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Research Article

An Endoreversible Thermodynamic Model Applied to the Convective Zone of the Sun

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Within the context of finite-time thermodynamics (FTTs) some models of convective atmospheric cells have been proposed to calculate the efficiency of the conversion of solar energy into wind energy and also for calculating the surface temperature of the planets of the solar system. One of these models is the Gordon and Zarmi (GZ) model, which consists in taking the sun-earth-wind system as a FTT-cyclic heat engine where the heat input is solar radiation, the working fluid is the earth's atmosphere and the energy in the winds is the work produced. The cold reservoir to which the engine rejects heat is the 3 K surrounding universe. In the present work we apply the GZ-model to investigate some features of the convective zone of the sun by means of a possible structure of successive convective cells along the well-established convective region of the sun. That is, from 0.714 R_S up to R_S being R_S the radius of the sun. Besides, we estimate the number of cells of the model, the possible size of the cells, their thermal efficiency, and also their average power output. Our calculations were made by means of two FTT regimes of performance: the maximum power regime and the maximum ecological function regime. Our results are in reasonable agreement with others reported in the literature.

1. Introduction

The problem of thermal balance between the planets of the solar system and the sun under a finite-time thermodynamics approach has been treated by several authors [1–8]. In some of these articles the question of the conversion of solar energy into wind energy is also treated. In particular, De Vos [3] demonstrated that cosmic radiation, starlight, and moonlight can be neglected for the thermal balance of any of the planets of the solar system and only the following quantities have an influence: the incident solar influx or solar constant I_{sc}, the planet's albedo ρ, and the greenhouse effect of the planet's atmosphere crudely evaluated by means of a coefficient γ. This coefficient can be taken as the normalized greenhouse effect introduced by Raval and Ramanathan in [9]. When only the global thermal balance between the sun and a planet is considered, one can roughly obtain the planet's surface temperature assumed as a uniform temperature T_p. If the conversion of solar energy into wind energy is to be modeled, it is necessary to involve at least two representative atmospheric temperatures for making the creation of work possible; that is, to take the planet's atmosphere as a working fluid that converts heat into mechanical work. In 1989, Gordon and Zarmi [1] introduced a FTT-model taking the sun-earth-wind system as a FTT-cyclic heat engine where the heat input is solar radiation, the working fluid is the earth's atmosphere, and the energy in the winds is the work produced; the cold reservoir to which the engine rejects heat is the 3 K surrounding universe. By means of this simplified model, Gordon and Zarmi were able to obtain reasonable values for the annual average power in the earth's winds and for the average maximum and minimum temperatures of the atmosphere, without resorting to detailed dynamic models of the earth's atmosphere, and without considering any other effect (such as earth's rotation, earth's orbital motion around the Sun, and ocean currents). Later, De Vos and Flater [2] extended the GZ model to take into account the wind energy dissipation by means of a maximum power criterion. This model was extended by De Vos and van der Wel [4, 5] by constructing a model based in convective Hadley cells. All the models used in [1–5] are endoreversible ones in the sense of FTT [10], that is, all irreversibilities are located in the exchanges between the engine and the external world. The GZ model was later studied under a nonendoreversible approach and by using the so-called ecological optimization criterion [6, 7]. This approach [11] consists of maximizing a function E that represents a good compromise between high-power output and low-entropy production. The function E is given by

$$E = P - T_{\text{ext}}\Delta S_u, \tag{1}$$

where P is the power output of the cycle, ΔS_u the total entropy production (system plus surroundings) per cycle, and T_{ext} is the temperature of the cold reservoir. This optimization criterion for the case of the so-called Curzon-Ahlborn cycle [12], for instance, leads to a cycle configuration such that for maximum E it produces around 75% of the maximum power and only about 25% of the entropy produced in the maximum power regime [13]. By means of employing this criterion in a nonendoreversible GZ model, the authors of [6] also found reasonable values for the annual average power of the winds and for the extreme temperatures of the earth's troposphere. Later, the nonendoreversible GZ model was applied to calculate the surface temperature of planets of the solar system [8], considering two regimes of performance: maximum power regime and maximum ecological function regime. In this work, we apply the GZ model to the convective zone of the sun which is located between 0.714 R_S and R_S [14]. Our FTT approach leads to a possible structure of the convective region of the sun consisting in approximately sixteen coupled cells. It is important to remark that these sixteen convective Carnotian cells are only a kind of idealized cells, thermodynamically equivalent to the complex structure of the actual convective zone of the sun. The paper is organized as follows: in Section 2, we present a brief review of the GZ model for the convective cells under both the maximum power and the ecological function regimes. In Section 3, we applied the GZ model to the convective zone of the sun and finally in Section 4 we present some concluding remarks.

2. Endoreversible GZ Model for Atmospheric Convection

The endoreversible GZ model is based on annual average quantities and thus it does not represent actual convective cells but a kind of annual virtual cell that takes into account the global thermodynamic restrictions over the convection as a dominant energy transfer mechanism in the air (which has a large Rayleigh number). Besides, this kind of model must only be taken as one that producing better upper bounds than those calculated by means of classical equilibrium thermodynamics, which is one of the main purposes of FTT.

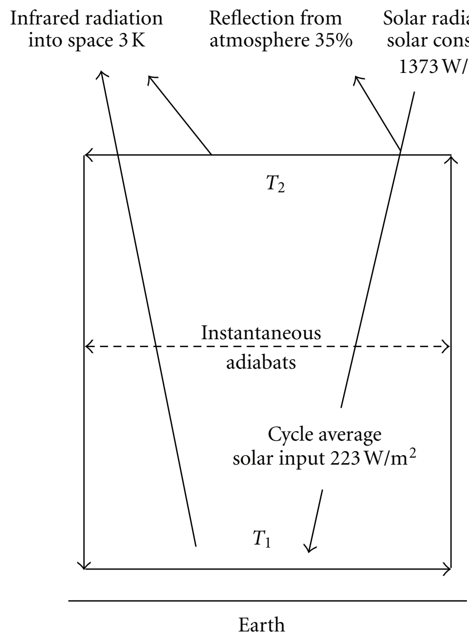

Figure 1: Scheme of a simplified solar-driven heat engine (taken from [1]).

2.1. Maximum Power Regime. In Figure 1, a schematic view of a simplified sun-earth-winds system as a heat engine cycle is depicted. This cycle consists of four branches: (1) two isothermal branches, one in which the atmosphere absorbs solar radiation at low altitudes and one in which the atmosphere rejects heat at high altitudes to the universe and (2) two intermediate instantaneous adiabats [10] with rising and falling currents. In [15], it was shown that a Curzon-Ahlborn FTT cycle in the endoreversible limit with instantaneous adiabats is reached for large compression ratios. In the GZ virtual cells, it is feasible to consider that this condition is fulfilled. According to GZ, this oversimplified Carnot-like engine corresponds very approximately to the global scale motion of wind in convective cells. Below, we use all of GZ model's assumptions.

For example, the work performed by the working fluid in one cycle W, the internal energy of the working fluid U, and the yearly average solar radiation flux q_s are expressed per unit area of the earth's surface. The temperatures of the four-branch cycle are taken as follows: T_1 is the working fluid temperature in the isothermal branch at the lowest altitude, where the working fluid absorbs solar radiation for half of the cycle. During the second half of the cycle, heat is rejected via black-body radiation from the working fluid at temperature T_2 (highest altitude of the cell) to the cold reservoir at temperature T_{ext} (the surrounding 3 K universe). In the GZ model, the objective is to maximize the work per cycle (average power) subjected to the endoreversibility constraint [10], that is,

$$\Delta S_{\text{int}} = \int_0^{t_o} \left\{ \frac{q_s(t) - \sigma\left[T^4(t) - T_{\text{ext}}^4(t)\right]}{T(t)} \right\} dt = 0, \tag{2}$$

where ΔS_{int} is the change of entropy per unit area, t_0 is the time of one cycle, σ is the Stefan-Boltzmann constant ($5.67\times 10^{-8}\,\text{W/m}^2\text{K}^4$), and q_s, and T are functions of time t, taken as [1]

$$T(t) = \begin{cases} T_1; & \text{if } 0 \le t \le \dfrac{t_0}{2}, \\ T_2; & \text{if } \dfrac{t_0}{2} \le t \le t_0, \end{cases}$$
$$q_s(t) = \begin{cases} 0; & \text{if } \dfrac{t_0}{2} \le t \le t_0, \\ \dfrac{I_{\text{sc}}(1-\rho)}{2}; & \text{if } 0 \le t \le \dfrac{t_0}{2}, \end{cases} \quad (3)$$

in the same way, $T_{\text{ext}} = 3\,\text{K}$ for $0 \le t \le t_0$, with I_{sc} the yearly average solar constant ($1373\,\text{W/m}^2$) and $\rho = 0.35$ [2], the effective average albedo of the earth's atmosphere. The GZ model maximizes the work per cycle W, taken from the first law of thermodynamics:

$$\Delta U = -W + \int_0^{t_0} \{q_s(t) - \sigma[T^4(t) - T_{\text{ext}}^4(t)]\}\,dt = 0, \quad (4)$$

by denoting average values as,

$$\overline{T} = \frac{T_1 + T_2}{2}, \qquad \overline{T^n} = \frac{T_1^n + T_2^n}{2}, \qquad \overline{q_s} = I_{sc}\frac{(1-\rho)}{4}, \quad (5)$$

where n is an integer with values $n = 3$ or 4. The factor of 1/4 arises from a factor of 1/2 to account for the day/night difference and a geometric factor of 1/2 to account for the earth's cross section, which is intercepted by solar radiation, as opposed to the corresponding hemispherical surface area of the earth. From (4) and (5) and taking into account the constraint given by (2), GZ construct the following Lagrangian L;

$$L = T^4(t) + \lambda\left[\frac{q_s(t)}{T(t)} - \sigma T^3(t)\right], \quad (6)$$

where λ is a Lagrange multiplier. The Euler-Lagrange formalism will be used, by using $\partial L(t)/\partial T(t) = 0$, GZ found the following values for the earth's atmosphere: $T_1 = 277\,\text{K}$, $T_2 = 192\,\text{K}$, and $P_{\max} = W_{\max}/t_0 = 17.1\,\text{W/m}^2$. These numerical values are not so far from "actual" values, which are $P \approx 7\,\text{W/m}^2$ [16], $T_1 = 290\,\text{K}$ (at ground level), and $T_2 \approx 195\,\text{K}$ (at an altitude of around 75–90 Km). However, as GZ assert, their power calculation must be taken as an upper bound due to several idealizations in their model. In [6], another endoreversible case was analyzed in which the tropopause layer with $T_{\text{ext}} = 200\,\text{K}$ was used as cold reservoir. In this case, the following Lagrangian was used:

$$L(t) = \overline{q_s + \sigma T_{\text{ext}}^4} - \sigma\overline{T^4} - \alpha\left[\frac{\overline{q_s}}{T_1} - \frac{\sigma\left(T_1^3 + T_2^3\right)}{2} - \sigma T_{\text{ext}}^4\left(\frac{1}{T_1} + \frac{1}{T_2}\right)\right], \quad (7)$$

with α a Lagrange multiplier. By numerically solving $\partial L(t)/\partial T(t) = 0$, they obtained $T_1 = 293.387\,\text{K}$ and $T_2 = 239.267\,\text{K}$, which are excellent values for convective cells restricted to the troposphere. If these temperature values are substituted in the expression for the average power (see [6])

$$P = \overline{q_s + \sigma T_{\text{ext}}^4} - \sigma\overline{T^4}, \quad (8)$$

a value of $P = 10.758\,\text{W/m}^2$ is obtained, which is a good value for the wind power [16].

2.2. Ecological Function Regime. As De Vos and Flater [2] state, no mechanism guarantees that the atmosphere maximizes the wind power. In fact, some authors [17–19] have recognized that the earth's atmosphere operates at nearly its maximum efficiency; thus, from an FTT point of view, an ecological-type criterion seems feasible. This is due to the properties of the E function, which at its maximum value represents an austere compromise between power and entropy production, additionally leading to a high efficiency [11, 13]. This ecological criterion, as previously occurred with the concepts of power output and efficiency [20], has also been used in the context of irreversible thermodynamics [21–23]. In particular, in [7] the so-called ecological criterion was applied to the GZ model. This criterion consists in maximizing equation (1). By means of the second law of thermodynamics, first, we calculate ΔS_u, the total entropy change per cycle (system plus surroundings),

$$\Delta S_u = \int_0^{t_0}\left\{\frac{-q_s(t) + \sigma[T^4(t) - T_{\text{ext}}^4(t)]}{T(t)}\right\}dt. \quad (9)$$

From (3), we obtain

$$\Delta S_u = \int_0^{t_0/2}\left\{-\frac{q_s(t)}{T_1} + \sigma\left(T_1^3 - \frac{T_{\text{ext}}^4}{T_1}\right)\right\}dt - \int_{t_0/2}^{t_0}\left\{\sigma\left(\frac{T_2^4 - T_{\text{ext}}^4}{T_{\text{ext}}}\right)\right\}dt. \quad (10)$$

Thus, the total entropy production is given by [7, 8],

$$\Sigma = \frac{\Delta S_u}{t_0} \approx \frac{\overline{q_s}}{T_1} + \frac{\sigma}{2}\left(T_1^3 + \frac{T_2^4}{T_{\text{ext}}}\right), \quad (11)$$

here, we have used the approximation $\overline{q_s} \gg \sigma T_{\text{ext}}^4 (223\,\text{W/m}^2 \gg 4.59\times 10^{-6}\,\text{W/m}^2)$ with $T_{\text{ext}} = 3\,\text{K}$. So, the ecological function E for this case is

$$E = \overline{q_s} - \sigma\overline{T^4} + \frac{T_{\text{ext}}\overline{q_s}}{T_1} - \frac{\sigma T_{\text{ext}}}{2}\left(T_1^3 + \frac{T_2^4}{T_{\text{ext}}}\right). \quad (12)$$

By using (12) and the constraint given by (2), we proposed the following Lagrangian function L_E:

$$L_E = \overline{q_s} - \sigma\overline{T^4} + \frac{T_{\text{ext}}\overline{q_s}}{T_1} - \frac{\sigma T_{\text{ext}}}{2} \times\left(T_1^3 + \frac{T_2^4}{T_{\text{ext}}}\right) - \alpha\left[\frac{\overline{q_s}}{T_1} - \sigma\overline{T^3}\right], \quad (13)$$

with α being the Lagrange multiplier. By substituting the values of $\overline{q_s}$, σ, and T_{ext} and numerically solving $\partial L(t)/\partial T(t) = 0$, we find $T_1 = 294.08\,\text{K}, T_2 = 109.54\,\text{K}$ and $P = 6.89\,\text{W/m}^2$, which are reasonable values for T_1 and P, but not for T_2. However, if we use as a cold reservoir, the tropopause layer with $T_{\text{ext}} = 200\,\text{K}$, we can now use the Lagrangian function: [24],

$$\begin{aligned} L_E = {} & \overline{q_s + \sigma T_{\text{ext}}^4} - \sigma \overline{T^4} + \left(\overline{q_s} + \frac{\sigma T_{\text{ext}}^4}{2} \right) \frac{T_{\text{ext}}}{T_1} \\ & - \frac{\sigma T_{\text{ext}}}{2} \left(T_1^3 + \frac{T_2^4}{T_{\text{ext}}} \right) - \frac{\sigma T_{\text{ext}}^4}{2} \\ & - \beta \left[\frac{\overline{q_s}}{T_1} - \frac{\sigma \left(T_1^3 + T_2^3 \right)}{2} + \sigma T_{\text{ext}}^4 \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \right], \end{aligned} \tag{14}$$

with β a Lagrange multiplier. By using again the Euler-Lagrange formalism, we numerically obtain $T_1 = 303\,\text{K}$, $T_2 = 219\,\text{K}$, and $P = 7\,\text{W/m}^2$ which are very good values, for T_1, T_2, and P. Besides, these values are restricted to typical values in the troposphere, where the climatic phenomena occurs. It is important to note that the power values ($6.89\,\text{W/m}^2$ and $7\,\text{W/m}^2$), which were calculated by the means of the ecological function, were deduced without considering the greenhouse effect (γ coefficient). When the later is taken into account, the values of P are bigger than $7\,\text{W/m}^2$ [7, 8]. These scenarios lead to larger upper bounds for the wind's power permitting an energy excess for other relevant dissipative processes such as ocean currents and biological structuring.

3. The GZ Model Applied to the Convective Zone of the Sun

The core of the sun goes from 0 to 0.2 R_S, where R_S (6.96×10^8 m [14]) is the radius of the sun. The radiative zone embraces the region between 0.2 R_S and 0.714 R_S and beyond that lies the convective zone. The later is estimated to have a width of approximately 0.286 R_S [14]. In (8) and (11) the input data were q_s and T_{ext}, the thermal energy and the temperature of the surrounding cold thermal bath for the earth's atmospheric cells, respectively. In the case of the convective zone of the sun, first we will use the maximum power criterion. In Figure 2, we show the heat fluxes balance for the convective zone of the sun. Then, by using (2), (3), (4), and (5) we obtain the following Lagrangian functional:

$$\begin{aligned} L(T_1, T_2, \lambda) = {} & \frac{q_s}{2} + \frac{\sigma T_{\text{ext}}^4}{2} - \frac{\sigma}{2} \left(T_1^4 + T_2^4 \right) \\ & - \lambda \left[\frac{q_s}{2T_1} - \frac{\sigma}{2} \left(T_1^3 + T_2^3 \right) + \frac{\sigma T_{\text{ext}}^4}{2} \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \right], \end{aligned} \tag{15}$$

where λ is a Lagrange multiplier, $T_{\text{ext}} = 3\,\text{K}$, $T_1 = 2.18 \times 10^6\,\text{K}$ [14] is the temperature of the spherical layer at 0.714 R_s and $q_s = \sigma T_1^4$ the input thermal energy at the lower layer of the convective zone. The energy transport through the sun can be considered as a "sandwich", that is, there are two regions in which radiation transports the energy separated by a region where convection transports it [25]. Strictly speaking, q_s

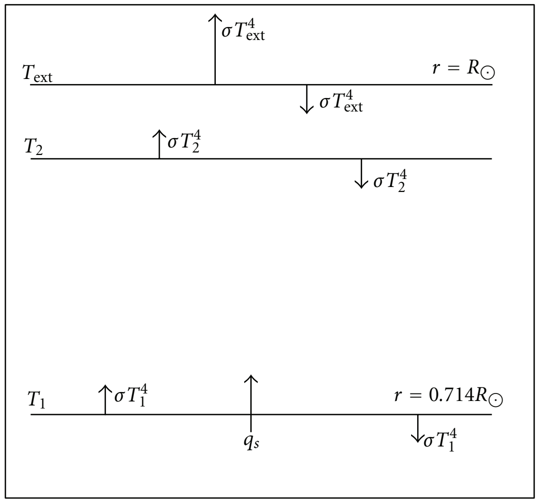

Figure 2: Schematic diagram of the energy fluxes present in the first internal convective cell. T_1 at 0.714 R_s is taken as the temperature of the first isothermal layer, $T_{\text{ext}} = 3\,\text{K}$ is taken as the cold reservoir temperature, and T_2 is taken as the upper shell temperature of the first convective cell. Short arrows indicate that emitted radiation is rapidly absorbed by opaque gases.

should be calculated by means of a diffusive model based on kinetic theory of gases [25]. However, for simplicity, in our thermodynamic model we take the 0.714 R_s layer at $T_1 \approx 2 \times 10^6\,\text{K}$ as a blackbody radiant system ($q_s = \sigma T_1^4$, see Figure 2). The radiation emitted by this layer is rapidly absorbed by the gases at the bottom of the convective zone. For the definition of T_2, see Figure 2. By using the Euler-Lagrange formalism over the Lagrangian of (15), that is, $\partial L(t)/\partial T(t) = 0$, we obtain the following equations:

$$\begin{aligned} T_1^5 - \frac{\lambda}{\sigma} \left(\frac{q_s}{4} + \frac{3\sigma}{4} T_1^4 + \frac{\sigma T_{\text{ext}}^4}{4} \right) &= 0, \\ T_2^5 - \lambda \left(\frac{3}{4} T_2^4 + \frac{T_{\text{ext}}^4}{4} \right) &= 0. \end{aligned} \tag{16}$$

By eliminating λ from these equations and by using the restriction given by (2), we obtain

$$T_2^4 \left(4T_1^3 T_2 + T_2^4 - 3T_1^4 \right) - T_{\text{ext}}^4 \left(T_1^4 + T_2^4 \right) = 0. \tag{17}$$

In this equation, the only unknown variable is T_2. Then, we numerically solve (17) to obtain T_2, the temperature of the upper bound for the first convective cell starting from $T_1 = 2.187761 \times 10^6\,\text{K}$ [14]. Our next step is to take the obtained T_2 value of the first cell as the temperature of the lower layer of the following successive cell. This new T_2 value is taken as T_1 in (17) and then we calculate a new T_2 for the second cell. For the following successive cells we use the same recursive procedure until to reach a final T_2 coinciding approximately with the well-known value of the average surface temperature of the sun, which is $T_S \approx 5780\,\text{K}$ [14].

In Table 1 we show that after 16 successive Carnotian convective cells we reach a final $T_2 \approx 6000\,\text{K}$. Table 1 shows

Table 1: Maximum power regime case: First column shows the normalized radial position of the hot layers corresponding to the sixteen virtual convective cells. The following columns give, respectively, second, the cell's widths; third, the hot isotherms; fourth, the cold isotherms; fifth, the average power output; sixth, thermal efficiency.

No	$r(T_1)/R_s$	$\Delta r = r(T_2) - r(T_1)$ (Km)	$T_1 \times 10^6$ (K)	$T_2 \times 10^6$ (K)	$\overline{W}(\mathrm{erg/cm^2 \cdot s})$	$\eta = \eta(T_1, T_2)$
1	0.714	61370.9	2.187761	1.51504	6.49519×10^{17}	0.307495
2	0.802177	42499.6	1.51504	1.04917	4.65789×10^{17}	0.307495
3	0.863239	29431.2	1.04917	0.726555	1.07122×10^{17}	0.307495
4	0.905525	20381.3	0.726555	0.503143	2.46361×10^{16}	0.307495
5	0.934809	14114.1	0.503143	0.348429	5.66582×10^{15}	0.307495
6	0.955088	9774.09	0.348429	0.241289	1.30303×10^{15}	0.307495
7	0.969131	6768.61	0.241289	0.167094	2.99671×10^{14}	0.307495
8	0.978856	4687.29	0.167094	0.115713	6.89186×10^{13}	0.307495
9	0.985591	3245.97	0.115713	0.0801319	1.585×10^{13}	0.307495
10	0.990254	2247.85	0.0801319	0.0554917	3.64521×10^{12}	0.307495
11	0.993484	1556.65	0.0554917	0.0384283	8.38353×10^{11}	0.307495
12	0.995721	1077.99	0.0384283	0.0266118	1.92829×10^{11}	0.307495
13	0.99727	746.511	0.0266118	0.0184288	4.43714×10^{10}	0.307495
14	0.998342	516.963	0.0184288	0.012762	1.02289×10^{10}	0.307495
15	0.999085	357.999	0.012762	0.00883776	2.37682×10^{9}	0.307495
16	0.999599	247.916	0.00883776	0.00612019	5.7099×10^{8}	0.307495

our results for Carnotian cells performing in the maximum power regime. As one can see in this Table (third column), the widths of the cells are decreasing toward the outer regions. The total width is around 0.280 R_s which is not so far of the value 0.286 R_s given by other sun models [26]. If we take as the mode of thermodynamic performance of the sun's convective cells the so-called maximum ecological regime [11], in a similar way as (15), then we obtain the following Lagrangian functional:

$$\begin{aligned} L_E(T_1, T_2, \lambda) = {} & \frac{q_s}{2}\left(1 - \frac{T_{\text{ext}}}{T_1}\right) + \sigma T_{\text{ext}}^4 \\ & - \sigma\left(T_1^4 + T_2^4\right) + \frac{\sigma T_{\text{ext}}}{2}\left(T_1^3 + T_2^3\right) \\ & - \frac{\sigma T_{\text{ext}}}{2}\left(\frac{T_1^4}{T_2} + \frac{T_2^4}{T_1}\right) - \frac{\sigma T_{\text{ext}}^5}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right) \\ & - \lambda\left[\frac{q_s}{2T_1} - \frac{\sigma}{2}\left(T_1^3 + T_2^3\right) - \frac{\sigma T_{\text{ext}}^4}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)\right]. \end{aligned} \tag{18}$$

By using the Euler-Lagrange formalism over the Lagrangian of (18), that is, $\partial L_E(t)/\partial T(t) = 0$ and following a similar procedure as in the case of (17), we obtain

$$\begin{aligned} & 8T_{\text{ext}}^4 T_1 T_2\left(T_1^4 + T_2^4\right) + 3T_{\text{ext}}^5\left(T_1^5 + T_2^5\right) \\ & \quad + T_{\text{ext}} T_2\left(4T_1^8 + 13T_1^5 T_2^3 - 16T_1^3 T_2^5 - 7T_2^8\right) \\ & \quad - T_1 T_2^5\left(32T_1^3 T_2 + 8T_2^4 - 24T_1^4\right) = 0. \end{aligned} \tag{19}$$

Similarly to (15), the only unknown variable in this equation is T_2. Following a similar numerical procedure as in the case of maximum power conditions, we can calculate a convective cell structure. In Table 2 we present the numerical results for the maximum ecological function. We can see in Table 2 that with 16 successive Carnotian convective cells we can reach a final $T_2 \approx 6000$ K.

Our results in Table 2 again show that the width of the cells decrease with increasing radius. The total width in this case is around 0.2859 R_s which is practically the value 0.286 R_s given by other sun models [26].

A remarkable fact observed in Tables 1 and 2 (third column) is that between the cell number 10 and 16, the vertical linear sizes are between 2247 Km and 247 Km, respectively. These are values near to those reported for the linear sizes of granules in [25], which are typically around 900–1000 Km, reaching their largest values up to 2000 Km in diameter. On the other hand, in the highest convective cell of our model, the average power has a value of 5.7×10^9 erg/cm^{2}s, which is of the order of the power reported in [25] for convection in the photosphere (which is 7×10^9 erg/cm^{2}s). Our highest cell overlaps with photosphere. This result is also of the order of the power reported for a mixing length theory of convection in [25], which is $10\text{–}20 \times 10^9$ erg/cm^{2}s. Clearly, our oversimplified model coincides with those reported in [25] in that the energy transported by convection must increase rapidly as we go below the surface region of the convective zone. Finally, it is very interesting that all 16 cells in Tables 1 and 2 have practically the same thermal efficiency, $\eta \approx 0.307$.

4. Concluding Remarks

In the present work we have used a simplified finite-time thermodynamic method to describe the global thermal properties of the convective zone of the sun. This method was previously used by Gordon and Zarmi to describe convective

TABLE 2: Maximum ecological regime case: first column shows the normalized radial position of the hot layers corresponding to the sixteen virtual convective cells. The following columns give, respectively, second, the cell's widths; third, the hot isotherms; fourth, the cold isotherms; fifth, the average power output; sixth, thermal efficiency.

No.	$r(T_1)/R_s$	$\Delta r = r(T_2) - r(T_1)$ (Km)	$T_1 \times 10^6$ (K)	$T_2 \times 10^6$ (K)	$\overline{W}(\text{erg/cm}^2\cdot\text{s})$	$\eta = \eta(T_1, T_2)$
1	0.714	61370.8	2.187761	1.51504	6.49519×10^{17}	0.307494
2	0.802176	42499.5	1.51504	1.04917	4.65788×10^{17}	0.307494
3	0.863239	29431.1	1.04917	0.726558	1.07123×10^{17}	0.307494
4	0.905525	20381.2	0.726558	0.503147	2.46362×10^{16}	0.307493
5	0.934808	14114.1	0.503147	0.348433	5.66589×10^{15}	0.307492
6	0.955087	9774.07	0.348433	0.241293	1.30306×10^{15}	0.307491
7	0.96913	6768.59	0.241293	0.167098	2.99682×10^{14}	0.307489
8	0.978855	4687.28	0.167098	0.115718	6.89224×10^{13}	0.307486
9	0.98559	3245.97	0.115718	0.0801366	1.58513×10^{13}	0.307482
10	0.990254	2247.85	0.0801366	0.0554965	3.64566×10^{12}	0.307476
11	0.993483	1556.65	0.0554965	0.0384331	8.38504×10^{11}	0.307468
12	0.99572	1077.98	0.0384331	0.0266166	1.9288×10^{11}	0.307456
13	0.997269	746.509	0.0266166	0.0184336	4.43885×10^{10}	0.307439
14	0.998341	516.961	0.0184336	0.0127669	1.02346×10^{10}	0.307414
15	0.999084	357.998	0.0127669	0.00884262	2.37873×10^{9}	0.307378
16	0.999599	247.916	0.00884262	0.00612506	5.71627×10^{8}	0.307326

motions of the air in the earth's atmosphere. These authors assert that this FTT-approach corresponds very approximately to the global scale motion of the wind in convective cells. However, it is necessary to remark that convective cells of this kind of FTT-models are only virtual cells performing by unit area and yearly averages. Thus, they only represent the global thermodynamic properties stemming from the first and second laws of thermodynamics; that is, kind of thermodynamically equivalent cells that only captures global average quantities and discards any other dynamical detail. Nevertheless, all these simplifications permit to obtain reasonable values for some thermal quantities associated to the convective zone of the sun. Our simplification is mainly based in taking several spherical virtual layers as black-body radiant surfaces, whose emitted radiation is rapidly absorbed by the opaque gases of the convective zone. This radiant energy is taken as the driver energy of convective cells.

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