Hindawi Publishing Corporation
International Journal of Population Research
Volume 2013, Article ID 207313, 7 pages
http://dx.doi.org/10.1155/2013/207313

Research Article

Longer Life and Higher Fertility in an OLG Economy with Production

Luciano Fanti

Department of Economics and Management, University of Pisa, Via Cosimo Ridolfi 10, 56124 Pisa, Italy

Correspondence should be addressed to Luciano Fanti; lfanti@ec.unipi.it

Received 4 July 2013; Revised 17 September 2013; Accepted 18 September 2013

Academic Editor: Alberto Davila

This paper investigates the effect of a change in life expectancy (i.e., longevity) on fertility in a standard OLG economy. The main result is that, in contrast with other papers, an increase in the longevity rate may increase the fertility rate as well. It is shown that such a result holds when the cost of rearing children in terms of goods and services (rather than in terms of forgone wages) matters. In particular, such a result depends on the relative "strength" of the capital in the technology as compared with the "strength" of the parsimony. Moreover it is shown, again in contrast with other papers, that with an unfunded social security system it is more likely that a longer life may increase the fertility. The latter result is even more likely in the presence of child subsidy policies, which are widespread in developed countries. In conclusion, we argue that in countries having a population with a high longevity, a high capital share, a large unfunded social security, and child subsidy policies (such as Italy), a further increase of longevity may increase fertility in the long run and thus partially alleviate the peril of the so-called "demographic bomb."

1. Introduction

In the recent years longevity and fertility have been primary policy concerns, especially in the developed countries in which the population aging affects public budget (e.g., pensions and health cares) and labour market. As a consequence, there is a vast recent literature considering the role of longevity in dynamic models. Among the recent papers, De La Croix and Licandro [1] consider the role of rising longevity on schooling decisions in the early stages of life. Zhang et al. [2] consider a model with public education and imperfect annuity markets, where a decline in mortality affects growth. Cipriani and Makris [3] consider the role of expectations of longevity on economic outcomes in an overlapping generations model where longevity of one generation depends on the average human capital level of the same generation. However while De la Croix and Licandro [1] and Zhang et al. [2] assume exogenous longevity and Cipriani and Makris [3] focus on an endogenous mechanism determining longevity, they abstract from the fertility issue and thus from the possible effects of longevity on fertility. The latter issue, although it seems to be crucial for determining the long run population growth, has been investigated by few papers.

Among these, the seminal paper by Ehrlich and Lui [4] found that rising longevity promotes growth by rising human capital investment in children and by reducing fertility. Also Yakita [5] shows that an increase of population aging always lowers fertility, even in the presence of a social security system (which has only the effect of weakening the negative effect of increased longevity on fertility, through a positive effect of pensions, i.e., contribution rates, on fertility). The economic intuition behind the negative relationship between longevity and fertility is that when the probability to enter the old age increases, savings also increase for future consumption during the old age, and since the number of children represents a "consumption" in the young age, such a number is reduced. To the extent that such a negative relationship becomes a common wisdom, it also becomes growing the concern for a "population implosion" in developed countries, characterised by a rapid increase in the life expectancy.

However I argue that the conventional wisdom that longevity reduces fertility does not necessarily holds true: in this paper I show that longevity (while always increases savings) may increase fertility as well, provided that technology and preferences parameters satisfy rather realistic conditions.

This paper is rather related to Yakita [5], who uses the standard OLG Diamond [6] model,[1] extended only with endogenous fertility,[2] while Ehrlich and Lui [4] departs substantially from such a standard model, for instance, introducing human capital choices, so that the relationship between longevity and fertility crucially depends on human capital and thus it is not directly comparable with the one occurring in the Yakita's model. Therefore the aim of this paper is to investigate the relationship between longevity and fertility in the basic OLG frame and compare the results with those obtained in a similar frame by Yakita [5].[3]

The crucial feature of this paper is the assumption that the cost of children is represented by a fixed amount of goods and services, different from Yakita [5], who assumes a time cost of children (i.e., the cost of rearing children is given by the forgone wage for the time dedicated to rear children). My assumption is often used in the literature (e.g., [7] and more recently [8]) alternatively to that of time cost of children, and then it may be worth to investigate from a theoretical point of view which are its implications on the link between longevity and fertility. Moreover, this assumption may also be justified noting that recent policies (particularly in the Northern-Europe countries) tend, through, for instance, publicly provided child care services and parental leaves, to allow for parents to continue to work. Thus, while the cost in terms of foregone wages tends to be reduced, the cost in terms of goods and services remains truly essential[4] (while such a cost is not considered in the Yakita's model).

Moreover, again in contrast with Yakita's results, it is found that when a PAYG social security system is implemented, fertility rates are reduced (in particular, the higher the size of such a system, the lower the fertility rate) and thus it is even more likely that the increase of longevity brings upon also an increase of fertility.[5] Again it is also shown that the presence of child subsidy policies, which are rather scattered in developed countries for favouring the fertility recovery, makes very likely the "virtuous" circle "longer life-higher fertility." In essence, the assumption of the children cost in terms of goods and services (instead of in terms of time) is responsible for the different results derived in the paper. To sum up, the paper has shown that, to the extent that the cost in terms of goods and services matters (while the cost given by forgone wages tends to be less important), the "vicious circle" longer life-lower fertility (leading to a "population implosion" in the long run, as evidenced by the preceding literature) may be broken, under rather plausible economic conditions (in terms, for instance, of distributive shares and parsimony rates). Indeed, the further longevity increase expected in the future in developed countries—which are plagued by a below-replacement fertility—may endogenously cause a fertility recovery, in contrast with a common wisdom and thus partially alleviate the peril of the so-called "demographic bomb."

The remainder of this paper is organized as follows. Section 2 introduces the basic model and Section 3 shows the steady-state results. Sections 4 and 5 extend the model with an unfunded social security and a child subsidy policy, respectively, derive the results for such cases, and compare them with those of the basic model. Concluding comments are in Section 6.

2. Model

The model is the standard OLG version of Diamond [6], incorporating an uncertain lifetime (e.g., Zhang and Zhang [9], Yakita [5]).[6]

The demographic structure is the following: in each period t, N_t persons are born, and they live for two periods (thus this structure is called "overlapping generations" because at each point in time, two generations (i.e., t and $t-1$) are alive and overlap). Alternatively, we might consider that each person who lives three periods is a child in the first period, is working in the second period, and retires in the third one (i.e., three generations $t+1$, t, and $t-1$ are alive and overlap). However, since it is usually assumed that during the first period the individual does not take any decisions, and his consumption is a fraction of that of his parents, then, for simplicity, only two generations are considered.

A representative individual lives two periods: a working period fixed with certainty and a retirement period whose length is uncertain. It is assumed, for the sake of simplicity, that (1) the individual is either alive or dead at the beginning of the retirement period; (2) the probability that the individual is alive is given by $(1-p)$.

Adult individuals belonging to generation t have a homothetic and separable utility function defined over consumption when young (c_t) and old (c_{t+1}) and from having children (n_t), as in Galor and Weil [10]. Note that the number of children is $n_t \geq 0$, that is, $N_t = n_t N_{t-1}$: each generation grows at the (endogenous) rate $n_t - 1$. Consequently, the total population $N_t + N_{t-1}$ grows also at the rate $n_t - 1$. Moreover, note that, for simplicity, the mortality rate has not been included in the analysis and that the model may also represent economies where population shrinks, that is, $n < 1$.

Furthermore it is assumed that the utility function is the standard logarithmic (function of the consumption of both periods, the number of children, and the longevity), respectively:

$$U_t = (1-\phi)\log c_t + \gamma(1-p)\log c_{t+1} + \phi \log n_t, \quad (1)$$

where γ is the subjective discount factor and $(1-\phi)$ and ϕ represent the relative preference with respect to the first-period consumption and the number of children, respectively. The cost of child rearing for each child is m, measured in terms of output (see for instance [7]). The labour supply (net of leisure) is constant and normalised to unity. The budget constraint of the young individual is

$$c_t = w_t - m n_t - s_t, \quad (2)$$

where w_t is the wage rate per working hour and s_t denotes savings. Following a simplified two-period version of Blanchard [11] model as in Yakita [5], I assume that (1) the insurance companies exchange a payment to the individuals of $((1 + r_{t+1})/(1 - p_a))s_t$ for the estate s_t accruing to the companies, where p_a is the average death probability;[7] (2) the death probability p is the same for all individuals ($p = p_a$). Therefore the budget constraint of the individuals when old is

$$c_{t+1} = \frac{1 + r_{t+1}}{1 - p} s_t. \tag{3}$$

The standard maximisation procedure by the individuals leads to the following choices of savings and number of children, respectively:

$$s_t = \frac{(1 - p)\gamma w_t}{1 + \phi + (1 - p)\gamma}, \qquad n_t = \frac{\phi w_t}{m[1 + \phi + (1 - p)\gamma]}. \tag{4}$$

As regards firms, I assume a representative firm producing at period t, which acts competitively. The constant returns to scale production function[8] is the standard Cobb-Douglas one: $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[9] where Y, K, and L are output, capital and the labour input, respectively (and $L = N$, since only the young people are working), $A > 0$ is a scale parameter, and $\alpha \in (0, 1)$ is the capital's weight in technology. The intensive form technology of production may be written as $y_t = AK_t^{\alpha}$, with $k_t := K_t/N_t$ and $y_t := Y_t/N_t$ being capital and output per-worker, respectively. Assuming total depreciation of capital at the end of each period, profits maximisation leads to the following marginal conditions:[10]

$$r_t = \alpha A K_t^{\alpha-1} - 1, \tag{5}$$

$$w_t = (1 - \alpha) A K_t^{\alpha}. \tag{6}$$

To have a better understanding of how the firm interacts with the agents from different generations,[11] we note that at time $t = 0$ the capital stock K_0 is already installed in the firm producing at $t = 0$. There is a crucial assumption on which the dynamics of the OLG growth model grounds, the so-called "one-period time-to-build": for all $t \geq 1$, capital K_t is productive at time t and is built from the savings of time $t - 1$. It is assumed that the representative firm that produces at time t exists during two periods, $t - 1$ and t,[12] in the first of which it "receives" the deposits of goods I_{t-1} from the young individuals (i.e., their savings). This deposit of goods produced at time $t - 1$ is transformed in the productive capital used in the production process at time $t : K_t = I_{t-1} = N_{t-1}s_{t-1}$. Since the individuals remain the owners of the stock of capital, then they will receive the profits of the firm when old.

Equilibrium in the goods market requires that demand is equal to supply, which is the same as requiring that investment is equal to saving. This means that at equilibrium, the link between two periods t and $t + 1$ is given by the accumulation rule for capital: savings of the young households are transformed into productive capital for the next period, that is, $K_{t+1} = I_t = N_t s_t$.

Therefore the market-clearing condition in goods as well as in capital markets is expressed, in intensive terms, as $n_t k_{t+1} = s_t$, which is the difference equation representing the dynamics of the OLG growth model with endogenous fertility. Substituting out for n, s, and w from (4) and (6), the steady-state solution (i.e., $k_{t+1} = k_t = k^*$) is given by the following long-run capital per-worker:[13]

$$k^* = \frac{(1 - p)\gamma m}{\phi}. \tag{7}$$

3. Comparative Static Analysis

Which are the effects of the longevity on the long-run rate of fertility? This simple question gives rise to interesting findings in our basic OLG model. Making use of (6) and (7) the long-run rate of fertility is determined by

$$n^*(p) = \frac{\phi(1 - \alpha) A[(1 - p)\gamma m/\phi]^{\alpha}}{m[1 + \phi + (1 - p)\gamma]}, \tag{8}$$

and the following proposition holds.

Proposition 1. *The increase of longevity reduces fertility if and only if (1):* $(1 - p) > \alpha(1 + \phi)/(1 - \alpha)\gamma$; *or, alternatively, (2):* $\alpha < \alpha^* = (1 - p)/[1 + \phi + (1 - p)\gamma]$; *or (3):* $\gamma > \alpha(1 + \phi)/[(1 - \alpha)(1 - p)]$.

Proof. The proof straightforwardly derives from

$$\frac{\partial n^*(p)}{\partial(1 - p)} \gtreqless 0 \Longleftrightarrow [\alpha(1 + \phi) - \gamma(1 - p)(1 - \alpha)] \gtreqless 0. \tag{9}$$

□

The simple observation of the condition (9) leads to the following remark: the higher (1) the weight of capital in the technology (i.e., the capital distributive share), (2) the existing longevity, (3) the preference for having children, and the lower (4) the parsimony (i.e., the rate of savings) are, the more likely a higher longevity causes a higher (instead of a lower, as predicted by the preceding literature (e.g., [5])) long run fertility.

In particular, by observing the inequality (2) of Proposition 1, in which at the left side member there is the weight of capital in technology and at the right side member there are the rate of savings and the preference for having children, we may conclude that whether the longevity either decreases or increases fertility ultimately depends only on preferences and technology. In particular, the final result depends on the comparison between the "strength" of the capital in the technology and the "strength" of the parsimony and the taste for children.

A simple quantitative exercise may illustrate that a longer life increases fertility in very plausible economic conditions. Let us assume, only for illustrative purposes, the following parametric set: $A = 5$ (simply a scale parameter when the production function is Cobb-Douglas and the utility is log-linear, as in de-la-Croix and Michel [12, p. 50]), $\gamma = 0.3$ (implying a quarterly discount factor of 0.99, assuming that one period equals 30 years, again as in de-la-Croix and Michel [12, p. 50]), $p = 0.25$, $m = 0.4$, $\phi = 0.5$ (implying that preference for children is the half of that for consumption), $\alpha = 0.33$. The long run fertility rate is $n = 1.38$, and the ratio cost of children/net wage is *0.242* (in line with observed children costs, see, e.g., endnote (4)). From Proposition 1 it is derived that $\alpha^* = 0.13$, so that for most cases of the capital distributive share, a longer life increases fertility: for instance, when p decreases to *0.2*, n increases to *1.396*.

4. Social Security, Longevity, and Fertility

Since the issue of the relationship between unfunded pension systems and demographic behaviours such as longevity and fertility is highly debated, it is important to investigate whether the results of the previous section also hold in the presence of an unfunded social security scheme. In particular, the problem to address is "whether parents increase the fertility rate in a growing economy with a pay-as-you-go social security system" (Yakita [5, p. 637-8]). Yakita found that (1) "a higher contribution rate tends to have a positive effect on the fertility rate", but (2) despite the positive effect of pensions noted at the point 1, the introduction of a social security system "will never overwhelm or even offset, the negative effect of increased life expectancy." (Yakita [5, p. 638-9]). In other words, Yakita has shown that social security enhances fertility rates and weakens, although it does not revert, the negative effect of increased longevity on fertility rates.

In order to investigate whether the above mentioned results also hold in the present model, a PAYG pension scheme is introduced in the model of the Section 2.

The sole modifications of the model are (1) the government runs a PAYG pension balanced budget policy in every period according to the following constraint:

$$(1-p)\,z_t = \theta w_t n_{t-1}, \tag{10}$$

where the left-hand side represents the social security expenditure (the individuals' probability of surviving to the second period of life multiplied for z_t, which is the benefit perceived by each pensioner at time t), and the right-hand side represents the tax receipts ($0 < \theta < 1$ is the (constant) contribution rate paid by the young-adult contributors, w is the wage rate and n is the rate of fertility).

(2) the budget constraint of the individuals when old is

$$c_{t+1}^2 = \frac{1+r_{t+1}}{1-p} s_t + z_{t+1}. \tag{11}$$

Now the maximization of (1) under the constraints (2), (11), and the government budget (10) gives the following saving function and the demand for children chosen by individuals:

$$s_t = \frac{w_t(1-\theta)\left[(1-p)\gamma m - \phi\theta\left(w_{t+1}/(1+r_{t+1})\right)\right]}{(1+(1-p)\gamma+\phi)\,m - \phi\theta\left(w_{t+1}/(1+r_{t+1})\right)}, \tag{12}$$

$$n_t = \frac{\phi w_t(1-\theta)}{(1+(1-p)\gamma+\phi)\,m - \phi\theta\left(w_{t+1}/(1+r_{t+1})\right)}. \tag{13}$$

Substituting out for s and n according to (12) and (13), respectively, exploiting again (5) and (6), and assuming individuals are perfect foresighted, the dynamic equilibrium sequence of capital boils down to the following long-run per-capita stock of capital:

$$k^*(p) = \frac{(1-p)\gamma m\alpha}{\phi\left[\alpha+\theta(1-\alpha)\right]}. \tag{14}$$

Therefore the long-run fertility rate (where the apex "ss" denotes the present case with social security) is

$$n^{*\mathrm{ss}} = \frac{\phi(1-\theta)\left[\alpha+\theta(1-\theta)\right](1-\alpha)A\left[(1-p)\gamma m\alpha/\phi\left[\alpha+\theta(1-\alpha)\right]\right]^{\alpha}}{m\left[\alpha(1+(1-p)\gamma+\phi)+\theta(1-\alpha)(1+\phi)\right]}. \tag{15}$$

The effect of the introduction of the social security system (i.e., the contribution rate θ) on the fertility rate results from the study of the derivative of (15) with respect to θ. Unfortunately the sign of the effect depends on a fourth-degree polynomial function of θ (not reported here for brevity) and therefore we resorted to numerical simulations to obtain the following result.

Result 1. The higher the size of the pension system (the contribution rate), the lower is the fertility rate.[14] This result contrasts with that found by Yakita and seems to be in accord with a certain common wisdom, which argues that "public pensions themselves are in part to blame for the fertility decline" (Cigno [13, page 37]).[15]

Armed with this result, now we study whether and how Proposition 1 is changed by the introduction of the PAYG social security system. Therefore the following proposition holds.

Proposition 2. *The increase of longevity reduces fertility if and only if* $\alpha < \alpha^{ss} = ((1-p)\gamma-\theta(1+\phi))/(1+\phi+(1-p)\gamma-\theta(1+\phi))$.

Proof. The proof straightforwardly derives from

$$\frac{\partial n^*(p)}{\partial(1-p)} \gtreqless 0 \Longleftrightarrow \left[-\alpha\left[1+\gamma(1-p)+\phi-\theta(1+\phi)\right] + \gamma(1-p)-\theta(1+\phi)\right] \gtreqless 0. \tag{16}$$

□

Proposition 3. *(a) When there is a social security system, a positive effect of a longer life on fertility rates is more likely than in the case of absence of pensions. (b) Moreover the higher the size of the pension system (i.e., the contribution rate), the more likely a positive effect of a longevity increase on fertility rates.*

Proof. The proof of the parts (a) and (b) straightforwardly derives, respectively, from (a) $\alpha^* > \alpha^{ss}$, for any $0 < \theta < 1$; (b) $\partial\alpha^{ss}/\partial\theta < 0$. □

Therefore, it has been shown that, contrary to the Yakita's findings, the presence of social security, and in particular a large social security system, favours the possibility that a longer life stimulates a higher fertility.

It is easy to see that a longer life always increases fertility in very plausible economic conditions, under a PAYG pension system. For illustrating this, we choose (in addition to the parameter set of the preceding section) $\tau = 0.3$ (similar to the Italian contribution rate). The long run fertility rate is $n = 0.88$ (in line with most European countries, being the replacement ratio $n = 1$). From Proposition 2 it is derived that $\alpha^{ss} = -0.018$, so that for whatever capital distributive share, a longer life always increases fertility: for instance, when p decreases to 0.2, n increases to 0.896. Therefore, the presence of a sizable PAYG pension system, which is a widespread feature of many developed countries which are experiencing dramatic fertility drops as well as longevity increases, practically ensures the result that a longer life stimulates a fertility recovery.

Moreover, in the next section, we check the robustness of the result that an increase of longevity increases fertility, by introducing a family policy rather widespread in developed countries for fighting against the dramatic fertility drop: a per child cash subsidy policy.[16]

5. Child Subsidy Policy and Longevity Effects on Fertility

The sole modifications of the model of Section 2 are (1) the government runs a child subsidy balanced budget policy in every period according to the following constraint:

$$vn_t = \tau w_t, \tag{17}$$

where the left-hand side represents the child subsidy expenditure (the constant per child cash subsidy (v) multiplied for the number of children n_t), and the right-hand side represents the tax receipts ($0 < \tau < 1$ is the (constant) tax rate on the wage of the young-adult); (2) the budget constraint of the young individual is

$$c_t^1 = w_t (1 - \tau) - (m - v) n_t - s_t. \tag{18}$$

In particular the government balances the child policy budget by fixing an appropriate tax rate, according to the rule

$$\tau = \frac{vn_t}{w_t}. \tag{19}$$

Again maximising (1) under the constraints (3) and (18) and taking into account (5), (6), and (19) we obtain[17] the following long-run per-capita stock of capital:

$$k^* (p) = \frac{(1 - p)\gamma (m - v)}{\phi} \tag{20}$$

and consequently the following long-run fertility rate (where the apex "cs" denotes the present case with child subsidy):

$$n^{*\text{cs}} = \frac{\phi (1 - \alpha) A[(1 - p)\gamma(m - v)/\phi]^{\alpha}}{(m - v)(1 + (1 - p)\gamma) + m\phi}. \tag{21}$$

Now we study whether and how Proposition 1 is changed by the introduction of the child subsidy policy. Therefore the following proposition holds.

Proposition 4. *The increase of longevity reduces fertility if and only if* $\alpha < \alpha^{cs} = (1 - p)\gamma(m - v)/[(m - v)(1 + (1 - p)\gamma) + m\phi]$.

Proof. The proof straightforwardly derives from

$$\frac{\partial n^* (p)}{\partial (1 - p)} \gtreqless 0 \iff [-\alpha [(m - v)\gamma (1 - p) - \phi [\gamma (m - v) - m] - (m - v) - \phi] + \gamma (1 - p)(m - v)] \gtreqless 0. \tag{22}$$

□

For illustrating quantitatively Proposition 4, we use the same parametric set of the preceding Section 2. From the simulation in Section 2 we know that without child subsidy the long run fertility rate is $n = 1.38$ and the critical capital share value is $\alpha^* = 0.13$. From Proposition 4 it is derived that if a child subsidy $v = 0.35$ is introduced then we obtain $n = 1.83$ and $\alpha^{cs} = 0.043$; that is, with a sizable child subsidy the critical share of capital is practically negligible. Therefore, we may state that in practice for whatever capital distributive share, a longer life always increases fertility: for instance, when p decreases to 0.2, n increases to 1.87.

6. Conclusions

This paper has explored, in a conventional OLG Diamond's model, the effects of a longer life on fertility, showing that an increase in longevity may, under rather plausible economic conditions, increase fertility rates and that this result is magnified by the presence of a PAYG social security system as well as child subsidy policies, which are rather popular in developed countries plagued by a fertility drop.

In this paper it is adopted, in line with a vast literature, a fixed cost of children in terms of goods and services, rather than a time cost in terms of forgone wages (adopted by another part of the literature) and it is argued that since family policies, especially in some European countries, tend to facilitate the compatibility of working and family life through, for instance, child care services, parental leaves, appropriate timetables, holidays, and so on, then the cost of children in terms of foregone wages tends to be reduced, while the cost in terms of goods and services remains fundamental. As a consequence the consideration of the sole time cost may lead to partial conclusions. Indeed this paper, by considering the polar case of the sole fixed cost, achieves opposite conclusions to those obtained in the literature considering the sole time cost (i.e., [5]).

The paper finds that, especially in countries (such as Italy) having a population with a high longevity, a high capital distributive share, a large PAYG social security system, and child subsidies policies, a further increase of longevity may increase the fertility rate in the long run.

This result may partially reduce the alarm for a "population implosion" in developed countries with rapid population aging and below-replacement fertility. To the extent that longevity and fertility are primary policy concerns, the result derived in this paper may be rather significant.

Finally, while the present results have been obtained in a closed economy, the investigation whether the "virtuous circle" higher longevity-higher fertility may also hold in small open developed economies would seem to be an interesting topic for future research.

Endnotes

1. While Yakita introduces in the Diamond's model also an externality à la Romer for generating endogenous rather than exogenous growth, we preserve the neoclassical exogenous growth Diamond's model. The findings of this paper does not depend on whether growth is endogenous or exogenous.
2. The endogenous choice of fertility is motivated according to the so-called weak altruism of parents towards their children; that is, parents choose the number of their children as a "consumption" good ([7, 10, 14]). By contrast, Ehrlich and Lui [4] assume that parents invest in the quantity (and in the human capital) of their children in order to secure old-age insurance.
3. Since a better understanding of the relationship longevity-fertility in as simple as possible models is a preliminary task, I have not embarked in developing more refined models: this allowed me, in the present paper, for focusing on a key ingredient as a responsible of opposite effects, such as the form of the cost of rearing children. Of course the effects of a longer life on fertility may be rather complicated in the cases in which, for instance, parents derive utility from their children's utility, leave unintended or voluntary bequests, invest in the children's human capital, and so on, but I abstracted from the vast literature which considers such cases.
4. For instance, in Italy this cost has been estimated to be, for each child and for each month, 252 €, 212 €, and 233 € for the age classes 0–5, 6–14, and 15–18 years, respectively (this amounts to say that the per child cost is around 25% of a labourer's average wage). Therefore, our assumption amounts to say that, loosely speaking, parents take into account only this cost when they decide whether to have or not a child, independently of whether they receive a high or a low wage (while they assume that the forgone work hours for child caring are included in the amount allowed, without forgone wages, by the family laws (e.g., parental leaves)).
5. This sharp contrast with the previous results show that the type of the cost of rearing children is crucial for determining the effect of a longer life on the long-run fertility rates.
6. I refer to the book of de-la-Croix and Michel [12, ch. 1] for a thorough analysis of the basic OLG growth model.
7. It is assumed that (i) the private annuity market is competitive and the companies are risk neutral; (ii) individuals are willing to invest their assets in such insurance companies, given the hypothesis of absence of bequests.
8. Note that with constant return to scale the use of a representative firm (instead of heterogeneous firms) is justified since the number of firms does not matter and does not affect production, given that firms adopt the same technology.
9. Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.
10. The price of output has been normalised to unity.
11. I thank an anonymous referee to have suggested to clarify the dynamic interaction between firms and agents from different generations.
12. This assumption is not strictly necessary, because the maximization target of the firm is essentially a static one, and thus we might, alternatively, also assume that firms live forever.
13. Note that, as known, in the standard Diamond's model under Cobb-Douglas production function and logarithmic utility function, the existence, uniqueness and stability of the equilibrium are always preserved and this also holds in the present model which extends the Diamond's model with endogenous fertility choices and exogenous longevity.
14. For the sake of precision, Result 1 holds provided that the share of capital is not rather unrealistically low. Indeed, simulations revealed that a positive effect of pensions on fertility might occur only if the capital share is significantly below 0.33 (which is the standard value of the capital share in most papers) and in any case only if the contribution rate is very low: for instance, even in the case of an unrealistically low value of the capital share about 0.15 the positive effect would only hold for contribution rates lower than 0.01 (for given realistic values of $\phi < 0.5$ and $(1 - p) < 0.8$).
15. As to the issue of the relationship between pension systems and low fertility in Europe, see, for instance, Cigno and Werding [15].
16. Child policies have been investigated, although in a different context, for instance, by Fanti and Gori [8, 16].
17. For economy of space, the standard passages, equal to those in Sections 2 and 4, are omitted.

References

[1] D. De La Croix and O. Licandro, "Life expectancy and endogenous growth," *Economics Letters*, vol. 65, no. 2, pp. 255–263, 1999.

[2] J. Zhang, J. Zhang, and R. Lee, “Rising longevity, education, savings, and growth,” *Journal of Development Economics*, vol. 70, no. 1, pp. 83–101, 2003.

[3] G. P. Cipriani and M. Makris, “A model with self-fulfilling prophecies of longevity,” *Economics Letters*, vol. 91, no. 1, pp. 122–126, 2006.

[4] I. Ehrlich and F. T. Lui, “Intergenerational trade, longevity and economic growth,” *Journal of Political Economy*, vol. 99, pp. 1029–1059, 1991.

[5] A. Yakita, “Uncertain lifetime, fertility and social security,” *Journal of Population Economics*, vol. 14, no. 4, pp. 635–640, 2001.

[6] P. A. Diamond, “National debt in a neoclassical growth model,” *American Economic Review*, vol. 55, pp. 1126–1150, 1965.

[7] Z. Eckstein, S. Stern, and K. I. Wolpin, “Fertility choice, land, and the Malthusian hypothesis,” *International Economic Review*, vol. 29, no. 2, pp. 353–361, 1988.

[8] L. Fanti and L. Gori, “Population and neoclassical economic growth: a new child policy perspective,” *Economics Letters*, vol. 104, no. 1, pp. 27–30, 2009.

[9] J. Zhang and J. Zhang, “Social security, intergenerational transfers, and endogenous growth,” *Canadian Journal of Economics*, vol. 31, no. 5, pp. 1225–1241, 1998.

[10] O. Galor and D. N. Weil, “The gender gap, fertility, and growth,” *American Economic Review*, vol. 86, no. 3, pp. 374–387, 1996.

[11] O. J. Blanchard, “Debt, deficits and finite horizon,” *Journal of Political Economy*, vol. 93, pp. 223–247, 1985.

[12] D. de la Croix and P. Michel, *A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations*, Cambridge University Press, New York, NY, USA, 2002.

[13] A. Cigno, “Low fertility in Europe: is the pension system the victim or the culprit?” *CESifo Forum*, vol. 8, no. 3, pp. 37–42, 2007.

[14] Z. Eckstein and K. I. Wolpin, “Endogenous fertility and optimal population size,” *Journal of Public Economics*, vol. 27, no. 1, pp. 93–106, 1985.

[15] A. Cigno and M. Werding, *Children and Pensions*, MIT Press, Cambridge, Mass, USA, 2007.

[16] L. Fanti and L. Gori, “A note on child policy and fertility in an overlapping generations small open economy: when the labour market institutions matter,” *International Journal of Population Research*, vol. 20, pp. 1–6, 2012.