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Research Article

Dark Energy from the Gas of Wormholes

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We assume the space-time foam picture in which the vacuum is filled with a gas of virtual wormholes. It is shown that virtual wormholes form a finite (of the Planckian order) value of the energy density of zero-point fluctuations. However such a huge value is compensated by the contribution of virtual wormholes to the mean curvature and the observed value of the cosmological constant is close to zero. A nonvanishing value appears due to the polarization of vacuum in external classical fields. In the early Universe some virtual wormholes may form actual ones. We show that in the case of actual wormholes vacuum polarization effects are negligible while their contribution to the mean curvature is apt to form the observed dark energy phenomenon. Using the contribution of wormholes to dark matter and dark energy we find estimates for characteristic parameters of the gas of wormholes.

1. Introduction

As is well known modern astrophysics (and, even more generally, theoretical physics) faces two key problems. Those are the nature of dark matter and dark energy. Recall that more than 90% of matter of the Universe has a nonbaryonic dark (to say, mysterious) form, while lab experiments still show no evidence for the existence of such matter. Both dark components are intrinsically incorporated in the most successful ΛCDM (Lambda cold dark matter) model which reproduces correctly properties of the Universe at very large scales (e.g., see [1] and references therein). We point out that ΛCDM predicts also the presence of cusps ($\rho_{\mathrm{DM}} \sim 1/r$) in centers of galaxies [2] and a too large number of galaxy satellites. Therefore other models are proposed, for example, like axions [3], which may avoid these. To be successful such models should involve a periodic self-interaction and therefore require a fine tuning, while in general the presence of standard nonbaryonic particles cannot solve the problem of cusps. Indeed, if we admit the existence of a self-interaction in the dark matter component, or some coupling to baryonic matter (which should be sufficiently strong to remove cusps), then we completely change properties of the dark matter component at the moment of recombination and destroy all successful predictions at very large scales. Recall that both warm and self-interacting dark matter candidates are rejected by the observing $\Delta T/T$ spectrum [1]. In other words, the two key observational phenomena (cores of dark matter in centers of galaxies [4–6] and $\Delta T/T$ spectrum) give a very narrow gap for dark matter particles which seems to require attracting some exotic objects in addition to standard nonbaryonic particles.

As it was demonstrated recently [7] the problem of cusps can be cured, if some part of nonbaryon particles is replaced by wormholes. Wormholes represent extremely heavy (in comparison to particles) objects which at very large scales behave exactly like nonbaryon cold particles, while at smaller scales (in galaxies) they strongly interact with baryons and form the observed [4–6] cored ($\rho_{\mathrm{DM}} \sim \mathrm{const}$) distribution. We note that stable wormholes violate necessarily the averaged null energy conditions which gives the basic argument against the existence of such objects. Without exotic matter stable wormholes may however exist in modified theories, for example, see [8] and references therein. In the case when the energy conditions hold a wormhole collapses into a couple of conjugated (of equal masses) blackholes which almost impossible to distinguish from standard primordial blackholes. However, the topological nontriviality of such objects retains and gravitational effects of a gas of wormholes considered in [9] and some results of [7] still remain valid which means that nontraversable wormholes can be used to smooth cusps in centers of galaxies. Thus, it worth expecting that wormholes may play an important role in the explanation of the dark matter phenomenon.

Saving the dark matter component ΛCDM requires the presence (~70%) of dark energy (of the cosmological constant). Moreover, there is evidence for the start of an acceleration phase in the evolution of the Universe [10–14]. In the present paper we use virtual wormholes to estimate the contribution of zero-point fluctuations in the value of the cosmological constant. The idea to relate virtual wormholes (or baby universes) and the cosmological constant is not new, in somewhat different context it was used by Coleman in [15] and developed in [16]. Our basic aim is to demonstrate that virtual wormholes form a finite value of the energy density of zero-point fluctuations.

It is necessary to point here out to the principle difference between actual and virtual wormholes. The principle difference is that a virtual wormhole exists only for a very small period of time and at very small scales and does not necessarily obey to the Einstein equations. It represents tunnelling event and therefore, the averaged null energy condition (ANEC) cannot forbid the origin of such an object. For the future we also note that a set of virtual wormholes may work as an actual wormhole opening thus the way for an artificial construction of wormhole-type objects in lab experiments.

In the present paper we describe a virtual wormhole as follows. From the very beginning we use the Euclidean approach (e.g., see [17] and the standard textbooks [18]). Then the simplest virtual wormhole is described by the metric ($\alpha = 1, 2, 3, 4$)

$$ds^2 = h^2(r)\,\delta_{\alpha\beta}dx^\alpha dx^\beta, \tag{1}$$

where

$$h(r) = 1 + \theta(a - r)\left(\frac{a^2}{r^2} - 1\right) \tag{2}$$

and $\theta(x)$ is the step function. Such a wormhole has vanishing throat length, while the step function at the junction may cause a problem in Einstein's equation or when a topological Euler term is involved. A more careful analysis needs to consider distributional curvature and so forth, see [19]. To avoid these difficulties we may consider from the very beginning a wormhole of a finite throat length $\sim 1/\beta$ where the step function is replaced with a smooth function (e.g., $\theta(x,\beta) = (\exp(\beta x)+1)^{-1}$). Then where it is necessary one may consider the limit $\beta \to \infty$ only in final expressions. This insures that the Bianchi identity holds and that the above metric remains inside of the domain of usual gravity. (Unexpectedly our approach (the use of step function) seems to irritate an essential part of physicists working with wormholes (e.g., PRD reviewers, moreover there is a claim that we study some other exotic objects which are not wormholes [20]). Therefore, it is necessary to clarify our position here. We are quite aware that at present state the physics of wormholes is certainly on the most speculative side. Therefore, we do not see the difference between specific forms of a smooth metric that one may use. For example, the simplest choice $h = (1 + a^2/r^2)$ gives the well-known metric which in 3-dimensions is called as the Bronnikov-Ellis metric, or the metric $h = (1 + (b/r) + (a^2/r^2))$ which includes an additional parameter (an arbitrary length of the handle) which is not called by any name but is not less trivial. In the case of actual wormholes the exact and correct form of metric may be established only upon understanding the nature and properties of exotic matter which may support such a metric as a stable solution to the Einstein equations. For this time has not come yet. In the case of virtual wormholes even this is not important; for in the complete theory one has to sum over all possible metrics which formally may be included in φ in (5). Our basic aim is to present sufficiently clear and simple model (let it be far from realistic) which retains basic qualitative features related to a nontrivial topology.)

In the region $r > a$, $h = 1$ and the metric (2) is flat, while the region $r < a$, with the obvious transformation $y^\alpha = (a^2/r^2)x^\alpha$, is also flat for $y > a$. Therefore, the regions $r > a$ and $r < a$ represent two Euclidean spaces glued at the surface of a sphere S^3 with the centre at the origin $r = 0$ and radius $r = a$. Such a space can be described with the ordinary double-valued flat metric in the region $r_\pm > a$ by

$$ds^2 = \delta_{\alpha\beta}dx_\pm^\alpha dx_\pm^\beta, \tag{3}$$

where the coordinates $x_\pm^\alpha$ describe two different sheets of space. Now, identifying the inner and outer regions of the sphere S^3 allows the construction of a wormhole which connects regions in the same space (instead of two independent spaces). This is achieved by gluing the two spaces in (3) by motions of the Euclidean space (the Poincare motions). If $R_\pm$ is the position of the sphere in coordinates $x_\pm^\mu$, then the gluing is the rule

$$x_+^\mu = R_+^\mu + \Lambda_\nu^\mu\left(x_-^\nu - R_-^\nu\right), \tag{4}$$

where $\Lambda_\nu^\mu \in O(4)$, which represents the composition of a translation and a rotation of the Euclidean space (Lorentz transformation). In terms of common coordinates such a wormhole represents the standard flat space in which the two spheres $S_\pm^3$ (with centers at positions $R_\pm$) are glued by the rule (4). We point out that the physical region is the outer region of the two spheres. Thus, in general, the wormhole is described by a set of parameters: the throat radius a, positions of throats $R_\pm$, and rotation matrix $\Lambda_\nu^\mu \in O(4)$.

In the present paper we assume the space-time foam picture in which the vacuum is filled with a gas of virtual wormholes. We show that virtual wormholes form a finite (of the Planckian order) value of the energy density of zero-point fluctuations. However such a huge value is compensated by the contribution of virtual wormholes to the mean curvature and the observed value of the cosmological constant should be close to zero.

To achieve our aim we, in Section 2, present the construction of the generating functional in quantum field theory. The main idea is that the partition function includes the sum over field configurations and the sum over topologies.

Where the sum over topologies is the sum over virtual wormholes described above. Such an approach gives a rather good leading approximation for calculation of the partition function and corresponds to the standard methods (e.g., Ritza method, etc.). In Section 3 we investigate properties of the two-point Green function. We show that the presence of the gas of virtual wormholes can be described by the topological bias exactly as it happens in the presence of actual wormholes [7, 9]. For limiting topologies when the density of virtual wormholes becomes infinite the Green function shows a good ultraviolet behavior which means that there exists a class of such systems when quantum field theories are free of divergencies. We demonstrate how the sum over topologies defines the mean value for the bias which takes the sense of a cutoff function in the space of modes. In Section 4 we explicitly demonstrate that for a particular set of virtual wormholes the bias defines not more than the projection operator on the subspace of functions obeying to the proper boundary conditions at wormhole throats. The projective nature of the bias means that wormholes merely cut some portion of degrees of freedom (modes). Phenomenologically it means that wormholes can be described by the presence of ghost fields which compensate the extra (cut by wormholes) modes. In Section 5 we show how the cutoff expresses via some dynamic parameters of wormholes. The exact definition of such parameters we leave it for the future investigation. In Section 6 we consider the origin of the cosmological constant. We demonstrate that the cosmological constant is determined by the contribution of the energy density of zero-point fluctuations and by the contribution of virtual wormholes to the mean curvature. We estimate contribution of virtual wormholes to the mean curvature and show also that wormholes lead to a finite (of the planckian order) value of $\langle T_{\mu\nu}\rangle$ which requires considering the contribution from the smaller and smaller wormholes with divergent density $n \to \infty$. We also present arguments of why in the absence of external classical fields the total value of the cosmological constant is exactly zero, while it acquires a nonvanishing value due to vacuum polarization effects (i.e., due to an additional distribution of virtual wormholes) in external fields. We also speculate the possibility of the formation of actual wormholes and in Section 7 we estimate their contribution to the dark energy. Finally in Section 8 we repeat basic results an discuss some perspectives.

2. Generating Function

The basic aim of this section is to construct the generating functional which can be used to get all possible correlation functions. Consider the partition function which includes the sum over topologies and the sum over field configurations

$$Z_{\text{total}} = \sum_{\tau}\sum_{\varphi} e^{-S}. \tag{5}$$

For the sake of simplicity we use from the very beginning the Euclidean approach. The action has the form

$$S = -\frac{1}{2}\left(\varphi\widehat{A}\varphi\right) + \left(J\varphi\right) \tag{6}$$

and we use the notions $(J\varphi) = \int J(x)\varphi(x)d^4x$. If we fix the topology of space by placing a set of wormholes with parameters ξ_i, then the sum over field configurations φ gives the well-known result

$$Z^*(J) = Z_0\left(\widehat{A}\right)e^{-(1/2)(J\widehat{A}^{-1}J)}, \tag{7}$$

where $Z_0(\widehat{A}) = \int[D\varphi]e^{(1/2)(\varphi\widehat{A}\varphi)}$ is the standard expression and $\widehat{A}^{-1} = A^{-1}(\xi)$ is the Green function for a fixed topology, that is, for a fixed set of wormholes $\xi_1, \dots, \xi_N$.

Consider now the sum over topologies τ. To this end we restrict the sum over the number of wormholes and integrals over parameters of wormholes:

$$\sum_{\tau} \longrightarrow \sum_{N}\int\prod_{i=1}^{N}d\xi_i = \int[DF], \tag{8}$$

where

$$F(\xi, N) = \frac{1}{N}\sum_{i=1}^{N}\delta(\xi - \xi_i) \tag{9}$$

and NF is the density of wormholes in the configuration space ξ. We also point out that in general the integration over parameters is not free (e.g., it obeys the obvious restriction $|\vec{R}_i^+ - \vec{R}_i^-| \geq 2a_i$). This defines the generating function as

$$Z_{\text{total}}(J) = \int[DF]\,Z_0\left(\widehat{A}\right)e^{-(1/2)(J\widehat{A}^{-1}J)}. \tag{10}$$

The sum over topologies assumes an additional averaging out for all mean values with the measure $d\mu_N = \rho(\xi, N)d^N\xi$, where

$$\rho(\xi, N) = \frac{Z_0\left(\widehat{A}(\xi, N)\right)}{Z_{\text{total}}(0)}, \tag{11}$$

which obey the obvious normalization condition $\sum_N\int d\mu_N = \sum_N \rho_N = 1$. The averaging out over topologies assumes the two stages. First we fix the total number of wormholes N and average over the parameters of wormholes ξ (i.e., over parameters of a static gas of wormholes in R^4). Then we sum over the number of wormholes N (the so-called big canonical ensemble).

The basic difficulty of the standard field theory is that the perturbation scheme based on (7) leads to divergent expressions. This remains true for every particular topology of space (i.e., for any particular finite set of wormholes), since there always exists a scale below which the space looks like the ordinary Euclidean space. What we expect is that the sum over all possible topologies will remove such a difficulty.

And indeed, the above measure (11) has the structure

$$Z_0\left(\widehat{A}(\xi, N)\right) = \exp\left(-\int\Lambda(\xi, N)\,d^4x\right), \tag{12}$$

where $\Lambda(\xi, N)$ is the cosmological constant related to the energy density of zero-point fluctuations calculated for a particular distribution of wormholes. (We recall that the total cosmological constant should include also the contribution from the mean curvature (55).) Any finite distribution of wormholes leads to the divergent expression $\Lambda(\xi, N) \rightarrow \infty$ and is suppressed (i.e., $\rho(\xi, N) \rightarrow 0$). However, the sum over all possible topologies assumes also the limiting topologies $n \rightarrow \infty$, where $n = N/V$ is the density of wormholes. In this limit wormhole throats degenerate into points and the minimal scale below which the space looks like the Euclidean space is merely absent. We point out that from the rigorous mathematical standpoint such limiting topologies cannot be described in terms of smooth manifolds, since they are not locally Euclidean and does not possess a finite set of maps. In mathematics similar objects are well known, for example, fractal sets. However, if a fractal set is obtained by cutting (by means of a specific rule or iterations) portions of space, our limiting topologies are obtained by gluing (identifying) some portions (or in the limit couples of points) of the Euclidean space. The basic feature of such topologies is that QFT becomes finite on such a set. Indeed, as we shall see a particular infinite distribution of wormholes can always be chosen in such a way that the energy of zero-point fluctuations becomes a finite $0 \leq \Lambda_{\infty}(\xi) < \infty$ (e.g., see the next section or the second term in (67)). In the sum over topologies only such limiting topologies do survive (i.e., $\rho_{\infty}(\xi) \neq 0$).

3. The Two-Point Green Function

From (7) we see that the very basic role in QFT plays the two point Green function. Such a Green function can be found from the equation

$$\widehat{A}G(x, x') = -\delta(x - x') \tag{13}$$

with proper boundary conditions at wormholes, which gives $G = A^{-1}$. Now let us introduce the bias function $N(x, x')$ as

$$G(x, y) = \int G_0(x, x') N(x', y) \, dx', \tag{14}$$

where $G_0(x, x')$ is the ballistic (or the standard Euclidean Green function) and the bias can be presented as

$$N(x, x') = \delta(x - x') + \sum_i b_i \delta(x - x_i), \tag{15}$$

where b_i are fictitious sources at positions x_i which should be added to obey the proper boundary conditions. We point out that the bias can be explicitly expressed via parameters of wormholes; that is, $N(x, x') = N(x, x', \xi_1, \ldots \xi_N)$. For the sake of illustration we consider first a particular example.

3.1. The Bias for a Particular Distribution of Wormholes (Rarefied Gas Approximation).

Consider now the bias for a particular set of wormholes. For the sake of simplicity we consider the case when $m = 0$. The Green function obeys the Laplace equation

$$-\Delta G(x, x') = \delta(x - x') \tag{16}$$

with proper boundary conditions at throats (we require G and $\partial G/\partial n$ to be continual at throats). The Green function for the Euclidean space is merely $G_0(x, x') = 1/4\pi^2(x - x')^2$ (and $G_0(k) = 1/k^2$ for the Fourier transform). In the presence of a single wormhole which connects two Euclidean spaces this equation admits the exact solution. For outer region of the throat S^3 the source $\delta(x - x')$ generates a set of multipoles placed in the center of sphere which gives the corrections to the Green function G_0 in the form (we suppose the center of the sphere at the origin)

$$\delta G = -\frac{1}{4\pi^2 x^2} \sum_{n=1}^{\infty} \frac{1}{n+1} \left(\frac{a}{x'}\right)^{2n} \left(\frac{x'}{x}\right)^{n-1} Q_n, \tag{17}$$

where $Q_n = (4\pi^2/2n) \sum_{l=0}^{n-1} \sum_{m=-l}^{l} Q_{nlm}^{*\prime} Q_{nlm}$ and $Q_{nlm}(\Omega)$ are four-dimensional spherical harmonics, for example, see [21]. In the present section we shall consider a dilute gas approximation and, therefore, it is sufficient to retain the lowest (monopole) term only. A single wormhole which connects two regions in the same space is a couple of conjugated spheres $S_{\pm}^3$ of the radius a with a distance $\vec{X} = \vec{R}_+ - \vec{R}_-$ between centers of spheres. So the parameters of the wormhole are (The additional parameter (rotation matrix Λ) is important only for multipoles of higher orders.) $\xi = (a, R_+, R_-)$. The interior of the spheres is removed and surfaces are glued together. Then the proper boundary conditions (the actual topology) can be accounted for by adding the bias of the source

$$\delta(x - x') \longrightarrow \delta(x - x') + b(x, x'). \tag{18}$$

In the approximation $a/X \ll 1$ (e.g., see for details [9]) the bias for a single wormhole takes the form

$$b_1(x, x', \xi) = \frac{a^2}{2} \left(\frac{1}{(R_- - x')^2} - \frac{1}{(R_+ - x')^2} \right) \times \left[\delta(\vec{x} - \vec{R}_+) - \delta(\vec{x} - \vec{R}_-) \right]. \tag{19}$$

This form for the bias is convenient when constructing the true Green function and considering the long-wave limit; however it is not acceptable in considering the short-wave behavior and vacuum polarization effects. Indeed, the positions of additional sources are in the physically nonadmissible region of space (the interior of spheres $S_{\pm}^3$). To account for the finite value of the throat size we should replace in (19) the point-like source with the surface density (induced on the throat); that is,

$$\delta(\vec{x} - \vec{R}_{\pm}) \longrightarrow \frac{1}{2\pi^2 a^3} \delta(|\vec{x} - \vec{R}_{\pm}| - a). \tag{20}$$

Such a replacement does not change the value of the true Green function; however, now all extra sources are in the physically admissible region of space.

In the rarefied gas approximation the total bias is additive; that is,

$$b_{\text{total}}(x, x') = \sum b_1(x, x', \xi_i) = N \int b_1(x, x', \xi) F(\xi) \, d\xi, \tag{21}$$

where NF is given by (9). For a homogeneous and isotropic distribution $F(\xi) = F(a, X)$, and then for the bias we find

$$b_{\text{total}}\left(x - x'\right) = \int \frac{1}{2\pi^2 a}\left(\frac{1}{R_-^2} - \frac{1}{R_+^2}\right)\delta\left(\left|\vec{x} - \vec{x}' - \vec{R}_+\right| - a\right) NF(\xi)\, d\xi. \tag{22}$$

Consider the Fourier transform $F(a, X) = \int F(a, k)e^{-ikX} \cdot (d^4k/(2\pi)^4)$ and using the integral $1/x^2 = \int(4\pi^2/k^2)e^{-ikx} \cdot (d^4k/(2\pi)^4)$ we find for $b(k) = \int b(x)e^{ikx}d^4x$ the expression

$$b_{\text{total}}(k) = N\int a^2\frac{4\pi^2}{k^2}\left(F(a,k) - F(a,0)\right)\frac{J_1(ka)}{ka/2}da. \tag{23}$$

(1) Example of a Finite Density of Wormholes. Consider now a particular (of a finite density) distribution of wormholes $F(a, X)$, for example,

$$NF(a, X) = \frac{n}{2\pi^2 r_0^3}\delta(a - a_0)\,\delta(X - r_0), \tag{24}$$

where $n = N/V$ is the density of wormholes. In the case $N = 1$ this function corresponds to a single wormhole with the throat size a_0 and the distance between throats $r_0 = |R_+ - R_-|$. We recall that the action (6) remains invariant under translations and rotations which straightforwardly leads to the above function. Then $NF(a, k) = \int NF(a, X)e^{ikx}d^4x$ reduces to $NF(a, k) = n(J_1(kr_0)/(kr_0/2))\delta(a - a_0)$. Thus from (23) we find

$$b(k) = -na^2\frac{4\pi^2}{k^2}\left(1 - \frac{J_1(kr_0)}{kr_0/2}\right)\frac{J_1(ka_0)}{ka_0/2}. \tag{25}$$

And for the true Green function we get

$$G_{\text{true}} = G_0(k)\,N(k) = G_0(k)\,(1 + b(k)). \tag{26}$$

In the short-wave limit (ka, $kr_0 \gg 1$) $b(k) \to 0$ and therefore $N(k) \to 1$. This means that at very small scales the space filled with a finite density of wormholes looks like the ordinary Euclidean space. In the long-wave limit $k \to 0$ we get $J_1(kr_0)/(kr_0/2) \approx 1 - (1/2)(kr_0/2)^2 + \cdots$ which gives $b(k) \approx -\pi^2 na^2r_0^2/2$, while in a more general case we find $b(k) \approx -\int(\pi^2/2)a^2r_0^2 n(a, r_0)da\,dr_0$, where $n(a, r_0)$ is the density of wormholes with a particular values of a and r_0, and for the bias function (15) we get

$$N(k) \longrightarrow 1 - \frac{\pi^2}{2}\int a^4 n(a, r_0)\frac{r_0^2}{a^2}da\,dr_0 \le 1. \tag{27}$$

In other words, in the long-wave limit (ka, $kr_0 \ll 1$) the presence of a particular set of virtual wormholes diminishes merely the value of the charge values.

(2) Limiting Topologies or Infinite Densities of Wormholes. Consider now the limiting distribution when the density of wormholes $n \to \infty$. Since every throat cuts the finite portion of the volume $(\pi^2/2)a^4$, this case requires considering the limit $a \to 0$. We assume that in this limit $a^2NF(a, X) \to \delta(a)\nu(X)$, where $\nu(X)$ is a finite specific distribution. Then (23) reduces to $b_{\text{total}}(k) = (4\pi^2/k^2)(\tilde{\nu}(k) - \tilde{\nu}(0))$ where $\tilde{\nu}(k) = \int \nu(X)e^{ikX}d^4X$ and the bias (15), (18) $N(k)$ becomes

$$N(k) = 1 - \frac{4\pi^2}{k^2}\left(\tilde{\nu}(0) - \tilde{\nu}(k)\right). \tag{28}$$

It is important that this limit still agrees with the rarefied gas approximation, for the basic gas parameter (i.e., the portion of volume cut by wormholes) tends to $\xi = \int(\pi^2/2)a^4F(a, X)d^4X da \to 0$. The above expression is obtained in the linear approximation only. Taking into account next orders (e.g., see [22] where the case of a dense gas is also considered) we find $N(k) = 1 + b_{\text{total}}(k) + b_{\text{total}}^2(k) + \cdots$ and the true Green function in a gas of wormholes becomes

$$G_{\text{true}} = G_0(k)\,N(k) = \frac{1}{k^2 + 4\pi^2\left(\tilde{\nu}(0) - \tilde{\nu}(k)\right)}. \tag{29}$$

In the long-wave limit $k \to 0$ the function $\tilde{\nu}(k)$ can be expanded as $\tilde{\nu}(k) \approx \tilde{\nu}(0) + (1/2)\tilde{\nu}''(0)k^2$ which also defines a renormalization of charge values $N(k) \to 1/(1 - 2\pi^2\tilde{\nu}''(0))$ which coincides with (27). However in this limiting case we have some freedom in the choice of $\tilde{\nu}(k)$, which we can use to assign $N(k)$ an arbitrary function of k. In other words we may get here a class of limiting topologies where Green functions $G_{\text{true}} = G_0(k)N(k)$ have a good ultraviolet behavior and quantum field theories in such spaces turn out to be finite. In particular, this will certainly be the case when

$$G_{\text{true}}\left(x - x' = 0\right) = \int\frac{1}{k^2 + 4\pi^2\left(\tilde{\nu}(0) - \tilde{\nu}(k)\right)}\frac{d^4k}{(2\pi)^4} < \infty. \tag{30}$$

This possibility however requires the further and more deep investigation.

3.2. Green Function, General Consideration. The action (6) remains invariant under translations $\vec{x}' = \vec{x} + \vec{c}$ with an arbitrary $\vec{c}$ which means that the measure (11) does not actually depend on the position of the center of mass of the gas of wormholes and, therefore, we may restrict ourself with homogeneous distributions $F(\xi)$ of wormholes in space only. Indeed, we may define $d^N\xi = d^N\xi' d^4c$, while the integration over d^4c gives the volume of R^4; that is, $\int d^4c = L^4 = V$ which disappears from (11) due to the denominator. (Technically, we may first restrict a portion of R^4 in (6) to a finite volume V and then in final expressions consider the limit $V \to \infty$ (which represents the standard tool in thermodynamics and QFT).) In what follows we shall omit the prime from ξ'.

Let us consider the Fourier representation $N(x, x', \xi) \to N(k, k', \xi)$ which in the case of a homogeneous distribution of wormholes gives $N(k, k') = N(k, \xi)\,\delta(k - k')$; then we find

$$G(k) = G_0(k)\,N(k, \xi) \tag{31}$$

and the Green function can be taken as

$$G = \frac{N(k,\xi)}{k^2 + m^2}. \tag{32}$$

Then for the total partition function we find

$$Z_{\text{total}}(J) = \int [DN(k)]\, e^{-I(N)} e^{-(1/2)(L^4/(2\pi)^4)\int((N(k)/(k^2+m^2))|J_k|^2)dk}, \tag{33}$$

where $[DN] = \prod_k dN_k$ and $\sigma(N)$ comes from the integration measure (i.e., from the Jacobian of transformation from $F(\xi)$ to $N(k)$)

$$e^{-I(N)} = \int [DF]\, Z_0(N(k,\xi))\, \delta(N(k) - N(k,\xi)). \tag{34}$$

We point out that $I(N)$ can be changed by means of adding to the action (6) of an arbitrary "nondynamical" constant term which depends only on topology $S \to S + \Delta S(N(k))$ (e.g., a topological Euler term). The multiplier $Z_0(N)$ defines the simplest measure for topologies. Now by means of using the expression (32) and (33) we find the two-point Green function in the form

$$G(k) = \frac{\overline{N}(k)}{k^2 + m^2}, \tag{35}$$

where $\overline{N}(k)$ is the cutoff function (the mean bias) which is given by (We recall that in this integral contribute only limiting topologies in which density of wormholes diverges ($n \to \infty$))

$$\overline{N}(k) = \frac{1}{Z_{\text{total}}(0)} \int [DN]\, e^{-I(N)} N(k). \tag{36}$$

At the present stage we still cannot evaluate the exact form for the cutoff function $\overline{N}(k)$ in virtue of the ambiguity of $\Delta S(N(k))$ pointed out. Such a term may include two parts. First part $\Delta_1 S$ describes the proper dynamics of wormholes and should be considered separately. Indeed, in general wormholes are dynamical self-gravitating objects which require considering the gravitational contribution to the action. Some part of such a contribution (mean curvature induced by wormholes) is discussed in Section 6. However, since a wormhole represents an extended nonlocal object, it possesses a rather complex dynamics and this problem requires the further investigation. The second part $\Delta_2 S$ may describe "external conditions" (e.g., an external classical field in (33)) for the mean topology. Actually the last term can be used to prescribe an arbitrary particular value for the cutoff function $\overline{N}(k) = f(k)$. Indeed, the "external conditions" can be accounted for by adding the term $\Delta_2 S = (\lambda, N) = \int \lambda(k) N(k) d^4k$, where $\lambda(k)$ plays the role of a specific chemical potential which implicitly depends on $f(k)$ through the equation

$$f(k) = \frac{1}{Z_{\text{total}}(\lambda, 0)} \int [DN]\, e^{(\lambda,N) - I(N)} N(k). \tag{37}$$

From (33) we see that the role of such a chemical potential may play the external current $\lambda(k) = -(1/2)(L^4/(2\pi)^4) \cdot G_0(k)|J_k^{\text{ext}}|^2$ or equivalently an external classical field $\varphi^{\text{ext}} = G_0(k) J_k^{\text{ext}}$. In quantum field theory such a term leads merely to a renormalization of the cosmological constant. By other words the mean topology (i.e., the cutoff function or mean distribution of wormholes) is driven by the cosmological constant Λ and vice versa.

4. Topological Bias as a Projection Operator

By the construction the topological bias $N(x, x')$ plays the role of a projection operator onto the space of functions (a subspace of functions on R^4) which obey the proper boundary conditions at throats of wormholes. This means that for any particular topology (for a set of wormholes) there exists the basis $\{f_i(x)\}$ in which it takes the diagonal form $N(x, x') = \sum N_i f_i(x) f_i^*(x')$ with eigenvalues $N_i = 0, 1$ (since $N_i^2 = N_i$). In this section we illustrate this simple fact (which is probably not obvious for readers) by the explicit construction of the reference system for a single wormhole when physical functions become (due to the boundary conditions) periodic functions of one of coordinates.

Indeed, consider a single wormhole with parameters ξ (i.e., $\xi = (a, R^+, R^-)$, where a is the throat radius and $R^\pm$ are positions of throats in space. (In general, there exists an additional parameter Λ^α_β which defines a rotation of one of throats before gluing. However, it does not change the subsequent construction. There always exists a diffeomorphic map of coordinates $x' = h(x)$ which sets such a matrix to unity).) Consider now a particular solution ϕ_0 to the equation $\Delta\phi_0 = 0$ (harmonic function) for R^4 in the presence of the wormhole, which corresponds to the situation when throats possess a unit charge/mass but those have the opposite signs. Now define the family of lines of force $x(s, x_0)$ which obey the equation $dx/ds = -\nabla\phi_0(x)$ with initial conditions $x(0) = x_0$. Physically, such lines correspond to lines of force for a two charged particles in positions $R^\pm$ with charges ± 1. We note that all points which lay on the trajectory $x(s, x_0)$ may be taken as initial conditions and they define the same line of force with the obvious redefinition $s \to s - s_0$. By other words we may take as a new coordinates the parameter s and portion of the coordinates orthogonal to the family of lines $x_0^\perp$. Coordinates $x_0^\perp$ can be taken as laying in the hyperplane R^3 which is orthogonal to the vector $\vec{d} = \vec{R}^- - \vec{R}^+$ and goes through the point $\vec{X}_0 = (\vec{R}^- + \vec{R}^+)/2$. (Instead of the construction used here one may use also another method. Indeed, consider two point charges, and then the function $\phi_0 = 1/(x - x_+)^2 - 1/(x - x_-)^2$ can be taken as a new coordinate. Wormhole appears when we identify (glue) surfaces $\phi_0 = \pm\omega$. We point out that such surfaces are not spheres, though they reduce to spheres in the limit $|x_+ - x_-| \to \infty$ or $\omega \to \infty$.)

Let $s^\pm(x_0^\perp)$ be the values of the parameter s at which the line intersects the throats $R^\pm$. Then instead of s we may consider a new parameter θ as $s(\theta) = s^- + (s^+ - s^-)\theta/2\pi$, so that when $\theta = 0, 2\pi$ the parameter s takes the values

$s = s^-$, s^+ respectively. The gluing procedure at throats means merely that we identify points at $\theta = 0$ and $\theta = 2\pi$ and all physical functions in the space R^4 with a single wormhole ξ become periodic functions of θ. Thus, the coordinate transformation $x = x(\theta, x_0^\perp)$ gives the map of the above space onto the cylinder with a specific metric $dl^2 = (d\vec{x}(\theta, x_0^\perp))^2 = g_{\alpha\beta}dy^\alpha dy^\beta$ (where $y = (\theta, x_0^\perp)$) whose components are also periodic in terms of θ. Now we can continue the coordinates to the whole space R^4 (to construct a cover of the fundamental region $\theta \in [0, 2\pi]$) simply admitting all values $-\infty < \theta < +\infty$ this, however, requires to introduce the bias

$$\frac{1}{\sqrt{g}}\delta\left(\theta-\theta'\right) \longrightarrow N\left(\theta-\theta'\right) = \sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{g}}\delta\left(\theta-\theta'+2\pi n\right), \tag{38}$$

since every point and every source in the fundamental region acquires a countable set of images in the nonphysical region (inside of wormhole throats). Considering now the Fourier transforms for θ we find

$$N\left(k,k'\right) = \sum_{n=-\infty}^{+\infty}\delta\left(k-n\right)\delta\left(k-k'\right). \tag{39}$$

We point out that the above bias gives the unit operator in the space of periodic functions of θ. From the standpoint of all possible functions on R^4 it represents the projection operator $\widehat{N}^2 = \widehat{N}(\xi)$ (taking an arbitrary function f we find that upon the projection $f_N = \widehat{N}f$ f_N becomes a periodic function of θ; that is, only periodic functions survive).

The above construction can be easily generalized to the presence of a set of wormholes. In the approximation of a dilute gas of wormholes we may neglect the influence of wormholes on each other (at least there always exists a sufficiently smooth map which transforms the family of lines of force for "independent" wormholes onto the actual lines). Then the total bias (projection) may be considered as the product

$$\begin{aligned} N_{\text{total}}\left(x,x'\right) = \int\left(\prod_i\sqrt{g_i}d^4y_i\right)N\left(\xi_1,x,y_1\right) \\ \times N\left(\xi_2,y_1,y_2\right)\cdots N\left(\xi_N,y_{N-1},x'\right), \end{aligned} \tag{40}$$

where $N(\xi_i, x, x')$ is the bias for a single wormhole with parameters ξ_i. Every such a particular bias $N(\xi_i, x, x')$ realizes projection on a subspace of functions which are periodic with respect to a particular coordinate $\theta_i(x)$, while the total bias gives the projection onto the intersection of such particular subspaces (functions which are periodic with respect to every parameter θ_i).

5. Cutoff

The projective nature of the bias operator $N(x, x')$ allows us to express the cutoff function $\overline{N}(k)$ via dynamic parameters of wormholes. Indeed, consider a box L^4 in R^4 and periodic boundary conditions which gives $k = 2\pi n/L$ (in final expressions we consider the limit $L \to \infty$, which gives $\sum_k \to (L^4/(2\pi)^4)\int d^4k$). And let us consider the decomposition for the integration measure in (33) as

$$I = I_0 + \sum\lambda_1(k)N(k) + \frac{1}{2}\sum\lambda_2\left(k,k'\right)N(k)N\left(k'\right) + \cdots, \tag{41}$$

where $\lambda_1(k)$ includes also the contribution from $Z_0(k)$. We point out that this measure plays the role of the action for the bias $N(k)$. Indeed, the variation of the above expression gives the equation of motions for the bias in the form

$$\sum_{k'}\lambda_2\left(k,k'\right)N\left(k'\right) = -\lambda_1(k), \tag{42}$$

which can be found by considering the proper dynamics of wormholes. We however do not consider the problem of the dynamic description of wormholes here and leave this for the future research. Moreover, we may expect that in the first approximation one may retain the linear term only. Indeed, this takes place when $N(k)$ is not a dynamic variable, or if we take into account that $N(k)$ is a collective variable. Then the projective nature of the bias $N(k) = 0, 1$ means that it can be phenomenologically expressed via some Fermionic ghost field $\Psi(k)$ (e.g., $N(k, k') = \Psi(k)\Psi^+(k')$) where the negative and positive frequency parts of the operator $\Psi(k)$ obey the anticommutation relations $\Psi^+(k)\Psi(k') + \Psi(k')\Psi^+(k) = \delta(k-k')$. In the absence of ghost particles $\Psi(k)|0\rangle = 0$ we get $N(k, k') = \delta(k - k')$; that is, $N(k) = 1$ and wormholes are absent. In the term of the ghost field the action becomes $I(N) = I_0 + (\Psi, \widehat{\lambda}_1\Psi) + \cdots$. Therefore in the leading approximation equations of motion take the linear form $\widehat{\lambda}_1\Psi = 0$.

Thus taken into account that $N(k) = 0, 1$ ($N^2 = N$) we find

$$\overline{N}(k) = \frac{1}{Z_{\text{total}}(k)}\sum_{N=0,1}e^{-\lambda_1(k)N(k)}N(k) = \frac{e^{-\lambda_1(k)}}{1+e^{-\lambda_1(k)}}. \tag{43}$$

The simplest choice gives merely $\lambda_1(k) = -\sum\ln Z_0(k)$, where the sum is taken over the number of fields and $Z_0(k)$ is given by $Z_0(k) = \sqrt{\pi/(k^2+m^2)}$. In the case of a set of massless fields we find $\overline{N}(k) = Z(k)/(1 + Z(k))$ where $Z(k) = (\sqrt{\pi}/k)^\alpha$ and α is the effective number of degrees of freedom (the number of boson minus fermion fields). To ensure the absence of divergencies one has to consider the number of fields $\alpha > 4$ [23]. However, such a choice gives the simplest estimate which, in general, cannot be correct. Indeed, while its behavior at very small scales (i.e., when exceeding the Planckian scales $Z(k) \lesssim 1$ and $\overline{N}(k) = Z(k)$) may be physically accepted, since it produces some kind of a cutoff, on the mass-shell $k^2 + m^2 \to 0$ it gives the behavior $\overline{N}(k) \to 1$ which is merely incorrect (e.g., from (27) we see that the true behavior should be $\overline{N}(k) \to \text{const} < 1$).

One may expect that the true cutoff function has a much more complex behavior. Indeed, some theoretical models in particle physics (e.g., string theory) have the property to be lower-dimensional at very small scales. The mean cutoff

$\overline{N}(k)$ gives the natural tool to describe a scale-dependant dimensional reduction [24, 25]. In fact, this function defines the spectral number of modes in the interval between k and $k + dk$ as

$$\int \overline{N}(k) \frac{d^4k}{(2\pi)^4} = \int \frac{\overline{N}(k) k^4}{(2\pi)^2} \frac{dk}{k}. \tag{44}$$

Hence we can define the effective spectral dimension D of space as follows:

$$k^4 \overline{N}(k) \sim k^D. \tag{45}$$

From the empirical standpoint the dimension $D = 4$ is verified at laboratory scales only, while the rigorous tool to define the spectral density of states (or the mean cutoff) can give the lattice quantum gravity, for example, see [26, 27] and references therein. And indeed, the spectral dimension for nonperturbative quantum gravity defined via Euclidean dynamical triangulations was calculated recently in [28]. It turns out that it runs from a value of $D = 3/2$ at short distance to $D = 4$ at large distance scales. We also point out that all observed dark matter phenomena can be explained by the fractal dimension $D \approx 2$ starting from scales $L \gtrsim (1 \div 5)$ Kpc, for example, [29–33].

6. Cosmological Constant

Let us consider the total Euclidean action [17]

$$I_E = -\frac{1}{16\pi G} \int (R - 2\Lambda_0) \sqrt{g} d^4x - \int L_m \sqrt{g} d^4x. \tag{46}$$

The variation of the above action leads to the Einstein equations

$$R_{ab} - \frac{1}{2} g_{ab} R + g_{ab} \Lambda_0 = 8\pi G T_{ab}, \tag{47}$$

where $T^{ab} = (1/2)(g)^{-1/2}(\delta L_m/\delta g_{ab})$ is the stress energy tensor and Λ_0 is a naked cosmological constant. In cosmology such equations are considered from the classical standpoint, which means that they involve characteristic scales $\ell \gg \ell_{\rm pl}$. However, the presence of virtual wormholes at Planckian scales defines some additional contribution in both parts of these equations which can be adsorbed into the cosmological constant. Therefore the total cosmological constant can be defined as

$$\Lambda_{\rm tot} = \Lambda_0 + \Lambda_m + \Lambda_R = \Lambda_0 + 2\pi G \langle T \rangle + \frac{1}{4} \langle R \rangle_w, \tag{48}$$

where $\langle T \rangle$ is the energy of zero-point fluctuations. (It includes also the contribution of zero-point fluctuations of gravitons.) That is, the mean vacuum value (we recall that in the standard QFT Λ_m is infinite, while wormholes form a finite value) and $\langle R \rangle_w = \Lambda_R$ is a contribution of wormholes into the mean curvature due to gluing (1).

6.1. Contribution of Virtual Wormholes into Mean Curvature. Consider a single wormhole whose metric is given by (1) $ds^2 = h^2(r)\delta_{\alpha\beta} dx^\alpha dx^\beta$; then the components of the Ricci tensor are (We are much obliged to the Referee who pointed out to the subtleties when working with a step function in the metric.)

$$\begin{aligned} R_{\alpha\beta} = 7 &- \left(\frac{h'}{h}\right)' \left(\delta_{\alpha\beta} + (N-2) n_\alpha n_\beta\right) \\ &- \frac{1}{r}\frac{h'}{h} \left((N-2)\Delta_{\alpha\beta} + \delta_{\alpha\beta}(N-1)\right) \\ &- \left(\frac{h'}{h}\right)^2 (N-2)\Delta_{\alpha\beta}, \end{aligned} \tag{49}$$

where $n^\nu = x^\nu/a$ is the unite normal vector to the throat surface, $\Delta_{\alpha\beta} = \delta_{\alpha\beta} - n_\alpha n_\beta$, $h' = \partial h/\partial r$, and N is the number of dimensions. The curvature scalar is

$$R = -\frac{(N-1)}{h^2} \left[2\frac{h''}{h} + 2\frac{1}{r}\frac{h'}{h}(N-1) + \left(\frac{h'}{h}\right)^2 (N-4)\right]. \tag{50}$$

Then substituting $h = 1 + (a^2/r^2 - 1)\theta$ in the above equation we find

$$\begin{aligned} -\frac{h^4}{N-1} R = 4h\frac{a^2}{r^3}\delta &+ \frac{2h}{r^{N-1}}\left(r^{N-1}\lambda\right)' \\ &+ (N-4)\left(\lambda^2 - 4\theta\frac{a^2}{r^3}\lambda\right) \\ &+ 4(N-4)\frac{a^2}{r^4}\theta(\theta-1), \end{aligned} \tag{51}$$

where θ is a smooth function which only in a limit becomes a step function, $\lambda = (1 - a^2/r^2)\delta$, and $\delta = -\theta'$. In the limit $\theta \to \theta(a-r)$ we find $\delta \to \delta(r-a)$ and the last two terms in (51) are negligible as compared to the first two terms, while in four dimensions the last two terms vanish. Then the curvature is concentrated on the throat of the wormhole where θ differs from the step function. In the limit of a vanishing throat size we have $h \to 1$ at the throat and we get

$$-R = \frac{4(N-1)}{a}\delta(r-a) + \frac{2(N-1)}{r^{N-1}}\left(r^{N-3}\left(r^2 - a^2\right)\delta\right)'. \tag{52}$$

In general all the terms together in (51) and analogous terms in the Ricci tensor provide that the Bianchi identity holds and the energy is conserved [19]. Therefore none of them can be dropped out. However it is easy to see the leading contribution to the integral over space comes from the first term only and in the limit of a vanishing throat size we find for a single wormhole

$$\frac{1}{4}\int R\sqrt{g} d^4x = -6\pi^2 a^2. \tag{53}$$

In the case of a set of wormholes (24) we find

$$\frac{1}{4}\int R\sqrt{g}d^4x = -6\pi^2\sum_j a_j^2 = -12\pi^2\int n(a)\,a^2 da\,d^4x = \int \Lambda_R d^4x, \quad (54)$$

where $2n(a) = \int n(a,r_0)dr_0$ is the density of wormhole throats with a fixed value of the throat size a. This defines the contribution to the cosmological constant from the mean curvature as

$$\Lambda_R = -12\pi^2\int n(a)\,a^2 da < 0. \quad (55)$$

We see that this quantity is always negative.

6.2. Stress Energy Tensor. In this section we consider the contribution from matter fields. In the case of a scalar field the stress energy tensor has the form

$$-T_{\alpha\beta}(x) = \partial_\alpha\varphi\partial_\beta\varphi - \frac{1}{2}g_{\alpha\beta}\left(\partial^\mu\varphi\partial_\mu\varphi + m^2\varphi^2\right). \quad (56)$$

Then the mean vacuum value of the stress energy tensor can be obtained directly from the two-point green function (32), (35) as

$$-\left\langle T_{\alpha\beta}(x)\right\rangle = \lim_{x'\to x}\left(\partial_\alpha\partial'_\beta - \frac{1}{2}g_{\alpha\beta}\left(\partial^\mu\partial'_\mu + m^2\right)\right)\left\langle G\left(x,x',\xi\right)\right\rangle. \quad (57)$$

By means of using the Fourier transform $G(x,x',\xi) = \int e^{-ik(x-x')}G(k,\xi)\,(d^4k/(2\pi)^4)$ and the expressions (32), (35) we arrive at

$$\left\langle T_{\alpha\beta}(x)\right\rangle = \frac{1}{4}g_{\alpha\beta}\int\left(1+\frac{m^2}{k^2+m^2}\right)\overline{N}(k)\,\frac{d^4k}{(2\pi)^4}, \quad (58)$$

where the property $\int k_\alpha k_\beta f(k^2)d^4k = (1/4)g_{\alpha\beta}\int k^2 f(k^2)d^4k$ has been used and $\overline{N}(k) = \langle N(k,\xi)\rangle$ is the cutoff function (36).

For the sake of simplicity we consider the massless case. Then by the use of the cutoff $\overline{N}(k) = \pi^{\alpha/2}/(\pi^{\alpha/2}+k^\alpha)$ from the previous section we get the finite estimate ($\alpha > 4$ is the effective number of the field helicity states)

$$\Lambda_m = 2\pi G\sum\int\frac{\pi^{\alpha/2}}{\pi^{\alpha/2}+k^\alpha}\frac{d^4k}{(2\pi)^4} = \frac{\pi G}{4}\Gamma\left(\frac{\alpha-4}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right)\sim 1, \quad (59)$$

where the sum is taken over the number of fields. Since the leading contribution comes here from very small scales, we may hope that this value will not essentially change if the true cutoff function changes the behavior on the mass-shell as $k \to 0$ (e.g., if we take $\lambda_1(k) = -\sum\ln Z_0(k) + \delta\lambda(k)$ with $\delta\lambda(k) \ll \ln Z_0(k)$ as $k \gg 1$).

To understand how wormholes remove divergencies, it will be convenient to split the bias function into two parts $N(k,\xi) = 1 + b(k,\xi)$, where 1 corresponds to the standard Euclidean contribution, while $b(k,\xi)$ is the contribution of wormholes. The first part gives the well-known divergent contribution of vacuum field fluctuations $8\pi G\langle T^0_{\alpha\beta}\rangle = \Lambda_* g_{\alpha\beta}$ with $\Lambda_* \to +\infty$, while the second part remains finite for any finite number of wormholes and, due to the projective nature of the bias described in the previous section, it partially compensates (reduces) the value of the cosmological constant; that is, $8\pi G\langle\Delta T_{\alpha\beta}\rangle = \delta\Lambda g_{\alpha\beta}$, where $\delta\Lambda = \sum_N \rho_N\delta\Lambda(N)$ and $\delta\Lambda(N)$ is a negative finite contribution of a finite set of wormholes.

Consider now the particular distribution of virtual wormholes (24) and evaluate their contribution to the cosmological constant which is given by $\delta\Lambda(N) = 2\pi G\int b(k)\,(d^4k/(2\pi)^4) = 2\pi G b_{\text{total}}(0)$. Then from the expressions (22) and (24) we get

$$b_{\text{total}}(0) = -\frac{n}{4\pi^4 a^3 r_0^3}\int\left(1-\frac{a^2}{R_-^2}\right)\delta\left(R_+ - a\right)\times\delta\left(\left|R_+ - R_-\right| - r_0\right)d^4R_- d^4R_+, \quad (60)$$

which gives

$$b_{\text{total}}(0) = -n\left(1 - f\left(\frac{a}{r_0}\right)\right), \quad (61)$$

where

$$f\left(\frac{a}{r_0}\right) = \frac{2}{\pi}\int_0^\pi\frac{a^2\sin^2\theta d\theta}{a^2+2ar_0\cos\theta + r_0^2}. \quad (62)$$

For $a/r_0 \ll 1$ (we recall that by the construction $a/r_0 \le 1/2$) this function has the value $f(a/r_0) \approx a^2/r_0^2$. Thus, for the contribution of wormholes we find

$$\delta\Lambda_m = -2\pi G\int n\left(a,r_0\right)\left(1 - f\left(\frac{a}{r_0}\right)\right)da\,dr_0 = -2\pi G n\left(1 - \langle f\rangle\right). \quad (63)$$

6.3. Vacuum Value of the Cosmological Constant. From the above expression we see that to get the finite value of the cosmological constant $\Lambda_m = \Lambda_* + \delta\Lambda_m < \infty$ one should consider the limit $n \to \infty$ (infinite density of virtual wormholes) which requires considering the smaller and smaller wormholes. From the other hand we have the obvious restriction $\int 2n(a,r_0)(\pi^2/2)a^4 da\,dr_0 < 1$, where $(\pi^2/2)a^4$ is the volume of one throat (wormholes cannot cut more than the total volume of space). (We also point out that in removing divergencies the leading role plays the zero-point energy. Indeed $\delta\Lambda_m \sim -2\pi G n$, while the mean curvature has the order $\Lambda_R \sim -a^2 n$ and for $a \ll \ell_{\text{pl}}$ we have $\delta\Lambda_m \gg \Lambda_R$. Moreover in the limit $n \to \infty$, one gets $a \to 0$ and therefore $\Lambda_R/\delta\Lambda_m \to 0$.) Therefore, in the leading order it seems to be sufficient to retain point-like wormholes only (i.e., consider the limit $a \to 0$). Then instead of (24) we may assume the vacuum distribution of virtual wormholes in the form

$$NF(a,X) = \frac{1}{a^2}\delta(a)\,\nu(X), \tag{64}$$

where $\nu(X) = \int a^2 NF(a,X)da$ and $\int (1/a^2)\nu(X)d^4X = n \to \infty$ has the meaning of the infinite density of point-like wormholes, while $\nu \sim a^2 n$ remains a finite. In this case the volume cut by wormholes vanishes $\int 2n(\pi^2/2)a^4 da\,dr_0 = a^2\int \nu(X)d^4X \to 0$ and the rarefied gas approximation works well. This defines the bias and the mean cutoff (here we define the Fourier transform $\widetilde{\nu}(k) = \int \nu(X)e^{ikX}d^4X$) as

$$\overline{N}(k) = 1 - \frac{4\pi^2}{k^2}\left(\widetilde{\nu}(0) - \widetilde{\nu}(k)\right). \tag{65}$$

The contribution to the mean curvature (55) can be expressed via the same function $\nu(k)$ as

$$\begin{aligned}\Lambda_R &= -12\pi^2\int n(a,r_0)\,a^2 da\,dr_0 \\ &= -12\pi^2\int \nu(X)\,d^4X = -12\pi^2\widetilde{\nu}(0).\end{aligned} \tag{66}$$

Thus, for the total cosmological constant we get the expression

$$\begin{aligned}\Lambda_{\text{tot}} &= \Lambda_0 \\ &\quad + 2\pi G\int\left(1 - \frac{4\pi^2}{k^2}\left(\widetilde{\nu}(0) - \widetilde{\nu}(k)\right)\right)\frac{d^4k}{(2\pi)^4} - 12\pi^2\widetilde{\nu}(0).\end{aligned} \tag{67}$$

We stress that all these terms should be finite. Indeed all distributions of virtual wormholes $\widetilde{\nu}(k)$ which lead to an infinite value of Λ_{tot} are suppressed in (5) by the factor $\sim e^{-\int \Lambda_{\text{tot}}d^4x}$, while the minimal value is reached when wormholes cut all of the volume of space and the action is merely $S = 0$. (Frankly speaking this statement is not rigorous. At first look the two last terms in (67) are independent and one may try to take $\widetilde{\nu}(0)$ an arbitrary big. If this were the case then the action would not possess the minimum at all. However $\widetilde{\nu}(0)$ cannot be arbitrary big, since it will violate the rarefied gas approximation and the linear expression (67) brakes down. Moreover, fermions give here a contribution of the opposite sign. The rigorous investigation of this problem requires the further studying and we present it elsewhere.) We also point out that here we considered the real scalar field as the matter source, while in the general case the stress energy tensor should include all existing Bose and Fermi fields (Fermi fields give a negative contribution to Λ_m).

The value of Λ_0 looks like a free parameter, which in quantum gravity runs with scales [27]. However at large scales its asymptotic value may be uniquely fixed by the simple arguments as follows. Indeed in quantum field theory properties of the ground state (vacuum) change when we imply an external classical fields. The same is true for the distribution of virtual wormholes $\widetilde{\nu}(k,J)$, for example, see (33) and (37) and, therefore, $\Lambda_{\text{tot}} = \Lambda_{\text{tot}}(J)$ which we describe in the next subsection. We recall that in gravity the role of the external current plays the stress energy tensor of matter fields $J = T_{ab}$. However one believes that in the absence of all classical fields the vacuum state should represent the most symmetric (Lorentz invariant) state which in our case corresponds to the Euclidean space. In order to be consistent with the Einstein equations this requires $\Lambda_{\text{tot}}(J = 0) = 0$ which uniquely fixes the value of Λ_0 in (67). We point out that from somewhat different considerations such a choice was advocated earlier in [15, 34, 35].

6.4. Vacuum Polarization in an External Field. Consider now topology fluctuations in the presence of an external current. In the presence of an external current J^{ext} the distribution of virtual wormholes changes $\widetilde{\nu}(k,J) = \widetilde{\nu}(k) + \delta\widetilde{\nu}(k,J)$. Indeed in (33) for the case of a weak external field the contribution of the external current into the action can be expanded as $\exp(-V(J)) \simeq 1 - V$, where

$$\begin{aligned}V &= -\frac{1}{2}\int J(x)\,G(x,y)\,J(y)\,d^4x\,d^4y \\ &= -\frac{1}{2}\frac{L^4}{(2\pi)^4}\int G_0(k)\,N(k)\left|J_k\right|^2 d^4k.\end{aligned} \tag{68}$$

Then using (36) we find $\overline{N}(k,J) = \overline{N}(k,0) + \delta N(k,J)$, where

$$\delta N(k,J) = \delta b(J) \simeq -\frac{1}{2}\frac{L^4}{(2\pi)^4}\int \sigma^2(k,p)\,G_0(p)\left|J_p\right|^2 d^4p \tag{69}$$

is the bias related to an additional distribution of virtual wormholes and $\sigma^2(k,p) = \overline{\Delta N^*(k)\Delta N(p)}$

$$\sigma^2(k,p) = \frac{1}{Z_{\text{total}}(0)}\int [DN]\,e^{-I(N)}\Delta N^*(k)\,\Delta N(p) \tag{70}$$

defines the dispersion of vacuum topology fluctuations (here $\Delta N = N - \overline{N}$). The exact definition of $\sigma^2(k,p)$ requires the further development of a fundamental theory. In particular, it can be numerically calculated in lattice quantum gravity [28]. However it can be shown that at scales $k,\ p \gg k_{\text{pl}}$ it reduces to $\sigma^2(k,p) \to \sigma_k^2\delta(k-p)$ and therefore

$$\delta b(J) = -\sigma_k^2\frac{4\pi^2}{2k^2}\left|J_k^{\text{ext}}\right|^2. \tag{71}$$

Now comparing this function with (23) we relate the additional distribution of virtual wormholes and the external classical field as

$$\frac{4\pi^2}{k^2}\left(\delta\widetilde{\nu}(0,J) - \delta\widetilde{\nu}(k,J)\right) = \frac{1}{2}\sigma_k^2\frac{4\pi^2}{k^2}\left|J_k^{\text{ext}}\right|^2, \tag{72}$$

where $\delta\widetilde{\nu}(k,J) = \int a^2\delta NF(a,k)da$. We point out that the above expression does not define the value $\delta\widetilde{\nu}(0,J)$ which requires an additional consideration. Moreover, in general the external field J does not possess a symmetry and therefore the correction $\langle \delta T_{\alpha\beta}(x)\rangle$ does not reduce to a single cosmological constant. However, such corrections always violate the averaged null energy condition [36, 37] and may be considered as some kind of dark energy or, by other words, it represents an exotic matter. Some portion of dark energy still has the form of the cosmological constant which defines a nonvanishing present day value (we recall that fermions give a contribution of the opposite sign)

$$\delta\Lambda_{\text{tot}} = -2\pi G \int \frac{4\pi^2}{k^2} \langle \delta\widetilde{\nu}(0,J) - \delta\widetilde{\nu}(k,J)\rangle \frac{d^4k}{(2\pi)^4} - 12\pi^2 \delta\widetilde{\nu}(0,J), \tag{73}$$

where $\langle \delta\widetilde{\nu}(k,J)\rangle$ denotes an averaging over rotations.

The only unknown parameter in (72) is the dispersion σ_k^2 which defines the intensity of topology fluctuations in the vacuum. It has also the sense of the efficiency coefficient which defines the portion of the energy of the external field spent on the formation of additional wormholes. Though the evaluation of σ_k^2 requires the further development of a fundamental theory, one may expect that $\sigma_k^2 = \overline{N}(k)(1 - \overline{N}(k))$, where $\overline{N}(k)$ is the mean cutoff. It is expected that $\overline{N}(k) \to 0$ as $k \gg k_{\text{pl}}$ and $\overline{N}(k) \to \overline{N}_0 \leq 1$. This means that $\sigma \to 0$ as $k \gg k_{\text{pl}}$ and $\sigma \to \sigma_0 \ll 1$ as $k \ll k_{\text{pl}}$, while it takes the maximum value $\sigma_{\max} \sim 1$ at Planckian scales $k \sim k_{\text{pl}}$. By other words, the most efficient transmission of the energy into wormholes takes place for wormholes of the Planckian size. In the case when external classical fields have characteristic scales $\lambda = 2\pi/k \gg \ell_{\text{pl}}$ in (72) the efficiency coefficient σ_k^2 and the cutoff $\overline{N}(k)$ become constant $\sigma \simeq \sigma_0$, $\overline{N}(k) \simeq \overline{N}_0$, while their ratio may be estimated as $\alpha\sigma_0^2/\overline{N}_0 = \Omega_{\text{DE}}/\Omega_b$, where α is the effective number of fundamental fields which contribute to $\delta\Lambda_{\text{tot}}$ and Ω_{DE}, Ω_b are dark energy and baryon energy densities, respectively. According to the modern picture this ratio gives $\Omega_{\text{DE}}/\Omega_b \approx 0.75/0.05 = 15$, while $\sigma_0^2/\overline{N}_0 \sim 1$ (as $\overline{N}_0 \ll 1$) and therefore this defines the estimate for the effective number of fundamental fields (helicity states) as $\alpha \sim 15$.

6.5. Speculations on the Formation of Actual Wormholes. As we already pointed out the additional distribution of virtual wormholes (72) reflects the symmetry of external classical fields and therefore it forms a homogeneous and isotropic background and perturbations. We recall that virtual wormholes represent an exotic form of matter. In the early Universe such perturbations start to develop and may form actual wormholes. The rigorous description of such a process represents an extremely complex and interesting problem which requires the further study. Some aspects of the behavior of the exotic density perturbations were considered in [7], while the simplest example of the formation of a wormhole-type object was discussed recently by us in [38]. Therefore we may expect that some portion of such a form of dark energy is reserved now in actual wormholes which we consider in the next section.

7. Dark Energy from Actual Wormholes

Consider now the contribution to the dark energy from the gas of actual wormholes. Unlike the virtual wormholes, actual wormholes do exist at all times and, therefore, a single wormhole can be viewed as a couple of conjugated cylinders $T_\pm^3 = S_\pm^2 \times R^1$. So that the number of parameters of an actual wormhole is less $\eta = (a, r_+, r_-)$, where a is the radius of $S_\pm^2$ and $r_\pm \in R^3$ is a spatial part of $R_\pm$.

Actual wormholes also produce two kinds of contribution to the dark energy. One comes from their contribution to the mean curvature which corresponds to an exotic stress energy momentum tensor. Such a stress energy momentum tensor reflects the dark energy reserved by additional virtual wormholes discussed in the previous section. Such energy is necessary to support actual wormholes as a solution to the Einstein equations. The second part comes from vacuum polarization effects by actual wormholes. The consideration in the previous section shows that for macroscopic wormholes the second part has the order $\langle \Delta T_{\alpha\beta}\rangle \sim 8\pi G n$ and is negligible as compared to the curvature $R \sim a^2 n$ (since macroscopic wormholes have throats $a \gg \ell_{\text{pl}}$). However, for the sake of completeness and for methodological aims we describe it as well.

For rigorous evaluation of dark energy of the second type we, first, have to find the bias $b_1(x, x', \eta)$ analogous to (19) for the topology $R^4/(T_+^3 \cup T_-^3)$. There are many papers treating different wormholes in this respect (e.g., see [36, 37] and references therein). However, in the present paper for an estimation we shall use a more simple trick.

7.1. Beads of Virtual Wormholes (Quantum Wormhole). Indeed, instead of the cylinders $T_\pm^3$ we consider a couple of chains (beads of virtual wormholes $T_\pm^3 \to \cup_n S_{\pm,n}^3$). Such an idea was first suggested in [39] and one may call such an object as quantum wormhole. Then the bias can be written straightforwardly

$$b_1(x, x', \eta) = \sum_{n=-\infty}^{+\infty} \frac{1}{4\pi^2 a} \left(\frac{1}{(R_{-,n} - x')^2} - \frac{1}{(R_{+,n} - x')^2} \right) \times \left[\delta\left(\left| \vec{x} - \vec{R}_{+,n} \right| - a \right) - \delta\left(\left| \vec{x} - \vec{R}_{-,n} \right| - a \right) \right], \tag{74}$$

where $R_{\pm,n} = (t_n, r_\pm)$ with $t_n = t_0 + 2\ell n$ and $\ell \geq a$ is the step. We may expect that upon averaging over the position $t_0 \in [-\ell, \ell]$ the bias for the beads will reproduce the bias for cylinders $T_\pm^3$ (at least it looks like a very good approximation). We point out that the averaging out $(1/2\ell) \int_{-\ell}^{\ell} dt_0$ and the sum $\sum_{n=-\infty}^{+\infty}$ reduces to a single integral $(1/2\ell) \int_{-\infty}^{\infty} dt$ of the zero term in (74). And moreover, the resulting total bias corresponds merely to a specific choice of the distribution function $F(\xi)$ in (21). Namely, we may take

$$NF(\xi) = \frac{1}{2\ell} \delta(t_+ - t_-) f(|r_+ - r_-|, a), \tag{75}$$

where $R_\pm = (t_\pm, r_\pm)$ and $f(s,a)$ is the distribution of cylinders, which can be taken as ($\tilde{n}$ is 3-dimensional density)

$$f(\eta) = \frac{\tilde{n}(a)}{4\pi r_0^2}\delta(s - r_0). \quad (76)$$

Using the normalization condition $\int NF(\xi)d\xi = N$ we find the relation $N = (1/2\ell)\tilde{n}V = nV$, where n is a 4-dimensional density of wormholes and $1/(2\ell)$ is the effective number of wormholes on the unit length of the cylinder (i.e., the frequency with which the virtual wormhole appears at the positions $r_\pm$). This frequency is uniquely fixed by the requirement that the volume which cuts the bead is equal to that which cuts the cylinder $(4/3)\pi a^3 = (\pi^2/2)a^4(1/2\ell)$ (i.e., $2\ell = (3\pi/8)a$ and $n = (8/3\pi a)\tilde{n}$). Thus, we can use directly expression (23) and find (compare to (25))

$$b(k) = -\int n(a)\,a^2\frac{4\pi^2}{k^2}\left(1 - \frac{\sin|\mathbf{k}|\,r_0}{|\mathbf{k}|\,r_0}\right)\frac{J_1(ka)}{ka/2}da, \quad (77)$$

where $k = (k_0, \mathbf{k})$. Here the first term merely coincides with that in (25) and, therefore, it gives the contribution to the cosmological constant $\delta\Lambda/(8\pi G) = -n/4 = -2\tilde{n}/(3\pi a)$, while the second term describes a correction which does not reduce to the cosmological constant and requires a separate consideration.

7.2. Stress Energy Tensor. From (57) we find that the stress energy tensor

$$-\left\langle \Delta T_{\alpha\beta}(x)\right\rangle = \int \frac{k_\beta k_\alpha - (1/2)\,g_{\alpha\beta}k^2}{k^2}b(k,\xi)\frac{d^4k}{(2\pi)^4} \quad (78)$$

reduces to the two functions

$$T_{00} = \varepsilon = \lambda_1 - \frac{1}{2}\mu,$$
$$T_{ij} = p\delta_{ij}, \qquad p = \frac{1}{3}\lambda_2 - \frac{1}{2}\mu, \quad (79)$$

where $\varepsilon + 3p = -\mu$ and $\lambda_1 + \lambda_2 = \mu$ and these functions are

$$\lambda_1 = -\int \frac{k_0^2}{k^2}b\frac{d^4k}{(2\pi)^4}, \qquad \lambda_2 = -\int \frac{|\mathbf{k}|^2}{k^2}b\frac{d^4k}{(2\pi)^4}. \quad (80)$$

By means of the use of the spherical coordinates $k_0^2/k^2 = \cos^2\theta$, $|\mathbf{k}|^2/k^2 = \sin^2\theta$, and $d^4k = 4\pi\sin^2\theta k^3 dk d\theta$ we get

$$\lambda_i = \frac{n(a,r_0)}{4\beta_i}\left(1 - 2\beta_i\left(\frac{a}{r_0}\right)^2 f_i\left(\frac{a}{r_0}\right)\right), \quad (81)$$

where $\beta_1 = 1$, $\beta_2 = 1/3$, and f_i is given by

$$f_{\binom{1}{2}}(y) = \frac{2}{\pi}\int_{-1}^{1}\int_0^\infty \sin(x\sin\theta)\frac{J_1(yx)}{yx/2}\begin{pmatrix}\cos^2\theta\\ \sin^2\theta\end{pmatrix}dx\,d\cos\theta. \quad (82)$$

For $a/r_0 \ll 1$ we find

$$f_{1,2}\left(\frac{a}{r_0}\right) \approx \left(1 + o_{1,2}\left(\frac{a}{r_0}\right)\right). \quad (83)$$

Thus, finally we find

$$\varepsilon \simeq -\frac{n}{4} = -\frac{2\tilde{n}}{3\pi a}, \qquad p \simeq \varepsilon\left(1 - \frac{4}{3}\left(\frac{a}{r_0}\right)^2\right), \quad (84)$$

which upon the continuation to the Minkowsky space gives the equation of state in the form (An arbitrary gas of wormholes splits in fractions with a fixed a and r_0.)

$$p = -\left(1 - \frac{4}{3}\left(\frac{a}{r_0}\right)^2\right)\varepsilon, \quad (85)$$

which in the case when $a/r_0 \ll 1$ behaves like a cosmological constant. However when $a \gg \ell_{\rm pl}$ such a constant is extremely small and can be neglected, while the leading contribution comes from the mean curvature.

7.3. Mean Curvature. In this subsection we consider the Minkowsky space. Then the simplest actual wormhole can be described by the metric analogous to (1), for example, see [7]

$$ds^2 = c^2dt^2 - h^2(r)\,\delta_{\alpha\beta}dx^\alpha dx^\beta, \quad (86)$$

where $h(r) = 1 + \theta(a - r)(a^2/r^2 - 1)$. To avoid problems with the Bianchi identity and the conservation of energy the step function should be also smoothed as in (1). The stress energy tensor which produces such a wormhole can be found from the Einstein equation $8\pi G T_\alpha^\beta = R_\alpha^\beta - (1/2)\delta_\alpha^\beta R$. Both regions $r > a$ and $r < a$ represent portions of the ordinary flat Minkowsky space and therefore the curvature is $R_i^k \equiv 0$. However on the boundary $r = a$ it has the singularity. Since the metric (86) does not depend on time we find

$$R_0^0 = R_\alpha^0 = 0, \qquad R_\alpha^\beta = \frac{2}{a}\delta(a - r)\left\{n_\alpha n^\beta + \delta_\alpha^\beta\right\} + \lambda_\alpha^\beta, \quad (87)$$

where $n^\alpha = n_\alpha = x^\alpha/r$ is the outer normal to the throat S^2, and λ_α^β are additional terms (e.g., see (49) and (51)) which in the leading order are negligible upon averaging over some portion of space $\Delta V \gtrsim a^3$. In the case of a set of wormholes this gives in the leading order

$$R_0^0 = R_\alpha^0 = 0, \qquad R_\alpha^\beta = \sum\frac{2}{a_i}\delta\left(a_i - |r - r_i|\right)\left\{n_{i\alpha}n_i^\beta + \delta_\alpha^\beta\right\}, \quad (88)$$

where a_i is the radius of a throat and r_i is the position of the center of the throat in space and $n_i^\alpha = (x^\alpha - r_i^\alpha)/|r - r_i|$. In the case of a homogeneous and isotropic distribution of such throats we find $R_\alpha^\beta = (1/3)R\delta_\alpha^\beta$ (averaging over spatial directions gives $\langle n_\alpha n^\beta\rangle = (1/3)\delta_\alpha^\beta$) where

$$R = -8\pi G T = \sum\frac{8}{a_i}\delta\left(a_i - |r - R_i|\right) = 32\pi\int a\tilde{n}(a)\,da, \quad (89)$$

where T stands for the trace of the stress energy momentum tensor which one has to add to the Einstein equations to support such a wormhole. It is clear that such a source violates the weak energy condition and, therefore, it reproduces the form of dark energy (i.e., $T = \varepsilon + 3p < 0$). If the density of such sources (and, resp., the density of wormholes) is sufficiently high, then this results in the observed [10–14] acceleration of the scale factor for the Friedmann space as $\sim t^{\alpha}$ with $\alpha = 2\varepsilon/3(\varepsilon + p) = 2\varepsilon/(2\varepsilon + (\varepsilon + 3p)) > 1$, for example, see also [40–42]. In terms of the 4-dimensional density of wormholes $n = (8/3\pi a)\widetilde{n}$ we get $R \sim a^2 n \gg 8\pi G n$ as $a \gg \ell_{\text{pl}}$ and, therefore, the leading contribution indeed comes from the mean curvature.

8. Estimates and Concluding Remarks

Now consider the simplest estimates. Actual wormholes seem to be responsible for the dark matter [7, 9]. Therefore, to get the estimate to the number density of wormholes is rather straightforward. First wormholes appear at scales when dark matter effects start to display themselves; that is, at scales of the order $L \sim (1 \div 5)\,\text{Kpc}$, which gives in that range the number density

$$\widetilde{n} \sim \frac{1}{L^3} \sim (3 \div 0.024) \times 10^{-65} \text{cm}^{-3}. \tag{90}$$

The characteristic size of throats can be estimated from (89) $\varepsilon_{\text{DE}} \sim (G)^{-1}\overline{\widetilde{n}a}$. Since the density of dark energy is $\varepsilon_{\text{DE}}/\varepsilon_0 = \Omega_{\text{DE}} \sim 0.75$, where ε_0 is the critical density, then we find the estimate

$$\overline{a} \sim \frac{2}{3}(1 \div 125) \times 10^{-3} R_{\odot} \Omega_{\text{DE}} h_{75}^2, \tag{91}$$

where $R_{\odot}$ is the Solar radius, $h_{75} = H/(75\,\text{km}/(\text{sec Mpc}))$ and H is the Hubble constant. We also recall that the background density of baryons ε_b generates a nonvanishing wormhole rest mass $M_w = (4/3)\pi\overline{a}^3 R^3 \varepsilon_b$ (where $R(t)$ is the scale factor of the Universe and therefore M_w remains constant), for example, see [7]. It produces the dark matter density related to the wormholes as $\varepsilon_{\text{DM}} \simeq M_w \widetilde{n}$. The typical mass of a wormhole M_w is estimated as

$$M_w \sim 1,7 \times (1 \div 125) \times 10^2 M_{\odot} \Omega_{\text{DM}} h_{75}^2, \tag{92}$$

where $M_{\odot}$ is the Solar mass. We point out that this mass has not the direct relation to the parameters of the gas of wormholes. However it defines the moment when wormhole throats separated from the cosmological expansion. The above estimate shows that if wormholes form due to the development of perturbations in the exotic matter, then this process should start much earlier than the formation of galaxies.

Thus, we see that virtual wormholes should indeed lead to the regularization of all divergencies in QFT which agrees with recent results [28]. Therefore, they form the local finite value of the cosmological constant. In the absence of external classical fields such a value should be exactly zero at macroscopic scales. A some nonvanishing value for the cosmological constant appears as the result of vacuum polarization effects in external fields. Indeed, external fields form an additional distribution of virtual wormholes which possess an exotic stress energy tensor (some kind of dark energy). Only some part of it forms the cosmological constant, while the rest reflects the symmetry of external fields and possesses inhomogeneities. We assume that during the evolution of our Universe inhomogeneities in the exotic matter develop and may form actual wormholes. Although this problem requires the further and more deep investigation we refer to [38] where the formation of a simplest wormhole-like object has been considered. In other words, such polarization energy is reserved now in a gas of actual wormholes. We estimated parameters of such a gas and believe that such a gas may indeed be responsible for both, dark matter and dark energy phenomena.

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