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Research Article

Finite-Time Combination-Combination Synchronization for Hyperchaotic Systems

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A new type of finite-time synchronization with two drive systems and two response systems is presented. Based on the finite-time stability theory, step-by-step control and nonlinear control method, a suitable controller is designed to achieve finite-time combination-combination synchronization among four hyperchaotic systems. Numerical simulations are shown to verify the feasibility and effectiveness of the proposed control technique.

1. Introduction

As a new subject in 1980s, chaos almost covers all the fields of science. It is known that chaos is an interesting nonlinear phenomenon which may lead to irregularity and unpredictability in the dynamic system, and it has been intensively studied in the last three decades. Since Pecora and Carroll proposed the PC method to synchronize two chaotic systems in 1990 [1, 2], the study of synchronization of chaotic systems has been widely investigated due to their potential applications in various fields, for instance, in chemical reactions, biological systems, and secure communication. Over the past decades, a variety of control approaches such as adaptive control [3], linear feedback control [4], active control [5], and backstepping control [6] have been proposed for various types of synchronization, which include complete synchronization [7], projective synchronization [8, 9], general synchronization [10], lag synchronization [11], and novel compound synchronization [12].

Most of the aforementioned works are based on the synchronization scheme which consists of one drive system and one response system and can be seen as one-to-one system. However, we found it not secure and flexible enough in many real world applications, for instance, in secure communication. Recently, Runzi et al. presented a new type of synchronization with two drive systems and one response system [13]. Then, Sun et al. extended multi-to-one system to multi-to-two systems and reported a new type of synchronization, namely, combination-combination synchronization, where synchronization is achieved between two drive systems and two response systems [14]. The type of synchronization can improve the security of communication; for instance, we can split the transmitted signals into several parts, then load each part in different drive systems, and then restore it to the original signals by combining the received signals of different response systems correctly.

Notice that the mentioned literatures mainly investigated the asymptotic synchronization of chaotic systems. However, in the view of practical application, optimizing the synchronization time is more important than achieving synchronization asymptotically [15–19]. Recently, based on the step-by-step control method, Wang et al. realized the finite-time synchronization of two chaotic systems by designing a proper controller [15]. The method has the ability to achieve global stability in finite time. In addition, the step-by-step technique has the advantage of reducing controller complexity.

Motivated by the previous discussion, this paper aims to study the finite-time synchronization between a combination of two drive systems and a combination of two response systems in drive-response synchronization scheme. We have applied the finite-time stability theory to our analysis to achieve finite-time combination-combination synchronization. The step-by-step control method and nonlinear control technique are adopted to synchronize four different

hyperchaotic systems. Numerical simulations are presented to verify the theoretical findings.

2. The Finite-Time Combination-Combination Synchronization Scheme

Consider the drive systems and response systems as follows:

$$\text{drive system 1} \qquad \dot{\mathbf{x}}_1 = f_1(\mathbf{x}_1), \tag{1}$$

$$\text{drive system 2} \qquad \dot{\mathbf{x}}_2 = f_2(\mathbf{x}_2), \tag{2}$$

$$\text{response system 1} \qquad \dot{\mathbf{y}}_1 = g_1(\mathbf{y}_1) + \mathbf{u}, \tag{3}$$

$$\text{response system 2} \qquad \dot{\mathbf{y}}_2 = g_2(\mathbf{y}_2) + \mathbf{u}^*, \tag{4}$$

where $\mathbf{x}_1 = (x_1, x_2, \ldots, x_n)^T$, $\mathbf{x}_2 = (y_1, y_2, \ldots, y_n)^T$, $\mathbf{y}_1 = (z_1, z_2, \ldots, z_n)^T$, and $\mathbf{y}_2 = (w_1, w_2, \ldots, w_n)^T$ are the state vectors of systems (1), (2), (3), and (4), respectively. $f_1, f_2, g_1, g_2 : \mathbf{R}^n \to \mathbf{R}^n$ are four continuous functions, and $\mathbf{u}, \mathbf{u}^* : \mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^n$ are two controllers of the response systems (3) and (4) which will be designed, respectively.

Definition 1. If there exist four constant matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D} \in \mathbf{R}^n$ and $\mathbf{C} \neq \mathbf{0}$, $\mathbf{D} \neq \mathbf{0}$ such that $\lim_{t \to t_s} \|\mathbf{A}\mathbf{x_1} + \mathbf{B}\mathbf{x_2} - \mathbf{C}\mathbf{y_1} - \mathbf{D}\mathbf{y_2}\| = 0$ and $\|\mathbf{A}\mathbf{x_1} + \mathbf{B}\mathbf{x_2} - \mathbf{C}\mathbf{y_1} - \mathbf{D}\mathbf{y_2}\| \equiv 0$ for $t \geq t_s$, where $\mathbf{x}_1 = (x_1, x_2, \ldots, x_n)^T$, $\mathbf{x}_2 = (y_1, y_2, \ldots, y_n)^T$, $\mathbf{y}_1 = (z_1, z_2, \ldots, z_n)^T$, and $\mathbf{y}_2 = (w_1, w_2, \ldots, w_n)^T$, one gets that the drive systems (1) and (2) are realized as finite-time combination-combination synchronization with the response systems (3) and (4), where $\|\cdot\|$ represents the matrix norm.

Lemma 2 (see [15]). *Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:*

$$\dot{V}(t) \leq -cV^{\eta}(t), \quad \forall t \geq t_0, V(t_0) \geq 0, \tag{5}$$

where $c > 0$ and $0 < \eta < 1$ are constants. Then, for any initial time t_0, $V(t)$ satisfies

$$\begin{gathered} V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), \quad t_0 \leq t \leq t_1, \\ V(t) \equiv 0, \quad \forall t \geq t_1, \end{gathered} \tag{6}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{7}$$

Thus, for any initial value $V(t_0)$, the system (5) has $V(t) = 0$ in $t_1 = t_0 + V^{1-\eta}(t_0)/c(1-\eta)$; that is, the system can achieve global stability in finite time.

3. Problem Statement and Control Scheme

3.1. Problem Statement. Consider two identity hyperchaotic Chen systems as the drive systems [20]:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + w_1, \\ \dot{y}_1 &= dx_1 - x_1 z_1 + cy_1, \\ \dot{z}_1 &= x_1 y_1 - bz_1, \\ \dot{w}_1 &= y_1 z_1 + rw_1, \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + w_2, \\ \dot{y}_2 &= dx_2 - x_2 z_2 + cy_2, \\ \dot{z}_2 &= x_2 y_2 - bz_2, \\ \dot{w}_2 &= y_2 z_2 + rw_2. \end{aligned} \tag{9}$$

Consider two identity hyperchaotic Lorenz systems as the response systems [21]:

$$\begin{aligned} \dot{x}_3 &= a_1(y_3 - x_3) + w_3 + u_1, \\ \dot{y}_3 &= c_1 x_3 - y_3 - x_3 z_3 + u_2, \\ \dot{z}_3 &= x_3 y_3 - b_1 z_3 + u_3, \\ \dot{w}_3 &= -y_3 z_3 + r_1 w_3 + u_4, \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{x}_4 &= a_1(y_4 - x_4) + w_4 + u_1^*, \\ \dot{y}_4 &= c_1 x_4 - y_4 - x_4 z_4 + u_2^*, \\ \dot{z}_4 &= x_4 y_4 - b_1 z_4 + u_3^*, \\ \dot{w}_4 &= -y_4 z_4 + r_1 w_4 + u_4^*, \end{aligned} \tag{11}$$

where $\mathbf{u} = (u_1, u_2, u_3, u_4)^T$ and $\mathbf{u}^* = (u_1^*, u_2^*, u_3^*, u_4^*)^T$ are two controllers of the response systems (10) and (11) which will be designed, respectively.

Without loss of generality, we choose $\mathbf{A} = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$, $\mathbf{B} = \mathrm{diag}(\beta_1, \beta_2, \beta_3, \beta_4)$, $\mathbf{C} = \mathrm{diag}(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$, and $\mathbf{D} = \mathrm{diag}(1, 1, 1, 1)$.

The objective of the synchronization scheme is to design a suitable controller $\mathbf{U}(x, y, z, w)$ such that $\lim_{t \to t_s} \|\mathbf{A}\mathbf{x_1} + \mathbf{B}\mathbf{x_2} - \mathbf{C}\mathbf{y_1} - \mathbf{D}\mathbf{y_2}\| = 0$ and $\|\mathbf{A}\mathbf{x_1} + \mathbf{B}\mathbf{x_2} - \mathbf{C}\mathbf{y_1} - \mathbf{D}\mathbf{y_2}\| \equiv 0$ for $t \geq t_s$, where $\mathbf{x}_1 = (x_1, x_2, \ldots, x_n)^T$, $\mathbf{x}_2 = (y_1, y_2, \ldots, y_n)^T$, $\mathbf{y}_1 = (z_1, z_2, \ldots, z_n)^T$, and $\mathbf{y}_2 = (w_1, w_2, \ldots, w_n)^T$; that is, the drive systems (8) and (9) are realized as finite-time combination-combination synchronization with the response systems (10) and (11).

3.2. The Control Scheme. Let $e_1 = \alpha_1 x_1 + \beta_1 x_2 - \gamma_1 x_3 - x_4$, $e_2 = \alpha_2 y_1 + \beta_2 y_2 - \gamma_2 y_3 - y_4$, $e_3 = \alpha_3 z_1 + \beta_3 z_2 - \gamma_3 z_3 - z_4$, and $e_4 = \alpha_4 w_1 + \beta_4 w_2 - \gamma_4 w_3 - w_4$. The controller to be designed is $U_1 = \gamma_1 u_1 + u_1^*$, $U_2 = \gamma_2 u_2 + u_2^*$, $U_3 = \gamma_3 u_3 + u_3^*$, and $U_4 = \gamma_4 u_4 + u_4^*$.

Thus, we can get the error system as follows:

$$\begin{aligned}
\dot{e}_1 &= -ae_1 + e_4 + a(\alpha_1 y_1 + \beta_1 y_2 - \gamma_1 x_3 - x_4) \\
&\quad - a_1(\gamma_1 y_3 - \gamma_1 x_3 + y_4 - x_4) + (\alpha_1 - \alpha_4) w_1 \\
&\quad + (\beta_1 - \beta_4) w_2 - (\gamma_1 - \gamma_4) w_3 - U_1, \\
\dot{e}_2 &= c_1 e_1 + ce_2 + (c+1)(\gamma_2 y_3 + y_4) + d(\alpha_2 x_1 + \beta_2 x_2) \\
&\quad - c_1[\alpha_1 x_1 + \beta_1 x_2 + (\gamma_2 - \gamma_1) x_3] \\
&\quad - (\alpha_2 x_1 z_1 + \beta_2 x_2 z_2 - \gamma_2 x_3 z_3 - x_4 z_4) - U_2, \\
\dot{e}_3 &= -be_3 + (b_1 - b)(\gamma_3 z_3 + z_4) \\
&\quad + (\alpha_3 x_1 y_1 + \beta_3 x_2 y_2 - \gamma_3 x_3 y_3 - x_4 y_4) - U_3, \\
\dot{e}_4 &= re_4 + (r - r_1)(\gamma_4 w_3 + w_4) \\
&\quad + (\alpha_4 y_1 z_1 + \beta_4 y_2 z_2 + \gamma_4 y_3 z_3 + y_4 z_4) - U_4.
\end{aligned} \tag{12}$$

Our aim is to design a suitable controller, such that the drive systems (8) and (9) are realized as combination-combination synchronization with the response systems (10) and (11) in finite time. Then, the problem is changed to design a suitable controller, such that the error system (12) achieves the finite-time stability at the origin.

The design plan and its steps are as follows.

Step 1. Choose

$$\begin{aligned}
U_4 &= (r - r_1)(\gamma_4 w_3 + w_4) \\
&\quad + (\alpha_4 y_1 z_1 + \beta_4 y_2 z_2 + \gamma_4 y_3 z_3 + y_4 z_4) + k_4 e_4 + e_4^{\alpha},
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
u_4 &= \frac{1}{\gamma_4}[(r - r_1)(\gamma_4 w_3 + w_4) \\
&\quad + (\alpha_4 y_1 z_1 + \beta_4 y_2 z_2 + \gamma_4 y_3 z_3 + y_4 z_4)], \\
u_4^* &= k_4 e_4 + e_4^{\alpha}, \quad k_4 > r, \ \alpha = \frac{q}{p}
\end{aligned} \tag{14}$$

is a proper rational number, and p is a positive odd number, $p > q$.

Substituting U_4 into the fourth equation of (12), we get

$$\dot{e}_4 = (r - k_4) e_4 - e_4^{\alpha}. \tag{15}$$

Choose a candidate Lyapunov function

$$V_4 = \frac{1}{2} e_4^2. \tag{16}$$

Thus, the derivative of V_4 along the solution of error equation (15) is

$$\begin{aligned}
\dot{V}_4 &= (r - k_4) e_4^2 - e_4^{\alpha+1} \le -e_4^{\alpha+1} \\
&= -2^{(\alpha+1)/2}\left(\frac{1}{2} e_4^2\right)^{(\alpha+1)/2} = -2^{(\alpha+1)/2} V_4^{(\alpha+1)/2}.
\end{aligned} \tag{17}$$

According to Lemma 2, the system (15) is finite-time stability, which implies that there exists $T_1 > 0$, such that $e_4 \equiv 0$ for $t \ge T_1$.

Step 2. Choose

$$\begin{aligned}
U_1 &= a(\alpha_1 y_1 + \beta_1 y_2 - \gamma_1 x_3 - x_4) \\
&\quad - a_1(\gamma_1 y_3 - \gamma_1 x_3 + y_4 - x_4) \\
&\quad + (\alpha_1 - \alpha_4) w_1 + (\beta_1 - \beta_4) w_2 - (\gamma_1 - \gamma_4) w_3 \\
&\quad + k_1 e_1 + e_1^{\beta},
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
u_1 &= \frac{1}{\gamma_1}[a(\alpha_1 y_1 + \beta_1 y_2 - \gamma_1 x_3 - x_4) \\
&\quad - a_1(\gamma_1 y_3 - \gamma_1 x_3 + y_4 - x_4) \\
&\quad + (\alpha_1 - \alpha_4) w_1 + (\beta_1 - \beta_4) w_2 \\
&\quad - (\gamma_1 - \gamma_4) w_3], \\
u_1^* &= k_1 e_1 + e_1^{\beta}, \quad k_1 > -a, \ 0 < \beta < 1.
\end{aligned} \tag{19}$$

For $t > T_1$, substituting U_1 into the first equation of (12), we get

$$\dot{e}_1 = -(k_1 + a) e_1 - e_1^{\beta}. \tag{20}$$

Choose a candidate Lyapunov function

$$V_1 = \frac{1}{2} e_1^2. \tag{21}$$

Thus, the derivative of V_1 along the solution of error equation (20) is

$$\begin{aligned}
\dot{V}_1 &= -(k_1 + a) e_1^2 - e_1^{\beta+1} \le -e_1^{\beta+1} \\
&= -2^{(\beta+1)/2}\left(\frac{1}{2} e_1^2\right)^{(\beta+1)/2} = -2^{(\beta+1)/2} V_1^{(\beta+1)/2}.
\end{aligned} \tag{22}$$

According to Lemma 2, the system (20) is finite-time stability, which implies that there exists $T_2 > 0$, such that $e_1 \equiv 0$ for $t \ge T_2$.

Step 3. Choose

$$\begin{aligned}
U_2 &= (c+1)(\gamma_2 y_3 + y_4) + d(\alpha_2 x_1 + \beta_2 x_2) \\
&\quad - c_1[\alpha_1 x_1 + \beta_1 x_2 + (\gamma_2 - \gamma_1) x_3] \\
&\quad - (\alpha_2 x_1 z_1 + \beta_2 x_2 z_2 - \gamma_2 x_3 z_3 - x_4 z_4) \\
&\quad + k_2 e_2 + e_2^{\gamma},
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
u_2 &= \frac{1}{\gamma_2}[(c+1)(\gamma_2 y_3 + y_4) + d(\alpha_2 x_1 + \beta_2 x_2) \\
&\quad - c_1[\alpha_1 x_1 + \beta_1 x_2 + (\gamma_2 - \gamma_1) x_3] \\
&\quad - (\alpha_2 x_1 z_1 + \beta_2 x_2 z_2 - \gamma_2 x_3 z_3 - x_4 z_4)] \\
u_2^* &= k_2 e_2 + e_2^{\gamma}, \quad k_2 > c, \ 0 < \gamma < 1.
\end{aligned} \tag{24}$$

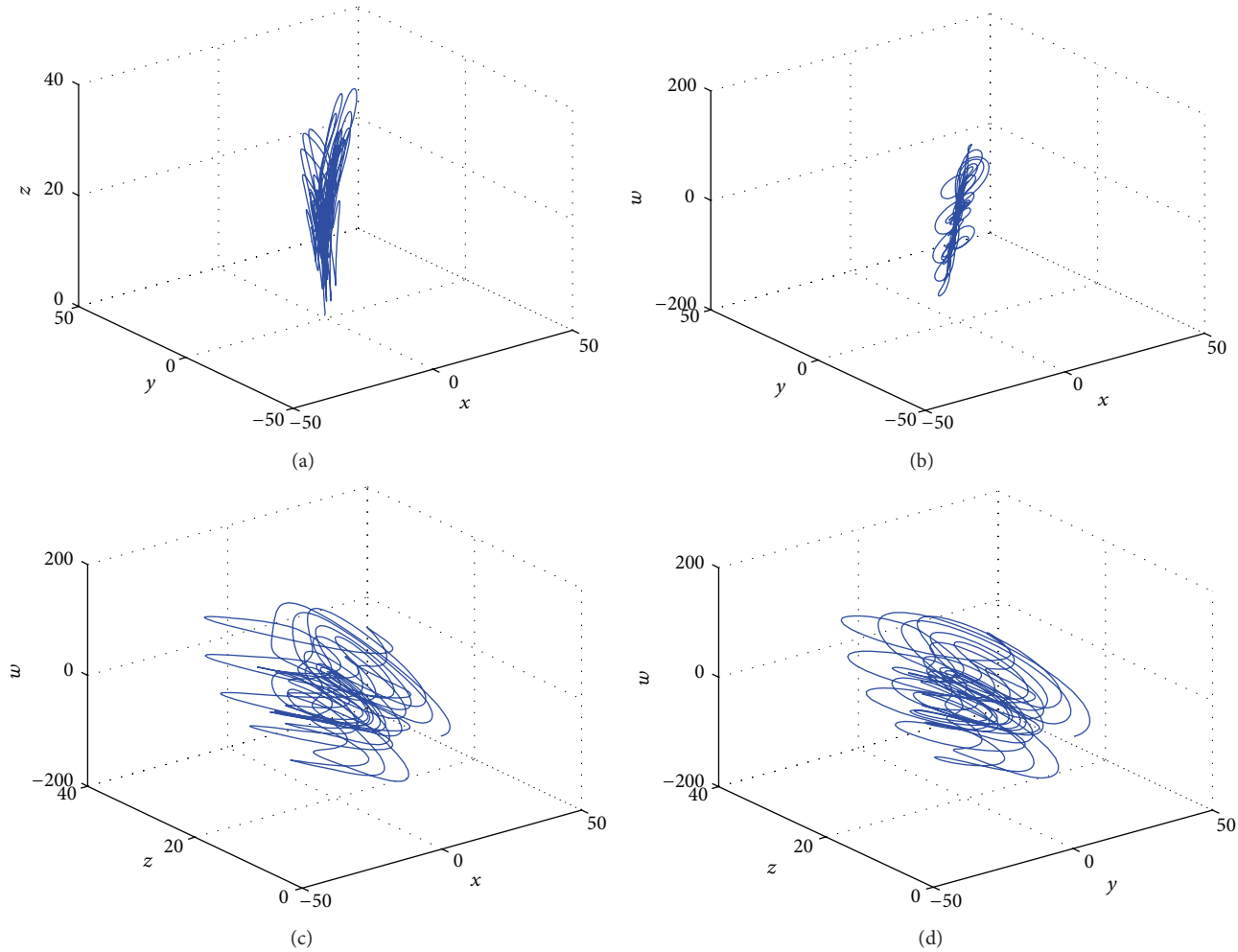

Figure 1: Hyperchaotic attractors of the hyperchaotic Chen system with $a = 35$, $b = 3$, $c = 12$, $d = 7$, and $r = 0.5$.

For $t > T_2$, substituting U_2 into the second equation of (12), we get

$$\dot{e}_2 = (-k_2 + c)\, e_2 - e_2^{\gamma}. \tag{25}$$

Choose a candidate Lyapunov function

$$V_2 = \frac{1}{2} e_2^2. \tag{26}$$

Thus, the derivative of V_2 along the solution of error equation (25) is

$$\begin{aligned} \dot{V}_2 &= (-k_2 + c)\, e_2^2 - e_2^{\gamma+1} \le -e_2^{\gamma+1} \\ &= -2^{(\gamma+1)/2} \left(\frac{1}{2} e_2^2\right)^{(\gamma+1)/2} = -2^{(\gamma+1)/2} V_2^{(\gamma+1)/2}. \end{aligned} \tag{27}$$

According to Lemma 2, the system (25) is finite-time stability, which implies that there exists $T_3 > 0$, such that $e_2 \equiv 0$ for $t \ge T_3$.

Step 4. Choose

$$\begin{aligned} U_3 = {} & (b_1 - b)(\gamma_3 z_3 + z_4) \\ & + (\alpha_3 x_1 y_1 + \beta_3 x_2 y_2 - \gamma_3 x_3 y_3 - x_4 y_4) \\ & + k_3 e_3 + e_3^{\delta}, \end{aligned} \tag{28}$$

where

$$\begin{aligned} u_3 = {} & \frac{1}{\gamma_3} \left[(b_1 - b)(\gamma_3 z_3 + z_4) \right. \\ & \left. + (\alpha_3 x_1 y_1 + \beta_3 x_2 y_2 - \gamma_3 x_3 y_3 - x_4 y_4) \right], \\ u_3^* = {} & k_3 e_3 + e_3^{\delta}, \quad k_3 > -b, \ 0 < \delta < 1. \end{aligned} \tag{29}$$

For $t > T_3$, substituting U_3 into the third equation of (12), we get

$$\dot{e}_3 = -(b + k_3)\, e_3 - e_3^{\delta}. \tag{30}$$

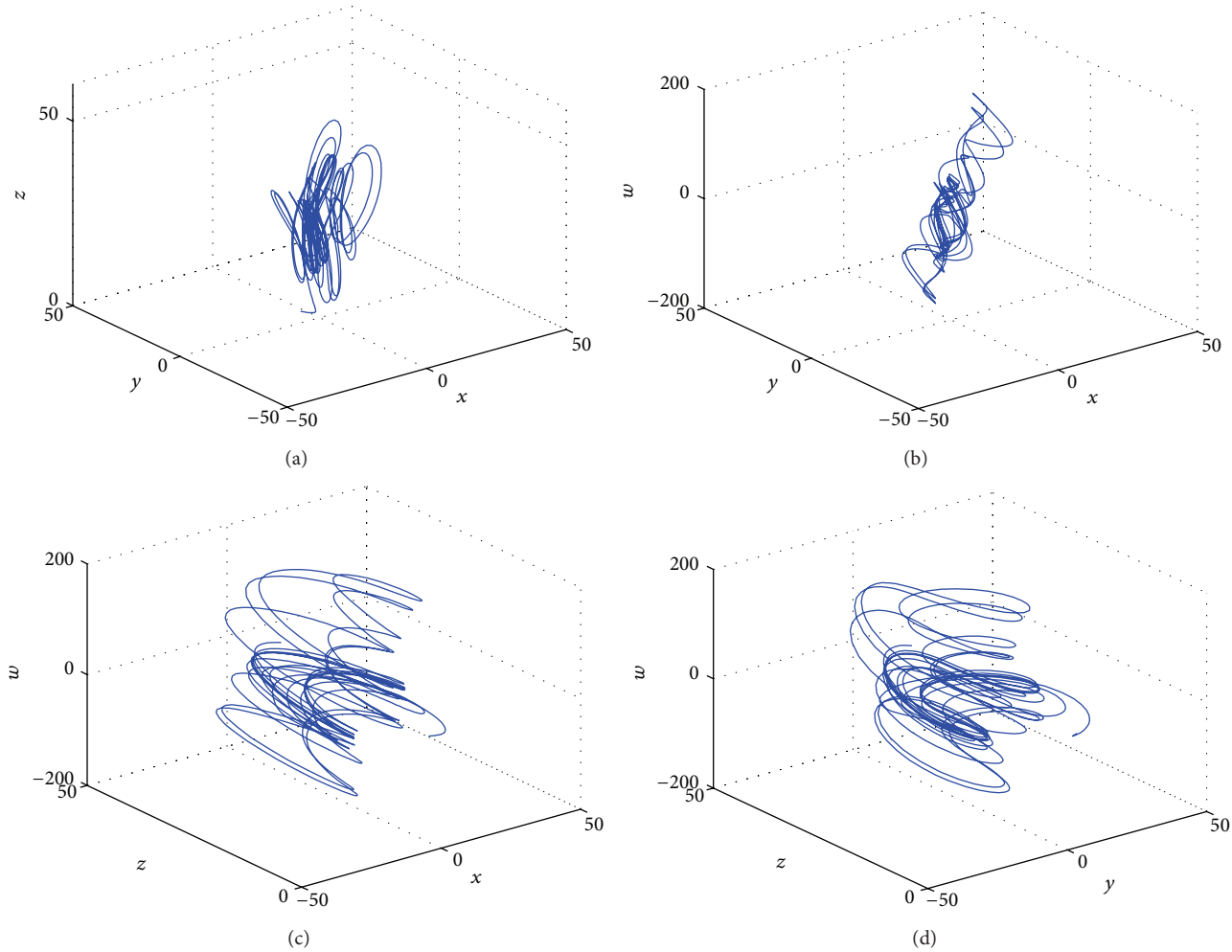

FIGURE 2: Hyperchaotic attractors of the hyperchaotic Lorenz system with $a_1 = 10$, $b_1 = 8/3$, $c_1 = 28$, and $r_1 = -1$.

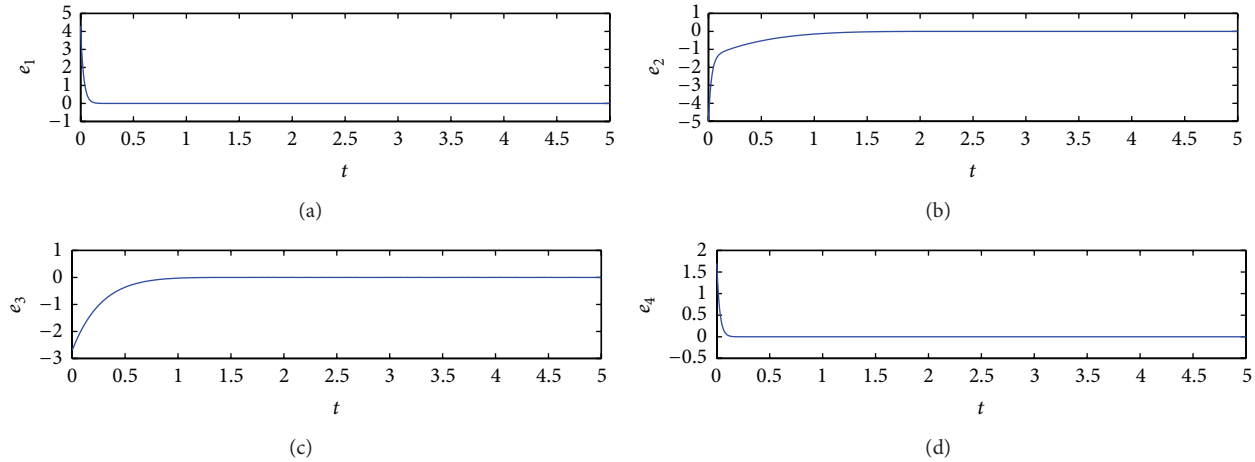

FIGURE 3: The errors between drive systems (8) and (9) and response systems (10) and (11).

Choose a candidate Lyapunov function

$$V_3 = \frac{1}{2}e_3^2. \tag{31}$$

Thus, the derivative of V_3 along the solution of error equation (30) is

$$\begin{aligned}\dot{V}_3 &= -(b+k_3)e_3^2 - e_3^{\delta+1} \le -e_3^{\delta+1} \\ &= -2^{(\delta+1)/2}\left(\frac{1}{2}e_3^2\right)^{(\delta+1)/2} = -2^{(\delta+1)/2}V_3^{(\delta+1)/2}.\end{aligned} \tag{32}$$

According to Lemma 2, the system (30) is finite-time stability, which implies that there exists $T_4 > 0$, such that $e_3 \equiv 0$ for $t \ge T_4$.

The controller is designed as follows:

$$\begin{aligned}U_1 &= a(\alpha_1 y_1 + \beta_1 y_2 - \gamma_1 x_3 - x_4) \\ &\quad - a_1(\gamma_1 y_3 - \gamma_1 x_3 + y_4 - x_4) \\ &\quad + (\alpha_1 - \alpha_4)w_1 + (\beta_1 - \beta_4)w_2 - (\gamma_1 - \gamma_4)w_3 \\ &\quad + k_1 e_1 + e_1^{\beta}, \\ U_2 &= (c+1)(\gamma_2 y_3 + y_4) + d(\alpha_2 x_1 + \beta_2 x_2) \\ &\quad - c_1[\alpha_1 x_1 + \beta_1 x_2 + (\gamma_2 - \gamma_1)x_3] \\ &\quad - (\alpha_2 x_1 z_1 + \beta_2 x_2 z_2 - \gamma_2 x_3 z_3 - x_4 z_4) \\ &\quad + k_2 e_2 + e_2^{\gamma}, \\ U_3 &= (b_1 - b)(\gamma_3 z_3 + z_4) \\ &\quad + (\alpha_3 x_1 y_1 + \beta_3 x_2 y_2 - \gamma_3 x_3 y_3 - x_4 y_4) + k_3 e_3 + e_3^{\delta}, \\ U_4 &= (r - r_1)(\gamma_4 w_3 + w_4) \\ &\quad + (\alpha_4 y_1 z_1 + \beta_4 y_2 z_2 + \gamma_4 y_3 z_3 + y_4 z_4) + k_4 e_4 + e_4^{\alpha}.\end{aligned} \tag{33}$$

According to what we discussed previously, we can obtain this conclusion that the error system (12) achieves finite-time stability under the control of the controller (33). Furthermore, the drive systems (8) and (9) are realized as combination-combination synchronization with the response systems (10) and (11) in finite time $T \le T_s$, where $T_s = T_1 + T_2 + T_3 + T_4$.

4. Numerical Simulation

To verify the effectiveness of the proposed finite-time synchronization method, we consider the hyperchaotic Chen system with the parameters $a = 35, b = 3, c = 12, d = 7$, and $r = 0.5$. The hyperchaotic attractor of the system is shown in Figure 1.

Consider the hyperchaotic Lorenz system with the parameters $a_1 = 10$, $b_1 = 8/3$, $c_1 = 28$, and $r_1 = -1$. The hyperchaotic attractor of the system is shown in Figure 2.

In the following simulation, we assume $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 1$, and $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 1$, and the initial states for the drive systems and response systems are arbitrarily given by $(x_1(0), y_1(0), z_1(0), w_1(0))^T = (0.1, 0.5, 1, 0.2)^T$, $(x_2(0), y_2(0), z_2(0), w_2(0))^T = (0.2, 1, 0.3, 2)^T$, $(x_3(0), y_3(0), z_3(0), w_3(0))^T = (-1, 1.5, 3, -2)^T$, and $(x_4(0), y_4(0), z_4(0), w_4(0))^T = (-3, 5, 1, 2.5)^T$. Then, and we choose the synchronization controller with $k_1 = 0$, $k_2 = 13$, $k_3 = 0$, $k_4 = 36$, $\alpha = \beta = \gamma = \delta = 2/3$. The synchronization evolution for this controller is shown in Figure 3.

The error vector is achieved to zero which implies that systems (8), (9) and (10), (11) have achieved finite-time combination-combination synchronization.

5. Conclusion

In this paper, the problem of finite-time combination-combination synchronization with two drive systems and two response systems was investigated. Based on the finite-time stability theory, the step-by-step control and nonlinear control approach, a suitable controller was introduced. The simulation results demonstrated that the proposed controller works well for synchronizing four hyperchaotic systems in finite time.

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