Research Article

Process of Identifying Stress Fields from Strain Fields in the Specimen with a Hole

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The paper is focused on the process of identifying stress fields from strain fields in the specimen with a hole. The experiment was realized on the specimen with a hole made from anisotropic material. The main attention is paid to the analysis of deformation in the areas of stress (near the hole). That geometry generates a heterogeneous strain field which has been measured during the test using a digital image correlation system. The advantage of using heterogeneous strain fields in the identification procedure is that a complex state of stress-strain can be analyzed at the same time. On the other hand the stress field cannot be directly computed from the test and a suitable identification procedure has to be developed. Here, the virtual fields method (VFM) adapted for plastic strain has been used to identify the hardening behaviour and the anisotropy of the material. The values obtained by the VFM have been compared with the results coming from a standard identification made with uniaxial tensile tests.

1. Introduction

In sheet metal forming an anisotropic behaviour of sheet metals is often observed when they undergo plastic deformation. Such anisotropy is due to the rolling process which introduces anisotropy in the texture of the material. The correct identification of the plastic behaviour of anisotropic sheet metal is a very important aspect that has to be taken into account in order to enhance the results of the numerical models.

Traditionally the identification is carried out using uniaxial tensile tests performed at different directions respect to the rolling one. However in recent years, thanks to the development in the full-fields measurement techniques, more complex specimens which generate heterogeneous stress-strain fields can be used to study the behaviour of the material during the plastic flow.

The parameters can be then identified using for instance an inverse approach which includes the finite element (FE) updating [1–3].

Another identification procedure is the virtual fields method (VFM) [4] which has been successfully applied in

many applications, including elastoplasticity [5, 6] and large strain plasticity [7].

The paper is focused on the process of identifying stress fields from strain fields in the specimen with a hole [8–10]. The main attention is paid to the analysis of deformation in the areas of stress (near the hole) [11–14]. The displacement and strain fields were measured using a stereocorrelation technique and the results have been used to identify the parameter with VFM. The Hill48 yielding criterion and the Swift hardening law were used as plasticity model.

The mentioned methods are applied to evaluate the properties of rolled sheets used for automotive production. The uncoated IF steel sheet was used for analysis (interstitial-free (IF)).

2. Geometry and Parameters of Specimen

This section assesses the properties of cold-rolled sheet metals used in pressing car body parts. A specimen with a hole and the geometry and the dimensions in millimetres as shown in

FIGURE 1: The geometry and dimensions (mm) of specimen with a hole.

TABLE 1: The mechanical properties of the specimen.

Dir.	Thick. (mm)	Rp0,2 (MPa)	Rm (MPa)	A80 (%)	r	n
0°	0,75	124	297	44,5	1,7	0,25
45°	0,75	139	305	42	1,6	0,23
90°	0,75	132	290	43,5	2,3	0,24

Figure 1 was chosen for the deformation analysis. Further in the paper the term anisotropic material will refer to a sheet metal material that is anisotropic in plastic deformation.

Table 1 shows the specimen material's mechanical properties as demonstrated in tensile tests in the direction of sheet metal rolling. The values obtained by experiment are shown for the rolling direction (0°).

The specimen was made of DC06 sheet metal of extra deep drawn steel with classic high values of normal anisotropy coefficient r , which is suitable for the production of stampings of complex shapes [15].

To measure the specimen with a hole, a cross-beam displacement of 0.01 mm/image was set for the first ten steps, a displacement of 0.1 mm/image for the next ten steps, and a displacement of 0.2 mm/image from step 20 until the specimen broke apart (Figure 2). Please note that in this chart the elastic area is divided into several steps so it is clearer. The maximum cross-beam displacement was 20 mm, which corresponds to a maximum force of $F_{\max} = 10,280$ N.

3. Results and Discussion

3.1. Analysis of Strain Fields Using Digital Image Correlation. The results were visualised for a cross-beam displacement of 3.8 mm using the Vic 3D software.

Figure 3 shows the strain fields ϵ_x , ϵ_y , and γ_{xy} as obtained in measurements for the specimen with a hole (Table 1) and processed with Vic 3D.

FIGURE 2: Diagram of force and cross-beam displacement for specimen with a hole.

3.2. Processing Results with Camfit. Program Camfit is a GUI MATLAB-based software that implements the virtual fields method. It can deal with linear elasticity (isotropic and orthotropic) as well as simple elastoplasticity. Camfit's purpose is to process the displacement fields measured using the virtual fields method. The theoretical basis of the virtual fields method can be found in the literature [4–6].

The values of the displacement fields u_x and u_y and the force values corresponding to each picture were exported from Vic 3D. The mask and the grid size were selected after loading values.

For reasons of task symmetry, the area shown along with the grid in Figure 4 was chosen for the specimens with a hole (Figure 1).

Figure 5 shows displacement fields u_x and u_y for specimen with a hole as obtained in measurements with a

FIGURE 3: The contours of strain fields (a) ϵ_x , (b) ϵ_y , and (c) γ_{xy} (–) obtained by Vic 3D for specimen with a hole.

FIGURE 4: Grid chosen for specimens with a hole.

cross-beam displacement of 3.8 mm. Figure 5(a) shows displacement fields u_x and u_y in the selected area and Figure 5(b) shows displacement fields u_x and u_y with the grid applied.

Figure 6 shows the strain fields ε_x , ε_y , and γ_{xy} as obtained by Camfit for a cross-beam displacement of 3.8 mm. The results can be compared with results obtained by Vic 3D (Figure 3).

In elastoplasticity range, Camfit uses only one constant virtual strain field, and as defined in the previous text, it depends on the test configuration. This can also be seen as the difference between averages, normal or shear force, measured and recalculated from the strains, and the constitutive parameters. The program first identifies the isotropic elastic constants (Young's modulus and Poisson's ratio) for specimen with a hole, as we can see in Figure 7. This enables us to define an elastic range for the test, thanks to a plot which reports external and internal virtual work. If linearity is correct, then the elastic area was well defined. Otherwise, it should be changed to arrive to acceptable linearity.

The second stage concerns the plastic parameters and there are four isotropic hardening models, which are based on the simple Prandtl-Reuss model. In this case, the power law model was chosen and the results are shown in Figure 8.

The values X_1 , X_2 , and X_3 are obtained by Camfit for elastoplastic area in the whole process of the measurement. The other parameter, which can be determined in the elastoplastic area, is yield stress.

Comparing the results in figure and those in Table 1, it is clear that the yield stress obtained by Camfit for the elastic-plastic area is, in essence, the same as the yield point obtained in a standardised tensile test. Other parameters of sheet metal material properties obtained in elastic-plastic strain shown in figure are the input data for identifying the stress fields in the elastic-plastic area.

3.3. Stress Fields Determination Using MATLAB. The parameters obtained by the Camfit program for the elastoplastic

area were then processed in Matlab to compute stress fields σ_x , σ_y , and τ_{xy} for anisotropic material [16].

The chosen plastic model is the Hill48 yield criterion [17] with isotropic hardening. In the plane stress case, the Hill48 criterion can be written as

$$\sigma_{eq} = (g + h) \sigma_x^2 - 2h \sigma_x \sigma_y + (f + h) \sigma_y^2 + 2n \tau_{xy}^2, \quad (1)$$

where f , g , h , and n are parameters which can be computed from the Lankford parameters at different orientations. If we assume that the equivalent plastic strain σ_{eq} is equivalent to the yield stress in the rolling direction, that is, 0° , it can be shown that [18]

$$g = \frac{1}{1 + R_0}; \quad f = \frac{1}{1 + R_{90}}; \quad h = \frac{R_0}{1 + R_0}; \quad (2)$$

$$n = \left(\frac{1}{2} + R_{45} \right) \left(1 + \frac{R_0}{R_{90}} \right) g,$$

where R_0 , R_{45} , and R_{90} are the values of the Lankford parameters measured at orientations 0° , 45° , and 90° , respectively. The hardening law is described using the Swift law:

$$\sigma_{eq} = K \cdot (\varepsilon_0 + \varepsilon_{eqpl})^n, \quad (3)$$

where ε_{eqpl} is the equivalent accumulated plastic strain, K , ε_0 , and n are the parameters to be identified by Camfit and denoted by X_1 , X_2 , and X_3 .

The stress fields obtained from strain fields calculated by Matlab for specimen with a hole for displacement 3.8 mm are illustrated in Figure 9.

Table 2 shows the measurement results as obtained by Camfit. Classic standardised tensile tests (for two specimens) had been carried out prior to the experiment. The yield stress for the given material and the material properties obtained by these tests are shown in Table 1. The results of yield stress

FIGURE 5: The displacement fields u_x and u_y for specimen with a hole.

FIGURE 6: The strain fields ϵ_x , ϵ_y , and γ_{xy} (–) obtained by Camfit for specimen with a hole.

FIGURE 7: Identification of parameters in elastic area for specimen with a hole.

Young's modulus = 139.4 GPa
Poisson's ratio = 0.26
Elastoplasticity with power law ($s_{eq} = X_1 * (e_p + X_2)^{X_3}$)
 $X_1 = 1284.2$ MPa
 $X_2 = 0.008$
 $X_3 = 0.478$
Initial yield stress = 125.7 MPa
Minimization: cost function = 62.734; iterations = 50

FIGURE 8: Parameters obtained in elastoplastic range by Camfit for specimen with a hole.

TABLE 2: Results obtained by Camfit and standardised tensile test.

Parameters in elastic range		Camfit Parameters in elastoplastic range (power law)				Tensile test Yield stress	
E [GPa]	μ	X_1	X_2	X_3	R_e	Test1	Test2
139,4	0,26	1284,2	0,008	0,478	125,7	124	128

FIGURE 9: The stress fields σ_x , σ_y , and τ_{xy} MPa obtained from strain fields for specimen with a hole.

obtained by Camfit were compared with the results obtained in the tensile tests (Test1 and Test2 in Table 2). Comparing the yield stress values obtained with Camfit and the yield stress values obtained in classic tensile tests shows a very good conformity, which proves the fact that the virtual fields method is suitable for determining material properties of sheet metal specimens.

4. Conclusions

The present paper is aimed at the process of identifying stress fields from strain fields in the specimen with a hole.

Digital image correlation (DIC) was used to analyze plastic strain development. DIC is one of the rapidly developing modern unconventional optical methods of experimental mechanics. The possibility of overall capture of the displacements on the surface of specimens permits a real picture to be obtained of how strain is distributed at the points of stress.

The material properties of the steel sheet metals used to produce specimens were obtained by standardised tensile tests and their values were compared with the results obtained by an experimental method of identifying sheet metal material properties. Camfit software was used in this experimental method and overall measurement was applied. Comparing the resultant values for the material's yield point shows a good conformity with the results obtained by Camfit, which uses the virtual work method to make calculations. The Camfit material parameters were then used in determining stress fields using the strain fields measured while applying hardening models suitable for isotropic materials. These methods enable the analysis of stress in the elastic and the plastic areas in tasks of concentrated stress while considering the material's anisotropy in plastic deformation.

This allows us to predict damage to thin-walled structural features primarily in the areas where stress may concentrate

and serves as a basis for assessing the service time of such features and it provides information for the optimisation of the manufacturing of such features by sheet metal cold pressing.

Conflict of Interests

With all responsibility the authors would like to declare that non of them is financially supported by producers or distributors of commercial programmes MATLAB, Camfit, and Vic 3D. Moreover, they do not have any other kind of interest to propagate and support these commercial programmes. In the paper they were used as a necessary tool for their simulations.

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