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Research Article

New Subclasses of Biunivalent Functions Involving Dziok-Srivastava Operator

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We introduce two new subclasses of biunivalent functions which are defined by using the Dziok-Srivastava operator. Furthermore, we find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses.

1. Introduction

Let A denote the class of all functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$. Also let S denote the class of all functions in A which are univalent in U.

Some of the important and well-investigated subclasses of the univalent function class S include, for example, the class $S^*(\beta)$ of starlike functions of order β in U and the class $K(\beta)$ of convex functions of order β in U. By definition, we have

$$\begin{aligned} S^*(\alpha) &= \left\{ f \in S : \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \beta, \right. \\ &\qquad \left. 0 \le \beta < 1, z \in U \right\}, \\ K(\alpha) &= \left\{ f \in S : \operatorname{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > \beta, \right. \\ &\qquad \left. 0 \le \beta < 1, z \in U \right\}. \end{aligned} \tag{2}$$

Ding et al. [1] introduced the following class $Q_\lambda(\beta)$ of analytic functions defined as follows:

$$\begin{aligned} Q_\lambda(\beta) = \left\{ f \in A : \operatorname{Re}\left((1-\lambda)\frac{f(z)}{z} + \lambda f'(z)\right) > \beta, \right. \\ \left. 0 \le \beta < 1, \lambda \ge 0 \right\}. \end{aligned} \tag{3}$$

It is easy to see that $Q_{\lambda_1}(\beta) \subset Q_{\lambda_2}(\beta)$ for $\lambda_1 > \lambda_2 \ge 0$. Thus, for $\lambda \ge 1$, $0 \le \beta < 1$, $Q_\lambda(\beta) \subset Q_1(\beta) = \{f \in A : \operatorname{Re} f'(z) > \beta, 0 \le \beta < 1\}$ and hence $Q_\lambda(\beta)$ is univalent class (see [2–4]).

It is well known that every function $f \in S$ has an inverse f^{-1}, defined by

$$\begin{aligned} f^{-1}(f(z)) &= z \quad (z \in U), \\ f(f^{-1}(w)) &= w \quad \left(|w| < r_0(f); r_0(f) \ge \frac{1}{4}\right), \end{aligned} \tag{4}$$

where

$$\begin{aligned} f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 \\ - (5a_2^3 - 5a_2 a_3 + a_4) w_4 + \cdots. \end{aligned} \tag{5}$$

A function $f \in A$ is said to be bi-univalent in U if both $f(z)$ and $f^{-1}(z)$ are univalent in U. Let Σ denote the class of bi-univalent functions in U given by (1). For a brief history and interesting examples in the class Σ see [5].

Brannan and Taha [6] (see also [7]) introduced certain subclasses of the bi-univalent function class Σ similar to the familiar subclasses $S^*(\beta)$ and $K(\beta)$ of starlike and convex functions of order β $(0 \le \beta < 1)$, respectively (see [8]). Thus, following Brannan and Taha [6] (see also [7]), a function $f \in A$ is in the class $S^*_\Sigma(\alpha)$ of strongly bi-starlike functions of order α $(0 < \alpha \le 1)$ if each of the following conditions is satisfied:

$$f \in \Sigma, \quad \left|\arg\left(\frac{zf'(z)}{f(z)}\right)\right| < \frac{\alpha\pi}{2} \quad (0 < \alpha \le 1, z \in U),$$
$$f \in \Sigma, \quad \left|\arg\left(\frac{zg'(w)}{g(w)}\right)\right| < \frac{\alpha\pi}{2} \quad (0 < \alpha \le 1, z \in U), \tag{6}$$

where g is the extension of f^{-1} to U. The classes $S^*_\Sigma(\alpha)$ and $K_\Sigma(\alpha)$ of bi-starlike functions of order α and biconvex functions of order α, corresponding, respectively, to the function classes $S^*(\beta)$ and $K(\beta)$, were also introduced analogously. For each of the function classes $S^*_\Sigma(\alpha)$ and $K_\Sigma(\alpha)$, they found nonsharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ (for details, see [6, 7]).

For function f given by (1) and g given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \tag{7}$$

the Hadamard product (or convolution) of f and g is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z). \tag{8}$$

For complex parameters $a_1, \ldots, a_q$ and $b_1, \ldots, b_s$ ($bj \notin \mathbb{Z}_0^- = \{0, -1, -2, \ldots\}$; $j = 1, \ldots, s$), the generalized hypergeometric function ${}_qF_s$ is defined by the following infinite series:

$${}_qF_s(a_1, \ldots, a_q; b_1, \ldots, b_s; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_q)_n}{(b_1)_n \cdots (b_s)_n} \frac{z^n}{n!}$$
$$(q \le s+1; q, s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \mathbb{N} = \{1, 2, 3, \ldots\}; z \in U), \tag{9}$$

where $(\theta)_n$ is the Pochhammer symbol (or shift factorial) defined, in terms of the Gamma function Γ, by

$$(\theta)_n = \frac{\Gamma(\theta + n)}{\Gamma(\theta)} = \begin{cases} 1, & (n = 0) \\ \theta(\theta+1)\cdots(\theta+n-1), & (n \in \mathbb{N}). \end{cases} \tag{10}$$

Correspondingly a function $h(a_1, \ldots, a_q; b_1, \ldots, b_s; z)$ is defined by

$$h(a_1, \ldots, a_q; b_1, \ldots, b_s; z) = z\, {}_qF_s(a_1, \ldots, a_q; b_1, \ldots, b_s; z) \quad (z \in U). \tag{11}$$

Dziok and Srivastava [9] (see also [10]) considered a linear operator

$$H(a_1, \ldots, a_q; b_1, \ldots, b_s) : A \longrightarrow A, \tag{12}$$

defined by the following Hadamard product:

$$H(a_1, \ldots, a_q; b_1, \ldots, b_s) f(z) = h(a_1, \ldots, a_q; b_1, \ldots, b_s; z) * f(z),$$
$$(q \le s+1; q, s \in \mathbb{N}_0; z \in U). \tag{13}$$

If $f \in A$ is given by (1), then we have

$$H(a_1, \ldots, a_q; b_1, \ldots, b_s) f(z) = z + \sum_{n=2}^{\infty} \Gamma_n[a_1; b_1] a_n z^n \quad (z \in U), \tag{14}$$

where

$$\Gamma_n[a_1; b_1] = \frac{(a_1)_n \cdots (a_q)_n}{(b_1)_n \cdots (b_s)_n} \frac{1}{n!} \quad (n \in \mathbb{N}). \tag{15}$$

To make the notation simple, we write

$$H_{q,s}[a_1; b_1; z] = H(a_1, \ldots, a_q; b_1, \ldots, b_s) f(z). \tag{16}$$

It easily follows from (14) that

$$z(H_{q,s}[a_1; b_1; z])' = a_1 H_{q,s}[a_1 + 1; b_1; z] - (a_1 - 1) H_{q,s}[a_1; b_1; z]. \tag{17}$$

The linear operator $H_{q,s}[a_1; b_1; z]$ is a generalization of many other linear operators considered earlier.

The object of the present paper is to introduce two new subclasses of the bi-univalent functions which are defined by using the Dziok-Srivastava operator and find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses of the function class Σ employing the techniques used earlier by Srivastava et al. [5] (see also [11]).

In order to derive our main results, we have to recall here the following lemma [12].

Lemma 1. *If $h \in P$, then $|c_k| \le 2$ for each k, where P is the family of all functions h analytic in U for which* Re $h(z) > 0$ $h(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots$ *for* $z \in U$.

Unless otherwise mentioned, we assume throughout this paper that $a_i, b_j \in \mathbb{C} \setminus \mathbb{Z}_0^-$, $i = 1, \ldots, s$, $j = 1, \ldots, q$, $q \le s+1$; $q, s \in \mathbb{N}_0$, $0 < \alpha \le 1$, $\lambda \ge 1$, $z \in U$, $\Gamma_n[a_1; b_1]$ is given by (15) and all powers are understood as principle values.

2. Coefficient Bounds of the Function Class $T_{q,s}^{\Sigma}[a_1;b_1,\alpha,\lambda]$

Definition 2. One says that a function $f(z)$ given by (1) is said to be in the class $T_{q,s}^{\Sigma}[a_1;b_1,\alpha,\lambda]$ if it satisfies the following condition:

$$f \in \Sigma, \quad \left|\arg\left((1-\lambda)\frac{H_{q,s}[a_1;b_1;z]}{z} + \lambda\left(H_{q,s}[a_1;b_1;z]\right)'\right)\right| < \frac{\alpha\pi}{2}, \tag{18}$$
$$\left|\arg\left((1-\lambda)\frac{g(w)}{w} + \lambda g'(w)\right)\right| < \frac{\alpha\pi}{2},$$

where the function g is given by

$$\begin{aligned} g(w) &= H_{q,s}^{-1}[a_1;b_1;z] \\ &= w - \Gamma_2[a_1;b_1]a_2w^2 \\ &\quad + \left(2(\Gamma_2[a_1;b_1])^2a_2^2 - \Gamma_3[a_1;b_1]a_3\right)w^3 \\ &\quad - \left(5(\Gamma_2[a_1;b_1])^3a_2^3 - 5\Gamma_2[a_1;b_1] \right. \\ &\quad \left. \times \Gamma_3[a_1;b_1]a_2a_3 + \Gamma_4[a_1;b_1]a_4\right)w^4 + \cdots. \end{aligned} \tag{19}$$

Remark 3. (i) For $q = 2$, $s = 1$, and $a_1 = a_2 = b_1 = 1$, we have $T_{2,1}^{\Sigma}[1,1;2;\alpha,\lambda] = B_{\Sigma}(\alpha,\lambda)$, where the class $B_{\Sigma}(\alpha,\lambda)$ was introduced and studied by Frasin and Aouf [11].

(ii) For $q = 2$, $s = 1$, and $a_1 = a_2 = b_1 = \lambda = 1$, we have $T_{2,1}^{\Sigma}[1,1;2;\alpha,1] = H_{\Sigma}(\alpha,\lambda)$, where the class $H_{\Sigma}(\alpha,\lambda)$ was introduced and studied by Srivastava et al. [5].

Theorem 4. *Letting $f(z)$ given by (1) be in the class $T_{q,s}^{\Sigma}[a_1;b_1,\alpha,\lambda]$, then*

$$|a_2| = \frac{2\alpha}{|\Gamma_2[a_1;b_1]|\sqrt{(\lambda+1)^2 + \alpha(1+2\lambda-\lambda^2)}}, \tag{20}$$

$$|a_3| = \frac{4\alpha^2}{|\Gamma_3[a_1;b_1]|(\lambda+1)^2} + \frac{2\alpha}{|\Gamma_3[a_1;b_1]|(2\lambda+1)}. \tag{21}$$

Proof. It follows from (18) that

$$(1-\lambda)\frac{H_{q,s}[a_1;b_1;z]}{z} + \lambda\left(H_{q,s}[a_1;b_1;z]\right)' = [p(z)]^2,$$
$$(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) = [q(w)]^2, \tag{22}$$

where $p(z)$ and $q(w)$ in P have the forms

$$p(z) = 1 + p_1z + p_2z^2 + p_3z^3 + \cdots, \tag{23}$$

$$q(w) = 1 + q_1w + q_2w^2 + q_3w^3 + \cdots. \tag{24}$$

Now, equating the coefficients in (22), we get

$$(\lambda+1)\Gamma_2[a_1;b_1]a_2 = \alpha p_1, \tag{25}$$

$$(2\lambda+1)\Gamma_3[a_1;b_1]a_3 = \alpha p_2 + \frac{\alpha(\alpha-1)}{2}p_1^2, \tag{26}$$

$$-(\lambda+1)\Gamma_2[a_1;b_1]a_2 = \alpha q_1, \tag{27}$$

$$\begin{aligned} &(2\lambda+1)\left(2(\Gamma_2[a_1;b_1])^2a_2^2 - \Gamma_3[a_1;b_1]a_3\right) \\ &\quad = \alpha q_2 + \frac{\alpha(\alpha-1)}{2}q_1^2. \end{aligned} \tag{28}$$

From (25) and (27), we get

$$p_1 = -q_1, \tag{29}$$

$$2(\lambda+1)^2(\Gamma_2[a_1;b_1])^2a_2^2 = \alpha^2\left(p_1^2 + q_1^2\right). \tag{30}$$

Now from (26), (28), and (30), we obtain

$$\begin{aligned} &2(2\lambda+1)(\Gamma_2[a_1;b_1])^2a_2^2 \\ &\quad = \alpha(p_2+q_2) + \frac{\alpha(\alpha-1)}{2}\left(p_1^2+q_1^2\right) \\ &\quad = \alpha(p_2+q_2) + \frac{\alpha(\alpha-1)}{2}\frac{2(\lambda+1)^2(\Gamma_2[a_1;b_1])^2a_2^2}{\alpha^2}. \end{aligned} \tag{31}$$

Therefore, we have

$$a_2^2 = \frac{\alpha^2(p_2+q_2)}{(\Gamma_2[a_1;b_1])^2\left[(\lambda+1)^2 + \alpha(1+2\lambda-\lambda^2)\right]}. \tag{32}$$

Applying Lemma 1 for the coefficients p_2 and q_2, we immediately have

$$|a_2| \le \frac{2\alpha}{|\Gamma_2[a_1;b_1]|\sqrt{(\lambda+1)^2 + \alpha(1+2\lambda-\lambda^2)}}. \tag{33}$$

This gives the bound on $|a_2|$ as asserted in (20).

Next, in order to find the bound on $|a_3|$, by subtracting (28) from (26) and using (29), we get

$$\begin{aligned} &2(2\lambda+1)\Gamma_3[a_1;b_1]a_3 - 2(2\lambda+1)(\Gamma_2[a_1;b_1])^2a_2^2 \\ &\quad = \alpha p_2 + \frac{\alpha(\alpha-1)}{2}p_1^2 - \left(\alpha q_2 + \frac{\alpha(\alpha-1)}{2}q_1^2\right) \\ &\quad = \alpha(p_2-q_2). \end{aligned} \tag{34}$$

It follows from (30) and (34) that

$$\begin{aligned} &2(2\lambda+1)\Gamma_3[a_1;b_1]a_3 \\ &\quad = \frac{\alpha^2(2\lambda+1)\left(p_1^2+q_1^2\right)}{(\lambda+1)^2} + \alpha(p_2-q_2), \end{aligned} \tag{35}$$

And, then,

$$a_3 = \frac{\alpha^2\left(p_1^2+q_1^2\right)}{2(\lambda+1)^2\Gamma_3[a_1;b_1]} + \frac{\alpha(p_2-q_2)}{2(2\lambda+1)\Gamma_3[a_1;b_1]}. \tag{36}$$

Applying Lemma 1 once again for the coefficients p_1, p_2, q_1, and q_2, we readily get

$$|a_3| \le \frac{4\alpha^2}{(\lambda+1)^2 |\Gamma_3[a_1;b_1]|} + \frac{2\alpha}{(2\lambda+1)|\Gamma_3[a_1;b_1]|}. \quad (37)$$

This completes the proof of Theorem 4. □

Remark 5. (i) Taking $q=2, s=1$, and $a_1=a_2=b_1=1$, in Theorem 4, we obtain the result obtained by Frasin and Aouf [11, Theorem 2.2].

(ii) Taking $q=2, s=1$, and $a_1=a_2=b_1=\lambda=1$, in Theorem 4, we obtain the result obtained by Srivastava et al. [5, Theorem 1].

3. Coefficient Bounds of the Function Class $T_{q,s}^{\Sigma}[a_1;b_1,\beta,\lambda]$

Definition 6. One says that a function $f(z)$ given by (1) is said to be in the class $T_{q,s}^{\Sigma}[a_1;b_1,\beta,\lambda]$ if it satisfies the following condition:

$$f \in \Sigma, \quad \operatorname{Re}\left\{(1-\lambda)\frac{H_{q,s}[a_1;b_1;z]}{z} + \lambda\left(H_{q,s}[a_1;b_1;z]\right)'\right\} > \beta, \quad (38)$$

$$\operatorname{Re}\left\{(1-\lambda)\frac{g(w)}{w} + \lambda g'(w)\right\} > \beta,$$

where the function g is defined by (19).

Remark 7. (i) For $q=2$, $s=1$, and $a_1=a_2=b_1=1$, we have $T_{2,1}^{\Sigma}[1,1;2;\beta,\lambda] = B_{\Sigma}(\beta,\lambda)$, where the class $B_{\Sigma}(\beta,\lambda)$ was introduced and studied by Frasin and Aouf [11].

(ii) For $q=2$, $s=1$, and $a_1=a_2=b_1=\lambda=1$, we have $T_{2,1}^{\Sigma}[1,1;2;\beta,1] = H_{\Sigma}(\beta,\lambda)$, where the class $H_{\Sigma}(\beta,\lambda)$ was introduced and studied by Srivastava et al. [5].

Theorem 8. *Letting $f(z)$ given by (1) be in the class $T_{q,s}^{\Sigma}[a_1;b_1,\beta,\lambda]$, $0 \le \beta < 1$ and $\lambda \ge 1$, then*

$$|a_2| = \frac{\sqrt{2(1-\beta)}}{|\Gamma_2[a_1;b_1]|\sqrt{2\lambda+1}}, \quad (39)$$

$$|a_3| = \frac{4(1-\beta)^2}{|\Gamma_3[a_1;b_1]|(\lambda+1)^2} + \frac{2(1-\beta)}{|\Gamma_3[a_1;b_1]|(2\lambda+1)}. \quad (40)$$

Proof. It follows from (38) that

$$(1-\lambda)\frac{H_{q,s}[a_1;b_1;z]}{z} + \lambda\left(H_{q,s}[a_1;b_1;z]\right)' = \beta + (1-\beta)p(z), \quad (41)$$

$$(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) = \beta + (1-\beta)q(w),$$

where $p(z)$ and $q(w)$ have the forms (23) and (24), respectively.

As in the proof of Theorem 4, by suitably comparing coefficients in (41), we get

$$(\lambda+1)\Gamma_2[a_1;b_1]a_2 = (1-\beta)p_1, \quad (42)$$

$$(2\lambda+1)\Gamma_3[a_1;b_1]a_3 = (1-\beta)p_2, \quad (43)$$

$$-(\lambda+1)\Gamma_2[a_1;b_1]a_2 = (1-\beta)q_1, \quad (44)$$

$$(2\lambda+1)\left(2(\Gamma_2[a_1;b_1])^2 a_2^2 - \Gamma_3[a_1;b_1]a_3\right) = (1-\beta)q_2. \quad (45)$$

From (42) and (44), we get

$$p_1 = -q_1, \quad (46)$$

$$2(\lambda+1)^2(\Gamma_2[a_1;b_1])^2 a_2^2 = (1-\beta)^2\left(p_1^2+q_1^2\right). \quad (47)$$

Also, from (43) and (45), we find that

$$2(2\lambda+1)(\Gamma_2[a_1;b_1])^2 a_2^2 = (1-\beta)(p_2+q_2). \quad (48)$$

Therefore, we have

$$|a_2^2| \le \frac{(1-\beta)}{(\Gamma_2[a_1;b_1])^2[2(2\lambda+1)]}(|p_2|+|q_2|). \quad (49)$$

Applying Lemma 1 for the coefficients p_2 and q_2, we immediately have

$$|a_2| \le \frac{\sqrt{2(1-\beta)}}{|\Gamma_2[a_1;b_1]|\sqrt{2\lambda+1}}. \quad (50)$$

This gives the bound on $|a_2|$ as asserted in (39).

Next, in order to find the bound on $|a_3|$, by subtracting (45) from (43), we get

$$2(2\lambda+1)\Gamma_3[a_1;b_1]a_3 - 2(2\lambda+1)(\Gamma_2[a_1;b_1])^2 a_2^2 = (1-\beta)(p_2-q_2), \quad (51)$$

or, equivalently,

$$a_3 = \frac{(\Gamma_2[a_1;b_1])^2 a_2^2}{\Gamma_3[a_1;b_1]} + \frac{(1-\beta)(p_2-q_2)}{2(2\lambda+1)\Gamma_3[a_1;b_1]}, \quad (52)$$

and, then from (47), we find that

$$a_3 = \frac{(1-\beta)^2\left(p_1^2+q_1^2\right)}{2(\lambda+1)^2\Gamma_3[a_1;b_1]} + \frac{(1-\beta)(p_2-q_2)}{2(2\lambda+1)\Gamma_3[a_1;b_1]}. \quad (53)$$

Applying Lemma 1 once again for the coefficients p_1, p_2, q_1, and q_2, we readily get

$$|a_3| \le \frac{4(1-\beta)^2}{(\lambda+1)^2|\Gamma_3[a_1;b_1]|} + \frac{2(1-\beta)}{(2\lambda+1)|\Gamma_3[a_1;b_1]|}. \quad (54)$$

This completes the proof of Theorem 8. □

Remark 9. (i) Taking $q = 2, s = 1$, and $a_1 = a_2 = b_1 = 1$, in Theorem 8, we obtain the result obtained by Frasin and Aouf [11, Theorem 3.2].

(ii) Taking $q = 2, s = 1$, and $a_1 = a_2 = b_1 = \lambda = 1$, in Theorem 8, we obtain the result obtained by Srivastava et al. [5, Theorem 2].

References

[1] S. S. Ding, Y. Ling, and G. J. Bao, "Some properties of a class of analytic functions," *Journal of Mathematical Analysis and Applications*, vol. 195, no. 1, pp. 71–81, 1995.

[2] M. P. Chen, "On the regular functions satisfying $(f(z)/z) > \alpha$," *Bulletin of the Institute of Mathematics. Academia Sinica*, vol. 3, no. 1, pp. 65–70, 1975.

[3] P. N. Chichra, "New subclasses of the class of close-to-convex functions," *Proceedings of the American Mathematical Society*, vol. 62, no. 1, pp. 37–43, 1976.

[4] T. H. MacGregor, "Functions whose derivative has a positive real part," *Transactions of the American Mathematical Society*, vol. 104, pp. 532–537, 1962.

[5] H. M. Srivastava, A. K. Mishra, and P. Gochhayat, "Certain subclasses of analytic and bi-univalent functions," *Applied Mathematics Letters*, vol. 23, no. 10, pp. 1188–1192, 2010.

[6] D. A. Brannan and T. S. Taha, "On some classes of bi-univalent functions," in *Mathematical Analysis and Its Applications*, S. M. Mazhar, A. Hamoui, and N. S. Faour, Eds., vol. 3 of *KFAS Proceedings Series*, pp. 53–60, Pergamon Press, Oxford, UK, 1985, see also *Studia Universitatis Babeş-Bolyai. Series Mathematica*, vol. 31, no. 2, pp. 70–77, 1986.

[7] T. S. Taha, *Topics in univalent function theory [Ph.D. thesis]*, University of London, London, UK, 1981.

[8] D. A. Brannan, J. Clunie, and W. E. Kirwan, "Coefficient estimates for a class of star-like functions," *Canadian Journal of Mathematics*, vol. 22, pp. 476–485, 1970.

[9] J. Dziok and H. M. Srivastava, "Classes of analytic functions associated with the generalized hypergeometric function," *Applied Mathematics and Computation*, vol. 103, no. 1, pp. 1–13, 1999.

[10] J. Dziok and H. M. Srivastava, "Certain subclasses of analytic functions associated with the generalized hypergeometric function," *Integral Transforms and Special Functions*, vol. 14, no. 1, pp. 7–18, 2003.

[11] B. A. Frasin and M. K. Aouf, "New subclasses of bi-univalent functions," *Applied Mathematics Letters*, vol. 24, no. 9, pp. 1569–1573, 2011.

[12] C. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, Germany, 1975.