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Research Article

EPQ Model for Trended Demand with Rework and Random Preventive Machine Time

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Economic production quantity (EPQ) inventory model for trended demand has been analyzed with rework facility and stochastic preventive machine time. Due to the complexity of the model, search method is proposed to determine the best optimal solution. A numerical example and sensitivity analysis are carried out to validate the proposed model. From the sensitivity analysis, it is observed that the rate of change of demand has significant impact on the optimal inventory cost. The model is very sensitive to the production and demand rate.

1. Introduction

An item that does not satisfy quality standards but can be attained after reprocess is termed as a recoverable item and the process is known as rework. It is observed that in an industrial sector, the rework reduced production cost and maintained quality standard of the item. Schrady [1] debated rework process. Khouja [2] formulated an economic lot-size and shipment policy by incorporating a fraction of defective items and direct rework. Koh et al. [3] and Dobos and Richter [4] discussed two production policies with options to order new products externally or recover old products. Chiu et al. [5] analyzed an imperfect rework process for EPQ model with repairable and scrapped items. Jamal et al. [6] advocated the policy for rework of defective items in the same cycle which was extended by Cárdenas-Barrón [7]. Widyadana and Wee [8] gave an analysis of these problems using an algebraic approach. Chiu [9] and Chiu et al. [10] discussed EPQ model by allowing shortages and considering service level constraint. Yoo et al. [11] discussed an EPQ model with imperfect production quality, imperfect inspection, and rework.

Meller and Kim [12], Sheu and Chen [13] and Tsou and Chen [14] studied Variants of EPQ model with preventive maintenance. Abboud et al. [15] analyzed an economic order quantity model by considering machine unavailability owing to preventive maintenance and shortage. Chung et al. [16] extended the previous model to compute an economic production quantity for deteriorating inventory model with stochastic machine unavailable time and shortage. Wee and Widyadana [17] revisited the previous model incorporating rework.

In this paper, we analyze an economic production quantity (EPQ) model with rework and random preventive maintenance time together when demand is increasing function of time. The consideration of random preventive maintenance time, rework, and trended demand in the model increases its applicability in the electronic and automobile industries. In this production system, produced items are inspected immediately. Defective items are stocked and reworked at the end of the production uptime. We will call these items as recoverable items. Out of these recoverable items, the fraction of the items will be labeled as "new" and rest will be scrapped. Preventive maintenance is performed at the end of the rework process, and the maintenance time is assumed to be random. When demand is increasing, shortages may occur which will be treated as lost sales in this study. It is observed that the rate of change of demand has significant impact on the optimal

Table 1: Sensitivity analysis of T_{1a} and total cost when preventive maintenance time is uniformly and exponentially distributed.

Parameter	Percentage change	Uniform distribution		Exponential distribution $\lambda = 20$	
		T_{1a}	TCT	T_{1a}	TCT
A	−40%	0.1118836	2217.023407	0.165363168	3105.82179
	−20%	0.1127598	2396.444313	0.168010377	3225.883231
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1145095	2751.208003	0.173362159	3460.599558
	40%	0.1153831	2926.585673	0.176064945	3575.33395
P	−40%	0.119	0.224	0.873011973	3769.650118
	−20%	0.116	0.220356527	0.280588937	3224.896639
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.0784111	2671.944386	0.123903512	3421.121046
	40%	0.0598742	2748.060997	0.09758743	3473.297234
P_1	−40%	0.1252239	2840.779326	0.179184671	3667.154126
	−20%	0.117738	2670.650277	0.17374104	3462.440106
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1110459	2512.591082	0.168712009	3266.965218
	40%	0.1093473	2469.539689	0.167344624	3212.637461
a	−40%	0.0468611	2285.690117	0.081365729	2483.982152
	−20%	0.0736194	2419.311543	0.118822843	2928.274545
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1801641	2784.239588	0.248801609	3743.389279
	40%	0.313401	3147.605249	0.385620567	4166.830717
b	−40%	0.1133496	2572.975288	0.170020919	3336.335153
	−20%	0.113492	2573.73768	0.170347969	3340.233353
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1137789	2575.262185	0.171008033	3348.022553
	40%	0.1139235	2576.024314	0.1713411	3351.913594
x	−40%	0.1075459	2440.729188	0.165578265	3180.040146
	−20%	0.1105098	2506.778232	0.168074836	3261.520565
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1169347	2644.037111	0.173391457	3427.954424
	40%	0.1204229	2715.549502	0.176225505	3513.094599
x_1	−40%	0.1131507	2451.633648	0.170247776	3220.242525
	−20%	0.1133924	2513.000758	0.170462207	3282.119979
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1138787	2636.131758	0.17089215	3406.270464
	40%	0.1141232	2697.89656	0.171107662	3468.544359
h	−40%	0.1163864	2035.802579	0.196138784	2497.162768
	−20%	0.114993	2306.766649	0.18157314	2935.554059
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1123112	2839.068303	0.162020977	3729.597492
	40%	0.11102	3100.535386	0.15486872	4096.262909
h_1	−40%	0.113733	2555.165605	0.171388527	3315.223296
	−20%	0.113684	2564.8349	0.171031253	3329.691775
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1135862	2584.160837	0.170325715	3358.53567
	40%	0.1135373	2593.817484	0.169977356	3372.911591

Table 1: Continued.

Parameter	Percentage change	Uniform distribution		Exponential distribution $\lambda = 20$	
		T_{1a}	TCT	T_{1a}	TCT
S	−40%	0.1119752	2570.475444	0.15263831	3147.25954
	−20%	0.1129966	2572.958931	0.162618107	3255.743808
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1140719	2575.549497	0.1774544	3418.940914
	40%	0.1143897	2576.310317	0.183312722	3483.929611
S_c	−40%	0.113643	2453.561123	0.170776446	3223.952673
	−20%	0.113639	2514.030563	0.170726744	3284.041215
	0	0.1136351	2574.499976	0.170676999	3344.12915
	20%	0.1136311	2634.969362	0.17062721	3404.216467
	40%	0.1136271	2695.438718	0.170577378	3464.303169

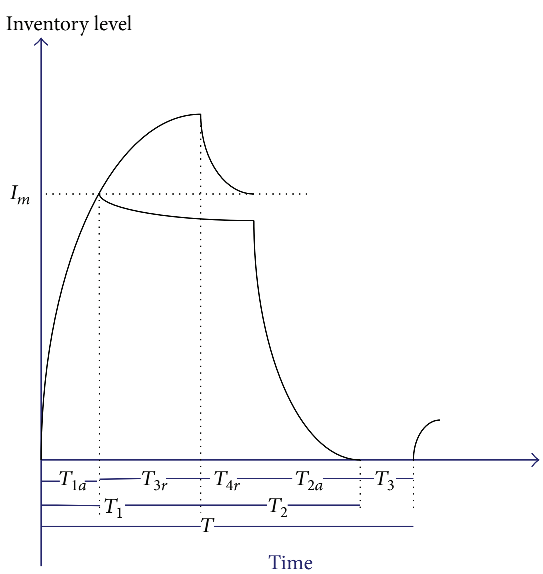

Figure 1: Inventory status of serviceable items with lost sales.

inventory cost. It is suggested that when demand is trended, preventive maintenance time should be controlled by recruiting well-qualified technicians. The uniform distribution and exponential distribution for preventive maintenance time are explored. The paper is organized as follows: Section 2 is about the mathematical development of the proposed problem. In Section 3, example and sensitivity are given. Conclusions are highlighted in Section 4.

2. Mathematical Model

Assumptions. (1) The inventory system under consideration deals with single item. (2) Standard quality items must be greater than the demand. (3) The production and rework rates are constant. (4) The demand rate, $R(t) = a(1 + bt)$, is increasing function of time, where $a > 0$ is scale demand and $0 < b < 1$ denotes the rate of change of demand. (5) Setup cost for rework process is zero or negligible. (6) Recoverable items are spawned during the production uptime, and scrapped items are produced during the rework uptime.

The status of the serviceable inventory is depicted in Figure 1. Production occurs during $[0, T_{1a}]$. In phase x defective items per unit time are to be reworked. The rework process starts at the end of the predetermined production uptime. The rework time ends at T_{3r} time period. The different production processes of the material and defective items result in different product rates. During the rework, some rejected and scrapped items will occur. LIFO policy is assumed for the production system. So, serviceable items during the rework uptime are utilized before the fresh items from the production in uptime. The new production run is started when the inventory level reaches zero at the end of T_{2a} time period. It may happen that the production may not start at T_{2a} time period because unavailability of the machine is randomly distributed with a probability density function $f(t)$. The nonavailability of machine may result in shortage during T_3 time period. The production will resume after the T_3 time period.

From the above description, the inventory level in a production uptime period is governed by the differential equation

$$\frac{dI_{1a}(t_{1a})}{dt_{1a}} = P - R(t_{1a}) - x, \quad 0 \le t_{1a} \le T_{1a}. \tag{1}$$

The inventory level in a rework uptime is

$$\frac{dI_{3r}(t_{3r})}{dt_{3r}} = P_1 - R(t_{3r}) - x_1, \quad 0 \le t_{3r} \le T_{3r}. \tag{2}$$

The inventory level in a production downtime is

$$\frac{dI_{2a}(t_{2a})}{dt_{2a}} = -R(t_{1a}), \quad 0 \le t_{2a} \le T_{2a}. \tag{3}$$

The inventory level in a rework downtime is

$$\frac{dI_{4r}(t_{4r})}{dt_{4r}} = -R(t_{4r}), \quad 0 \le t_{4r} \le T_{4r}. \tag{4}$$

Under the assumption of LIFO production system, the inventory level of good items depletes at a constant rate during rework uptime and downtime. The inventory level is governed by

$$\frac{dI_{3a}(t_{3a})}{dt_{3a}} = 0, \quad 0 \le t_{3a} \le T_{3r} + T_{4r}. \tag{5}$$

Using $I_{1a}(0) = 0$, the solution of (1) is

$$I_{1a}(t_{1a}) = (P - a - x)\,t_{1a} - \frac{ab}{2}t_{1a}^2, \quad 0 \le t_{1a} \le T_{1a} \tag{6}$$

which is the inventory level during $[0, T_{1a}]$. Hence, the total inventory in a production uptime is

$$\begin{aligned} TI_{1a} &= \int_0^{T_{1a}} I_{1a}(t_{1a})\,dt_{1a} \\ &= (P - a - x)\frac{T_{1a}^2}{2} - \frac{ab}{6}T_{1a}^3. \end{aligned} \tag{7}$$

Using $I_{3r}(0) = 0$, $I_{4r}(0) = 0$, the total inventory of serviceable items for the rework uptime and rework downtime is

$$\begin{aligned} TI_{3r} &= (P_1 - a - x_1)\frac{T_{3r}^2}{2} - \frac{ab}{6}T_{3r}^3, \\ TI_{4r} &= a\left[\frac{T_{4r}^2}{2} + \frac{b}{3}T_{4r}^3\right], \end{aligned} \tag{8}$$

respectively.

Using $I_{2a}(I_{2a}) = 0$, the total inventory level of a production downtime is

$$TI_{2a} = a\left[\frac{T_{2a}^2}{2} + \frac{b}{3}T_{2a}^3\right]. \tag{9}$$

The maximum inventory is

$$I_m = I_{1a}(T_{1a}) = (P - a - x)\,T_{1a} - \frac{ab}{2}T_{1a}^2 \tag{10}$$

and hence, the total inventory in a rework uptime is

$$TI_{3a} = I_m(T_{3r} + T_{4r}). \tag{11}$$

Now, let us analyze the inventory level of recoverable items (Figure 2).

The inventory level of recoverable items in a production uptime is governed by the differential equation

$$\frac{dI_{r1}(t_{r1})}{dt_{r1}} = x, \quad 0 \le t_{r1} \le T_{1a}. \tag{12}$$

Since initially there are no recoverable items, that is, $I_{r1}(0) = 0$, the solution of (12) is

$$I_{r1}(t_{r1}) = xt_{r1}, \quad 0 \le t_{r1} \le T_{1a}. \tag{13}$$

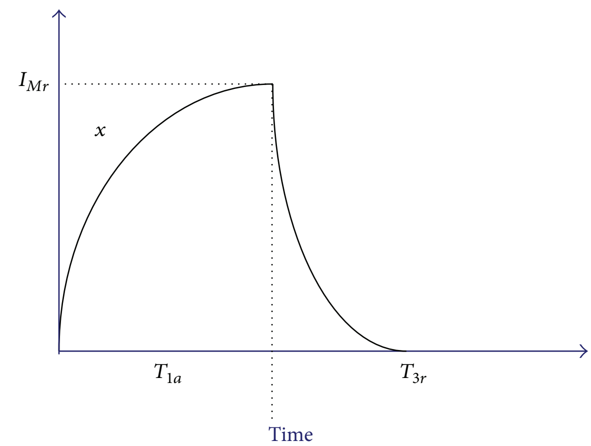

Figure 2: Inventory status of recoverable items.

Hence, the total inventory of recoverable items in a production uptime is

$$TTI_{r1} = \frac{xt_{1a}^2}{2} \tag{14}$$

and the maximum recoverable inventory is

$$I_{Mr} = I_{r1}(T_{1a}) = xT_{1a}. \tag{15}$$

The inventory level of recoverable item in the rework uptime is modeled as

$$\frac{dI_{r3}(t_{r3})}{dt_{r3}} = -P_1, \quad 0 \le t_{r3} \le T_{3r}. \tag{16}$$

Using $I_{r3}(T_{3r}) = 0$, the inventory level of recoverable item in rework uptime is

$$I_{r3}(t_{r3}) = P_1(T_{3r} - t_{r3}), \quad 0 \le t_{r3} \le T_{3r}. \tag{17}$$

Hence, the total inventory of recoverable item in the rework uptime is

$$TTI_{r3} = \frac{P_1T_{3r}^2}{2}. \tag{18}$$

The number of recoverable inventories is

$$I_{Mr} = I_{r3}(0) = P_1T_{3r}. \tag{19}$$

Hence,

$$T_{3r} = \frac{I_{Mr}}{P_1}. \tag{20}$$

Substituting I_{Mr} from (15), we get

$$T_{3r} = \frac{xT_{1a}}{P_1}. \tag{21}$$

Hence, the total recoverable inventory is

$$I_w = TTI_{r1} + TTI_{r3} = \frac{xT_{1a}^2}{2}\left(1 + \frac{x}{P_1}\right). \tag{22}$$

The inventory level at the beginning of the production downtime is equal to the inventory level at the end of the production uptime; that is,

$$I_{1a}(T_{1a}) = I_{2a}(0). \tag{23}$$

Therefore,

$$T_{2a} \approx \frac{1}{a}\left[(P - a - x)T_{1a} - \frac{ab}{2}T_{1a}^2\right]. \tag{24}$$

When $t_{3r} = T_{3r}$ and $t_{4r} = 0$, the inventory level for serviceable item in rework process satisfies

$$(P_1 - a - x_1)T_{3r} - \frac{ab}{2}T_{3r}^2 = a\left[T_{4r} - \frac{b}{2}T_{4r}^2\right]. \tag{25}$$

Neglecting T_{4r}^2 (because $0 < T_{4r} < 1$), we get

$$T_{4r} \approx \frac{1}{a}(P_1 - a - x_1)\frac{x}{P_1}T_{1a}. \tag{26}$$

The total production inventory cost is the sum of the production set up cost, inventory cost of serviceable item, inventory cost of recoverable item, and scrap cost:

$$\begin{aligned} TC = A &+ h\left[TI_{1a} + TI_{3r} + TI_{2a} + TI_{4r} + TI_{3a}\right] \\ &+ h_1 I_w + S_C x_1 T_{3r} \end{aligned} \tag{27}$$

and the total cycle time is

$$T = T_{1a} + T_{3r} + T_{2a} + T_{4r}. \tag{28}$$

Hence, the total cost per unit time without lost sales is given by

$$TCT_{\mathrm{NL}} = \frac{TC}{T}. \tag{29}$$

The optimal production uptime for the EPQ system without lost sales can be obtained by setting

$$\frac{dTCT_{\mathrm{NL}}(T_{1a})}{dT_{1a}} = 0. \tag{30}$$

When unavailability time of a machine is longer than the production downtime duration, lost sales will occur. So the total inventory cost is

$$\begin{aligned} E(TC) = A &+ h\left[TI_{1a} + TI_{3r} + TI_{2a} + TI_{4r} + TI_{3a}\right] \\ &+ h_1 I_w + S_C x_1 T_{3r} + S_L \\ &\times \int_{t=T_{2a}+T_{4r}}^{\infty} R(t)\left(t - (T_{2a} + T_{4r})\right) f(t)\,dt \end{aligned} \tag{31}$$

and the total cycle time for lost sales is

$$\begin{aligned} E(T) = T_{1a} &+ T_{3r} + T_{2a} + T_{4r} \\ &+ \int_{t=T_{2a}+T_{4r}}^{\infty} \left(t - (T_{2a} + T_{4r})\right) f(t)\,dt. \end{aligned} \tag{32}$$

Hence, the total cost per unit time for lost sales is

$$E(TCT) = \frac{E(TC)}{E(T)}. \tag{33}$$

We discuss lost sales scenario for two distributions, namely uniform distribution and exponential distribution.

2.1. Uniform Distribution. Define the probability distribution function $f(t)$, when the preventive maintenance time t is distributed uniformly as follows:

$$f(t) = \begin{cases} \dfrac{1}{\tau}, & 0 \le t \le \tau \\ 0, & \text{otherwise.} \end{cases} \tag{34}$$

Substituting $f(t)$ in (33) gives the total cost per unit time for uniform distribution as

$$\begin{aligned} &TCT_U \\ &= \Bigg(A + h\left[TI_{1a} + TI_{3r} + TI_{2a} + TI_{4r} + TI_{3a}\right] + h_1 I_w \\ &\quad + S_C x_1 T_{3r} + S_L \int_0^{\tau} \left(\frac{a(1+bt)}{\tau}\right)\left(t - (T_{2a} + T_{4r})\right) dt\Bigg) \\ &\quad \times \Bigg(T_{1a} + T_{3r} + T_{2a} + T_{4r} + \left(\frac{1}{\tau}\right) \\ &\quad \times \int_{t=T_{2a}+T_{4r}}^{\tau} \left(t - (T_{2a} + T_{4r})\right) dt\Bigg)^{-1} \end{aligned} \tag{35}$$

substituting all the time variables in (35) in terms of T_{1a}, the objective function; TCT_u is a function of T_{1a} only. The optimum value of T_{1a} can be computed by setting

$$\frac{dTCT_U(T_{1a})}{dT_{1a}} = 0. \tag{36}$$

To derive the best solution from nonlost sales and lost sales scenarios, we propose the following steps [17].

Step 1. Calculate (30), (24), and (26) and set $T_{\mathrm{sb}} = T_{2a} + T_{4r}$.

Step 2. If $T_{\mathrm{sb}} < \tau$, then the obtained solution is not feasible, and go to Step 3; otherwise the solution is obtained.

Step 3. Set $T_{\mathrm{sb}} = \tau$. Find $T_{1\mathrm{aub}}$ using (26) and (24). Calculate $TCT_{\mathrm{NL}}(T_{1\mathrm{aub}})$ using (29).

Step 4. Calculate (36), (24), and (26) and set $T_{\mathrm{sb}} = T_{2a} + T_{4r}$.

Step 5. If $T_{\mathrm{sb}} \ge \tau$, then $T_{1a}^* = T_{1\mathrm{aub}}$ and the corresponding total cost is $TCT_{\mathrm{NL}}(T_{1\mathrm{aub}})$; otherwise, calculate $TCT_U(T_{1a})$.

Step 6. If $TCT_{\mathrm{NL}}(T_{1\mathrm{aub}}) \le TCT_U(T_{1a})$, then $T_{1a}^* = T_{1\mathrm{aub}}$: otherwise $T_{1a}^* = T_{1a}$.

2.2. Exponential Distribution. Define the probability distribution function $f(t)$, when the preventive maintenance time t is distributed exponential with mean $1/\lambda$ as

$$f(t) = \lambda e^{-\lambda t}, \quad \lambda > 0. \tag{37}$$

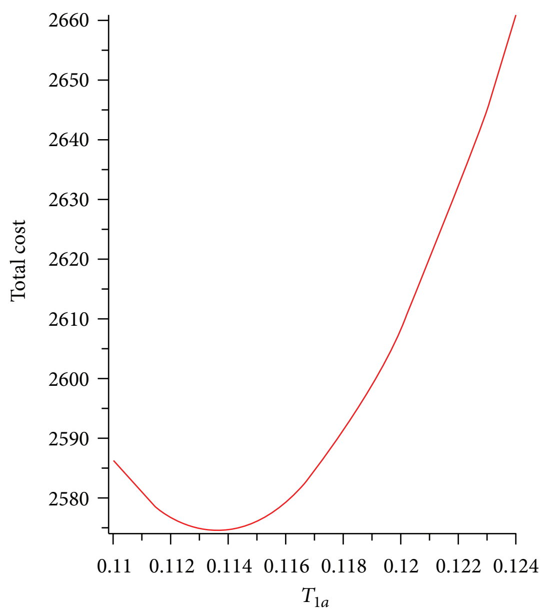

FIGURE 3: Convexity of total cost.

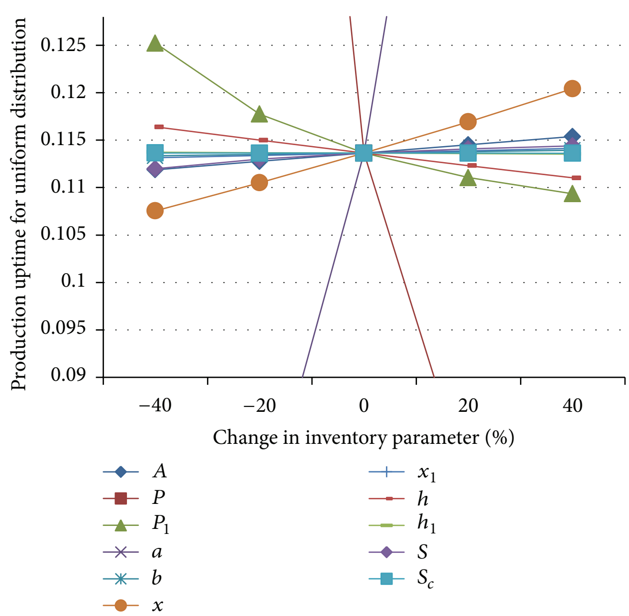

FIGURE 4: Sensitivity analysis of production uptime for uniform distribution.

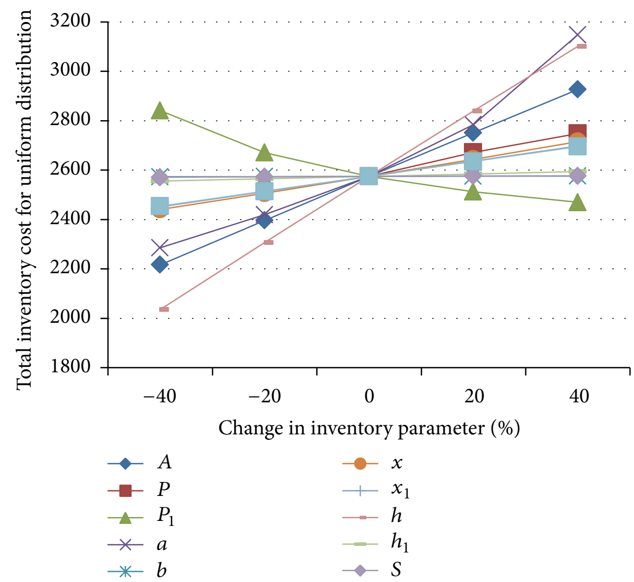

FIGURE 5: Sensitivity analysis of total cost for uniform distribution.

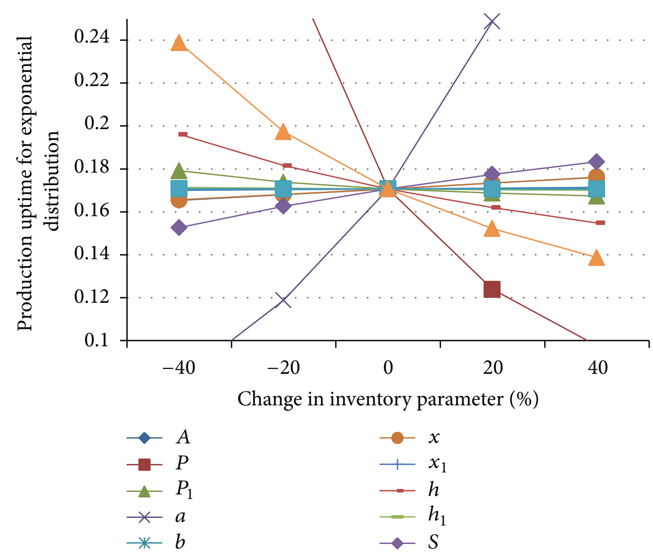

FIGURE 6: Sensitivity analysis of production uptime for exponential distribution.

Here, the total cost per unit time for the lost sale S_L is

$$TCT_E = \Bigg(A + h\left[TI_{1a} + TI_{3r} + TI_{2a} + TI_{4r} + TI_{3a}\right] + h_1 I_w + S_C x_1 T_{3r} + S_L \int_{t=T_{2a}+T_{4r}}^{\infty} R(t)\left(t - \left(T_{2a}+T_{4r}\right)\right)\lambda e^{-\lambda t} dt \Bigg) \times \left(T_{1a} + T_{3r} + T_{2a} + T_{4r} + \left(\frac{1}{\lambda}\right) e^{-\lambda (T_{2a}+T_{4r})} \right)^{-1}. \tag{38}$$

Arguing as in (Section 2.1), we can obtain optimum total cost. The high nonlinearity of the cost functions (29), (35), and (38) does not guarantee that the optimal solution is global. However, using parametric values, convexity of the objective function is established.

3. Numerical Examples and Sensitivity Analysis

Consider, following parametric values to study the working of the proposed problem. Let A = \$200 per production cycle, P = 10,000 units per unit time, a = 5000 units per unit time, b = 10%, x = 500 units per unit time; x_1 = 400 units per unit time, h = \$5 per unit per unit time. h_1 =

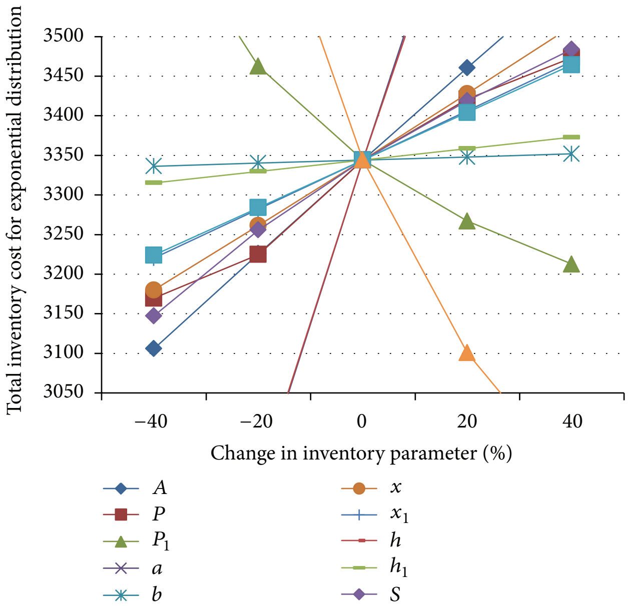

Figure 7: Sensitivity analysis of total cost for exponential distribution.

\$3 per unit per unit time, S_L = \$10 per unit, S_C = \$12 per unit, and the preventive maintenance time is uniformly distributed over the interval $[0, 0.1]$ [17]. Using the solution procedure outlined, the optimal production uptime is T_{1a} = 41.5 days and the corresponding minimum total cost per unit time is TCT_U = \$2575. This establishes that some lost sales reduce the total cost per unit time. The convexity of TCT_U is established in Figure 3.

The sensitivity analysis is carried out by changing each of the parameters by −40%, −20%, +20%, and +40%. The optimal production uptime T_{1a} and the optimal total cost per unit time for inventory parameters under consideration are shown in Table 1.

Figures 4 and 6 depict sensitivity analysis of production uptime, T_{1a}, with respect to all the inventory parameters considered in the modeling when preventive maintenance time follows uniform distribution/exponential distribution. It is observed that production uptime is slightly sensitive to changes in P and a and moderately sensitive to changes in b and τ, with little impact due to changes in the other inventory parameters. T_{1a} has negative impact with the increase in the production rate, P, and positive impact when scale demand, a, and rate of demand, b, increase.

The optimal total cost per unit time is slightly sensitive to changes in a, P, x, and L and moderately sensitive to changes in A, b, τ, S_C, x_1, and S_L. No change is observed in the optimal total cost per unit time for the remaining inventory parameters. The optimal total cost per unit time is inversely related to P and P_1 and directly related to other inventory parameters (see Figures 5 and 7).

4. Conclusions

In this research, rework of imperfect quality and random preventive maintenance time are incorporated in economic production quantity model when demand increases with time. The random preventive maintenance time is distributed uniformly and exponentially. The models are validated by the example. The sensitivity analysis suggests that the optimal total cost per unit time is sensitive to changes in the production rate, the demand rate, and the product defect rate in both the uniform and the exponential distributed preventive maintenance time. To combat increasing demand, the management should adopt the latest machinery which decreases defective production rate, reducing rework, and as a consequence, the machine's production uptime can be utilized to its utmost. Further research can be carried out to study the effect of deterioration of units.

Notations

I_{1a}: Serviceable inventory level in a production uptime
I_{2a}: Serviceable inventory level in a production downtime
I_{3a}: Serviceable inventory level in a rework uptime
I_{3r}: Serviceable inventory level from rework uptime
I_{4r}: Serviceable inventory level from rework process in rework downtime
I_{r1}: Recoverable inventory level in a production uptime
I_{r3}: Recoverable inventory level in a rework uptime
TI_{1a}: Total serviceable inventory in a production uptime
TI_{2a}: Total serviceable inventory in a production downtime
TI_{3a}: Total serviceable inventory in a rework uptime
TI_{3r}: Total serviceable inventory from a rework uptime
TI_{4r}: Total serviceable inventory from rework process in a rework downtime
TTI_{r1}: Total recoverable inventory level in a production uptime
TTI_{r3}: Total recoverable inventory level in a rework uptime
T_{1a}: Production uptime
T_{2a}: Production downtime
T_{3r}: Rework uptime
T_{4r}: Rework downtime
T_{sb}: Total production downtime
T_{1aub}: Production uptime when the total production downtime is equal to the upper bound of uniform distribution parameter
I_m: Inventory level of serviceable items at the end of production uptime
I_{Mr}: Maximum inventory level of recoverable items in a production uptime
I_w: Total recoverable inventory

P: Production rate
P_1: Rework process rate
$R = R(t)$: Demand rate; $a(1+bt), a > 0, 0 < b < 1$
x: Product defect rate
x_1: Product scrap rate
A: Production setup cost
h: Serviceable items holding cost
h_1: Recoverable items holding cost
S_C: Scrap cost
S_L: Lost sales cost
TC: Total inventory cost
T: Cycle time
TCT: Total inventory cost per unit time for lost sales model
TCT_{NL}: Total inventory cost per unit time for without lost sales model
TCT_U: Total inventory cost per unit time for lost sales model with uniform distribution preventive maintenance time
TCT_E: Total inventory cost per unit time for lost sales model with exponential distribution preventive maintenance time.

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