Hindawi Publishing Corporation
ISRN Applied Mathematics
Volume 2013, Article ID 146026, 10 pages
http://dx.doi.org/10.1155/2013/146026

Research Article

Hybrid Multiattribute Group Decision Making Based on Intuitionistic Fuzzy Information and GRA Method

Jian Guo

School of Management Science and Engineering, Central University of Finance and Economics, Beijing 100081, China

Correspondence should be addressed to Jian Guo; txmguojian2002@126.com

Received 14 May 2013; Accepted 30 July 2013

Academic Editors: M.-H. Hsu and T. Y. Kam

Hybrid multiple attribute group decision making involves ranking and selecting competing courses of action available using attributes to evaluate the alternatives. The decision makers assessment information can be expressed in the form of real number, interval-valued number, linguistic variable, and the intuitionistic fuzzy number. All these evaluation information can be transformed to the form of intuitionistic fuzzy numbers. A combined GRA with intuitionistic fuzzy group decision-making approach is proposed. Firstly, the hybrid decision matrix is standardized and then transformed into an intuitionistic fuzzy decision matrix. Then, intuitionistic fuzzy averaging operator is utilized to aggregate opinions of decision makers. Intuitionistic fuzzy entropy is utilized to obtain the entropy weights of the criteria, respectively. After intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution are calculated, the grey relative relational degree of alternatives is obtained and alternatives are ranked. In the end, a numerical example illustrates the validity and applicability of the proposed method.

1. Introduction

Multiattribute decision making is an important issue in modern society, which is to select an appropriate option from a set of feasible alternatives with respect to the features of all predefined attributes. It often involves multiple decision makers, multiple selection criteria, and subjective and imprecise assessments. The attribute values given by the decision maker (or expert) over the alternatives under each attribute may not be all described by exact numbers, and sometimes they may take the following forms, such as exact numerical values, interval numbers, triangular fuzzy numbers, linguistic labels, and intuitionistic fuzzy numbers. Quite a number of research work have been done to solve the multiattribute decision making problems where the attributes take one of the former forms over the last decades [1–4].

In some real-life situations, a decision maker's (DM's) preferences for alternatives may not be expressed accurately due to the fact that DM may not possess a precise level of knowledge and the DM is unable to express the degree to which one alternative is better than others. In such cases, the DM may provide his/her preferences with a degree of doubt. Intuitionistic fuzzy set introduced by Atanassov [5–9] which is a generalization of the concept of Zadeh's fuzzy set [10] and is more suitable to deal with these cases than fuzzy sets. Intuitionistic fuzzy set is characterized by a membership function and a nonmembership function as well as a third function that is called the hesitation degree and thus can depict the fuzzy character of data more detailedly and comprehensively than fuzzy set which is only characterized by a membership function. This third function is useful to express the lack of knowledge and the hesitancy concerning both membership and nonmembership of an element to a set. Expression of hesitation is particularly helpful for decision makers. Intuitionistic fuzzy set has been proven to be highly useful to deal with uncertainty and vagueness, and it is a very suitable tool to be used to describe the imprecise or uncertain decision information. So far, a number of literatures have discussed the topic of IFVs theory which has been widely used in many fields such as multiattribute decision making [11–20], medical diagnosis [21–23], pattern recognition [24–29], and clustering analysis [30]. Many research achievement have been made to enrich the IFSs theory from different points of view, including interval-valued intuitionistic fuzzy sets (IVIFSs) [31, 32], intuitionistic fuzzy entropy measures [33–43], distance and similarity measures [44–47], approaches for ranking intuitionistic fuzzy values (IFVs) or interval-valued intuitionistic fuzzy values (IVIFVs) [48–53], and intuitionistic fuzzy aggregation operators [54–57]. In the previously mentioned literature, the characteristics of the alternatives with respect to a set of criteria are represented by IFSs. However, requiring a DM to express his/her opinions against alternatives as IFSs is not always appropriate due to the variety and complexity of decision-making problems. In most practical cases, it is more convenient and reasonable for a DM to express the attribute values of alternatives in different data types, such as exact values, intervals, IFVs, and linguistic terms, naturally producing a hybrid MADM problem.

Several different transformation techniques have been developed for converting hybrid decision matrix into a unified form that needs to be further handled to rank alternatives. Liang et al. [58] studied the hybrid multiple attribute decision-making problem in which the information about attribute weights was completely unknown. They turned a hybrid decision matrix with intervals and fuzzy values into a real decision matrix. Then a model was presented to seek objective weight based on entropy, and subjective weight and objective weight were integrated into general weight. Wang and Cui [4] investigated a hybrid multiple attribute decision-making problem with precision number, interval number, and fuzzy number. Herrera et al. [59] developed a fuzzy evaluation schema to deal with heterogeneous information and presented a transformation functions which can unify linguistic, numerical, and interval-valued information into the common format of 2-tuple fuzzy linguistic representation. Martínez et al. [60] developed an aggregation process for dealing with nonhomogeneous information composed of numerical, interval-valued, and linguistic information and took the 2-tuple fuzzy linguistic representation model as a base model. Si and Wei [61] introduced a hybrid multiattribute decision-making method which transformed a hybrid decision matrix with exact values, intervals, and linguistic terms into an intuitionistic fuzzy decision matrix and proposed an intuitionistic fuzzy optimization model to obtain the comprehensive evaluation value of each alternative, which is represented by intuitionistic fuzzy numbers. Liang et al. [62] proposed a new TOPSIS decision-making approach for hybrid multiattribute group decision-making problem with linguistic information and intuitionistic fuzzy values, and they defined a new conversion function to transfer multi-granularity linguistic information into intuitionistic fuzzy numbers. Guo and Li [63] developed an attitudinal-based method for constructing intuitionistic fuzzy information according to the attribute values expressed in different data types in hybrid MADM. Chen et al. [64] proposed an approach based on fuzzy preference relationship to solve the multiattribute decision-making problem whose attributes weights were known, and attributes values included real number, interval number, and fuzzy number.

It is of great importance to transform hybrid multi-attribute decision matrix into intuitionistic fuzzy decision matrix which is more flexible to handle vagueness or uncertainty and can avoid the loss and distortion of the original decision information. In this way, the final decision results are more reliable. In this paper we present a multiattribute decision-making method based on intuitionistic fuzzy sets. The existing related research does not focus on this aspect. The attribute values given by the decision makers can be expressed in the form of precision numbers, interval numbers, and even linguistic variables. We can transform the hybrid decision matrix into an intuitionistic fuzzy decision matrix. The importance of decision makers can be expressed in linguistic terms.

Grey relational analysis method was originally developed by Deng [65] and has been widely used in many multiple attribute decision making problems. It has been proven to be suitable for solving problems with complicated interrelationships between multiple factors and variables [66–68]. In this method, the performance of the alternatives are translated into comparable sequence first, and then the ideal target sequence is defined. The grey relational coefficient between each sequence and the ideal target sequence is calculated. The grey relational degree is calculated in the last, and the alternative which has the largest grey relational degree is the best one.

This paper is organized as follows. In Section 2, we give a brief review of basic concepts of intuitionistic fuzzy sets. In Section 3, intuitionistic fuzzy transformation techniques are presented to transform three other types of attribute value into unified IFVs. In Section 4, we outline a group decision-making mode based on hybrid decision-making problem with exact values, intervals, and linguistic variables. In Section 5, a numerical example is given to illustrate the effectiveness of the proposed approach. Finally, some concluding remarks are pointed out.

2. Preliminaries

2.1. Related Definitions of IFSs

Definition 1 (see [5]). Let X be a universe of discourse. An intuitionistic fuzzy set A in X is an object having the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}, \tag{1}$$

where the functions $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of membership and degree of nonmembership of the element $x \in X$ to the set $A \subseteq X$ with the condition that

$$\begin{gathered} 0 \le \mu_A(x) \le 1, \quad 0 \le \nu_A(x) \le 1, \\ 0 \le \mu_A(x) + \nu_A(x) \le 1. \end{gathered} \tag{2}$$

We call a third parameter $\pi_A(x)$ the intuitionistic index of the element x in the set A

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x). \tag{3}$$

It is the degree of indeterminacy membership of the element $x \in X$ to the set A. It is obvious that for every $x \in X$, $0 \le \pi_A(x) \le 1$.

Let $\alpha = (\mu_\alpha, \nu_\alpha)$, $\beta = (\mu_\beta, \nu_\beta)$ be two intuitionistic fuzzy numbers, $\lambda > 0$, then

$$\begin{aligned}
&(1)\ \alpha \oplus \beta = \left(\mu_\alpha + \mu_\beta - \mu_\alpha \cdot \mu_\beta, \nu_\alpha \nu_\beta\right),\\
&(2)\ \alpha \otimes \beta = \left(\mu_\alpha \mu_\beta, \nu_\alpha + \nu_\beta - \nu_\alpha \cdot \nu_\beta\right),\\
&(3)\ \lambda\alpha = \left(1 - (1-\mu_\alpha)^\lambda, \nu_\alpha^\lambda\right),\\
&(4)\ \alpha^\lambda = \left(\mu_\alpha^\lambda, 1 - (1-\nu_\alpha)^\lambda\right).
\end{aligned} \tag{4}$$

Definition 2 (see [54]). Let $\alpha_i = (\mu_i, \nu_i)$ $i = (1, 2, \dots, n)$ be a collection of intuitionistic fuzzy values, the intuitionistic fuzzy weighted averaging operator is defined as

$$\begin{aligned}
\mathrm{IFWA}_\omega (\alpha_1, \alpha_2, \dots, \alpha_n) &= \omega_1\alpha_1 \oplus \omega_2\alpha_2 \oplus \cdots \oplus \omega_n\alpha_n\\
&= \left(1 - \prod_{i=1}^{n} (1-\mu_i)^{\omega_i}, \prod_{i=1}^{n} \nu_i^{\omega_i}\right),
\end{aligned} \tag{5}$$

where ω_i is the weight of α_i, $\omega_i \in [0,1]$, and $\sum_{i=1}^{n} \omega_i = 1$.

Definition 3 (see [44]). A real function $d : \mathrm{IFS}_S(X) \to [0,1]$ is called a distance for IFS_S, if d satisfies the following properties:

(1) $0 \le d(A,B) \le 1$,

(2) $d(A,B) = 0$ iff $A = B$, for all $A, B \in \mathrm{IFS}_S(X)$,

(3) $d(A,B) = d(B,A)$, for all $A, B \in \mathrm{IFS}_S(X)$,

(4) if $A \subseteq B \subseteq C$, then $d(A,C) \ge \max(d(A,B), d(B,C))$, for all $A, B, C \in \mathrm{IFS}_S(X)$.

Definition 4 (see [44]). Suppose that A and B are two intuitionistic fuzzy sets in X, $X = (x_1, x_2, \dots, x_n)$. The Hamming distance between intuitionistic fuzzy sets A and B is $d_{\mathrm{IFS}}(A,B)$

$$\begin{aligned}
d_{\mathrm{IFS}}(A,B) = \frac{1}{2} \big(&\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right|\\
&+ \left|\pi_A(x_i) - \pi_B(x_i)\right|\big).
\end{aligned} \tag{6}$$

2.2. Entropy of Intuitionistic Fuzzy Sets. Entropy as a measure of the fuzziness of a fuzzy set was first mentioned by Zadeh [69]. Although this is called entropy due to an intrinsic similarity of equation to the one in Shannon entropy form, the two functions measure is different in types of uncertainty [34]. The Shannon entropy is a measure of uncertainty associated with the prediction of outcomes in a random experiment, but the entropy of fuzzy set describes the degree of fuzziness in fuzzy set. Entropy, as a very important notion for measuring fuzziness degree or uncertain information in fuzzy set theory, has been investigated widely by many researchers from different points of view. De Luca and Termini [70] first introduced a nonprobabilistic entropy for fuzzy sets and formulated the axiomatic requirements with which an entropy measure should comply. Burillo and Bustince [33] introduced the notions of entropy of IFSs to measure the degree of IFSs. Szimidt and Kacprzyk [34] proposed a nonprobabilistic-type entropy measure which was a result of a geometric interpretation of intuitionistic fuzzy sets. Hung and Yang [36] gave their axiomatic definitions of entropy of IFSs and IVFSs by exploiting the concept of probability. Many authors also proposed different entropy formulas for IFS [37–39].

In this section we will introduce a formula given by Wang et al. to calculate the entropy of an IFS.

Definition 5 (see [34]). A real-valued function $E: \mathrm{IFS}_S(X) \to [0,1]$ is called an entropy for $\mathrm{IFS}_S(X)$, if it satisfies the following axiomatic requirements:

(1) $E(A) = 0$ if and only if A is a crisp set,

(2) $E(A) = 1$ if and only if $\mu_A(x) = \nu_A(x)$ for each $\forall x \in X$,

(3) $E(A) = E(A^c)$ for each $\forall A \in \mathrm{IFS}_S(X)$,

(4) $E(A) \le E(B)$ if A is less fuzzy than B, that is, $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for $\mu_B(x) \le \nu_B(x)$ or $\mu_A(x) \ge \mu_B(x)$ and $\nu_A(x) \le \nu_B(x)$ for $\mu_B(x) \ge \nu_B(x)$.

Definition 6 (see [37]). Let $X = (x_1, x_2, \dots, x_n)$, $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ be an IFS; the entropy of IFS A given by Wang et al. is

$$E(A) = \frac{1}{n} \sum_{i=1}^{n} \frac{\min(\mu_A(x), \nu_A(x)) + \pi_A(x)}{\max(\mu_A(x), \nu_A(x)) + \pi_A(x)}. \tag{7}$$

Specially, for an intuitionistic fuzzy number $\alpha = \langle \mu_A(x), \nu_A(x)\rangle$, the intuitionistic fuzzy entropy is given as follows:

$$E(\alpha) = \frac{\min(\mu_A(x), \nu_A(x)) + \pi_A(x)}{\max(\mu_A(x), \nu_A(x)) + \pi_A(x)}. \tag{8}$$

3. Intuitionistic Fuzzy Transformation Techniques

Consider a hybrid MADM problem involving four different data types: exact values, intervals, IFVs, and linguistic terms. Let $A = \{A_1, A_2, \dots, A_n\}$ be a finite set of alternatives, and let $C = \{c_1, c_2, \dots, c_m\}$ be a set of attributes with a weight vector $w = (w_1, w_2, \dots, w_m)^T$, where $w_j \ge 0$ $(j = 1, 2, \dots, m)$ and $\sum_{j=1}^{m} w_j = 1$. Let $R^k = (a_{ij}^{(k)})_{n \times m}$ are a hybrid decision matrix, where $a_{ij}^{(k)}$ can be exact values, intervals, IFVs, or linguistic terms. In terms of the ideas previously mentioned, we need to transform three other types of attribute values in R^k into unified IFVs. In the following discussion, we will explore the transformation techniques for each of the aforementioned data types.

3.1. Conversion between Exact Value and IFNs. The values of different attributes have different dimensions. Thus, the real number in the hybrid decision needs to be standardized in order to eliminate interference in the final results.

Generally there are two kinds of attributes, the benefit type and the cost. The higher the benefit type value is, the better it is. While in the cost type, it is the opposite.

For the benefit type, the standardizing formulae are listed as follows:

$$b_{ij}^{(k)} = \frac{a_{ij}^{(k)}}{\sqrt{\sum_{i=1}^{m} \left(a_{ij}^{(k)}\right)^2}}. \tag{9}$$

For the cost type, the standardizing formulae are listed as follows:

$$b_{ij}^{(k)} = \frac{\left(1/a_{ij}^{(k)}\right)}{\sqrt{\sum_{i=1}^{m} \left(1/a_{ij}^{(k)}\right)^2}}. \tag{10}$$

Standardized precise number can be transformed into intuitionistic fuzzy numbers $\alpha_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)}, \pi_{ij}^{(k)})$

$$\mu_{ij}^{(k)} = b_{ij}^{(k)}, \qquad \nu_{ij}^{(k)} = 1 - b_{ij}^{(k)}, \qquad \pi_{ij}^{(k)} = 0. \tag{11}$$

3.2. Conversion between Intervals and IFNs. For the benefit type, the standardizing formulae are listed as follows:

$$b_{ij}^{L(k)} = \frac{a_{ij}^{L(k)}}{\sqrt{\sum_{i=1}^{m} \left(a_{ij}^{U(k)}\right)^2}}, \qquad b_{ij}^{U(k)} = \frac{a_{ij}^{U(k)}}{\sqrt{\sum_{i=1}^{m} \left(a_{ij}^{L(k)}\right)^2}}. \tag{12}$$

For the cost type, the standardizing formulae are listed as follows:

$$b_{ij}^{L(k)} = \frac{\left(1/a_{ij}^{U(k)}\right)}{\sqrt{\sum_{i=1}^{m} \left(\left(1/a_{ij}^{L(k)}\right)\right)^2}}, \qquad b_{ij}^{U(k)} = \frac{\left(1/a_{ij}^{L(k)}\right)}{\sqrt{\sum_{i=1}^{m} \left(\left(1/a_{ij}^{U(k)}\right)\right)^2}}. \tag{13}$$

Standardized interval number can be transformed into intuitionistic fuzzy numbers $\alpha_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)}, \pi_{ij}^{(k)})$

$$\mu_{ij}^{(k)} = b_{ij}^{L(k)}, \qquad \nu_{ij}^{(k)} = 1 - b_{ij}^{U(k)}, \qquad \pi_{ij}^{(k)} = b_{ij}^{U(k)} - b_{ij}^{L(k)}. \tag{14}$$

3.3. Conversion between Linguistic Variables and IFNs. A linguistic variable is a variable whose values are words or sentences in natural or artificial language [71, 72]. It is very useful in dealing with situations which are too complex or too ill-defined to be described properly in conventional quantitative expressions. For example, the subjective judgment of decision makers for the ratings of alternatives with respect to qualitative criteria and the importance of the decision makers can all be expressed as linguistic variables such as very good, good, fair, poor, and very poor. Such linguistic variables can be converted into IFNs. Converting linguistic data in MADM under uncertainty into IFVs is clearly significant because the flexibility in handling vagueness or uncertainty of the latter can avoid the loss and distortion of the original decision information and thus guarantee the mildness of fuzzy MADM and the reliability of the final decision results. The linguistic variables for the importance of the decision makers can be expressed in IFNs in Table 1. The ratings of alternatives with respect to qualitative criteria can be converted into IFNs in Table 2.

Table 1: Linguistic variables for the importance of decision makers.

Linguistic variables	IFNs
Very importance	(0.90, 0.10, 0.00)
Importance	(0.75, 0.20, 0.05)
Medium	(0.50, 0.45, 0.05)
Unimportance	(0.35, 0.60, 0.05)
Very unimportance	(0.10, 0.90, 0.00)

Table 2: Conversion between linguistic variables and IFNs.

Linguistic variables	IFNs
Extremely good (EG)/extremely high (EH)	(1.00, 0.00, 0.00)
Very very good (VVG)/very very high (VVH)	(0.90, 0.10, 0.00)
Very good (VG)/very high (VH)	(0.80, 0.10, 0.10)
Good (G)/high (H)	(0.70, 0.20, 0.10)
Medium good (MG)/medium high (MH)	(0.60, 0.30, 0.10)
Fair (F)/Medium (M)	(0.50, 0.40, 0.10)
Medium poor (MP)/medium low (ML)	(0.40, 0.50, 0.10)
Poor (P)/low (L)	(0.25, 0.60, 0.15)
Very poor (VP)/very low (VL)	(0.10, 0.75, 0.15)
Very very poor (VVP)/very very low (VVL)	(0.10, 0.90, 0.00)

4. Hybrid Multiattribute Decision-Making Model Based on Intuitionistic Fuzzy Entropy

In this section, we discuss how to utilize the intuitionistic fuzzy entropy to identify criterion weight vector in hybrid multiattribute decision making.

Hybrid multiattribute decision-making problems are defined on a set of alternatives, from which the decision maker has to select the best alternative according to some criteria. Suppose that there exists an alternative set $A = \{A_1, A_2, \ldots, A_n\}$ which consists of n alternatives, the decision maker will choose the best one from A according to an attribute set $C = \{c_1, c_2, \ldots, c_m\}$ which includes m criteria. For convenience, we denote the weight vector of attribute by $w = (w_1, w_2, \ldots, w_m)^T$, where $w_j \geq 0$ $(j = 1, 2, \ldots, m)$ and $\sum_{j=1}^{m} w_j = 1$.

Step 1. Construct the hybrid decision matrix of each decision maker. The hybrid decision matrix involves four different data types: exact values, intervals, IFVs, and linguistic terms, we can transform a hybrid decision matrix into intuitionistic fuzzy matrix according to the intuitionistic fuzzy transformation technique.

Assume that the rating of alternative A_i with respect to attribute c_j given by the kth experts e_k can be expressed in $\alpha_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)}, \pi_{ij}^{(k)})$. Hence, a hybrid multiattribute group decision-making problem can be concisely expressed in matrix format as follows:

$$R^{(k)} = \left(\alpha_{ij}^{(k)}\right)_{n\times m} = \begin{bmatrix} \alpha_{11}^{(k)} & \alpha_{12}^{(k)} & \cdots & \alpha_{1m}^{(k)} \\ \alpha_{21}^{(k)} & \alpha_{22}^{(k)} & \cdots & \alpha_{2m}^{(k)} \\ \vdots & \vdots & & \vdots \\ \alpha_{n1}^{(k)} & \alpha_{n2}^{(k)} & \cdots & \alpha_{nm}^{(k)} \end{bmatrix}, \quad (15)$$

where $\alpha_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)}, \pi_{ij}^{(k)})$.

Step 2. Calculate the weight with respect to the kth decision maker e_k. Determine the weights of decision makers. Let $D_k = (\mu_k, \nu_k, \pi_k)$ be an intuitionistic fuzzy number for rating of the kth decision maker. Then the weight of the kth decision maker can be obtained as [17]:

$$\lambda_k = \frac{\left(\mu_k + \pi_k\left(\mu_k/\left(\mu_k+\nu_k\right)\right)\right)}{\sum_{k=1}^{t}\left(\mu_k + \pi_k\left(\mu_k/\left(\mu_k+\nu_k\right)\right)\right)} \quad \text{where } \sum_{k=1}^{t}\lambda_k = 1. \quad (16)$$

Step 3. Compose the aggregated weighted intuitionistic fuzzy decision matrix. In this step, the aggregated weighted intuitionistic fuzzy decision matrix R is composed by considering the aggregated intuitionistic fuzzy decision matrix (i.e., (15) produced in Step 1) and the vector of the decision maker weights. The aggregated intuitionistic fuzzy decision matrix (AIFDM) was calculated by applying the intuitionistic fuzzy weighed averaging (IFWA) operator. By considering weights λ_k $(k = 1, 2, \ldots, t)$ of decision makers, elements γ_{ij} of the AIFDM can be calculated using IFWA as follows:

$$\begin{aligned} \gamma_{ij} &= \mathrm{IFWA}_{\lambda_t}\left(\alpha_{ij}^{(1)}, \alpha_{ij}^{(2)}, \ldots, \alpha_{ij}^{(t)}\right) \\ &= \lambda_1\alpha_{ij}^{(1)} \oplus \lambda_2\alpha_{ij}^{(2)} \oplus \cdots \oplus \lambda_t\alpha_{ij}^{(t)} \\ &= \left(1 - \prod_{k=1}^{t}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_t}, \prod_{k=1}^{t}\nu_{ij}^{(k)\lambda_t}\right), \end{aligned} \quad (17)$$

where

$$R = \left(\gamma_{ij}\right)_{n\times m} = \left(\left(\mu'_{ij}, \nu'_{ij}\right)\right)_{n\times m}. \quad (18)$$

Step 4. Determine the entropy weights of the selection criteria. In this step, all criteria may not be assumed to be of equal importance. W represents a set of grades of importance. Let w_j be the weights of the criteria. In order to obtain w_j, intuitionistic fuzzy entropy will be used firstly:

$$H_j = \frac{1}{n}\sum_{i=1}^{n}\frac{\min\left(\mu'_{ij}, \nu'_{ij}\right) + \pi'_{ij}}{\max\left(\mu'_{ij}, \nu'_{ij}\right) + \pi'_{ij}}. \quad (19)$$

The entropy weights of the jth criteria can be calculated as follows:

$$w_j = \frac{1 - H_j}{m - \sum_{j=1}^{m} H_j}. \quad (20)$$

Step 5. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS) based on intuitionistic fuzzy numbers. Both solutions are vectors of IFN elements, and they are derived from the AWIFDM matrix as follows. Then r^+ and r^- are equal to

$$\begin{aligned} r^+ &= \left(\left(\mu_1'^+, \nu_1'^+\right), \left(\mu_2'^+, \nu_2'^+\right), \ldots, \left(\mu_m'^+, \nu_m'^+\right)\right), \\ r^- &= \left(\left(\mu_1'^-, \nu_1'^-\right), \left(\mu_2'^-, \nu_2'^-\right), \ldots, \left(\mu_m'^-, \nu_m'^-\right)\right), \end{aligned} \quad (21)$$

where

$$\begin{aligned} \left(\mu_j'^+, \nu_j'^+\right) &= \left(\max_i \mu'_{ij}, \min_i \nu'_{ij}\right), \quad j = 1, 2, \ldots, m, \\ \left(\mu_j'^-, \nu_j'^-\right) &= \left(\min_i \mu'_{ij}, \max_i \nu'_{ij}\right), \quad j = 1, 2, \ldots, m. \end{aligned} \quad (22)$$

Step 6. Calculate the grey relational coefficient of each evaluation value from PIS and NIS using the following equations, respectively. The grey relational coefficient of each evaluation value from PIS and NIS are defined as

$$\begin{aligned} \xi_{ij}^+ = &\left(\min_{1\le i\le n}\min_{1\le j\le m} d\left(\gamma_{ij}, r_j^+\right) + \tau\max_{1\le i\le n}\max_{1\le j\le m} d\left(\gamma_{ij}, r_j^+\right)\right) \\ &\times\left(d\left(\gamma_{ij}, r_j^+\right) + \tau\max_{1\le i\le n}\max_{1\le j\le m} d\left(\gamma_{ij}, r_j^+\right)\right)^{-1}, \\ &i = 1, 2, \ldots, n, \quad j = 1, 2 \ldots, m, \\ \xi_{ij}^- = &\left(\min_{1\le i\le n}\min_{1\le j\le m} d\left(\gamma_{ij}, r_j^-\right) + \tau\max_{1\le i\le n}\max_{1\le j\le m} d\left(\gamma_{ij}, r_j^-\right)\right) \\ &\times\left(d\left(\gamma_{ij}, r_j^-\right) + \tau\max_{1\le i\le n}\max_{1\le j\le m} d\left(\gamma_{ij}, r_j^-\right)\right)^{-1}, \\ &i = 1, 2, \ldots, n, \quad j = 1, 2 \ldots, m, \end{aligned} \quad (23)$$

where $\tau \in [0, 1]$. Generally, $\tau = 0.5$ is applied.

Step 7. Calculate the degree of grey relational coefficient of each alternative from PIS and NIS as follows:

$$\xi_i^+ = \sum_{j=1}^{m} w_j \xi_{ij}^+, \quad i = 1, 2, \ldots, n,$$
$$\xi_i^- = \sum_{j=1}^{m} w_j \xi_{ij}^-, \quad i = 1, 2, \ldots, n. \tag{24}$$

Step 8. Calculate the relative relational degree of each alternative from the PIS by using the formula as follows:

$$\xi_i = \frac{\xi_i^+}{\xi_i^+ + \xi_i^-}, \quad i = 1, 2, \ldots, n. \tag{25}$$

Step 9. Rank alternatives. Rank alternatives in accordance with the values of ξ_i, $i = 1, 2, \ldots, n$ in descending order and select the alternative with the highest ξ_i.

5. Example

A company intends to develop a new software project, and software develop suppliers are required to provide four alternatives, that is, A_1, A_2, A_3, A_4, and six attributes are taken into account, including software quality (c_1), expected return (c_2), development cost (c_3), development time (c_4), hardware integrity (c_5), and user requirements satisfaction (c_6).

This is a hybrid MADM problem involving four different data types: exact values, intervals, linguistic terms, and intuitionistic fuzzy number. Attribute development cost and development time are a cost criterion, and the others are benefit ones. To solve this issue, we apply the developed method to the ranking and selection of the alternative software project below. When the attribute software quality and user requirements satisfaction are expressed in the types of linguistic terms, expected return and development cost are intervals types, development time is the exact values, and hardware integrity is expressed in the forms of intuitionistic fuzzy numbers. Three experts (e_1, e_2, e_3) are involved in the software development project selection process. In the selection process, each expert expresses his/her preferences depending on the nature of the alternatives and on his/her own knowledge over them. The hybrid decision matrix R^1, R^2, R^3 given by the expert e_1, e_2, e_3 is shown in Tables 3, 4, and 5.

Table 3: Hybrid decision matrix R^1 given by the expert e_1.

	c_1	c_2	c_3	c_4	c_5	c_6
A_1	VH	[60, 70]	[40, 50]	3	(0.45, 0.35)	VH
A_2	H	[70, 80]	[45, 53]	4.25	(0.65, 0.25)	M
A_3	MH	[75, 85]	[48, 58]	4	(0.55, 0.20)	VH
A_4	VH	[72, 83]	[42, 52]	3.25	(0.75, 0.15)	MH

Table 4: Hybrid decision matrix R^2 given by the expert e_2.

	c_1	c_2	c_3	c_4	c_5	c_6
A_1	H	[62, 75]	[41, 50]	3.5	(0.50, 0.30)	H
A_2	H	[70, 80]	[42, 53]	4	(0.60, 0.25)	MH
A_3	MH	[60, 70]	[45, 50]	3.75	(0.55, 0.15)	VH
A_4	VVH	[68, 75]	[35, 48]	2.75	(0.65, 0.20)	H

Table 5: Hybrid decision matrix R^3 given by the expert e_3.

	c_1	c_2	c_3	c_4	c_5	c_6
A_1	VH	[70, 80]	[45, 58]	3.25	(0.45, 0.15)	H
A_2	VH	[75, 85]	[43, 50]	4	(0.55, 0.15)	H
A_3	H	[62, 75]	[45, 55]	4.25	(0.35, 0.20)	M
A_4	H	[68, 78]	[38, 48]	3	(0.45, 0.25)	VH

Step 1. Transform the hybrid decision matrix of each decision maker into intuitionistic fuzzy decision matrix. The exact value and interval in the hybrid decision matrix given by the decision maker shown in Tables 3–5 are standardized and then transformed into an intuitionistic fuzzy number. The linguistic evaluation shown in Tables 3–5 is converted into IFNs by using Table 1. Then, the intuitionistic fuzzy decision matrices $R^{(k)}$ $(k = 1, 2, 3)$ of each decision maker shown in Tables 6, 7, and 8 are formed.

Step 2. Determine the weights of decision makers. The importance of the decision makers in the group decision making process is shown in Table 9. These linguistic variables used can be converted into IFNs by utilizing Table 2. In order to obtain the weights λ_k $(k = 1, 2, 3)$ of the decision makers, and formula (16) is used:

$$\begin{aligned}
\lambda_1 &= 0.90 \times \left(\left(0.75 + 0.05 \left(\frac{0.75}{0.75 + 0.20} \right) \right) + 0.90 + \left(0.50 + 0.10 \left(\frac{0.50}{0.50 + 0.40} \right) \right) \right)^{-1} \\
&= 0.4009, \\
\lambda_2 &= \left(0.75 + 0.05 \left(\frac{0.75}{0.75 + 0.20} \right) \right) \times \left(\left(0.75 + 0.05 \left(\frac{0.75}{0.75 + 0.20} \right) \right) + 0.90 + \left(0.50 + 0.10 \left(\frac{0.50}{0.50 + 0.40} \right) \right) \right)^{-1} \\
&= 0.3516, \\
\lambda_2 &= \left(0.50 + 0.10 \left(\frac{0.50}{0.05 + 0.40} \right) \right) \times \left(\left(0.75 + 0.05 \left(\frac{0.75}{0.75 + 0.20} \right) \right) + 0.90 + \left(0.50 + 0.10 \left(\frac{0.50}{0.50 + 0.40} \right) \right) \right)^{-1} \\
&= 0.2475.
\end{aligned} \tag{26}$$

Table 6: Intuitionistic decision matrix $R^{(1)}$.

	c_1	c_2	c_3	c_4	c_5	c_6
A_1	(0.80, 0.10, 0.10)	(0.37, 0.51, 0.12)	(0.45, 0.32, 0.23)	(0.59, 0.41, 0.00)	(0.45, 0.35, 0.20)	(0.80, 0.10, 0.10)
A_2	(0.70, 0.20, 0.10)	(0.43, 0.44, 0.13)	(0.43, 0.39, 0.18)	(0.41, 0.59, 0.00)	(0.65, 0.25, 0.10)	(0.50, 0.40, 0.10)
A_3	(0.60, 0.30, 0.10)	(0.49, 0.37, 0.14)	(0.33, 0.53, 0.14)	(0.44, 0.56, 0.00)	(0.55, 0.20, 0.25)	(0.80, 0.10, 0.10)
A_4	(0.80, 0.10, 0.10)	(0.44, 0.42, 0.14)	(0.43, 0.35, 0.22)	(0.54, 0.46, 0.00)	(0.75, 0.15, 0.10)	(0.60, 0.30, 0.10)

Table 7: Intuitionistic decision matrix $R^{(2)}$.

	c_1	c_2	c_3	c_4	c_5	c_6
A_1	(0.70, 0.20, 0.10)	(0.38, 0.47, 0.15)	(0.42, 0.37, 0.21)	(0.48, 0.52, 0.00)	(0.50, 0.30, 0.20)	(0.60, 0.30, 0.10)
A_2	(0.70, 0.20, 0.10)	(0.50, 0.36, 0.14)	(0.33, 0.50, 0.17)	(0.42, 0.58, 0.00)	(0.60, 0.25, 0.15)	(0.80, 0.10, 0.10)
A_3	(0.80, 0.10, 0.10)	(0.44, 0.43, 0.13)	(0.42, 0.42, 0.16)	(0.45, 0.55, 0.00)	(0.55, 0.15, 0.30)	(0.70, 0.20, 0.10)
A_4	(0.90, 0.10, 0.00)	(0.42, 0.47, 0.11)	(0.44, 0.26, 0.30)	(0.62, 0.38, 0.00)	(0.65, 0.20, 0.15)	(0.70, 0.20, 0.10)

Table 8: Intuitionistic decision matrix $R^{(3)}$.

	c_1	c_2	c_3	c_4	c_5	c_6
A_1	(0.80, 0.10, 0.10)	(0.42, 0.46, 0.12)	(0.38, 0.40, 0.22)	(0.54, 0.46, 0.00)	(0.45, 0.15, 0.40)	(0.70, 0.20, 0.10)
A_2	(0.80, 0.10, 0.10)	(0.45, 0.42, 0.13)	(0.44, 0.37, 0.19)	(0.44, 0.56, 0.00)	(0.55, 0.15, 0.30)	(0.70, 0.20, 0.10)
A_3	(0.70, 0.20, 0.10)	(0.49, 0.38, 0.13)	(0.34, 0.51, 0.15)	(0.41, 0.59, 0.00)	(0.35, 0.20, 0.45)	(0.50, 0.40, 0.10)
A_4	(0.70, 0.20, 0.10)	(0.41, 0.47, 0.12)	(0.46, 0.29, 0.25)	(0.59, 0.41, 0.00)	(0.45, 0.25, 0.30)	(0.80, 0.10, 0.10)

Table 9: The importance of decision makers.

	Linguistic variables
d_1	Very important
d_2	Important
d_3	Medium

Step 3. Construct the aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers. Utilizing formula (17), we obtain the intuitionistic fuzzy decision matrix R by aggregating all the intuitionistic fuzzy decision matrices $R^{(k)}$ $(k = 1, 2, 3)$. The intuitinistic fuzzy decision matrix R is shown in Table 10.

Step 4. Obtain the entropy weights of the criteria. Utilizing formula (19) to calculate the intuitionistic fuzzy entropy H_j $(j = 1, 2, \ldots, 6)$,

$$H_1 = 0.2863, \quad H_2 = 0.8972,$$
$$H_3 = 0.8662, \quad H_4 = 0.7714, \tag{27}$$
$$H_5 = 0.5628, \quad H_6 = 0.3693.$$

Then, using formula (20) to obtain the entropy weights,

$$w_1 = 0.3176, \quad w_2 = 0.0458,$$
$$w_3 = 0.0596, \quad w_4 = 0.1017, \tag{28}$$
$$w_5 = 0.1946, \quad w_6 = 0.2807.$$

Step 5. The intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution were obtained as follows:

$$\begin{aligned} r^+ &= ((0.83, 0.12, 0.05), (0.47, 0.14, 0.39), (0.44, 0.30, 0.26), \\ &\quad (0.58, 0.42, 0.00), (0.66, 0.19, 0.15), (0.72, 0.17, 0.11)), \\ r^- &= ((0.71, 0.18, 0.11), (0.39, 0.48, 0.13), (0.37, 0.48, 0.15), \\ &\quad (0.42, 0.58, 0.00), (0.47, 0.27, 0.46), (0.68, 0.21, 0.11)). \end{aligned} \tag{29}$$

Step 6. Calculate the grey relational coefficients of each alternative from the PIS and the NIS, respectively:

$$\begin{aligned} \xi^+ &= \left(\xi_{ij}^+\right)_{4\times 6} \\ &= \begin{bmatrix} 0.7680 & 0.6789 & 0.5919 & 0.6178 & 0.5000 & 1.0000 \\ 0.6594 & 0.9287 & 0.7528 & 1.0000 & 0.7971 & 0.8364 \\ 0.6154 & 1.0000 & 1.0000 & 0.9313 & 0.5572 & 0.9633 \\ 1.0000 & 0.7722 & 0.5072 & 0.5485 & 0.9616 & 0.8898 \end{bmatrix}, \\ \xi^- &= \left(\xi_{ij}^-\right)_{4\times 6} \\ &= \begin{bmatrix} 0.7559 & 1.0000 & 0.7798 & 0.8303 & 1.0000 & 0.8364 \\ 0.9022 & 0.7162 & 0.6085 & 0.5485 & 0.5729 & 1.0000 \\ 1.0000 & 0.6789 & 0.5072 & 0.5716 & 0.6831 & 0.8639 \\ 0.6154 & 0.8267 & 1.0000 & 1.0000 & 0.5000 & 0.9280 \end{bmatrix}. \end{aligned} \tag{30}$$

Step 7. According to the above step, the attribute weight vector is known as $w = (0.3176, 0.0458, 0.0596, 0.1017, 0.1946,$

Table 10: Intuitionistic decision matrix R.

	c_1	c_2	c_3	c_4	c_5	c_6
A_1	(0.77, 0.13, 0.10)	(0.39, 0.48, 0.13)	(0.42, 0.35, 0.23)	(0.54, 0.46, 0.00)	(0.47, 0.27, 0.26)	(0.72, 0.17, 0.11)
A_2	(0.73, 0.17, 0.10)	(0.46, 0.40, 0.14)	(0.40, 0.42, 0.18)	(0.42, 0.58, 0.00)	(0.61, 0.22, 0.17)	(0.68, 0.21, 0.11)
A_3	(0.71, 0.18, 0.11)	(0.47, 0.14, 0.39)	(0.37, 0.48, 0.15)	(0.44, 0.56, 0.00)	(0.51, 0.18, 0.31)	(0.71, 0.18, 0.11)
A_4	(0.83, 0.12, 0.05)	(0.43, 0.45, 0.12)	(0.44, 0.30, 0.26)	(0.58, 0.42, 0.00)	(0.66, 0.19, 0.15)	(0.69, 0.20, 0.11)

and 0.2807), then the degree of grey relational coefficient of each alternative from PIS and NIS can be computed by

$$\xi_1^+ = 0.7511, \quad \xi_2^+ = 0.7884, \quad \xi_3^+ = 0.7744, \quad \xi_4^+ = 0.8759,$$
$$\xi_1^- = 0.8462, \quad \xi_2^- = 0.8036, \quad \xi_1^- = 0.8125, \quad \xi_1^- = 0.7524. \tag{31}$$

Step 8. Calculate the relative relational degree of each alternative as

$$\xi_1 = 0.4702, \quad \xi_2 = 0.4952, \quad \xi_3 = 0.4880, \quad \xi_4 = 0.5379. \tag{32}$$

Step 9. Rank the alternatives. The relative relational degree of alternatives is determined, and then four alternatives are ranked as $A_4 > A_2 > A_3 > A_1$. The alternatives A_4 is selected as appropriate alternatives.

6. Conclusions

Multiattribute decision making is a complex process, as it often involves multiple decision makers making subjective and imprecise assessments in relation to multiple alternatives and multiple evaluation criteria. This paper presents a hybrid decision-making problem, in which the attribute values given by the decision maker may be described in different forms such as exact values, intervals, linguistic variables, and intuitionistic fuzzy number. The importance of the decision makers can all be expressed as linguistic variables. All these other types of attribute values can be transformed into intuitionistic fuzzy numbers. Also intuitionistic fuzzy averaging operator is utilized to aggregate opinions of decision makers. Intuitionistic fuzzy entropy is utilized to obtain the entropy weights of the criteria, respectively. GRA method is applied to the ranking and selection of alternatives. After intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution are calculated based on the hamming distance, the grey relative relational degree of alternatives is obtained and alternatives are ranked.

References

[1] Z. S. XU, *Uncertain Attribute Decision Making: Methods and Applications*, Tsinghua University Press, Beijing, China, 2004, Chinese.

[2] Y. Q. Xia and Q. Z. Wu, "A technique of order preference by similarity to ideal solution for hybrid multiple attribute decision making problems," *Journal of Systems Engineering*, vol. 19, no. 6, pp. 630–634, 2004 (Chinese).

[3] C.-M. Ding, F. Li, and H. Qi, "Technique of hybrid multiple attribute decision making based on similarity degree to ideal solution," *Systems Engineering and Electronics*, vol. 29, no. 5, pp. 737–740, 2007 (Chinese).

[4] W. Wang and M. Cui, "Hybrid multiple attribute decision making model based on entropy," *Journal of Systems Engineering and Electronics*, vol. 18, no. 1, pp. 72–75, 2007 (Chinese).

[5] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[6] K. T. Atanassov, *Intuitionistic Fuzzy Sets Theory and Applications*, vol. 35, Physica, Heidelberg, Germany, 1999.

[7] K. T. Atanassov, "More on intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 33, no. 1, pp. 37–45, 1989.

[8] K. T. Atanassov, "New operations defined over the intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 61, no. 2, pp. 137–142, 1994.

[9] K. T. Atanassov, "Two theorems for intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 110, no. 2, pp. 267–269, 2000.

[10] L. A. Zadeh, "Fuzzy sets," *Information and Computation*, vol. 8, pp. 338–353, 1965.

[11] K. Atanassov, G. Pasi, and R. Yager, "Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making," *International Journal of Systems Science*, vol. 36, no. 14, pp. 859–868, 2005.

[12] D.-F. Li, "Multiattribute decision making models and methods using intuitionistic fuzzy sets," *Journal of Computer and System Sciences*, vol. 70, no. 1, pp. 73–85, 2005.

[13] H.-W. Liu and G.-J. Wang, "Multi-criteria decision-making methods based on intuitionistic fuzzy sets," *European Journal of Operational Research*, vol. 179, no. 1, pp. 220–233, 2007.

[14] L. Lin, X.-H. Yuan, and Z.-Q. Xia, "Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets," *Journal of Computer and System Sciences*, vol. 73, no. 1, pp. 84–88, 2007.

[15] Z. Xu, "Intuitionistic preference relations and their application in group decision making," *Information Sciences*, vol. 177, no. 11, pp. 2363–2379, 2007.

[16] Z. Xu and R. R. Yager, "Dynamic intuitionistic fuzzy multi-attribute decison making," *International Journal of Approximate Reasoning*, vol. 48, no. 1, pp. 246–262, 2008.

[17] F. E. Boran, S. Genc, M. Kurt, and D. Akay, "A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method," *Expert Systems with Applications*, vol. 36, no. 8, pp. 11363–11368, 2009.

[18] J. Ye, "Fuzzy decision making method based on the weighted correlation coefficient under intuitionistic fuzzy environment," *European Journal of Operational Research*, vol. 205, no. 1, pp. 202–204, 2010.

[19] S. F. Zhang and S. Y. Liu, "A GRA-based intuitionistic fuzzy multi-criteria group decision making method for personnel selection," *Expert Systems with Applications*, vol. 38, no. 8, pp. 11401–11405, 2011.

[20] J. H. Park, I. Y. Park, Y. C. Kwun, and X. Tan, "Extension of the TOPSIS method for decision making problems under interval-valued intuitionistic fuzzy environment," *Applied Mathematical Modelling*, vol. 35, no. 5, pp. 2544–2556, 2011.

[21] S. K. De, R. Biswas, and A. R. Roy, "An application of intuitionistic fuzzy sets in medical diagnosis," *Fuzzy Sets and Systems*, vol. 117, no. 2, pp. 209–213, 2001.

[22] E. Szmidt and J. Kacprzyk, "Intuitionistic fuzzy sets in some medical applications," in *Computational Intelligence. Theory and Applications*, vol. 2206 of *Lecture Notes in Computer Science*, pp. 148–151, 2001.

[23] E. Szmidt and J. Kacprzyk, "A similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning," in *Proceedings of the 7th International Conference on Artificial Intelligence and Soft Computing (ICAISC '04)*, vol. 3070 of *Lecture Notes in Computer Science*, pp. 388–393, June 2004.

[24] L. Dengfeng and C. Chuntian, "New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions," *Pattern Recognition Letters*, vol. 23, no. 1–3, pp. 221–225, 2002.

[25] Z. Liang and P. Shi, "Similarity measures on intuitionistic fuzzy sets," *Pattern Recognition Letters*, vol. 24, no. 15, pp. 2687–2693, 2003.

[26] W.-L. Hung and M.-S. Yang, "Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance," *Pattern Recognition Letters*, vol. 25, no. 14, pp. 1603–1611, 2004.

[27] W. Wang and X. Xin, "Distance measure between intuitionistic fuzzy sets," *Pattern Recognition Letters*, vol. 26, no. 13, pp. 2063–2069, 2005.

[28] C. Zhang and H. Fu, "Similarity measures on three kinds of fuzzy sets," *Pattern Recognition Letters*, vol. 27, no. 12, pp. 1307–1317, 2006.

[29] I. K. Vlachos and G. D. Sergiadis, "Intuitionistic fuzzy information—applications to pattern recognition," *Pattern Recognition Letters*, vol. 28, no. 2, pp. 197–206, 2007.

[30] Z. Xu, J. Chen, and J. Wu, "Clustering algorithm for intuitionistic fuzzy sets," *Information Sciences*, vol. 178, no. 19, pp. 3775–3790, 2008.

[31] K. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.

[32] K. T. Atanassov, "Operators over interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 64, no. 2, pp. 159–174, 1994.

[33] P. Burillo and H. Bustince, "Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets," *Fuzzy Sets and Systems*, vol. 78, no. 3, pp. 305–316, 1996.

[34] E. Szmidt and J. Kacprzyk, "Entropy for intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 118, no. 3, pp. 467–477, 2001.

[35] X. D. Liu, S. H. Zheng, and F. L. Xiong, "Entropy and subsethood for general interval-valued intuitionistic fuzzy sets," in *Fuzzy Systems and Knowledge Discovery*, vol. 3613 of *Lecture Notes on Artificial Intelligence*, pp. 42–52, 2005.

[36] W.-L. Hung and M.-S. Yang, "Fuzzy entropy on intuitionistic fuzzy sets," *International Journal of Intelligent Systems*, vol. 21, no. 4, pp. 443–451, 2006.

[37] Y. Wang and Y.-J. Lei, "A technique for constructing intuitionistic fuzzy entropy," *Control and Decision*, vol. 22, no. 12, pp. 1390–1394, 2007 (Chinese).

[38] W. Zeng, F. Yu, X. Yu, and B. Cui, "Entropy of intuitionistic fuzzy set based on similarity measure," in *Proceedings of the 3rd International Conference on Innovative Computing Information and Control (ICICIC '08)*, vol. 254, p. 398, IEEE Computer Society, Washington, DC, USA, 2008.

[39] H. Zhang, W. Zhang, and C. Mei, "Entropy of interval-valued fuzzy sets based on distance and its relationship with similarity measure," *Knowledge-Based Systems*, vol. 22, no. 6, pp. 449–454, 2009.

[40] Q.-S. Zhang and S.-Y. Jiang, "A note on information entropy measures for vague sets and its applications," *Information Sciences*, vol. 178, no. 21, pp. 4184–4191, 2008.

[41] Q.-S. Zhang, S. Jiang, B. Jia, and S. Luo, "Some information measures for interval-valued intuitionistic fuzzy sets," *Information Sciences*, vol. 180, no. 24, pp. 5130–5145, 2010.

[42] T.-Y. Chen and C.-H. Li, "Determining objective weights with intuitionistic fuzzy entropy measures: a comparative analysis," *Information Sciences*, vol. 180, no. 21, pp. 4207–4222, 2010.

[43] C.-P. Wei, P. Wang, and Y.-Z. Zhang, "Entropy, similarity measure of interval-valued intuitionistic fuzzy sets and their applications," *Information Sciences*, vol. 181, no. 19, pp. 4273–4286, 2011.

[44] E. Szmidt and J. Kacprzyk, "Distances between intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 114, no. 3, pp. 505–518, 2000.

[45] E. Szmidt and J. Kacprzyk, "A new concept of a similarity measure for intuitionistic fuzzy sets and its use in group decision making," *Lecture Notes in Computer Science*, vol. 3558, pp. 272–282, 2005.

[46] Z. Xu, "Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making," *Fuzzy Optimization and Decision Making*, vol. 6, no. 2, pp. 109–121, 2007.

[47] W. Zeng and H. Li, "Relationship between similarity measure and entropy of interval valued fuzzy sets," *Fuzzy Sets and Systems*, vol. 157, no. 11, pp. 1477–1484, 2006.

[48] S. M. Chen and J. M. Tan, "Handling multicriteria fuzzy decision-making problems based on vague set theory," *Fuzzy Sets and Systems*, vol. 67, no. 2, pp. 163–172, 1994.

[49] D. H. Hong and C.-H. Choi, "Multicriteria fuzzy decision-making problems based on vague set theory," *Fuzzy Sets and Systems*, vol. 114, no. 1, pp. 103–113, 2000.

[50] E. Szmidt and J. Kacprzyk, "A new approach to ranking alternatives expressed via intuitionistic fuzzy sets," in *Computational Intelligence in Decision and Control*, D. Ruan et al., Ed., pp. 265–270, World Scientific, Singapore, 2008.

[51] E. Szmidt and J. Kacprzyk, "Amount of information and its reliability in the ranking of Atanassov's intuitionistic fuzzy alternatives," in *Recent Advances in Decision Making*, E. Rakus-Andersson, R. R. Yager, and N. Ichalkaranje, Eds., vol. 222, pp. 7–19, Springer, Berlin, Germany, 2009.

[52] Z. Wang, K. W. Li, and W. Wang, "An approach to multiattribute decision making with interval-valued intuitionistic fuzzy assessments and incomplete weights," *Information Sciences*, vol. 179, no. 17, pp. 3026–3040, 2009.

[53] Z. Xu, "A method based on distance measure for interval-valued intuitionistic fuzzy group decision making," *Information Sciences*, vol. 180, no. 1, pp. 181–190, 2010.

[54] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 6, pp. 1179–1187, 2007.

[55] Z.-S. Xu, "Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making," *Control and Decision*, vol. 22, no. 2, pp. 215–219, 2007 (Chinese).

[56] Z. Xu and J. Chen, "On geometric aggregation over interval-valued intuitionistic fuzzy information," in *Proceedings of the 4th International Conference on Fuzzy Systems and Knowledge Discovery (FSKD '07)*, pp. 466–471, Haikou, China, August 2007.

[57] Z. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *International Journal of General Systems*, vol. 35, no. 4, pp. 417–433, 2006.

[58] C. Y. Liang, J. Wu, W. X. Lu, and Y. Ding, "A new method on hybrid multiple attribute decision making problem for choosing the supplier," *Chinese Journal of Management Science*, vol. 14, no. 6, pp. 71–76, 2006 (Chinese).

[59] F. Herrera, L. Martínez, and P. J. Sánchez, "Managing non-homogeneous information in group decision making," *European Journal of Operational Research*, vol. 166, no. 1, pp. 115–132, 2005.

[60] L. Martínez, J. Liu, D. Ruan, and J.-B. Yang, "Dealing with heterogeneous information in engineering evaluation processes," *Information Sciences*, vol. 177, no. 7, pp. 1533–1542, 2007.

[61] Y.-J. Si and F.-J. Wei, "Hybrid multi-attribute decision making based on the intuitionistic fuzzy optimum selecting model," *Systems Engineering and Electronics*, vol. 31, no. 12, pp. 2893–2897, 2009.

[62] C. Y. Liang, E. Q. Zhang, X. W. Qing, and Q. Lu, "A method of multi-attribute group decision making with incomplete hybrid assessment information," *Chinese Journal of Management Science*, vol. 17, no. 4, pp. 126–132, 2009 (Chinese).

[63] K. Guo and W. Li, "An attitudinal-based method for constructing intuitionistic fuzzy information in hybrid MADM under uncertainty," *Information Sciences*, vol. 208, no. 15, pp. 28–38, 2012.

[64] X.-W. Chen, W.-S. Wang, G.-B. Song, and D.-Y. Song, "Hybrid multiattribute decision making based on fuzzy preference relation," *Systems Engineering and Electronics*, vol. 34, no. 3, pp. 529–533, 2012.

[65] J. L. Deng, "Introduction to grey system theory," *The Journal of Grey System*, vol. 1, no. 1, pp. 1–24, 1989.

[66] G.-W. Wei, "GRA method for multiple attribute decision making with incomplete weight information in intuitionistic fuzzy setting," *Knowledge-Based Systems*, vol. 23, no. 3, pp. 243–247, 2010.

[67] G.-W. Wei, "Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making," *Expert Systems with Applications*, vol. 38, no. 9, pp. 11671–11677, 2011.

[68] M.-S. Kuo and G.-S. Liang, "Combining VIKOR with GRA techniques to evaluate service quality of airports under fuzzy environment," *Expert Systems with Applications*, vol. 38, no. 3, pp. 1304–1312, 2011.

[69] L. A. Zadeh, "Probability measures of fuzzy events," *Journal of Mathematical Analysis and Applications*, vol. 23, pp. 421–427, 1968.

[70] A. de Luca and S. Termini, "A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory," *Information and Computation*, vol. 20, pp. 301–312, 1972.

[71] M. Delgado, J. L. Verdegay, and M. A. Vila, "Linguistic decision-making models," *International Journal of Intelligent Systems*, vol. 7, no. 5, pp. 479–492, 1992.

[72] V.-N. Huynh and Y. Nakamori, "A satisfactory-oriented approach to multiexpert decision-making with linguistic assessments," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 35, no. 2, pp. 184–196, 2005.