$

where $m_1 = \sqrt{N^2 - i\omega \mathrm{Pe}}$ and $m_2 = \sqrt{S^2 + H^2 + i\omega \mathrm{Re}}$.

Equations (11) and (12) are solved, and the solution, for the fluid velocity and temperature are given as follows:

$$u(y,t) = \left\{ M_1 e^{(m_2 y)/L} + \left(M_1 - \frac{\lambda}{m_2{}^2} \right) e^{-(m_2 y)/L} + \frac{\lambda}{m_2{}^2} + \frac{\mathrm{Gr}\,\sin(m_1 y)}{(m_1{}^2 L + m_2{}^2)\sin(m_1)} \right\} e^{i\omega t}, \tag{14}$$

$$\theta(y,t) = \frac{\sin(m_1 y)}{\sin m_1} e^{i\omega t}, \tag{15}$$

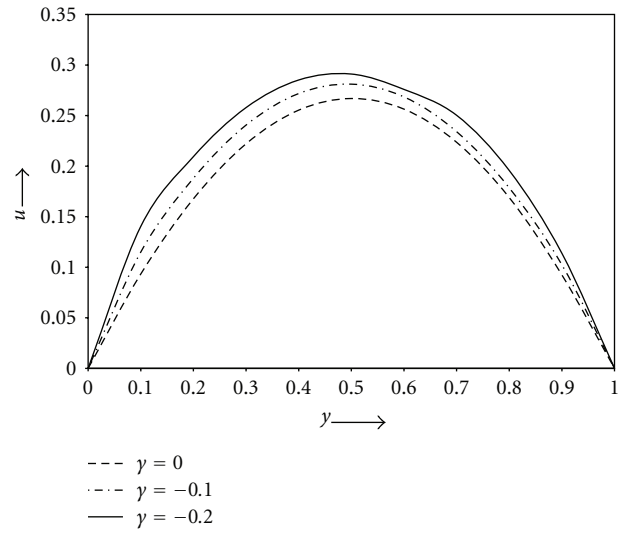

Figure 4: Variation of u against y when $H = 1.5$, $N = 2.5$.

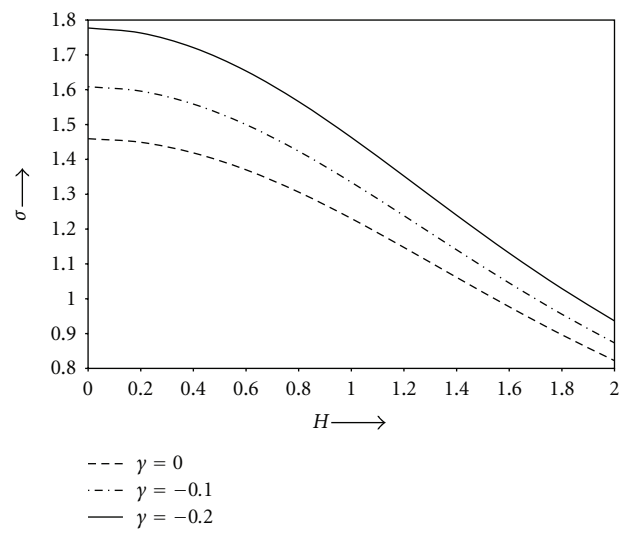

Figure 5: Variation of σ against H when $N = 1.5$.

where $M_1 = ((\lambda/m_2{}^2)e^{-m_2/L} - (\lambda/m_2{}^2) - \mathrm{Gr})/(m_1{}^2 L + m_2{}^2))/(e^{m_2/L} - e^{-m_2/L})$ and $L = 1 + i\omega\gamma$.

The nondimensional shear stress σ at the wall $y = 0$ is given by

$$\sigma = \frac{\overline{\sigma}}{(\mu_1 U/a)} = \left[\frac{\partial u}{\partial y} + \gamma \frac{\partial^2 u}{\partial y \partial t} \right]_{y=0}. \tag{16}$$

The rate of heat transfer across the channel's wall is given as

$$Nu = -\frac{\partial \theta}{\partial y} = -\frac{m_1 \cos(m_1)}{\sin(m_1)} e^{i\omega t}. \tag{17}$$

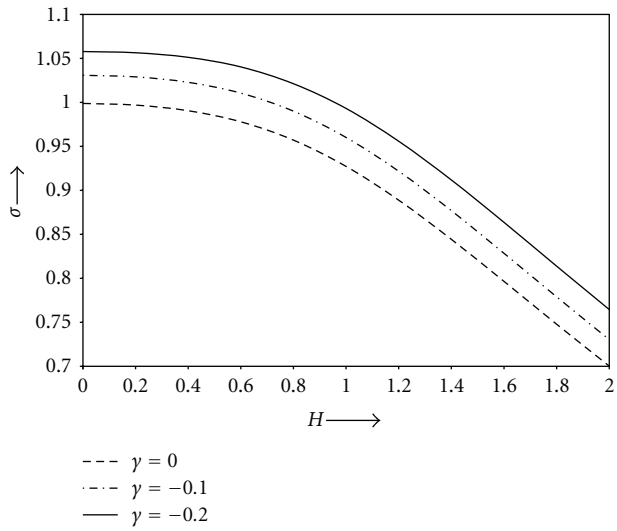

Figure 6: Variation of σ against H when $N = 2.5$.

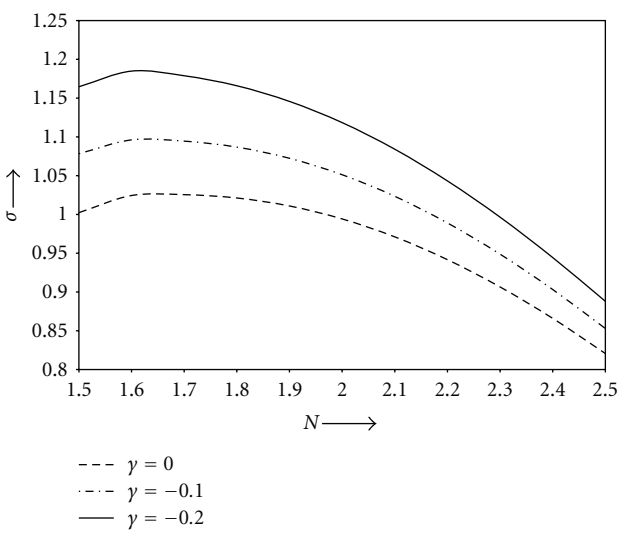

Figure 7: Variation of s against N when $H = 1.5$.

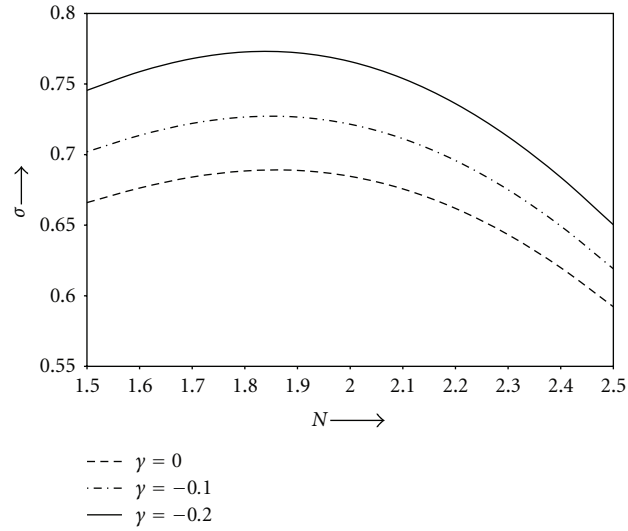

Figure 8: Variation of s against N when $H = 2.5$.

4. Discussions and Conclusion

The purpose of this study is to bring out the effects of the viscoelastic parameter γ on the governing flow with the combination of the other flow parameters. The corresponding results for Newtonian fluid can be deduced from the above results by setting $\gamma = 0$, and it is worth mentioning here that these results coincide with that of Makinde and Mhone [8]. We have considered the real parts of the results throughout for numerical validation. The velocity profile u against y is plotted in Figures 2–4 to observe the viscoelastic effects for various sets of values of Hartmann number H and radiation parameter N ($H = 0.5$, $N = 1.5$; $H = 0.5$, $N = 2.5$; $H = 1.5$, $N = 2.5$) with fixed values of other flow parameters, namely, $\mathrm{Pe} = 2$, $\mathrm{Re} = 2$, $s = 1$, $t = 0$, $\mathrm{Gr} = 2$, $\lambda = 1$, and $\omega = 1$. It is evident from Figures 2–4 that the velocity profile is parabolic in nature, and the values of velocity u increase with the increasing values of the viscoelastic parameter $|\gamma|$ ($\gamma = 0, -0.10, -0.20$) in comparison with the Newtonian fluid. It is also noted from the figures that the behaviours of the velocity profiles remain the same with the increasing values of the viscoelastic parameter $|\gamma|$ when (i) the values of the radiation parameter N increase with the fixed values of the magnetic field parameter H (Figures 2 and 3), (ii) the values of the magnetic field parameter H increase with fixed values of the radiation parameter N (Figures 3 and 4), and (iii) both the values of H and N increase (Figures 2 and 4).

Figures 5, 6, 7, and 8 exhibit the effects of the viscoelastic parameter $|\gamma|$ on skin friction σ against magnetic field parameter H and radiation parameter N, respectively with $\mathrm{Pe} = 2$, $\mathrm{Re} = 2$, $s = 1$, $t = 0$, $\mathrm{Gr} = 2$, $\lambda = 1$, and $\omega = 1$. Figures 5 and 6 show that for radiation parameter $N = 1.5$ and $N = 2.5$, the shear stress decreases with the increasing values of H for both Newtonian and non-Newtonian fluids, while shear stress increases for increasing values of $|\gamma|$ ($\gamma = 0, -0.10, -0.20$) in comparison with Newtonian fluid. From Figures 7 and 8, it is evident that the shear stress σ increases firstly and then decreases with the increasing values of N for both Newtonian and non-Newtonian cases. Also, Figures 7 and 8 depict that the shear stress increases with the increasing values of the viscoelastic parameter $|\gamma|$ in comparison with Newtonian fluid when the values of the magnetic field parameter H increase.

It has also been observed that the temperature field is not significantly affected by the viscoelastic parameter.

References

[1] S. D. Nigam and S. N. Singh, "Heat transfer by laminar flow between parallel plates under the action of transverse magnetic field," *Quarterly Journal of Mechanics and Applied Mathematics*, vol. 13, no. 1, pp. 85–97, 1960.

[2] V. M. Soundalgekar and J. P. Bhat, "Oscillatory MHD channel flow and heat transfer," *International Journal of Pure & Applied Mathematical*, vol. 15, pp. 819–828, 1971.
[3] K. Vajravelu, "Exact periodic solution of a hydromagnetic flow in a horizontal channel," *Journal of Applied Mechanics*, vol. 55, no. 4, pp. 981–983, 1988.
[4] H. A. Attia and N. A. Kotb, "MHD flow between two parallel plates with heat transfer," *Acta Mechanica*, vol. 117, pp. 215–220, 1996.
[5] R. Moreau, *Magnetohydrodynamics*, Kluwar Academic, Dordrecht, The Netherlands, 1990.
[6] A. Raptis, C. Massalas, and G. Tzivanidis, "Hydromagnetic free convection flow through a porous medium between two parallel plates," *Physics Letters A*, vol. 90, no. 6, pp. 288–289, 1982.
[7] T. K. Aldoss, M. A. Al-Nimr, M. A. Jarrah, and B. Al-Shaer, "Magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium," *Numerical Heat Transfer A*, vol. 28, no. 5, pp. 635–645, 1995.
[8] O. D. Makinde and P. Y. Mhone, "Heat transfer to MHD oscillatory flow in a channel filled with porous medium," *Romanian Journal of Physics*, vol. 50, pp. 931–938, 2005.
[9] H. Markovitz and B. D. Coleman, "Incompressible second-order fluids," *Advances in Applied Mechanics*, vol. 8, pp. 69–101, 1964.
[10] B. D. Coleman and W. Noll, "An approximation theorem for functionals, with applications in continuum mechanics," *Archive for Rational Mechanics and Analysis*, vol. 6, no. 1, pp. 355–370, 1960.
[11] A. C. L. Cogley, W. G. Vinvent, and E. S. Giles, "Differential approximation for radiative heat transfer in non-linear equations-grey gas near equilibrium," *American Institute of Aeronautics and Astronautics*, vol. 6, pp. 551–553, 1968.