

## Research Article

# Electronic-Rotational Coupling in Cl-*para*-H<sub>2</sub> Van der Waals Dimers

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Received 31 July 2012; Accepted 12 November 2012

Academic Editor: Hari P. Saha

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We examine the interaction between an open-shell chlorine atom and a *para*-H<sub>2</sub> molecule in the region of configuration space that corresponds to a weakly bound Cl-*para*-H<sub>2</sub> van der Waals dimer. By constructing and diagonalizing the Hamiltonian matrix that represents the coupled Cl atom electronic and H<sub>2</sub> rotational degrees of freedom, we obtain one-dimensional energy curves for the Cl-*para*-H<sub>2</sub> system in this region of configuration space. We find that the dimer exhibits fairly strong electronic-rotational coupling when the Cl-H<sub>2</sub> distance  $R$  is close to  $R = 5.5a_0$ ; however, this coupling does not modify substantially the positions and depths of the van der Waals wells in the dimer's  $E(R)$  curves. An approximation in which the *para*-H<sub>2</sub> fragment is treated in the strict  $j = 0$  limit thus appears to yield an accurate representation of those states of the weakly bound Cl-*para*-H<sub>2</sub> dimer that correlate with  $j = 0$  H<sub>2</sub> in the  $R \rightarrow \infty$  limit.

## 1. Introduction

Experimental studies [1] of the infrared absorption spectra of solid *para*-H<sub>2</sub> matrices that contain chlorine atoms as substitutional impurities indicate that Cl-H<sub>2</sub> interactions raise the transition energy associated with the  $^2P_{1/2} \leftarrow ^2P_{3/2}$  spin-orbit (SO) transitions of the Cl impurities. In these systems, the H<sub>2</sub> molecules in the Cl atom's first "solvation shell" reside in the van der Waals region of the Cl-H<sub>2</sub> potential energy surface [2]. A detailed analysis of the matrix-induced blue shift of the SO transition for Cl atoms embedded in solid *para*-H<sub>2</sub> would thus provide insight into the shape of the Cl-H<sub>2</sub> potential energy surface in this region of configuration space. This in turn could help us better understand the dynamics of the  $\text{Cl} + \text{H}_2 \rightarrow \text{HCl} + \text{H}$  reaction, which has long been considered a benchmark system in chemical reaction dynamics [3]. For example, theoretical studies [4] of the HCl/DCl product branching ratio of the  $\text{Cl} + \text{HD}$  reaction suggest that the van der Waals region of the potential energy surface plays a key role in controlling the reaction's dynamics at low collision energies.

In the *para*-H<sub>2</sub> matrix, the Cl atom's SO transition is blue shifted by about  $60 \text{ cm}^{-1}$  by Cl-H<sub>2</sub> interactions [1], which

amounts to a shift of about  $5 \text{ cm}^{-1}$  for each of the twelve H<sub>2</sub> molecules in the Cl atom's first solvation shell. The matrix-induced blue shift of the Cl SO transition can be qualitatively understood as arising from subtle differences in the van der Waals interactions of the ground and excited SO states of the Cl atom with nearby H<sub>2</sub> molecules. For a simulation to reproduce quantitatively the observed blue shift, we might therefore anticipate that the potential energy surfaces used in the simulation should be accurate to better than  $1 \text{ cm}^{-1}$ . Theoretical studies of the matrix-induced shift would thus represent a fairly stringent test of the Cl-H<sub>2</sub> potential energy surfaces involving both the ground and excited SO states of the Cl atom.

Previous simulations [5, 6] of open-shell atomic impurities in *para*-H<sub>2</sub> matrices have employed a simplified model for the *para*-H<sub>2</sub> matrix, a model in which the H<sub>2</sub> molecules are treated as spherical particles. This is equivalent to assuming that the H<sub>2</sub> molecules in the matrix remain in the pure  $j = 0$  ground rotational state, with their orientational degrees of freedom completely unperturbed by anisotropic interactions with the open-shell impurity. However, a careful study of the B-H<sub>2</sub> and Al-H<sub>2</sub> van der Waals dimers [7] indicated that the binding energies of these dimers increased

by 15% to 20% (or several  $\text{cm}^{-1}$ ) when the  $j = 0$  restriction for the  $\text{H}_2$  monomer was relaxed and  $j > 0$  rotational states were allowed to mix into the  $\text{H}_2$  molecule's rotational wave function. Similar behavior in the  $\text{Cl-H}_2$  dimer might raise concerns that the effective  $\text{Cl-H}_2$  potential energy functions obtained from a pure  $j = 0$  approximation for the  $\text{H}_2$  molecule would be insufficiently accurate for a quantitative study of the matrix-induced blue shift of the  $\text{Cl SO}$  transition. In such a case, more sophisticated treatments [8] of the  $\text{H}_2$  molecules' rotational degrees of freedom might be needed.

In this work, we examine electronic-rotational coupling in the  $\text{Cl-para-H}_2$  van der Waals dimer, using the potential energy surfaces for the dimer presented in [9] as the foundation for our study. We find some evidence for moderately strong electronic-rotational coupling in the low-energy repulsive region of the  $\text{Cl-H}_2$  potential energy surface. This coupling does not, however, substantially change the positions or depths of the van der Waals minima for dimers composed of a  $\text{Cl}$  atom and a  $\text{para-H}_2$  molecule. This suggests that treating the  $\text{H}_2$  molecules in  $\text{Cl}$ -doped solid  $\text{para-H}_2$  as pure  $j = 0$  molecules should be a reasonably good approximation.

## 2. Spin-Orbit States of an Isolated Cl Atom

We begin our investigation by reviewing the effects of  $\text{SO}$  coupling in an isolated chlorine atom. We first construct a basis set of antisymmetrized many-electron functions for the atom's  $3p$  subshell; we will use this basis set to evaluate the matrix elements of the atom's effective  $\text{SO}$  operator. For future convenience, we choose these antisymmetrized many-electron functions to be Slater determinants of Cartesian  $3p$  spin-orbitals in a space-fixed coordinate system. We use  $p_x$  and  $\bar{p}_x$ , respectively, to represent a one-electron Cartesian  $p_x$  orbital paired with either the spin-up or spin-down electron spin function. For specificity, we note that the  $p_x$  function is positive along the positive  $x$  axis. The six many-electron functions that span the subspace of interest are these Slater determinants:

$$P_x = (5!)^{-1/2} \det |p_x p_y \bar{p}_y p_z \bar{p}_z| \quad (1)$$

$$P_y = (5!)^{-1/2} \det |p_x \bar{p}_x p_y p_z \bar{p}_z| \quad (2)$$

$$P_z = (5!)^{-1/2} \det |p_x \bar{p}_x p_y \bar{p}_y p_z| \quad (3)$$

$$\bar{P}_x = (5!)^{-1/2} \det |\bar{p}_x p_y \bar{p}_y p_z \bar{p}_z| \quad (4)$$

$$\bar{P}_y = (5!)^{-1/2} \det |p_x \bar{p}_x \bar{p}_y p_z \bar{p}_z| \quad (5)$$

$$\bar{P}_z = (5!)^{-1/2} \det |p_x \bar{p}_x p_y \bar{p}_y \bar{p}_z|. \quad (6)$$

The  $\text{Cl}$  atom's  $\text{SO}$  operator is written as

$$\hat{V}_{\text{SO}} = \left( \frac{C_{\text{SO}}}{\hbar^2} \right) \times \hat{\mathbf{L}} \cdot \hat{\mathbf{S}}, \quad (7)$$

where  $\hat{\mathbf{L}}$  and  $\hat{\mathbf{S}}$  are the total orbital and spin angular momentum operators for the five electrons in the  $3p$  subshell

and the constant  $C_{\text{SO}}$  is two-thirds of the energy gap between the lower and upper  $\text{SO}$  energy levels of the  $\text{Cl}$  atom; this energy gap is  $\Delta E_{\text{SO}} = 882.35 \text{ cm}^{-1}$  [10]. In a basis set consisting of these six Slater determinants,  $\hat{V}_{\text{SO}}$  has the matrix representation

$$V_{\text{SO}} = C_{\text{SO}} \times \begin{pmatrix} 0 & i/2 & 0 & 0 & 0 & -1/2 \\ -i/2 & 0 & 0 & 0 & 0 & i/2 \\ 0 & 0 & 0 & 1/2 & -i/2 & 0 \\ 0 & 0 & 1/2 & 0 & -i/2 & 0 \\ 0 & 0 & i/2 & i/2 & 0 & 0 \\ -1/2 & -i/2 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

where the basis set is ordered  $\{P_x, P_y, P_z, \bar{P}_x, \bar{P}_y, \bar{P}_z\}$  from left to right (and top to bottom). In the same basis set, the operator corresponding to the  $z$ -axis projection of the total angular momentum of the  $3p$  subshell,

$$\hat{J}_z = \hat{L}_z + \hat{S}_z, \quad (9)$$

has the matrix representation

$$J_z = \hbar \times \begin{pmatrix} 1/2 & -i & 0 & 0 & 0 & 0 \\ i & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2 & -i & 0 \\ 0 & 0 & 0 & i & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}. \quad (10)$$

Because  $\hat{V}_{\text{SO}}$  and  $\hat{J}_z$  commute with one another, we can simultaneously diagonalize the  $V_{\text{SO}}$  and  $J_z$  matrices. The normalized many-electron functions that do this are

$$\psi_{g,3/2} = \frac{(-iP_x + P_y)}{\sqrt{2}} \quad (11)$$

$$\psi_{g,1/2} = \frac{(2P_z - \bar{P}_x - i\bar{P}_y)}{\sqrt{6}} \quad (12)$$

$$\psi_{g,-1/2} = \frac{(iP_x + P_y + 2i\bar{P}_z)}{\sqrt{6}} \quad (13)$$

$$\psi_{g,-3/2} = \frac{(i\bar{P}_x + \bar{P}_y)}{\sqrt{2}} \quad (14)$$

$$\psi_{e,1/2} = \frac{(P_z + \bar{P}_x + i\bar{P}_y)}{\sqrt{3}} \quad (15)$$

$$\psi_{e,-1/2} = \frac{(iP_x + P_y - i\bar{P}_z)}{\sqrt{3}}. \quad (16)$$

The four functions labeled with the subscript  $g$  are degenerate, with energy  $E = -C_{\text{SO}}/2$ . They represent the four components of the  $\text{Cl}$  atom's  $^2P_{3/2}$  lower  $\text{SO}$  energy level; the fraction listed in each function's subscript, when multiplied by  $\hbar$ , gives that function's  $\hat{J}_z$  eigenvalue. The two functions labeled with the subscript  $e$  are degenerate components of the  $\text{Cl}$  atom's  $^2P_{1/2}$  upper  $\text{SO}$  energy level and have energy  $E = C_{\text{SO}}$ ; these functions have  $\hat{J}_z$  eigenvalues of  $\pm\hbar/2$ , as indicated

by their respective subscripts. Later, we will use the symbol  $j_z$  to represent the subscripts attached to the six many-electron SO wave functions listed above; the  $\hat{J}_z$  eigenvalue for each function is thus  $\hbar j_z$ .

Next we examine the charge densities associated with the Cl atom's lower and upper SO energy levels. The charge density  $\rho$  associated with one of the six electronic wave functions  $\psi$  listed above is obtained by forming the product  $|\psi|^2 = \psi^* \psi$ , integrating over the five electrons' spin coordinates and the spatial coordinates of four of the five electrons, and multiplying by five. The final multiplication by five accounts for the fact that the many-electron function  $\psi$  is normalized so that integrating  $|\psi|^2$  over all five electrons' spatial coordinates gives the value one, while integrating the charge density  $\rho$  over all space should give the value five.

To emphasize the shapes of these charge densities, we note that the charge density associated with the Slater determinant  $P_q$ , which corresponds to a singly occupied  $p$  orbital aligned with the space-fixed  $q$  Cartesian axis and doubly occupied  $p$  orbitals aligned with the other two space-fixed Cartesian axes, can be pictured as a  $|p_q|^2$  charge density "hole" superimposed on the isotropic charge density function associated with a filled six-electron  $3p$  subshell. We will represent this hole-plus-filled-subshell charge distribution using the symbol  $H(p_q)$ .

After computing the charge densities associated with the six SO wave functions listed above and writing them in terms of this symbol, we obtain

$$\rho_{g,\pm 3/2} = \frac{[H(p_x) + H(p_y)]}{2} \quad (17)$$

$$\rho_{g,\pm 1/2} = \frac{[H(p_x) + H(p_y) + 4H(p_z)]}{6} \quad (18)$$

$$\rho_{e,\pm 1/2} = \frac{[H(p_x) + H(p_y) + H(p_z)]}{3}. \quad (19)$$

This shows that the Cl atom's upper SO energy level has an isotropic charge density distribution, while the charge densities associated with the lower SO energy level are anisotropic. The  $\psi_{g,\pm 3/2}$  wave functions have higher levels of electron density along the space-fixed Cartesian  $z$ -axis and lower levels of electron density in the  $(x, y)$  plane; the situation is reversed for the  $\psi_{g,\pm 1/2}$  wave functions.

### 3. Cl-H<sub>2</sub> without Spin-Orbit Coupling

Now we consider a Cl-H<sub>2</sub> dimer in which the Cl atom is located at the origin of the space-fixed  $(x, y, z)$  coordinate system and the H<sub>2</sub> center of mass is located on the positive space-fixed  $z$  axis. We use  $\alpha$  and  $\beta$  to represent the polar and azimuthal angles, respectively, of the H-H covalent bond in the space-fixed  $(x, y, z)$  coordinate system. The H-H covalent bond length is denoted  $r$ , and  $R$  is the Cl-H<sub>2</sub> distance. In this section, we neglect the effect of SO coupling in the Cl atom.

**3.1. Characteristics of the Cl-H<sub>2</sub> Potential Energy Surface.** First we set the azimuthal angle to  $\beta = 0$ , so that the H<sub>2</sub>

molecule resides in the  $(x, z)$  plane. If we neglect effects related to SO coupling, then there are three Cl-H<sub>2</sub> adiabatic electronic states, each of which is doubly degenerate because of the spin-1/2 nature of the Cl-H<sub>2</sub> dimer [2, 9, 11]. Two of these states have  $A'$  spatial symmetry with respect to the  $(x, z)$  plane; these are states in which the  $p$ -orbital "hole" of the Cl atom resides in the  $(x, z)$  plane. The third state has  $A''$  spatial symmetry; in this state, the Cl atom's  $p$ -orbital hole is aligned with the  $y$ -axis.

The two  $A'$  adiabatic electronic states, which we label  $1A'$  and  $2A'$  in order of increasing energy, can be viewed as linear combinations of diabatic states associated with the Slater determinants  $P_x$  and  $P_z$ . The mixing angle  $\theta$  defines these linear combinations [2, 9, 11]:

$$|1A'\rangle = \sin \theta |P_x\rangle + \cos \theta |P_z\rangle, \quad (20)$$

$$|2A'\rangle = \cos \theta |P_x\rangle - \sin \theta |P_z\rangle. \quad (21)$$

The mixing angle can be computed by applying an approximate diabaticization procedure to the adiabatic  $A'$  electronic wave functions obtained from a conventional *ab initio* calculation [2, 9, 11, 12].

When the H<sub>2</sub> polar angle  $\alpha$  is either  $\alpha = 0$  or  $\alpha = \pi/2$ , then symmetry considerations make the electronic states corresponding to the  $P_x$  and  $P_z$  Slater determinants good adiabatic electronic states of the Cl-H<sub>2</sub> system. For these geometries, the mixing angle  $\theta$  must be either  $\theta = 0$  or  $\theta = \pi/2$ . For  $\theta = 0$ , the  $P_z$  state is lower in energy than the  $P_x$  state, while for  $\theta = \pi/2$ , the  $P_x$  state is lower in energy than the  $P_z$  state. In Figure 1, we show how the Cl-H<sub>2</sub> interaction energy obtained in [9] for these two adiabatic states depends on the Cl-H<sub>2</sub> distance  $R$ , both for geometries with  $\alpha = 0$  (in which the H-H bond is collinear with the Cl-H<sub>2</sub> distance) and for geometries with  $\alpha = \pi/2$  (in which the H-H bond is perpendicular to the Cl-H<sub>2</sub> distance). In these plots, the H-H covalent bond length is held fixed at  $r = 1.449a_0$ , its  $\nu = 0$  vibrationally averaged value.

Note that for collinear approach of the H<sub>2</sub> molecule, the adiabatic  $P_x$  and  $P_z$  states undergo a crossing near  $R = 6.1a_0$ . At that point, the mixing angle  $\theta$  changes discontinuously from  $\theta = \pi/2$  at large  $R$  to  $\theta = 0$  at small  $R$ . We can explain this behavior using a simple picture of the Cl-H<sub>2</sub> interaction based on electrostatic and overlap contributions. At large  $R$ , it is more favorable for the "hole" in the Cl atom's  $3p$  subshell to be oriented at right angles to the covalent bond of the incoming H<sub>2</sub> molecule; this leads to the most favorable quadrupole-quadrupole interaction between the H<sub>2</sub> molecule and the Cl atom. At small  $R$ , however, repulsive overlap interactions become more important; these are minimized if the Cl atom's hole orients itself along the axis that connects the Cl nucleus and the H<sub>2</sub> center of mass.

Now we investigate how the Cl-H<sub>2</sub> interaction changes as the H<sub>2</sub> molecule rotates from  $\alpha = 0$  to  $\alpha = \pi/2$ , with both  $R$  and  $r$  held fixed. As before, we neglect SO coupling and we set  $r$  to its  $\nu = 0$  vibrationally averaged value of  $r = 1.449a_0$ . Figure 2(a) shows, for the two  $A'$  adiabatic electronic states, how the Cl-H<sub>2</sub> interaction energy depends on  $\alpha$  at  $R = 7a_0$ . In Figure 2(b) we plot, as functions of  $\alpha$ , the contributions that the  $P_x$  and  $P_z$  diabatic states make to the

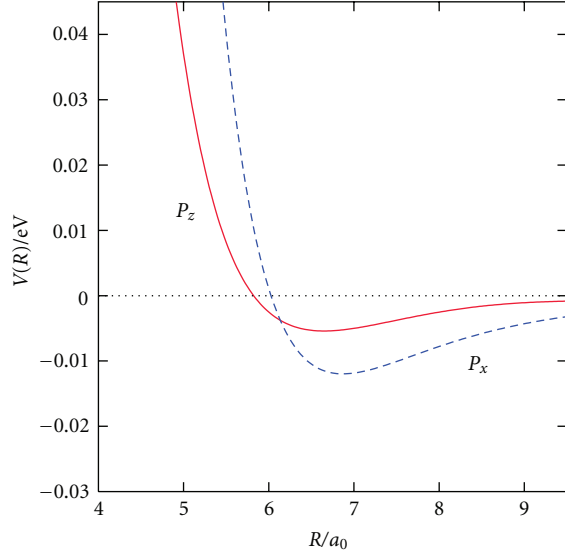
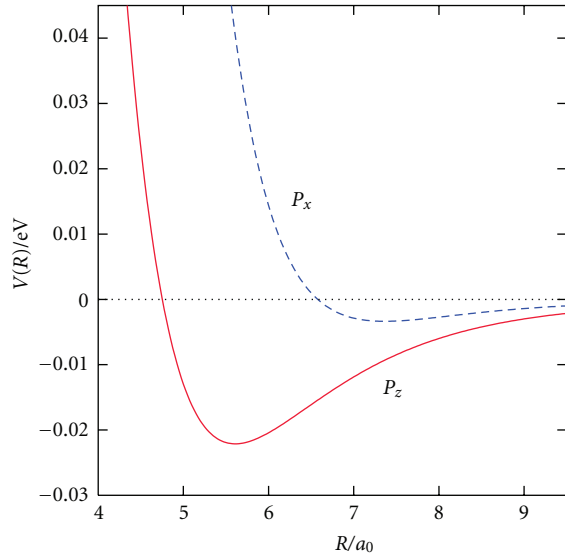
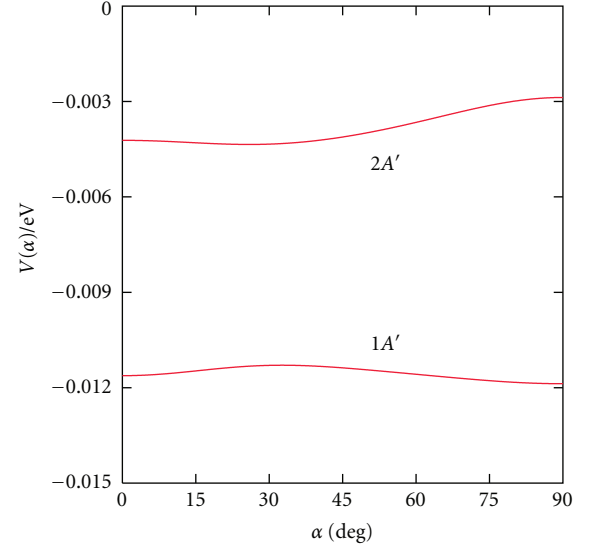
(a) Collinear  $\alpha = 0$ (b) T-shaped  $\alpha = \pi/2$ 

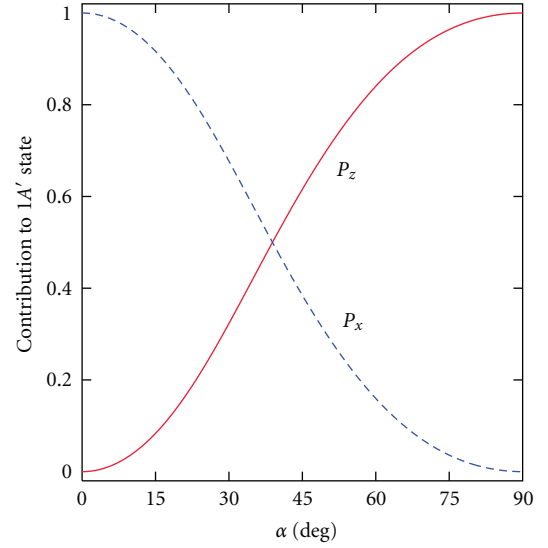
FIGURE 1: (a) Adiabatic Cl-H<sub>2</sub> potential energy surfaces, as functions of the Cl-H<sub>2</sub> distance  $R$ , for (a) collinear and (b) T-shaped  $C_{2v}$  geometries. Solid lines indicate the potential energy for the Cl electronic configuration in which the 3 $p$  hole is aligned with the space-fixed  $z$ -axis; dashed lines indicate the potential energy for the Cl electronic configuration in which the 3 $p$  hole is aligned with the space-fixed  $x$ -axis. In both panels,  $r$  is held fixed at  $r = 1.449a_0$ .

1 $A'$  adiabatic state; as we see from (20), these contributions are simply  $\sin^2\theta$  and  $\cos^2\theta$ . Figure 2(b) demonstrates that for the 1 $A'$  state, the hole in the Cl atom's 3 $p$  subshell rotates in space as  $\alpha$  changes so that the Cl atom maintains a favorable electrostatic interaction with the H<sub>2</sub> quadrupole.

**3.2. Electronic-Rotational Coupling in Cl-H<sub>2</sub> Dimers.** Now we lift the restriction that  $\beta = 0$  and investigate how the rotational energy levels of the H<sub>2</sub> molecule are perturbed by



(a)



(b)

FIGURE 2: (a) Adiabatic  $A'$  potential energy surfaces for planar Cl-H<sub>2</sub> geometries shown as functions of the H<sub>2</sub> polar angle  $\alpha$  for  $R = 7a_0$  and  $r = 1.449a_0$ . (b) Contributions made by the Cl atom's  $P_x$  (dashed line) and  $P_z$  (solid line) Slater determinants to the lowest-energy 1 $A'$  electronic state shown as functions of the H<sub>2</sub> polar angle  $\alpha$  for  $R = 7a_0$  and  $r = 1.449a_0$ .

its interaction with the Cl atom. We continue to neglect SO coupling in the Cl atom.

We construct a basis set for the coupled rotational and electronic degrees of freedom of the Cl-H<sub>2</sub> dimer that is a direct product of functions describing the Cl atom's electronic state and functions describing the H<sub>2</sub> molecule's rotation. For the electronic portion of the direct product basis, we use the six Slater determinants defined in (1) through (6); this will simplify the subsequent incorporation of SO coupling. For the rotational portion of the direct

product basis, we use the spherical harmonics  $Y_{j,m_j}(\alpha, \beta)$  with even values of the  $H_2$  rotational quantum number  $j$ .

In this work, we limit the possible  $j$  values to  $j = 0$  and  $j = 2$ ; this leads to a basis set with 36 orthonormal direct product electronic-rotational basis functions. In the absence of SO coupling, however, the  $36 \times 36$  Hamiltonian matrix splits into two uncoupled  $18 \times 18$  blocks that are, respectively, associated with the spin-up and spin-down Slater determinants. The two  $18 \times 18$  blocks have a common set of eigenvalues.

We could exploit the cylindrical symmetry of the Cl- $H_2$  system to further factorize the Hamiltonian matrix. This would involve rewriting the electronic portion of the basis set in terms of complex linear combinations of  $P_x$  and  $P_y$  that are eigenfunctions of the  $\hat{L}_z$  operator, with eigenvalues  $\hbar\lambda_z$ :

$$\frac{(P_x + iP_y)}{\sqrt{2}} \rightarrow \lambda_z = +1, \quad (22)$$

$$\frac{(P_x - iP_y)}{\sqrt{2}} \rightarrow \lambda_z = -1. \quad (23)$$

Because of the system's cylindrical symmetry, the quantum number  $\Lambda = \lambda_z + m_j$ , which, after multiplication by  $\hbar$ , is the  $z$ -axis projection of the sum of the electronic orbital angular momentum and the  $H_2$  rotational angular momentum, is rigorously good in the absence of SO coupling. If we were to express the SO-free Hamiltonian matrix in terms of the complex basis functions of (22) and (23), the two  $18 \times 18$  blocks of the matrix would therefore factor into uncoupled subblocks corresponding to the allowed values of  $\Lambda$ . Although we do not employ this approach here, we will use  $\Lambda$  to classify the overall symmetry of the SO-free Cl- $H_2$  wave functions. For the basis set employed here,  $\Lambda$  ranges from  $-3$  to  $+3$  in steps of one. In analogy with the nomenclature for diatomic molecules, we will denote Cl- $H_2$  wave functions with  $|\Lambda| = 0, 1, 2$ , or  $3$  as  $\sigma$ ,  $\pi$ ,  $\delta$ , or  $\phi$  states, respectively.

For a fixed value of the Cl- $H_2$  distance  $R$ , the elements of the  $36 \times 36$  electronic-rotational Hamiltonian matrix can be written as

$$\langle P_q Y_{j,m_j} | \hat{V}(\alpha, \beta; R) + \left(\frac{B}{\hbar^2}\right) \hat{J}^2 | P_{q'} Y_{j',m_{j'}} \rangle, \quad (24)$$

where  $q$  and  $q'$  represent space-fixed Cartesian directions,  $B$  is the  $H_2$  rotational constant (here taken to have the value  $59.06 \text{ cm}^{-1}$ ), and  $\hat{V}(\alpha, \beta; R)$  describes the Cl- $H_2$  interaction. The analogous matrix element in which the electronic portions of the basis functions are represented by  $\bar{P}_q$  and  $\bar{P}_{q'}$  has the same value as the matrix element in (24). Matrix elements that involve a spin-up and a spin-down Slater determinant are zero in the absence of SO coupling.

Because the Slater determinants  $P_q$  and  $P_{q'}$  are orthogonal unless  $q = q'$ , the kinetic energy portion of (24) can be evaluated quite easily; it is given by

$$Bj(j+1)\delta_{q,q'}\delta_{j,j'}\delta_{m_j,m_{j'}}. \quad (25)$$

After writing out explicitly the integration over the  $H_2$  angles  $\alpha$  and  $\beta$ , the potential energy portion of (24) becomes

$$\int_{-1}^1 d(\cos \alpha) \int_0^{2\pi} d\beta Y_{j,m_j}^*(\alpha, \beta) F_{q,q'}(\alpha, \beta; R) Y_{j',m_{j'}}(\alpha, \beta), \quad (26)$$

where

$$F_{q,q'}(\alpha, \beta; R) = \langle P_q | \hat{V}(\alpha, \beta; R) | P_{q'} \rangle. \quad (27)$$

Here, we compute the integrals in (26) by numerical quadrature, using the 240-point spherical  $t$ -design specified in [13].

To evaluate  $F_{q,q'}(\alpha, \beta; R)$ , we introduce a rotated system of Cartesian axes, which we denote  $(u, v, z)$ . The  $u$  and  $v$  axes rotate in the space-fixed  $(x, y)$  plane as the  $H_2$  azimuthal angle  $\beta$  changes, so that the  $H_2$  molecule always resides in the  $(u, z)$  plane. The Slater determinants  $P_x$  and  $P_y$ , defined in the space-fixed coordinate system, are related to their analogues  $P_u$  and  $P_v$ , defined in the rotated coordinate system, through

$$P_x = P_u \cos \beta - P_v \sin \beta, \quad (28)$$

$$P_y = P_u \sin \beta + P_v \cos \beta. \quad (29)$$

Equipped with this relationship, we can now evaluate  $F_{q,q'}(\alpha, \beta; R)$ , where  $q$  and  $q'$  are directions in the space-fixed coordinate system, in terms of integrals over  $P_u$ ,  $P_v$ , and  $P_z$ . Doing this, we obtain

$$F_{x,x}(\alpha, \beta; R) = \cos^2 \beta \langle P_u | \hat{V} | P_u \rangle + \sin^2 \beta \langle P_v | \hat{V} | P_v \rangle, \quad (30)$$

$$F_{y,y}(\alpha, \beta; R) = \sin^2 \beta \langle P_u | \hat{V} | P_u \rangle + \cos^2 \beta \langle P_v | \hat{V} | P_v \rangle, \quad (31)$$

$$F_{x,y}(\alpha, \beta; R) = \sin \beta \cos \beta (\langle P_u | \hat{V} | P_u \rangle - \langle P_v | \hat{V} | P_v \rangle), \quad (32)$$

$$F_{x,z}(\alpha, \beta; R) = \cos \beta \langle P_u | \hat{V} | P_z \rangle, \quad (33)$$

$$F_{y,z}(\alpha, \beta; R) = \sin \beta \langle P_u | \hat{V} | P_z \rangle, \quad (34)$$

where, for the sake of brevity, we have suppressed the  $(\alpha, \beta; R)$  dependence of  $\hat{V}$ . Note that  $\langle P_u | \hat{V} | P_v \rangle$  and  $\langle P_v | \hat{V} | P_z \rangle$  are both zero by reasons of symmetry.

The quantities  $\langle P_u | \hat{V} | P_u \rangle$ ,  $\langle P_v | \hat{V} | P_v \rangle$ , and  $\langle P_u | \hat{V} | P_z \rangle$  are the same as those in (2) through (4) of [9]. They depend on  $\alpha$  and  $R$  but are independent of  $\beta$ . As a consequence, Cl- $H_2$  electronic-rotational basis functions with different values of  $m_j$  are coupled only by the trigonometric functions of  $\beta$  that are shown explicitly in (30) through (34).  $F_{x,x}$  and  $F_{y,y}$  couple  $H_2$  rotational states with  $|m_j - m_{j'}| = 0$  or  $2$ ,  $F_{x,y}$  couples  $H_2$  rotational states with  $|m_j - m_{j'}| = 2$ , and  $F_{x,z}$  and  $F_{y,z}$  couple  $H_2$  rotational states with  $|m_j - m_{j'}| = 1$ .  $F_{z,z}$  has no dependence on  $\beta$  and therefore does not couple basis functions with different  $m_j$  values.

Constructing and diagonalizing the  $36 \times 36$  electronic-rotational Hamiltonian matrix at a series of  $R$  values gives

the Cl-H<sub>2</sub> curves shown in Figure 3, which we now discuss in some detail. The curves fall into three distinct groups, labeled as A, B, and C. The two curves in group A correlate smoothly with the  $j = 0$  rotational level of the H<sub>2</sub> molecule as  $R \rightarrow \infty$ , while those in groups B and C correlate with  $j = 2$  states of the H<sub>2</sub> molecule in this limit. All of the B curves, at large  $R$ , correlate with electronic-rotational basis functions that involve the Cl atom's  $P_z$  Slater determinant. The C curves, on the other hand, correlate at large  $R$  with electronic-rotational basis functions involving the  $P_x$  and  $P_y$  Slater determinants of the Cl atom.

We begin by considering the two curves (one  $\sigma$  and one  $\pi$ ) in group A. Both curves are attractive at long range, but the minimum of the  $\sigma$  curve is deeper and occurs at a smaller  $R$  value. The minimum for the  $\sigma$  curve occurs at  $R_{\min} = 5.85a_0$ , where  $E = -15.9$  meV. The minimum for the  $\pi$  curve is at  $R_{\min} = 7.08a_0$ , where  $E = -6.5$  meV. The  $\sigma$  curve correlates at large  $R$  with the  $P_z$  Slater determinant of the Cl atom's  $3p$  subshell; for this Slater determinant, the  $3p$  hole of the Cl atom is oriented along the space-fixed  $z$  axis, which is the direction of approach of the incoming H<sub>2</sub> molecule. The  $\pi$  curve, by contrast, correlates at large  $R$  values with the  $P_x$  and  $P_y$  Slater determinants, for which the Cl atom's  $3p$  hole is oriented in the  $(x, y)$  plane. The  $\sigma$  curve thus minimizes short-range repulsive overlap interactions between Cl and H<sub>2</sub> and permits closer approach of the incoming H<sub>2</sub> molecule.

Group B includes three curves: one  $\sigma$ , one  $\pi$ , and one  $\delta$ . These curves all correlate smoothly with the  $P_z$  Slater determinant as  $R \rightarrow \infty$ , so the Cl atom's  $3p$  hole is aligned with the space-fixed  $z$  axis for large  $R$ . For the  $\delta$  curve, the H<sub>2</sub> rotational state at large  $R$  corresponds to the ( $j = 2, m_j = \pm 2$ ) spherical harmonic. In this state, the H<sub>2</sub> molecule can be viewed classically as rotating in the  $(x, y)$  plane. This orientation of the rotating H<sub>2</sub> molecule both maximizes its attractive electrostatic quadrupole-quadrupole interaction with the Cl atom and minimizes short-range Cl-H<sub>2</sub> repulsive interactions at small  $R$  values. Consequently, of the three group B curves, the  $\delta$  curve has the deepest minimum.

The  $\pi$  curve from group B correlates at large  $R$  with electronic-rotational basis functions that are direct products of the H<sub>2</sub> ( $j = 2, m_j = \pm 1$ ) spherical harmonics and the Cl atom's  $P_z$  Slater determinant. As  $R \rightarrow \infty$ , therefore, the  $\pi$  character of this curve is associated with the rotational motion of the H<sub>2</sub> molecule. Figure 3 shows, however, that this curve undergoes an avoided crossing with the group A  $\pi$  curve near  $R = 5.5a_0$ .

To understand the nature of this avoided crossing in more detail, we project the wave function for the group A  $\pi$  curve onto two pairs of electronic-rotational direct product basis functions: (1) the two functions  $|P_x Y_{0,0}\rangle$  and  $|P_y Y_{0,0}\rangle$  and (2) the two functions  $|P_z Y_{2,1}\rangle$  and  $|P_z Y_{2,-1}\rangle$ . The first pair of functions describes the group A  $\pi$  curve at large  $R$  values, while the second pair of functions describes the group B  $\pi$  curve at large  $R$  values. Figure 4 shows the  $R$  dependence of the contributions that these two pairs of basis functions make to the  $\pi$  curve of group A.

It is clear that, for the entire range of  $R$  values shown in Figure 4, the makeup of the group A  $\pi$  curve is dominated by these two pairs of basis functions; summed together, their

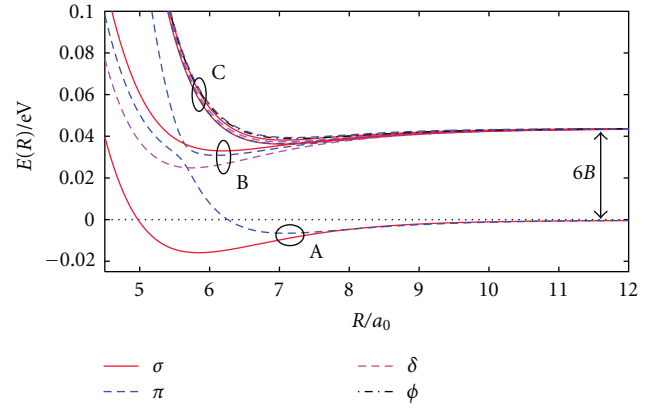


FIGURE 3: Coupled electronic-rotational Cl-H<sub>2</sub> energy curves that correlate with the  $j = 0$  and  $j = 2$  states of H<sub>2</sub> shown as functions of the Cl-H<sub>2</sub> distance  $R$ . Spin-orbit effects are neglected, and the H<sub>2</sub> bond length is held fixed at  $r = 1.449a_0$ .

contributions represent over 99% of the curve's character. At large  $R$  values, the curve can be described entirely by the  $|P_x Y_{0,0}\rangle$  and  $|P_y Y_{0,0}\rangle$  direct product functions, demonstrating that the  $\pi$  character of the state arises from the Cl atom's  $3p$  subshell. At small  $R$  values, however, the curve is described entirely by the  $|P_z Y_{2,1}\rangle$  and  $|P_z Y_{2,-1}\rangle$  direct product functions; here, the  $\pi$  character of the state arises from the rotational motion of the H<sub>2</sub> molecule. The crossover in the description of the  $\pi$  curve of group A reflects a strong interaction between these two zero-order states, an interaction that arises from the  $F_{x,z}$  and  $F_{y,z}$  functions of (33) and (34). The result is to strongly couple the electronic and rotational degrees of freedom of the Cl-H<sub>2</sub> dimer near  $R = 5.5a_0$ .

We have also taken the group A  $\sigma$  curve and projected it onto the direct product basis function  $|P_z Y_{0,0}\rangle$  to assess the contribution that this zero-order state makes to the curve. Figure 4 indicates that this zero-order state provides a very good description of the group A  $\sigma$  curve, accounting for more than 90% of its character over the entire range of  $R$  values shown there. This demonstrates that the group A  $\sigma$  curve can be closely approximated by the interaction between a pure  $j = 0$  *para*-H<sub>2</sub> molecule and a Cl atom described by the  $P_z$  Slater determinant.

#### 4. Cl-H<sub>2</sub> Including Spin-Orbit Coupling

Now we ask how the results summarized in Figures 3 and 4 change when we include the effects of SO coupling. To do this, we simply add the appropriate matrix elements of the SO operator (see (8)) to the  $36 \times 36$  Hamiltonian matrix expressed in the direct product basis set, and then compute the eigenvalues of the new matrix at a series of  $R$  values. The calculations of [9] indicate that, in the Cl-H<sub>2</sub> van der Waals region, the triatomic system's SO matrix elements are nearly independent of the Cl-H<sub>2</sub> geometry and have values nearly identical to those for an isolated Cl atom. This justifies the approximation implicit in our calculations, namely that the

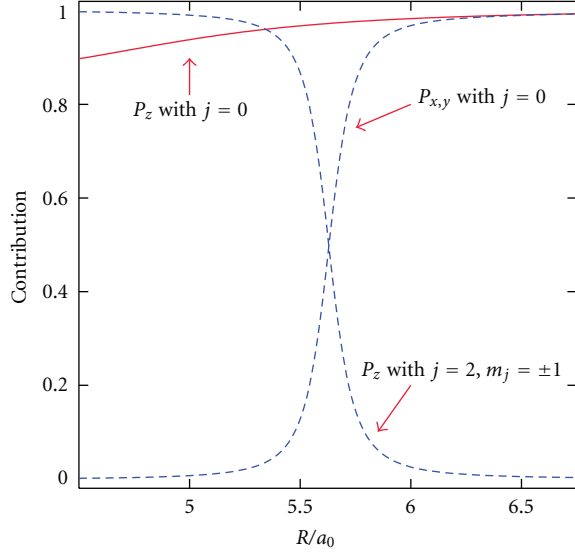


FIGURE 4: Solid line: the contribution made by the  $|P_z Y_{0,0}\rangle$  basis function to the lowest  $\sigma$  Cl-H<sub>2</sub>  $E(R)$  curve shown in Figure 3. Dashed lines: contributions made by the  $|P_{x,y} Y_{0,0}\rangle$  pair of basis functions and the  $|P_z Y_{2,\pm 1}\rangle$  pair of basis functions to the lowest  $\pi$  Cl-H<sub>2</sub>  $E(R)$  curve shown in Figure 3.

TABLE 1: Combinations  $(j_z, m_j)$  of azimuthal angular momentum quantum numbers that can couple together to produce a given positive  $\Omega$  value.

$\Omega = 1/2$	$\Omega = 3/2$	$\Omega = 5/2$	$\Omega = 7/2$
$(+3/2, -1)$	$(+3/2, 0)$	$(+3/2, +1)$	$(+3/2, +2)$
$(+1/2, 0)$	$(+1/2, +1)$	$(+1/2, +2)$	
$(-1/2, +1)$	$(-1/2, +2)$		
$(-3/2, +2)$			

$V_{SO}$  matrix for the free Cl atom is a good approximation of the effects of SO coupling in the Cl-H<sub>2</sub> dimer. For a given value of  $R$ , and in the absence of accidental degeneracies, the new Hamiltonian matrix has 18 distinct doubly degenerate eigenvalues. Kramers [14] explained that a system with a single unpaired electron would always, in the absence of an external magnetic field, have doubly degenerate energy levels; the two states that make up a single doubly degenerate energy level are sometimes called a Kramers pair.

As before, the system's cylindrical symmetry introduces a rigorously good quantum number that, when multiplied by  $\hbar$ , gives the  $z$ -axis projection of the system's total (electronic spin plus electronic orbital plus H<sub>2</sub> rotational) angular momentum. Here we call this quantum number  $\Omega$ ; its value is given by  $\Omega = j_z + m_j$ , and for the basis set employed here, the possible values of  $\Omega$  range from  $-7/2$  to  $7/2$  in steps of one. Although we have not exploited the system's cylindrical symmetry to block-diagonalize the Hamiltonian matrix, we use  $\Omega$  to classify the symmetry of the Cl-H<sub>2</sub> wave functions and to gain some physical insight into the shapes of the system's energy curves  $E(R)$ .

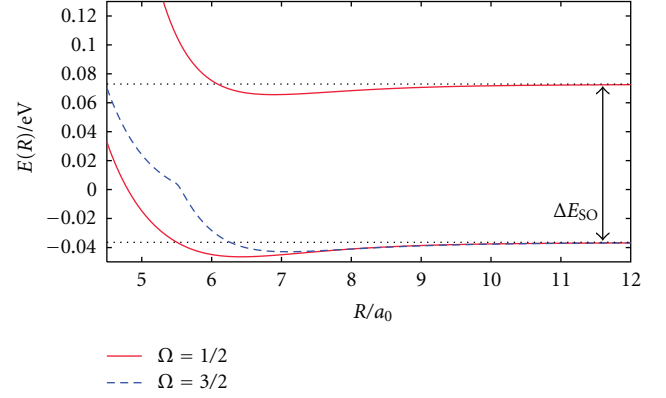


FIGURE 5: Coupled electronic-rotational Cl-H<sub>2</sub> energy curves that correlate with the  $j = 0$  rotational state of H<sub>2</sub> shown as functions of the Cl-H<sub>2</sub> distance  $R$ . Spin-orbit effects are included; the horizontal dashed lines indicate the asymptotic energies of the Cl-H<sub>2</sub> dimer as  $R \rightarrow \infty$  for the lower and upper spin-orbit levels of the Cl atom.

In Table 1, we list the combinations of the  $j_z$  and  $m_j$  quantum numbers that can generate each of the four positive allowed  $\Omega$  values. The corresponding negative  $\Omega$  values can be obtained by reversing the sign of both  $j_z$  and  $m_j$ . In the following discussion, we consider only the states that have positive  $\Omega$  values, as the states with negative  $\Omega$  values have identical energies. We will refer to this table and to the charge densities of the two SO levels of an isolated Cl atom ((17) through (19)), as we discuss the role that SO coupling has on the Cl-H<sub>2</sub> energy curves.

As Table 1 indicates, the  $j = 0$  state of the H<sub>2</sub> molecule, which has  $m_j = 0$  as well, can either (1) form a Cl-H<sub>2</sub> state with  $\Omega = 3/2$  by combining with the  $\psi_{g,+3/2}$  state of the Cl atom or (2) form a Cl-H<sub>2</sub> state with  $\Omega = 1/2$  by combining with the  $\psi_{g,+1/2}$  state or the  $\psi_{e,+1/2}$  state of the Cl atom. Consequently, there are three distinct Cl-H<sub>2</sub> curves that correlate as  $R \rightarrow \infty$  with the  $j = 0$  ground rotational state of the H<sub>2</sub> molecule. Two of the three curves are associated with the Cl atom's lower SO energy level, and one is associated with the atom's upper SO energy level. These curves, shown in Figure 5, are the most important ones for understanding the interaction between Cl and an approaching *para*-H<sub>2</sub> molecule.

All three curves exhibit relatively shallow van der Waals minima. The  $\Omega = 1/2$  state that correlates with the Cl atom's lower SO energy level has the deepest minimum, which occurs at  $R = 6.46a_0$ . At this  $R$  value, the depth of the Cl-H<sub>2</sub> curve, measured with respect to the  $R = \infty$  asymptote, is 10.0 meV. The other two states have shallower minima, which occur at larger  $R$  values: the minimum for the  $\Omega = 1/2$  state that correlates with the upper SO energy level is at  $R = 6.89a_0$ , while that for the  $\Omega = 3/2$  state is at  $R = 7.09a_0$ . The depths of these minima, measured with respect to the  $R = \infty$  asymptotes, are, respectively, 7.3 meV and 6.5 meV. The relative locations and depths of the three minima can

be understood by examining the charge densities of the Cl electronic states that are associated with each curve.

For the  $\Omega = 1/2$  curve that correlates with the lower SO energy level, the Cl atom is described at large  $R$  by the  $\psi_{g,+1/2}$  electronic wave function. This Cl atomic state has a depletion of electron density along the  $z$  axis (see (18)), which is the direction of approach of the incoming  $H_2$  molecule. Consequently, of the three curves shown in Figure 5, this curve allows the  $H_2$  molecule to approach the Cl atom most closely. For the  $\Omega = 3/2$  curve, on the other hand, the Cl atom is described at large  $R$  by the  $\psi_{g,+3/2}$  electronic wave function; as (17) shows, the electron density for this state of the Cl atom is depleted in the  $(x, y)$  plane and built up along the  $z$  axis. Of the three curves shown in Figure 5, this curve has its minimum at the largest  $R$  value. Finally, for the  $\Omega = 1/2$  curve that correlates with the upper SO energy level, the Cl atom's large- $R$  character is that of the  $\psi_{e,+1/2}$  state. In this state, the atom's  $3p$  "hole" is completely orientationally delocalized (see (19)). This leads to a minimum that occurs at an  $R$  value between the minima for the two other curves.

We now ask, for the two  $\Omega = 1/2$  curves depicted in Figure 5, how the approach of the incoming  $H_2$  molecule distorts the electronic structure of the Cl atom. To answer this question, we take the Cl- $H_2$  wave functions for the two curves and project them onto either the  $\psi_{g,+1/2}$  (for the lower curve) or the  $\psi_{e,+1/2}$  (for the upper curve) Cl atomic wave function. In computing these projections, we sum over all of the  $H_2$  fragment's rotational states. Figure 6(a) shows, as a function of the Cl- $H_2$  distance  $R$ , the fractional contributions that these two pure atomic SO states make to the corresponding Cl- $H_2$  dimer wave functions. For large  $R$ , the Cl electronic structure is essentially unperturbed by the  $H_2$  molecule. As  $R$  decreases below about  $6a_0$ , however, the impinging  $H_2$  molecule begins to mix some excited SO character into the wave function of the lower curve, and some ground SO character into the wave function of the upper curve.

For these two  $\Omega = 1/2$  curves, Cl- $H_2$  interactions also perturb the rotational degrees of freedom of the  $H_2$  fragment at small  $R$  values. This leads to a mixing of some  $j = 2$  character into the  $H_2$  molecule's rotational wave function. To measure this perturbation, we project the two Cl- $H_2$   $\Omega = 1/2$  wave functions onto the  $j = 0$   $H_2$  molecular rotational wave function. In computing these projections, we sum over all of the Cl fragment's SO states. Figure 6(b) shows how the contribution that the  $H_2$   $j = 0$  state makes to the overall dimer wave function changes with  $R$ . For the range of  $R$  values shown here, the character of both  $\Omega = 1/2$  curves is dominated by the  $j = 0$   $H_2$  rotational state.

Figures 7 through 9 complete our survey of the Cl- $H_2$  energy curves associated with the lower SO energy level. In these figures, the curves whose large- $R$  asymptotic values are roughly 7 meV all correlate, as  $R \rightarrow \infty$ , with the  $H_2$  molecule's  $j = 2$  rotational energy level. For the  $\Omega = 1/2$ ,  $5/2$ , and  $7/2$  states (Figures 7 and 9), these curves fall into two groups distinguished mainly by the position of their short-range repulsive walls. The curves whose repulsive walls occur at smaller  $R$  values are associated with the Cl atom's  $\psi_{g,\pm 1/2}$  states, which have a depletion of electron density along the  $z$

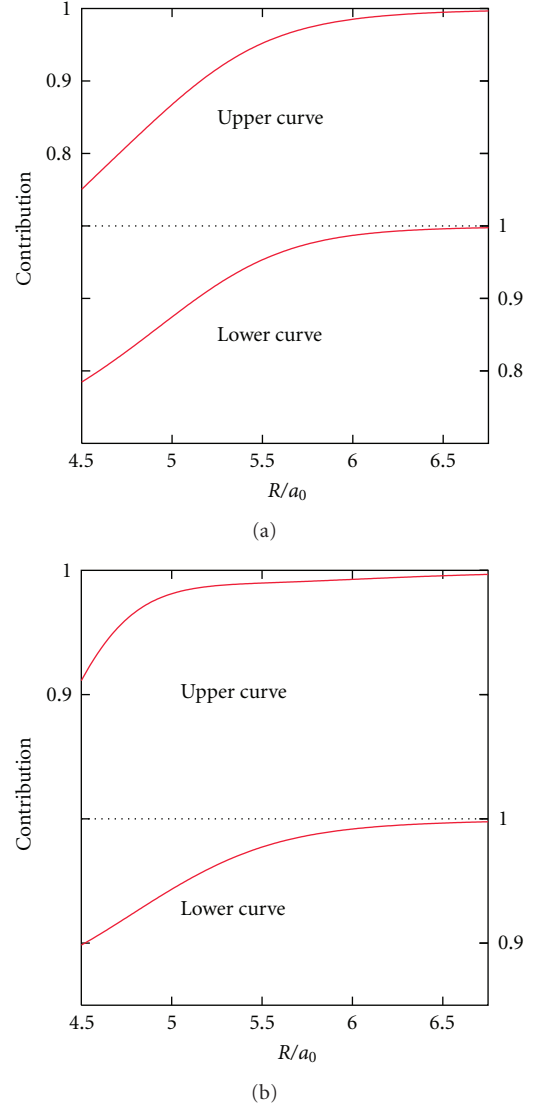


FIGURE 6: (a) The contribution made by the Cl atom's respective  $\psi_{g,+1/2}$  or  $\psi_{e,+1/2}$  spin-orbit electronic state to the two  $\Omega = 1/2$  curves shown in Figure 5. (b) The contribution made by the  $H_2$   $j = 0$  rotational state to the Cl- $H_2$  wave function for the two  $\Omega = 1/2$  curves shown in Figure 5.

axis. The curves whose repulsive walls occur at larger  $R$  values are associated with the Cl atom's  $\psi_{g,\pm 3/2}$  states, which do not exhibit depleted electron density along the  $z$  axis.

The  $\Omega = 3/2$  curves depicted in Figure 8 exhibit more complicated behavior. The uppermost curve shown there is associated with the Cl atom's  $\psi_{g,+3/2}$  state, and thus its repulsive wall begins to manifest itself at larger  $R$  values. The other three curves, however, participate in a pair of avoided crossings near  $R = 5.5a_0$ , close to where the avoided crossing observed in Figure 3 occurred. The unusual "kink" seen in Figure 5 is associated with this pair of avoided crossings.

Figure 10 shows this pair of avoided crossings at higher magnification. The three curves, in order of increasing energy, correlate in the large  $R$  limit with (1) the  $\psi_{g,+3/2}$  Cl

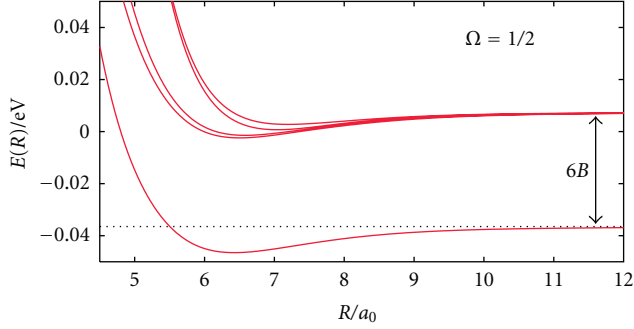


FIGURE 7: All of the Cl-H<sub>2</sub> dimer's  $\Omega = 1/2$  coupled electronic-rotational states that correlate with the lower spin-orbit level of the Cl atom are shown as functions of the Cl-H<sub>2</sub> distance  $R$ . The horizontal dashed line indicates the dimer's asymptotic energy, as  $R \rightarrow \infty$ , for the lower spin-orbit level of the Cl atom and the  $j = 0$  state of H<sub>2</sub>.

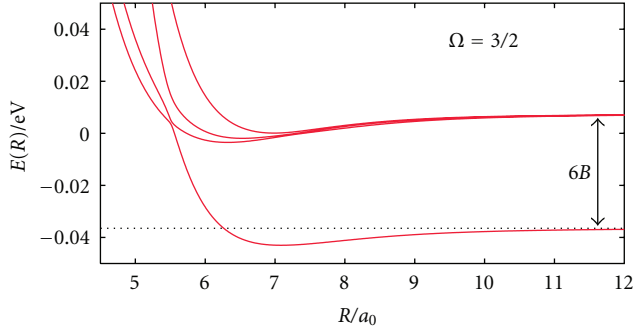


FIGURE 8: All of the Cl-H<sub>2</sub> dimer's  $\Omega = 3/2$  coupled electronic-rotational states that correlate with the lower spin-orbit level of the Cl atom are shown as functions of the Cl-H<sub>2</sub> distance  $R$ . The horizontal dashed line indicates the dimer's asymptotic energy, as  $R \rightarrow \infty$ , for the lower spin-orbit level of the Cl atom and the  $j = 0$  state of H<sub>2</sub>.

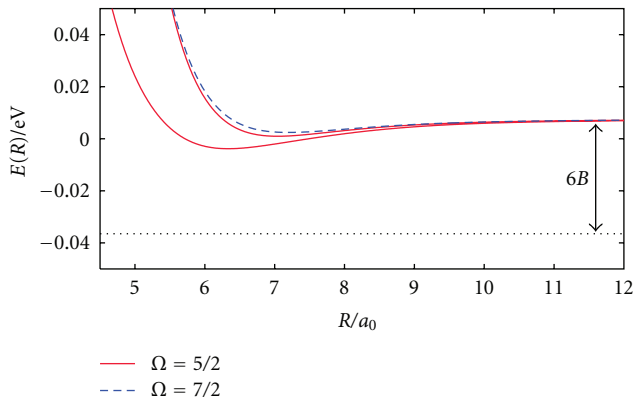


FIGURE 9: All of the Cl-H<sub>2</sub> dimer's  $\Omega = 5/2$  and  $\Omega = 7/2$  coupled electronic-rotational states that correlate with the lower spin-orbit level of the Cl atom are shown as functions of the Cl-H<sub>2</sub> distance  $R$ . The horizontal dashed line indicates the dimer's asymptotic energy, as  $R \rightarrow \infty$ , for the lower spin-orbit level of the Cl atom and the  $j = 0$  state of H<sub>2</sub>.

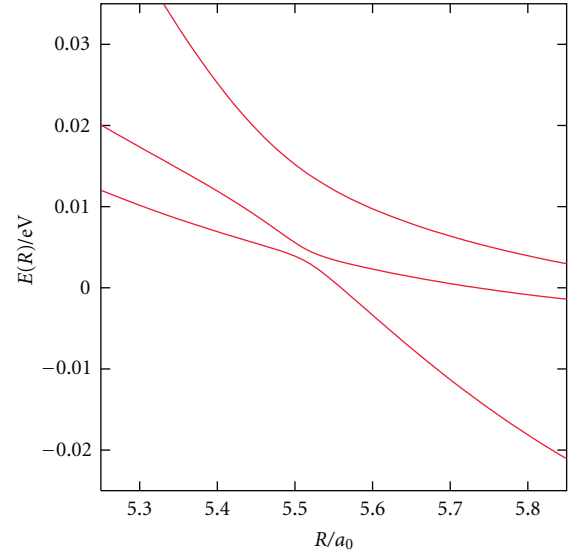


FIGURE 10: A magnified view of three  $\Omega = 3/2$  curves shown in Figure 8. This figure shows the pair of avoided crossings near  $R = 5.5a_0$  that affect the  $\Omega = 3/2$  coupled electronic-rotational states that correlate with the lower Cl spin-orbit level.

atomic state coupled with the ( $j = 0, m_j = 0$ ) rotational state of H<sub>2</sub>; (2) the  $\psi_{g,-1/2}$  Cl atomic state coupled with the ( $j = 2, m_j = 2$ ) rotational state of H<sub>2</sub>; and (3) the  $\psi_{g,+1/2}$  Cl atomic state coupled with the ( $j = 2, m_j = 1$ ) rotational state of H<sub>2</sub>. The pair of avoided crossings shown in Figure 10 is analogous to the avoided crossing between the group A and group B  $\pi$  curves shown in Figure 3. The additional complexity observed in Figure 10 arises from the fact that, as Table 1 shows, three different values of the H<sub>2</sub> molecule's  $m_j$  quantum number can combine with an appropriate SO level of the Cl atom to form a dimer state with  $\Omega = 3/2$ . Although the avoided crossings shown in Figure 10, like that shown in Figure 3, demonstrate that there are regions of configuration space where the Cl-H<sub>2</sub> dimer exhibits strong electronic-rotational coupling, these regions of configuration space are at moderately high energies on the repulsive wall of the dimer's potential energy surface.

Finally, in Figure 11, we show all of the curves that correlate at large  $R$  with the Cl atom's upper SO level. Only one of these curves, with  $\Omega = 1/2$ , correlates with the  $j = 0$  state of the H<sub>2</sub> molecule; the others correlate with the H<sub>2</sub> molecule's  $j = 2$  excited rotational state. All six curves shown in this figure have very similar shapes; this is because the upper SO level of the Cl atom, with its orientationally delocalized  $3p$  hole, appears isotropic to the incoming H<sub>2</sub> molecule.

We close by computing the Cl-H<sub>2</sub> interaction curves  $E(R)$ , including the effects of SO coupling, under the assumption that the H<sub>2</sub> molecule retains pure  $j = 0$  character at all values of  $R$ . We do this simply by removing all of the direct product basis functions with  $j > 0$  from the basis set used to express the Cl-H<sub>2</sub> dimer wave function; this leads to a  $6 \times 6$  matrix representation of the Cl-H<sub>2</sub> Hamiltonian that has three doubly degenerate eigenvalues,

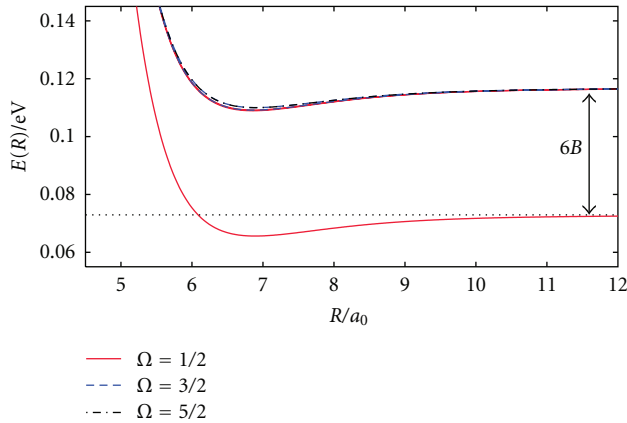
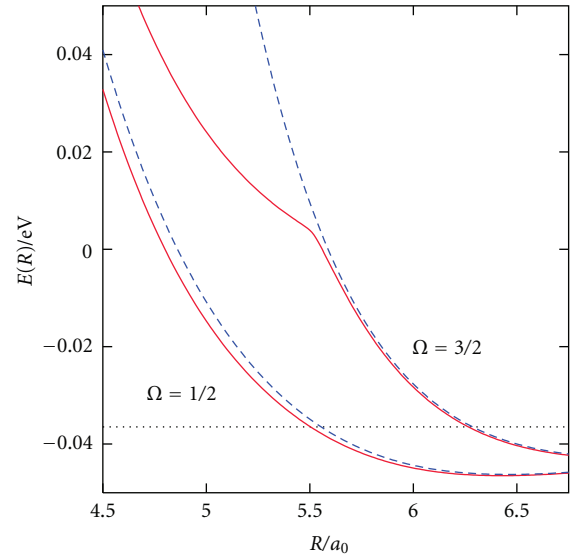


FIGURE 11: All of the Cl-H<sub>2</sub> dimer's coupled electronic-rotational states that correlate with the upper spin-orbit level of the Cl atom are shown as functions of the Cl-H<sub>2</sub> distance  $R$ . The horizontal dashed line indicates the dimer's asymptotic energy, as  $R \rightarrow \infty$ , for the upper spin-orbit level of the Cl atom and the  $j = 0$  state of H<sub>2</sub>.

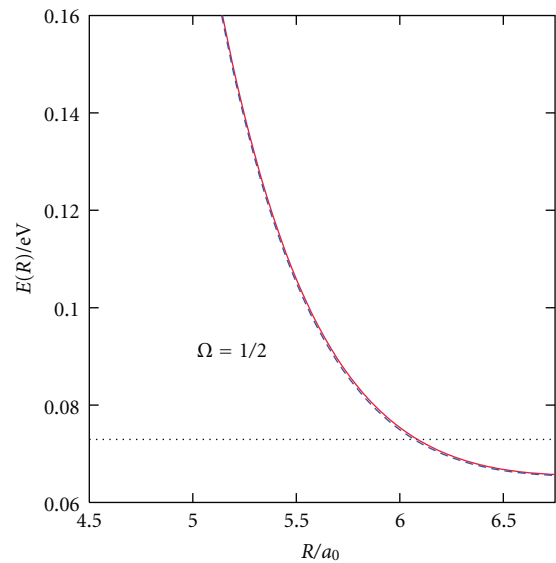
or three Kramers pairs. These three eigenvalues represent the interaction curves  $E(R)$  obtained using the pure  $j = 0$  approximation for the H<sub>2</sub> fragment. In Figure 12, we show these three curves and compare them with the analogous curves computed using the larger  $36 \times 36$  Hamiltonian matrix that includes  $j = 2$  states of the H<sub>2</sub> fragment. The three curves obtained from the larger Hamiltonian matrix were first shown in Figure 5.

We see from Figure 12 that the depths and positions of the van der Waals minima for these three curves change very little when we impose the pure  $j = 0$  approximation for the H fragment. For example, the depth of the  $\Omega = 1/2$  curve that correlates with the Cl atom's lower SO state (shown in Figure 12(a)) changes by about 0.5% when we impose this approximation. This is in sharp contrast to the case for the B-H<sub>2</sub> and Al-H<sub>2</sub> dimers [7], for which the zero-point-corrected binding energies change by 15% and 22%, respectively, when the H<sub>2</sub> fragment is frozen in its  $j = 0$  state. It appears that the anisotropy of the underlying diabatic potential energy surfaces is substantially larger for the B-H<sub>2</sub> and Al-H<sub>2</sub> systems than for the Cl-H<sub>2</sub> system; this leads to a greater perturbation of the H<sub>2</sub> molecule's rotational wave function in the region of the van der Waals minima of the B-H<sub>2</sub> and Al-H<sub>2</sub> systems.

For the two  $\Omega = 1/2$  curves shown in Figure 12, we see that the pure  $j = 0$  approximation reproduces the actual Cl-H<sub>2</sub> interaction curves very well; the main difference seems to be that the  $\Omega = 1/2$  curve that correlates with the lower SO level of the Cl atom has a slightly stiffer repulsive wall if the H<sub>2</sub> molecule is held fixed in its  $j = 0$  rotational state. For the  $\Omega = 3/2$  curve that correlates with the lower SO level of the Cl atom, the pure  $j = 0$  approximation is of course completely incapable of reproducing the true Cl-H<sub>2</sub> interaction for  $R$  values below  $R = 5.5a_0$ ; this is the region of configuration space where strong electronic-rotational coupling leads to the avoided crossings shown in Figure 10,



(a)



(b)

FIGURE 12: A comparison of the three Cl-H<sub>2</sub> curves that correlate with  $j = 0$  H<sub>2</sub> molecules when the H<sub>2</sub> molecule is either frozen in its  $j = 0$  state (dashed lines) or allowed to mix some  $j = 2$  character into its rotational wave function (solid lines). (a) shows, as functions of the Cl-H<sub>2</sub> distance  $R$ , curves for the two states that correlate with the lower spin-orbit state of the Cl atom; (b) shows curves for the state that correlates with the upper spin-orbit state of the Cl atom. The horizontal dashed lines show the respective asymptotic dimer energies as  $R \rightarrow \infty$ . The two curves shown in (b) are nearly indistinguishable from one another.

and the  $j = 0$  approximation is simply inapplicable there. However, this region of configuration space is relatively high on the repulsive wall of the  $\Omega = 3/2$  curve; because the experimental studies of Cl-doped solid *para*-H<sub>2</sub> are carried out at  $T \approx 2$  K, it is unlikely that the Cl atom's H<sub>2</sub> neighbors spend a significant portion of time exploring this portion of the  $\Omega = 3/2$  potential energy curve.

## 5. Summary

We have computed a series of one-dimensional energy curves  $E(R)$  for the Cl-*para*-H<sub>2</sub> dimer by diagonalizing the dimer's combined electronic-rotational Hamiltonian matrix. We express the Hamiltonian matrix in terms of a direct product basis set that facilitates the inclusion of SO coupling effects. The gross features of the dimer's  $E(R)$  curves, such as the depths and positions of the curves' van der Waals minima, can be rationalized by considering the charge densities associated with the lower and upper SO levels of the Cl atom.

We find that electronic-rotational coupling in the Cl-H<sub>2</sub> dimer is strong for Cl-H<sub>2</sub> distances near  $R = 5.5a_0$ . In the system where SO effects are ignored, this coupling gives rise to an avoided crossing between two Cl-H<sub>2</sub> states with  $\pi$  symmetry: one that correlates at large  $R$  with the  $j = 0$  H<sub>2</sub> rotational level and one that correlates with the  $j = 2$  rotational level. Once SO effects are included, this avoided crossing becomes a pair of avoided crossings associated with three of the Cl-H<sub>2</sub> dimer's  $\Omega = 3/2$  states: one of which correlates at large  $R$  with the  $j = 0$  H<sub>2</sub> rotational level and two of which correlate with the  $j = 2$  rotational level.

By removing  $j = 2$  H<sub>2</sub> rotational states from the direct product basis set used to express the Cl-H<sub>2</sub> Hamiltonian, we can assess how the assumption that the *para*-H<sub>2</sub> fragment retains its pure  $j = 0$  identity might affect the shape of the Cl-*para*-H<sub>2</sub> energy curves. We find that the positions and depths of the system's van der Waals minima are virtually unaffected when the H<sub>2</sub> fragment is restricted to its pure  $j = 0$  rotational state. This suggests that the H<sub>2</sub> molecules in Cl-doped solid *para*-H<sub>2</sub> can probably be treated as pure  $j = 0$  objects without a significant loss of accuracy.

## Acknowledgments

R. J. Hinde thanks D. T. Anderson and K. R. Brown for helpful discussions. This work was supported by the US National Science Foundation through Grant CHE-0848841.

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