

Research Article

Analysis of Radiative Radial Fin with Temperature-Dependent Thermal Conductivity Using Nonlinear Differential Transformation Methods

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Radiative radial fin with temperature-dependent thermal conductivity is analyzed. The calculations are carried out by using differential transformation method (DTM), which is a seminumerical-analytical solution technique that can be applied to various types of differential equations, as well as the Boubaker polynomials expansion scheme (BPES). By using DTM, the nonlinear constrained governing equations are reduced to recurrence relations and related boundary conditions are transformed into a set of algebraic equations. The principle of differential transformation is briefly introduced and then applied to the aforementioned equations. Solutions are subsequently obtained by a process of inverse transformation. The current results are then compared with previously obtained results using variational iteration method (VIM), Adomian decomposition method (ADM), homotopy analysis method (HAM), and numerical solution (NS) in order to verify the accuracy of the proposed method. The findings reveal that both BPES and DTM can achieve suitable results in predicting the solution of such problems. After these verifications, we analyze fin efficiency and the effects of some physically applicable parameters in this problem such as radiation-conduction fin parameter, radiation sink temperature, heat generation, and thermal conductivity parameters.

1. Introduction

Extended surfaces are extensively used in various industrial applications. An extensive review on this topic is presented by Kraus et al. [1]. Fins are very frequently encountered in many engineering applications to enhance heat transfer. Numerous contributions have been made in the heat transfer analysis of the fins. Constant thermophysical properties and uniform heat transfer coefficient are often assumed in the determination of the temperature distribution along an extended surface. The mathematical complexity of the conservation energy equation is reduced by this assumption and therefore well-established closed form analytical solutions can be obtained for a number of cases. If a large temperature

difference exists within a fin, the thermal conductivity may not be constant. Furthermore, in general, the heat transfer coefficient may vary along a fin. The heat transfer coefficient may be a function of the spatial coordinate only along a fin or may depend on the local temperature difference between the fin surface and the surrounding fluid. Kundu [2] analytically carried out the thermal analysis and optimization of longitudinal and pin fins of uniform thickness subject to fully wet, partially wet, and fully dry surface conditions. Moreover, Kundu [3] analytically analyzed the performance and optimization of longitudinal and pin fins of step reduction in local cross-section (SRC) profile subject to combined heat and mass transfer. Coşkun and Atay [4, 5] used variational iteration method to analyze convective

straight and radial fins with temperature-dependent thermal conductivity. Sharqawy and Zubair [6] carried out an analysis to study the efficiency of straight fins with different configurations when subjected to simultaneous heat and mass transfer mechanisms. Domairry and Fazeli [7] solved nonlinear straight fin differential equations by homotopy analysis method (HAM) to evaluate the temperature distribution within the fin. Arslanturk [8] obtained correlation equations for optimum design of annular fins with temperature-dependent thermal conductivity. Kulkarni and Joglekar [9] proposed and implemented a numerical technique based on residue minimization to solve the nonlinear differential equation, which governs the temperature distribution in straight convective fins having temperature-dependent thermal conductivity. Khani et al. [10] used HAM to evaluate the analytical approximate solutions and efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient. Kundu [11] described an analytical method for temperature and heat transfer characteristics of an annular step fin (ASF) with the simultaneous heat and mass transfer mechanisms. Bouaziz and Aziz [12] introduced a new concept called the double optimal linearization method (DOLM) to derive simple and accurate expressions for predicting the thermal performance of a convective-radiative fin with temperature-dependent thermal conductivity. Khani and Aziz [13] used HAM to develop analytical solutions for the thermal performance of a straight fin of trapezoidal profile when both thermal conductivity and heat transfer coefficient are temperature-dependent.

The differential transformation method (DTM) is a seminumerical-analytical method. DTM, which is based on the Taylor series expansion, was first proposed by Zhou [14] in 1986 for the solution of linear and nonlinear initial value problems that appear in electrical circuits. This method obtains a solution in the form of a polynomial. Indeed, this fact can be seen in Section 4, where the concept of differential transforms is briefly described. It may be pointed out that, later, this method has been successfully used in a series of literature [15–21] dealing with many engineering problems.

There are recent studies on the application of DTM to the heat transfer problems in the literature. Chu and Chen [22] applied a hybrid method of differential transformation and finite difference method to solve a transient heat conduction problem which had complex nonlinear terms. Chu and Lo [23] applied the differential transformation technique for transforming and discretizing the governing equations as well as the boundary conditions and provided two numerical examples. Lo and Chen [24] proposed an alternative numerical method to investigate hyperbolic heat conduction problems using the hybrid differential transfer/control-volume method. Joneidi et al. [25] provided analytical solution to fin efficiency of convective straight fins with temperature-dependent thermal conductivity by DTM. Jang et al. [26] investigated and characterized a two-dimensional thermal conductive boundary value problem with discontinuous boundary and initial conditions. Rashidi et al. [27] applied DTM to find the analytic solution for the problem of mixed convection about an inclined flat plate embedded in a porous

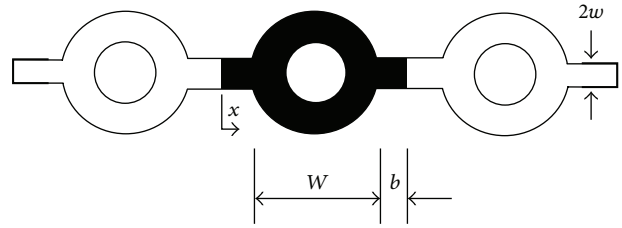


FIGURE 1: A heat pipe/fin radiating element.

medium. Yaghoobi and Torabi [28] applied DTM to solve the problems of convective and convective-radiative cooling of a lumped system with temperature-dependent specific heat.

More recently, Ganji et al. [29] applied DTM to the problem of convective-radiative straight fins with temperature-dependent thermal conductivity. But they considered zero dimensionless convective and radiative sink temperatures and without heat generation.

This paper is an analytical study of the thermal performance of a radiative fin with variation of thermal conductivity with temperature. The problem considers nonzero dimensionless radiation sink temperature, θ_s , and dimensionless heat generation, Q , which is the novelty of the present work. The resulting nonlinear differential equation is solved by DTM to evaluate the temperature distribution within the fin. Accordingly, the appropriate convergence study and comparison with previously published related articles, the results obtained using variational iteration method (VIM) [4, 30], Adomian decomposition method (ADM) [31], homotopy analysis method (HAM) [32], and numerical solution (NS), were employed in order to verify the accuracy of the proposed method. Using the temperature distribution, we express the efficiency of the fin in terms of radiation-conduction fin parameter, ψ , and thermal conductivity parameter, β . Because a broad range of governing parameters are investigated, the results should be useful in a number of engineering applications.

2. Description of the Problem

A typical heat pipe/fin space radiator is shown in Figure 1. Both surfaces of the fin are radiating to the outer space at a very low temperature, which is assumed equal to zero absolute. The fin has temperature-dependent thermal conductivity, k , which depends on temperature linearly, and fin is diffuse-grey with emissivity ϵ . The tube surfaces' temperature and the base temperature T_b of the fin are constant, and the convective exchange between the fin and the heat pipe is neglected. The temperature distribution within the fin is assumed to be one-dimensional, because the fin is assumed to be thin. Hence, only fin tip length b is considered as the computational domain.

The energy balance equation for a differential element of the fin [2–5] is given by

$$2w \frac{d}{dx} \left[k(T) \frac{dT}{dx} \right] - 2\epsilon\sigma (T^4 - T_s^4) + q = 0, \quad (1)$$

where $k(T)$ and σ are thermal conductivity and the Stefan-Boltzmann constant, respectively.

The thermal conductivity of the fin material is assumed to be a linear function of temperature [4–7] according to

$$k(T) = k_0 [1 + \lambda (T - T_a)], \quad (2)$$

where k_0 is the thermal conductivity at the T_a temperature of the fin and λ is the measure of variation of the thermal conductivity with temperature.

We introduce the following dimensionless parameters:

$$\begin{aligned} \theta &= \frac{T}{T_b}, & \theta_a &= \frac{T_a}{T_b}, & \theta_s &= \frac{T_s}{T_b}, \\ \xi &= \frac{x}{b}, & \beta &= \lambda T_b, & \psi &= \frac{\varepsilon \sigma b^2 T_b^3}{k_0 w}, & Q &= \frac{b^2 q}{T_b k_0}. \end{aligned} \quad (3)$$

The formulation of the fin problem [8–11] reduces to the following equation:

$$\frac{d}{d\xi} \left[(1 + \beta(\theta - \theta_a)) \frac{d\theta}{d\xi} \right] - \psi (\theta^4 - \theta_s^4) + Q = 0, \quad (4)$$

$$0 \leq \xi \leq 1,$$

with the following boundary conditions:

$$\left. \frac{d\theta}{d\xi} \right|_{\xi=0} = 0, \quad (5a)$$

$$\theta|_{\xi=1} = 1. \quad (5b)$$

3. Fin Efficiency

The heat transfer rate from the surfaces of a fin is found by applying the Stefan-Boltzmann law, namely [31],

$$Q_f = \int_0^b 2W\varepsilon\sigma T^4 dx. \quad (6)$$

Fin efficiency is defined as the ratio of energy radiated away by the fin to the energy that would be radiated if the entire fin was at the base temperature [33]:

$$Q_{f,ideal} = 2Wb\varepsilon\sigma T_b^4. \quad (7)$$

Employing the dimensionless parameters in (3), fin efficiency is expressed as

$$\eta = \frac{Q_f}{Q_{f,ideal}} = \frac{1}{b} \int_0^b \frac{T^4}{T_b^4} dx = \int_0^1 \theta^4 d\xi. \quad (8)$$

4. Fundamentals of Differential Transformation Method

Let $x(t)$ be analytic in a domain D , and let $t = t_i$ represent any point in D . The function $x(t)$ is then represented by a power

series whose center is located at t_i . A Taylor series expansion function of $x(t)$ about t_i takes the form

$$x(t) = \sum_{j=0}^{\infty} \frac{(t - t_i)^j}{j!} \left[\frac{d^j x(t)}{dt^j} \right]_{t=t_i}, \quad \forall t \in D. \quad (9)$$

The particular case of (9) when $t_i = 0$ is the Maclaurin series of $x(t)$ and is expressed as

$$x(t) = \sum_{j=0}^{\infty} \frac{(t)^j}{j!} \left[\frac{d^j x(t)}{dt^j} \right]_{t=0}. \quad (10)$$

As explained in [34] the differential transformation of the function $x(t)$ is defined as follows:

$$X(j) = \frac{(H)^j}{j!} \left[\frac{d^j x(t)}{dt^j} \right]_{t=0}, \quad (11)$$

where $x(t)$ is the original function and $X(j)$ is the transformed function (commonly referred to as the T-function). The differential spectrum of $X(j)$ is confined within the interval $t \in [0, H]$, where H is a constant. The differential inverse transform of $X(j)$ is defined as follows:

$$x(t) = \sum_{j=0}^{\infty} \left(\frac{t}{H} \right)^j X(j). \quad (12)$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. Values of the function $X(j)$ are referred to as discretely; that is, $X(0)$ is known as the zero discrete, $X(1)$ is the first discrete, and $X(j)$ is the j th discrete. The more discretely are available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of T -function $X(j)$, and its value is given by the sum of the T -function with $(t/H)^j$ as its coefficient. In real applications, with the right choice of constant H , the larger the values of argument j are, the more rapid the discretely of spectrum are reduced. The function $x(t)$ is expressed by a finite series and (12) can be written as

$$x(t) = \sum_{j=0}^n \left(\frac{t}{H} \right)^j X(j), \quad (13)$$

where $n + 1$ is total number of polynomial terms used in the DTM. Mathematical operations performed by differential transform method are listed in Table 1.

5. Solution Protocols

5.1. Solution Using the Differential Transformation Method DTM. Now we apply differential transformation method to

TABLE 1: The fundamental operations of differential transform method.

Original function	Transformed function
$x(t) = \alpha f(t) \pm \beta g(t)$	$X(j) = \alpha F(j) \pm \beta G(j)$
$x(t) = \frac{df(t)}{dt}$	$X(j) = (j+1)F(j+1)$
$x(t) = \frac{d^2 f(t)}{dt^2}$	$X(j) = (j+1)(j+2)F(j+2)$
$x(t) = t^m$	$X(j) = \delta(j-m) = \begin{cases} 1 & j = m \\ 0 & j \neq m \end{cases}$
$x(t) = \exp(\lambda t)$	$X(j) = \frac{\lambda^j}{j!}$
$x(t) = f(t)g(t)$	$X(j) = \sum_{l=0}^j F(l)G(j-l)$

(4). Taking the differential transform of (4) with respect to ξ , and considering $H = 1$ according to Table 1, we gives

$$\begin{aligned}
& (1 - \beta\theta_a)(j+2)(j+1)\Theta(j+2) \\
& + \beta \left(\sum_{l=0}^j \Theta(l)(j+2-l)(j+1-l)\Theta(j+2-l) \right) \\
& + \beta \left(\sum_{l=0}^j (l+1)\Theta(l+1)(j+1-l)\Theta(j+1-l) \right) \\
& - \psi \left(\sum_{m=0}^j \Theta(j-m) \right. \\
& \quad \times \left(\sum_{v=0}^m \Theta(m-v) \left(\sum_{u=0}^v \Theta(v-u)\Theta(u) \right) \right) \Big) \\
& + (\psi\theta_s^4 + Q)\delta(j) = 0.
\end{aligned} \tag{14}$$

From boundary condition in (5a), and employing the second formulation from Table 1, we obtain

$$\Theta(1) = 0. \tag{15}$$

Supposing that

$$\Theta(0) = A \tag{16}$$

and using (14)–(16) one can obtain $\Theta(j+2)$ as follows:

$$\Theta(2) = -\frac{1}{2} \frac{Q - \psi A^4 + \psi\theta_s^4}{1 + \beta A - \beta\theta_a}, \tag{17a}$$

$$\Theta(3) = 0, \tag{17b}$$

$$\begin{aligned}
\Theta(4) = & -\frac{1}{24} \left((Q - \psi A^4 + \psi\theta_s^4) \right. \\
& \times (4\psi A^4 - 4\psi A^3\beta\theta_a + \psi A^4\beta \\
& \left. + 3\beta Q + 3\beta\psi\theta_s^4) \right) (1 + \beta A - \beta\theta_a)^{-3}
\end{aligned} \tag{17c}$$

$$\begin{aligned}
\Theta(5) &= 0 \\
&\vdots
\end{aligned} \tag{17d}$$

where A is constant, and we will calculate it with considering another boundary condition in (5b) in point $\xi = 1$.

The above process is continuous. Substituting (17a), (17b), (17c), and (17d) into the main equation based on DTM, it can be obtained that the closed form of the solutions is

$$\begin{aligned}
\theta(\xi) = & A - \frac{1}{2} \frac{Q - \psi A^4 + \psi\theta_s^4}{1 + \beta A - \beta\theta_a} \xi^2 \\
& - \frac{1}{24} \left((Q - \psi A^4 + \psi\theta_s^4) \right. \\
& \times (4\psi A^4 - 4\psi A^3\beta\theta_a + \psi A^4\beta \\
& \left. + 3\beta Q + 3\beta\psi\theta_s^4) \right) (1 + \beta A - \beta\theta_a)^{-3} \\
& \times \xi^4 + \dots
\end{aligned} \tag{18}$$

To obtain the value of A , we substitute the boundary condition from (5b) into (18) in point $\xi = 1$. So, we have

$$\begin{aligned}
\theta(1) = & A - \frac{1}{2} \frac{Q - \psi A^4 + \psi\theta_s^4}{1 + \beta A - \beta\theta_a} \\
& - \frac{1}{24} \left((Q - \psi A^4 + \psi\theta_s^4) \right. \\
& \times (4\psi A^4 - 4\psi A^3\beta\theta_a + \psi A^4\beta \\
& \left. + 3\beta Q + 3\beta\psi\theta_s^4) \right) (1 + \beta A - \beta\theta_a)^{-3} + \dots \\
& = 1.
\end{aligned} \tag{19}$$

Solving (19) gives the value of A . The resultant equation can be solved by using Newton-Raphson iterative technique for determination of unknown tip temperature A . We employed the Maple's built-in *Roots* command which numerically approximates the roots of an algebraic function using the specified method, such as Newton-Raphson, bisection, secant, and fixed-point iteration methods, and returns the specified outputs. This command uses the Newton-Raphson method by default. Also, the estimated initial value was picked as a starting number near the solution using the graph to locate sufficiently close number to the root.

As an example, let us assume that $\beta = 0.4$, $\psi = 1$, $\theta_a = 0.2$, $\theta_s = 0.2$, and $Q = 0.1$. Therefore, the value of A , applying $n = 30$ which will be used in this paper, will be obtained as follows:

$$A = 0.8294001254. \quad (20)$$

Substituting this obtained A parameter in (18), the temperature profile of fin for this special case will be as follows:

$$\begin{aligned} \theta(\xi) = & 0.8294001254 + 0.1484360740\xi^2 \\ & + 0.019031859\xi^4 + 0.002675563\xi^6 + \dots \end{aligned} \quad (21)$$

The VIM solution of (4), with $\theta_a = \theta_s = Q = 0$, is obtained in the following form [4]:

$$\begin{aligned} \theta_{\text{VIM}} \cong & A + \frac{1}{2}A^4\psi\xi^2 - \frac{1}{2}A^5\beta\psi\xi^2 - \frac{1}{24}A^7(-4 + 3A\beta)\psi^2\xi^4 \\ & + \frac{1}{20}A^{10}\psi^3\xi^6 + \frac{1}{112}A^{13}\psi^4\xi^8 + \frac{1}{1440}A^{16}\psi^5\xi^{10}. \end{aligned} \quad (22)$$

And the ADM solution of (4), with $\theta_a = \theta_s = Q = 0$, is obtained in the following form [31]:

$$\begin{aligned} \theta_{\text{ADM}} \cong & A + \frac{1}{2}A^4\psi\xi^2 - \frac{1}{2}\beta A^5\psi\xi^2 + \frac{1}{6}A^7\psi^2\xi^4 + \frac{1}{2}\beta^2 A^6\psi\xi^2 \\ & - \frac{11}{24}\beta A^8\psi^2\xi^4 + \frac{13}{180}A^{10}\psi^3\xi^6 - \frac{1}{2}\beta^3 A^7\psi\xi^2 \\ & + \frac{25}{24}\beta^2 A^9\psi^2\xi^4 - \frac{17}{45}\beta A^{11}\psi^3\xi^6 + \frac{71}{2520}A^{13}\psi^4\xi^8. \end{aligned} \quad (23)$$

5.2. Solution Using the Boubaker Polynomials Expansion Scheme (BPES). The Boubaker polynomials expansion scheme (BPES) [34–52] is a resolution protocol which has been successfully applied to several applied-physics and mathematics problems. The BPES protocol ensures the validity of the related boundary conditions regardless of main equation features. The Boubaker polynomials expansion scheme (BPES) is based on the Boubaker polynomials first derivatives properties:

$$\begin{aligned} \left. \sum_{q=1}^N B_{4q}(x) \right|_{x=0} &= -2N \neq 0, \\ \left. \sum_{q=1}^N B_{4q}(x) \right|_{x=r_q} &= 0, \\ \left. \sum_{q=1}^N \frac{dB_{4q}(x)}{dx} \right|_{x=0} &= 0, \\ \left. \sum_{q=1}^N \frac{dB_{4q}(x)}{dx} \right|_{x=r_q} &= \sum_{q=1}^N H_q, \end{aligned} \quad (24)$$

with $H_n = B'_{4n}(r_n) = ((4r_n[2-r_n^2] \times \sum_{q=1}^n B_{4q}(r_n)/B_{4(n+1)}(r_n)) + 4r_n^3)$.

Several solutions have been proposed through the BPES in many fields such as numerical analysis [34–37], theoretical physics [38–41], mathematical algorithms [42], heat transfer [43], homodynamic [44, 45], material characterization [46], fuzzy systems modeling [47–50], and biology [51, 52].

In reference to (4), the BPES is applied to the system

$$\begin{aligned} \frac{d}{d\xi} \left[(1 + \beta(\theta(\xi) - \theta_a)) \frac{d\theta}{d\xi} \right] \\ - \psi(\theta(\xi)^4 - \theta_s^4) + Q = 0, \quad \xi \in [0; 1], \\ \frac{d\theta(\xi)}{d\xi} \Big|_{\xi=0} = 0, \\ \theta(\xi) \Big|_{\xi=1} = 1 \end{aligned} \quad (25)$$

through setting the expression

$$\theta(\xi) = \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}(r_k\xi)}{dt}, \quad (26)$$

where B_{4k} are the $4k$ -order Boubaker polynomials, r_k are B_{4k} minimal positive roots, N_0 is a prefixed integer, and $\lambda_k|_{k=1 \dots N_0}$ are unknown pondering real. The BPES solution is obtained through the following steps.

- (i) Integrate, for a given value of N_0 , the whole expressions given in the left side of (4) along the given domain. By introducing expression (26) in (25), boundary conditions become redundant since they already verified by the proposed expansion (26). Consecutively, and by majoring the quadrature terms, the problem is transformed in a linear system with unknown real variables $\lambda_k|_{k=1 \dots N_0}$. Consequently, it comes for (4) and (25)-(26) that

$$\begin{pmatrix} \vartheta_{1;1} & \dots & \dots & \vartheta_{1;N_0} \\ \dots & \vartheta_{2;2} & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vartheta_{N_0;1} & \dots & \dots & \vartheta_{N_0;N_0} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{N_0} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{N_0} \end{pmatrix}, \quad (27)$$

with the matrix standard form

$$\begin{aligned} [\vartheta] \times [\lambda] &= [B], \\ \text{with } [\vartheta] &= \begin{pmatrix} \vartheta_{1;1} & \dots & \dots & \vartheta_{1;N_0} \\ \dots & \vartheta_{2;2} & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vartheta_{N_0;1} & \dots & \dots & \vartheta_{N_0;N_0} \end{pmatrix}, \\ [\lambda] &= \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{N_0} \end{pmatrix}, \quad [B] = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{N_0} \end{pmatrix}. \end{aligned} \quad (28)$$

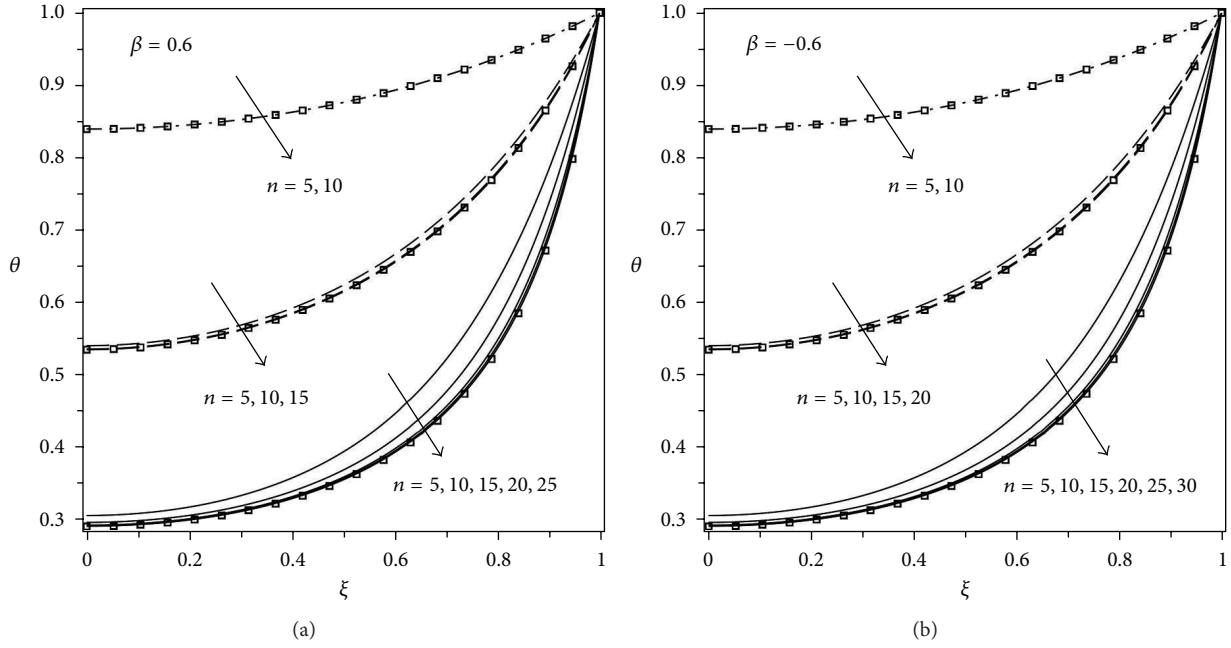


FIGURE 2: Convergence test of the dimensionless temperature with DTM, for $\psi = 1$ (dash-dot line), $\psi = 10$ (dash line), $\psi = 100$ (solid line), and NS (box symbol).

The system (28) is hence reduced to approximately $(64N_0)^3$ arithmetical operations and solved using the Householder algorithm [53, 54].

(ii) Incremente N_0 .

(iii) Test the convergence of the coefficients $\lambda_k^{(\text{Sol.})}|_{k=1 \dots N_0}$ until stability of the solution.

The final result is hence (obtained for $N_0 = 233$)

$$\theta_{\text{BPES}}(\xi) = \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k^{(\text{Sol.})} \times \frac{dB_{4k}(r_k \xi)}{dt}. \quad (29)$$

6. Results and Discussion

In this section, we divide our study into three subsections. Firstly, the convergence rate of the DTM and BPES is checked. Secondly, comparison with previously published related articles is employed in order to verify the accuracy of the proposed method. Finally, some figures are introduced regarding the effects of physically applicable parameters such as thermal conductivity parameter, β , dimensionless temperature whose $k(T)$ is constant, θ_a , dimensionless radiation sink temperature, θ_s , radiation-conduction parameter, ψ , and dimensionless heat generation parameter, Q , on the temperature distribution within the fin. Moreover, for all numerical results which are reported here, the following values of variables are used unless otherwise indicated by graphs or the table:

$$\begin{aligned} \beta &= 0.4, & \psi &= 1, & \theta_a &= 0.2, \\ \theta_s &= 0.2, & Q &= 0.1. \end{aligned} \quad (30)$$

6.1. Convergence Study. Opposite to the convergence of the Boubaker Polynomials Expansion Scheme (BPES), which is intrinsically tested (via the value of N_0 , Section 5.2), the proposed differential transformation method DTM adopts an iterative procedure to obtain the high-order Taylor series. Since the Taylor series is an infinite series, the differential transformation should theoretically comprise an infinite series. However, the present results indicate that in practice a small n -value (i.e., $n = 30$) is sufficient to provide an accurate solution.

Figure 2 depicts the convergence of the dimensionless temperature for six different cases. From Figure 2 it is clearly visible that for two cases of $\beta = -0.6$ and $\beta = 0.6$ when $\psi = 1$, more than 10 terms are needed to obtain the value of the θ . Also, it is seen that if $\beta = -0.6$ and $\psi = 100$ are chosen to plot the temperature within the fin, at least 30 terms are required to obtain an accurate value of the θ . As more terms are taken, the θ converges to its exact value. Therefore, the numerical results from the DTM approach which are presented in the next sections were obtained by taking sufficient terms $n = 30$ to the temperature solution. It should be noted that the final results from DTM are in good agreement with the Runge-Kutta-Fehlberg method which is a well-tested numerical solution. Moreover, since the problem is a highly nonlinear radiative equation, an exact analytical solution cannot be found for (4).

6.2. Validation of the Results. To check the accuracy of the present solution, by considering $\theta_a = \theta_s = Q = 0$ our problem is converted to a simpler case which was studied in four pioneering works [4, 30–32] in this field. The results obtained from DTM analysis are compared with VIM [4, 30]

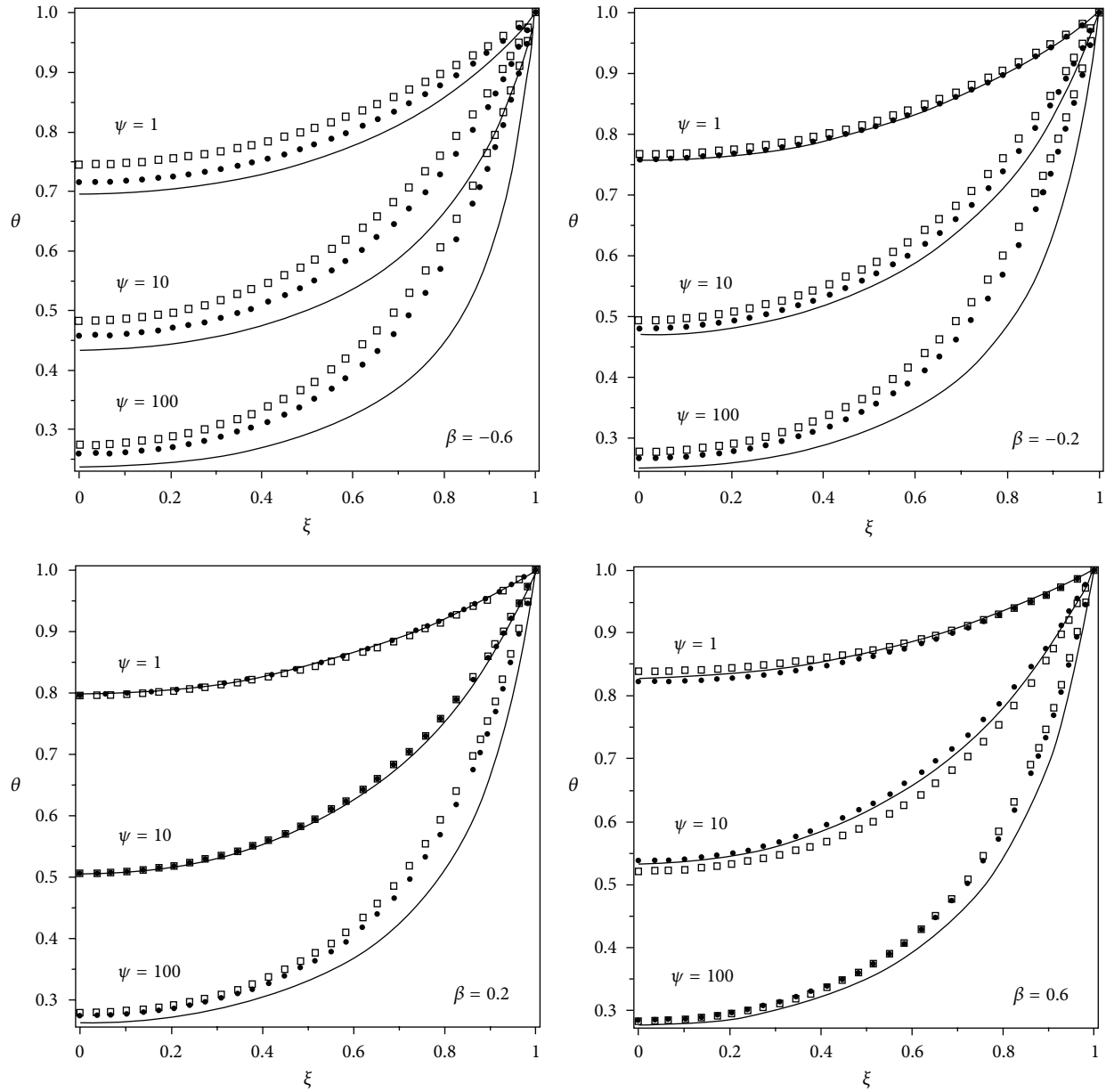


FIGURE 3: Comparison for dimensionless temperature variation for various β and ψ between DTM (solid line), VIM (box symbol), and ADM (solid-circle symbol).

and ADM [31] in Figure 3. As it can be seen, this method leads to acceptable results compared with those methods.

For the case of different values for thermal conductivity and radiation-conduction fin parameter, results of the present analysis are tabulated against the ADM, VIM, and again with numerical solution (NS), of the fourth-fifth order Runge-Kutta-Fehlberg method using the Maple package, in Table 2. A very interesting agreement between the results is observed, which confirms the validity of the DTM.

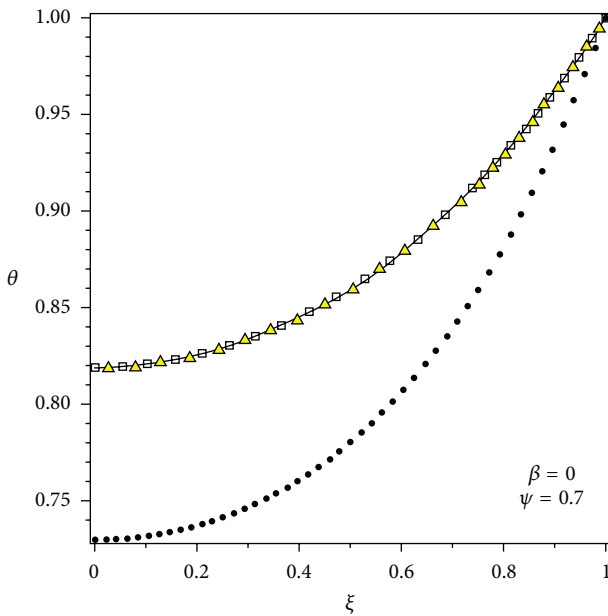
In Figure 4, the efficiency of the DTM and BPES solution technique for $\beta = 0$ can be illustrated with comparison respect to HAM [32] and NS. Also by means of these comparisons, it can be shown that DTM and BPES are a better alternatives in the solution of such problems.

6.3. Results of Present Study. Figure 5 shows the behavior of fin tip temperature, A , relative to the thermal conductivity parameters, β , and the radiation-conduction fin parameter, ψ . Figure 5 clearly demonstrates that increasing in the values of thermal conductivity parameter produces increase in values of fin tip temperature.

Figure 6 illustrates the dimensionless temperature distributions along the fin surface with β varying from -0.3 to 0.3 . The curve marked $\beta = 0$ represents the case when the thermal conductivity is a constant and its value is k_0 . The curves with $\beta > 0$ correspond to fin materials whose thermal conductivity increases as temperature increases. The converse is true of curves with $\beta < 0$. As the parameter β increases, the average thermal conductivity of the material increases,

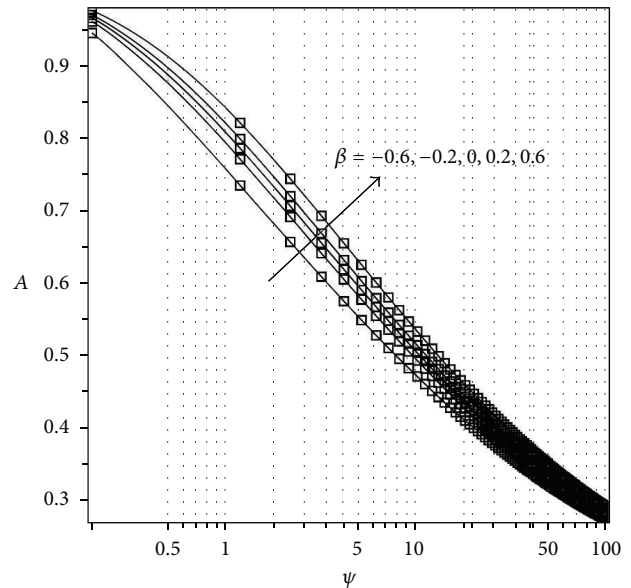
TABLE 2: The results of DTM, VIM, ADM, and NS for dimensionless temperature.

ξ	$\beta = 0.4, \psi = 1$				$\beta = 0.2, \psi = 0.5$			
	DTM	VIM [4]	ADM [31]	NS	DTM	VIM [4]	ADM [31]	NS
0	0.8133693583	0.8145587262	0.8122259487	0.8133695812	0.8679116912	0.8677934850	0.8678227415	0.8679120939
0.05	0.8137822645	0.8149299130	0.8126321774	0.8137824427	0.8682139355	0.8680864947	0.8681245987	0.8682140666
0.10	0.8150223528	0.8160457268	0.8138530912	0.8150225764	0.8691214181	0.8689665327	0.8690309943	0.8691217405
0.15	0.8170937494	0.8179129642	0.8158953383	0.8170939665	0.8706363930	0.8704366340	0.8705444008	0.8706367938
0.20	0.8200033838	0.8205430743	0.8187698837	0.8200036083	0.8727626384	0.8725018855	0.8726689432	0.8727630106
0.25	0.8237610702	0.8239523423	0.8224918404	0.8237612930	0.8755054875	0.8751694686	0.8754104067	0.8755058710
0.30	0.8283796228	0.8281621511	0.8270802450	0.8283798480	0.8788718740	0.8784487205	0.8787762466	0.8788722707
0.35	0.8338750084	0.8331993221	0.8325577851	0.8338752350	0.8828703906	0.8823512130	0.8827756075	0.8828707739
0.40	0.8402665394	0.8390965420	0.8389504951	0.8402667643	0.8875113635	0.8868908496	0.8874193466	0.8875117414
0.45	0.8475771116	0.8458928803	0.8462874349	0.8475773321	0.8928069419	0.8920839837	0.8927200706	0.8928073243
0.50	0.8558334915	0.8536344030	0.8546003708	0.8558337131	0.8987712062	0.8979495558	0.8986921830	0.8987715631
0.55	0.8650666613	0.8623748942	0.8639234800	0.8650668754	0.9054202946	0.9045092538	0.9053519505	0.9054206440
0.60	0.8753122271	0.8721766901	0.8742931088	0.8753124399	0.9127725525	0.9117876937	0.9127175865	0.9127729094
0.65	0.8866109039	0.8831116398	0.8857476040	0.8866111038	0.9208487049	0.9198126293	0.9208093621	0.9208490358
0.70	0.8990090866	0.8952622030	0.8983272589	0.8990092815	0.9296720572	0.9286151827	0.9296497443	0.9296723561
0.75	0.9125595243	0.9087226999	0.9120743995	0.9125597131	0.9392687258	0.9382301051	0.9392635667	0.9392690380
0.80	0.9273221155	0.9236007249	0.9270336526	0.9273222778	0.9496679059	0.9486960683	0.9496782441	0.9496681990
0.85	0.9433648484	0.9400187469	0.9432524344	0.9433650219	0.9609021785	0.9600559865	0.9609240279	0.9609023769
0.90	0.9607649133	0.9581159096	0.9607817046	0.9607650173	0.9730078657	0.9723573730	0.9730343160	0.9730082160
0.95	0.9796100255	0.9780500563	0.9796770290	0.9796103729	0.9860254404	0.9856527331	0.9860460201	0.9860261076
1	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000

FIGURE 4: Comparison for dimensionless temperature variation for $\beta = 0$ and $\psi = 0.7$ using DTM (solid line), BPES (triangle), NS (box symbol), and HAM (solid-box symbol).

and as expected, the result is a gradual increase in the local temperature.

In Figure 7 we illustrate the effect of the radiation-conduction parameter, ψ , on the temperature distribution

FIGURE 5: Variation of dimensionless fin tip temperature for various β with DTM (solid line) and NS (box symbol).

in the fin. As the radiative transport becomes stronger, the radiative cooling becomes more effective, which in turn causes the lowering of temperatures in the fin.

Figure 8 shows that as the θ_a increases, the temperature within the fin decreases. This is because of the fact that, as θ_a becomes lower, the $k(T)$ becomes more sensitive

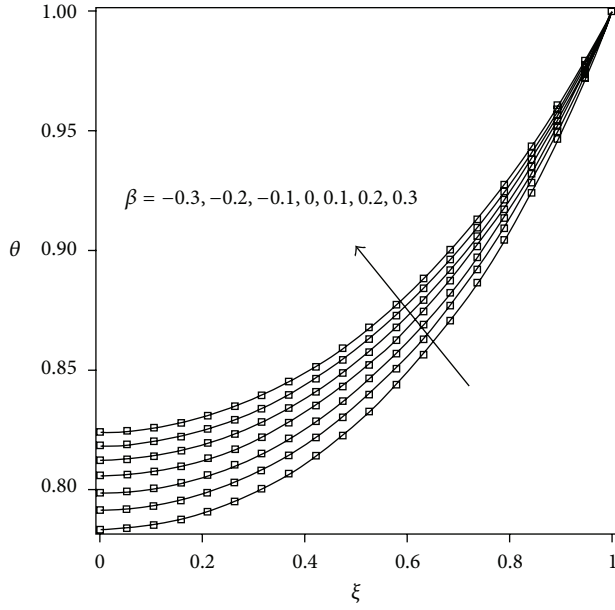


FIGURE 6: Dimensionless temperature in fin for various β with DTM (solid line) and NS (box symbol).

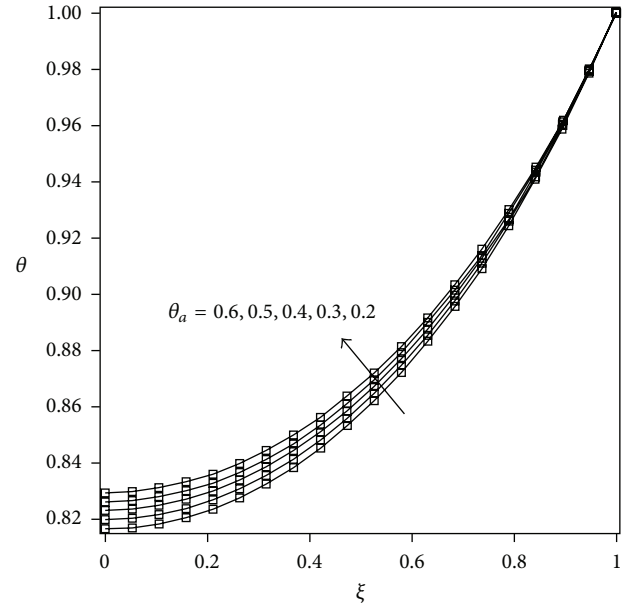


FIGURE 8: Dimensionless temperature in fin for various θ_a with DTM (solid line) and NS (box symbol).

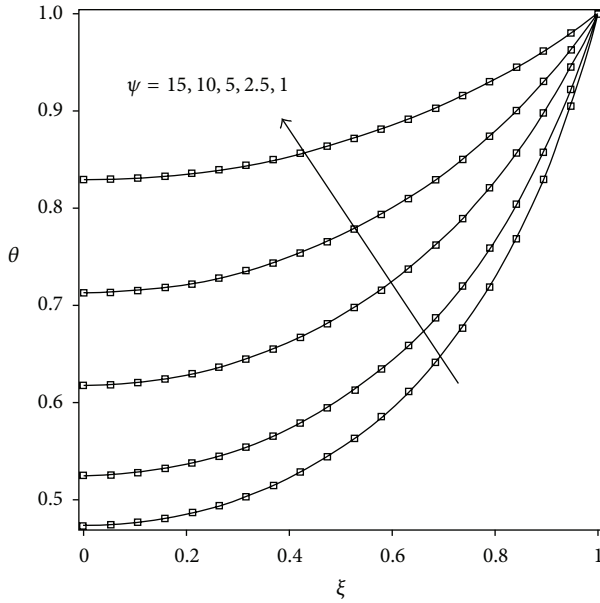


FIGURE 7: Dimensionless temperature in fin for various ψ with DTM (solid line) and NS (box symbol).

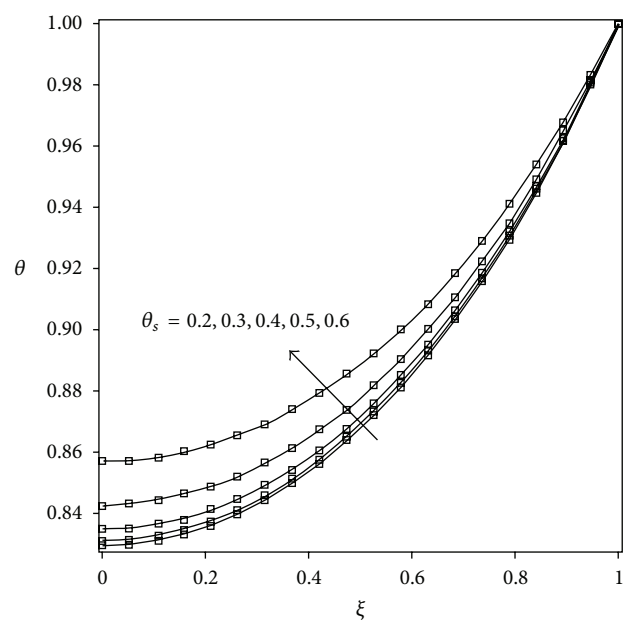


FIGURE 9: Dimensionless temperature in fin for various θ_s with DTM (solid line) and NS (box symbol).

to the temperature. Moreover, Figure 9 displays the effect of the radiation sink temperature, θ_s , on the temperature distribution in the fin. As the radiation sink temperature increases, the radiative heat loss decreases, resulting in higher temperatures in the material. Figure 10 allows us to assess the effect of the dimensionless heat generation in (4), which varies from 0.2 to 0.6. As expected, an increase in the value of Q causes an increase in the value of θ within the fin.

Figure 11 shows the behavior of the fin efficiency relative to ψ and β . Figure 11 clearly demonstrates that increasing

in the values of radiation-conduction parameter produces a decrease in the value of the fin efficiency. In addition, the fin efficiency increases as the thermal conductivity parameter, β , increases. To explain the effect of parameter β , we note that while the temperature increases as β increases (see Figure 6), the temperature of the fin becomes closer to T_b , and therefore, from (6), Q_f increases resulting in an increase in the value of η . For all results reported in Figures 5–11 the DTM results were checked against NS which were in excellent agreement

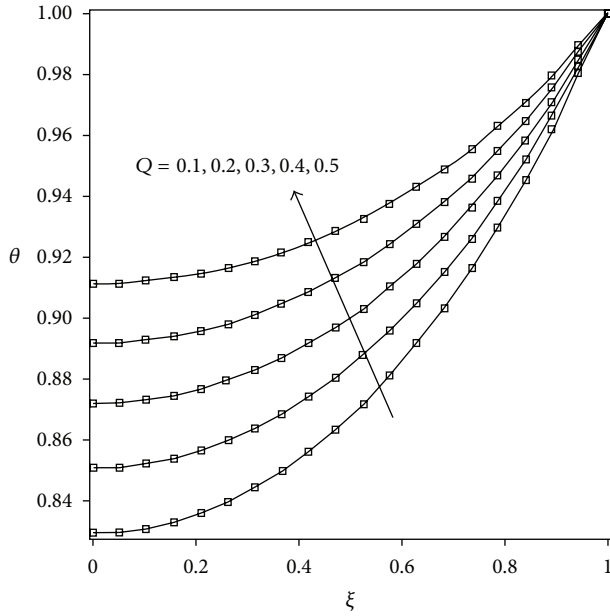


FIGURE 10: Dimensionless temperature in fin for various Q with DTM (solid line) and NS (box symbol).

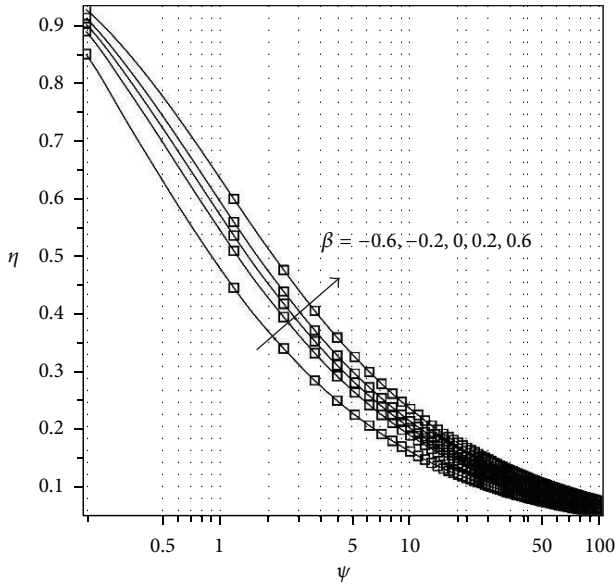


FIGURE 11: Variation of the fin efficiency with ψ and β with DTM (solid line) and NS (box symbol).

with each other. It would be useful to mention that DTM was tested for moving convective-radiative fins [55] against numerical solution, and results of both methods were in good agreement with each other.

7. Conclusion

In this study, the DTM and BPES have been utilized to derive approximate explicit analytical solution for nonlinear radiative radial fin heat transfer problem with temperature-dependent thermal conductivity and heat generation. The

figures and table clearly show that these methods provide excellent approximations to the solution of this nonlinear equation with high accuracy. As the radiation-conduction parameter increases the effect is to lower the fin temperature. As expected, it was observed that increasing in the value of β or Q increases the temperature distribution through the fin. The collection of temperature graphs should be useful in the study and design of a variety of engineering systems where fins are attached to realize heat transfer enhancement.

Nomenclature

- b : Fin tip length, m
- D : Domain
- H : Constant
- K : Temperature-dependent thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$
- K_0 : Thermal conductivity at the base temperature, $\text{W m}^{-1} \text{K}^{-1}$
- q : Volumetric heat generation, W m^{-3}
- Q : Dimensionless volumetric heat generation
- Q_f : Heat transfer rate from the surfaces of a fin
- T : Temperature, K
- T_b : Fin's base temperature, K
- T_s : Radiation sink temperature, K
- $X(k)$: Transformed analytical function
- $x(t)$: Original analytical function
- w : Semithickness of the fin, m.

Greek Symbols

- β : Thermal conductivity parameter
- ε : Emissivity
- η : Fin efficiency
- λ : Slope of the thermal conductivity-temperature curve, K^{-1}
- σ : Stefan-Boltzmann constant, $\text{W m}^{-2} \text{K}^{-4}$
- θ : Dimensionless temperature
- θ_s : Dimensionless radiation sink temperature
- ψ : Radiation-conduction fin parameter.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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