

Research Article

Collapse of a Relativistic Self-Gravitating Star with Radial Heat Flux: Impact of Anisotropic Stresses

Ranjan Sharma and Shyam Das

Department of Physics, P.D. Women's College, Jalpaiguri, West Bengal 735101, India

Correspondence should be addressed to Ranjan Sharma; rsharma@iucaa.ernet.in

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We develop a simple model for a self-gravitating spherically symmetric relativistic star which begins to collapse from an initially static configuration by dissipating energy in the form of radial heat flow. We utilize the model to show how local anisotropy affects the collapse rate and thermal behavior of gravitationally evolving systems.

1. Introduction

In cosmology and astrophysics, there exist many outstanding issues relating to a dynamical system collapsing under the influence of its own gravity. In view of Cosmic Censorship Conjecture, the general relativistic prediction is that such a collapse must terminate into a space-time singularity covered under its event horizon though there are several counter examples where it has been shown that a naked singularity is more likely to be formed (see [1] and references therein). In astrophysics, the end stage of a massive collapsing star has long been very much speculative in nature [1, 2]. From classical gravity perspective, to get a proper understanding of the nature of collapse and physical behavior of a collapsing system, construction of a realistic model of the collapsing system is necessary. This, however, turns out to be a difficult task because of the highly nonlinear nature of the governing field equations. To reduce the complexity, various simplifying methods are often adopted and the pioneering work of Oppenheimer and Snyder [3] was a first step in this direction when collapse of a highly idealized spherically symmetric dust cloud was studied. Since then, various attempts have been made to develop realistic models of gravitationally collapsing systems to understand the nature and properties of collapsing objects. It got a tremendous impetus when Vaidya [4] presented a solution describing the exterior gravitational field of a stellar body with outgoing radiation

and Santos [5] formulated the junction conditions joining the interior space time of the collapsing object to the Vaidya exterior metric [4]. These developments have enabled many investigators to construct realistic models of gravitationally evolving systems and also to analyze critically relevance of various factors such as shear, density inhomogeneity, local anisotropy, electromagnetic field, viscosity, and so forth, on the physical behaviour of collapsing bodies [6–52]. In the absence of any established theory governing gravitational collapse, such investigations have been found to be very useful to get a proper understanding about systems undergoing gravitational collapse.

The aim of the present work is to develop a simple model of a collapsing star and investigate the impact of pressure anisotropy on the overall behaviour of the collapsing body. Anisotropic stresses may occur in astrophysical objects for various reasons which include phase transition, density inhomogeneity, shear, and electromagnetic field [10, 53, 54]. In [53], it has been shown that influences of shear, electromagnetic field, and so forth on self-bound systems can be absorbed if the system is considered to be anisotropic, in general. Local anisotropy has been a well-motivated factor in the studies of astrophysical objects and its role on the gross features of static stellar configurations have been investigated by many authors (see, e.g., [10, 11, 55–59] and references therein). For dynamical systems, though pressure anisotropy is, in general, incorporated in the construction,

very few works have been reported till date where impacts of anisotropy have been discussed explicitly [12–14, 26, 60]. Appropriate junction conditions for an anisotropic fluid collapsing on the background space time described by the Vaidya metric have been obtained in [27]. Considering a spheroidal geometry of Vaidya and Tikekar [61], Sarwe and Tikekar [50] have analyzed the impact of geometry vis-a-vis matter composition on the collapse of stellar bodies which begin to collapse from initial static configurations possessing equal compactness. Sharma and Tikekar [51, 52] have investigated the evolution of nonadiabatic collapse of a shear-free spherically symmetric object with anisotropic stresses on the background of space-time obtained by introducing an inhomogeneous perturbation in the Robertson-Walker space-time.

In the present work, we have developed a model describing a shear-free spherically symmetric fluid distribution radiating away its energy in the form of radial heat flux. The star begins its collapse from an initially static configuration whose energy-momentum tensor describing the material composition has been assumed to be anisotropic, in general. To develop the model of the initial static star, we have utilized the Finch and Skea [62] ansatz which has earlier been found to be useful to develop physically acceptable models capable of describing realistic stars in equilibrium [63–66]. The background space-time for a static configuration for the given ansatz has a clear geometrical interpretation as may be found in [65]. By assuming a particular form of the anisotropic parameter, we have solved the relevant field equations and constructed a model for the initial static stellar configuration which could either be isotropic or anisotropic in nature. The solution provided by Finch and Skea [62] is a subclass of the solution provided here. Since, the solution presented here provides a wider range of values of the anisotropic parameter, it enables us to examine the impact of anisotropic stresses on the evolution of a large class of initial static configurations.

Our paper has been organized as follows. In Section 2, we have presented the basic equations governing the system undergoing non-adiabatic radiative collapse. In Section 3, by assuming a particular anisotropic profile, we have solved the relevant field equations to develop a model for the initial static star. In Section 4, by stipulating the boundary conditions across the surface separating the stellar configuration from the Vaidya [4] space-time, we have solved the surface equation which governs the evolution of the initial static star that begins to collapse when the equilibrium is lost. In Section 5, we have analyzed the impact of anisotropy on gravitationally collapsing systems by considering evolution of initial static stars which could either be isotropic or anisotropic. Impact of anisotropy on the evolution of temperature has been analyzed in Section 6. Finally, some concluding remarks have been made in Section 7.

2. Equations Governing the Collapsing System

We write the line element describing the interior space-time of a spherically symmetric star collapsing under the influence

of self-gravity (in standard coordinates $x^0 = t, x^1 = r, x^2 = \theta$, and $x^3 = \phi$) as

$$ds_-^2 = -A_0^2(r) dt^2 + f^2(t) [B_0^2(r) dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)], \quad (1)$$

where, $A_0(r)$, $B_0(r)$, and $f(t)$ are yet to be determined. Note that in (1), if we set $f(t) = 1$, the metric corresponds to a static spherically symmetric configuration.

We assume that the matter distribution of the collapsing object is an imperfect fluid described by an energy-momentum tensor of the form

$$T_{\alpha\beta} = (\rho + p_t) u_\alpha u_\beta + p_t g_{\alpha\beta} + (p_r - p_t) \chi_\alpha \chi_\beta + q_\alpha u_\beta + q_\beta u_\alpha, \quad (2)$$

where ρ represents the energy density p_r and p_t , respectively, denote fluid pressures along the radial and transverse directions; u^α is the 4-velocity of the fluid; χ^α is a unit space-like 4-vector along the radial direction; and $q^\alpha = (0, q^1, 0, 0)$ is the heat flux vector which is orthogonal to the velocity vector so that $u^\alpha u_\alpha = -1$ and $u^\alpha q_\alpha = 0$.

The Einstein's field equations governing the evolution of the system are then obtained as (we set $G = c = 1$)

$$8\pi\rho = \frac{1}{f^2} \left[\frac{1}{r^2} - \frac{1}{r^2 B_0^2} + \frac{2B_0'}{r B_0^3} \right] + \frac{3\dot{f}^2}{A_0^2 f^2}, \quad (3)$$

$$8\pi p_r = \frac{1}{f^2} \left[-\frac{1}{r^2} + \frac{1}{B_0^2 r^2} + \frac{2A_0'}{r A_0 B_0^2} \right] - \frac{1}{A_0^2} \left[2\frac{\ddot{f}}{f} + \frac{\dot{f}^2}{f^2} \right], \quad (4)$$

$$8\pi p_t = \frac{1}{f^2} \left[\frac{A_0''}{A_0 B_0^2} + \frac{A_0'}{r A_0 B_0^2} - \frac{B_0'}{r B_0^3} - \frac{A_0' B_0'}{A_0 B_0^3} \right] - \frac{1}{A_0^2} \left[2\frac{\ddot{f}}{f} + \frac{\dot{f}^2}{f^2} \right], \quad (5)$$

$$8\pi q^1 = -\frac{2A_0' \dot{f}}{A_0^2 B_0^3 f^3}. \quad (6)$$

In (3)–(6), a “prime” denotes differentiation with respect to r and a “dot” denotes differentiation with respect to t . Making use of (4) and (5), we define the anisotropic parameter of the collapsing object as

$$\begin{aligned} \Delta(r, t) &= 8\pi(p_r - p_t) \\ &= \frac{1}{f^2} \left[-\frac{A_0''}{A_0 B_0^2} + \frac{A_0'}{r A_0 B_0^2} + \frac{B_0'}{r B_0^3} + \frac{A_0' B_0'}{A_0 B_0^3} \right. \\ &\quad \left. + \frac{1}{r^2 B_0^2} - \frac{1}{r^2} \right]. \end{aligned} \quad (7)$$

We rewrite (3)–(5) as

$$8\pi\rho = \frac{8\pi\rho_s}{f^2} + \frac{3\dot{f}^2}{A_0^2 f^2}, \quad (8)$$

$$8\pi p_r = \frac{8\pi(p_r)_s}{f^2} - \frac{1}{A_0^2} \left[2\frac{\ddot{f}}{f} + \frac{\dot{f}^2}{f^2} \right], \quad (9)$$

$$8\pi p_t = \frac{8\pi(p_t)_s}{f^2} - \frac{1}{A_0^2} \left[2\frac{\ddot{f}}{f} + \frac{\dot{f}^2}{f^2} \right], \quad (10)$$

where ρ_s , $(p_r)_s$ and $(p_t)_s$ denote the energy-density, radial pressure, and tangential pressure, respectively, of the initial static star.

3. Interior Space-Time of the Initially Static Configuration

In our construction, we assume that an initially static star (with $f(t) = 1$ in (1)), described by metric potentials $A_0(r)$, $B_0(r)$, and anisotropy $\Delta_s(r)$, starts collapsing if, for some reasons, it loses its equilibrium. To develop a model of the initially static configuration, we first assume that the anisotropic parameter is separable in its variables so that $\Delta(r, t) = \Delta_s(r)/f^2(t)$. Equation (7) then reduces to

$$\Delta_s(r) = \left[-\frac{A_0''}{A_0 B_0^2} + \frac{A_0'}{r A_0 B_0^2} + \frac{B_0'}{r B_0^3} + \frac{A_0' B_0'}{A_0 B_0^3} + \frac{1}{r^2 B_0^2} - \frac{1}{r^2} \right], \quad (11)$$

which is independent of t . Equation (11) can only be solved if any two of the unknown functions (A_0 , B_0 , and $\Delta_s(r)$) are specified. To develop a physically reasonable model of the initial static configuration, we first utilize the Finch and Skea [62] ansatz for the metric potential B_0 given by

$$B_0^2(r) = \left(1 + \frac{r^2}{R^2} \right), \quad (12)$$

where R is the curvature parameter describing the geometry of the configuration. In the static case, the ansatz (12) has a clear geometric characterization and has been found to yield realistic models for compact stellar objects [64–66]. Substituting (12) in (11), we obtain

$$\begin{aligned} & \left(\frac{1-x^2}{R^2 x^4} \right) \frac{d^2 A_0}{dx^2} - 2 \left(\frac{1-x^2}{R^2 x^5} \right) \frac{dA_0}{dx} \\ & + \left[\left(\frac{1-x^2}{R^2 x^4} \right) - \Delta_s \right] A_0 = 0, \end{aligned} \quad (13)$$

where we have used the following transformation:

$$x^2 = \left(1 + \frac{r^2}{R^2} \right). \quad (14)$$

To solve (13), we assume a particular anisotropic profile

$$\Delta_s(x) = \frac{\alpha(x^2 - 1)(2 - x^2)}{R^2 x^6}, \quad (15)$$

where α is the measure of anisotropy. The motivation for choosing the particular form of the anisotropy parameter is the following: (1) it is physically reasonable as the anisotropy vanishes at the center ($r = 0$, i.e., $x = 1$) as expected, and (2) it provides a solution of (13) in closed form. Note that $\alpha = 0$ corresponds to an initial static star which is isotropic in nature. Substituting (15) in (13), we get

$$\frac{d^2 A_0}{dx^2} - \frac{2}{x} \frac{dA_0}{dx} + \left[1 - \frac{\alpha(x^2 - 2)}{x^2} \right] A_0 = 0, \quad (16)$$

whose solution is obtained as

$$\begin{aligned} A_0(x) = & P x^{3/2} J_{(1/2)\sqrt{9-8\alpha}}(-ix\sqrt{-1+\alpha}) \\ & + Q x^{3/2} Y_{(1/2)\sqrt{9-8\alpha}}(-ix\sqrt{-1+\alpha}), \end{aligned} \quad (17)$$

where P and Q are integration constants, $J_{(1/2)\sqrt{9-8\alpha}}(-ix\sqrt{-1+\alpha})$ is the Bessel function of first kind of order $(1/2)\sqrt{9-8\alpha}$, and $Y_{(1/2)\sqrt{9-8\alpha}}(-ix\sqrt{-1+\alpha})$ is the Bessel function of second kind of order $(1/2)\sqrt{9-8\alpha}$. It is obvious that the solution is valid for $\alpha < 1$. At $\alpha = 1$, the Bessel function encounters a singularity and, therefore, we shall deal with the $\alpha = 1$ case separately.

Special Cases

Case 1 ($\alpha = 0$ ($\Delta_s(r) = 0$)). That is, initial static configuration is isotropic in nature. Equation (13) reduces to

$$\frac{d^2 A_0}{dx^2} - \frac{2}{x} \frac{dA_0}{dx} + A_0 = 0, \quad (18)$$

whose solution is found to be

$$A_0(x) = [(C - Dx) \cos x + (Cx + D) \sin x], \quad (19)$$

where C and D are integration constants. This particular solution has been found previously in [62]. However, the solution (19) can be obtained directly from the solution (17) by substituting $\alpha = 0$ and setting $C = -\sqrt{2/\pi}Q$ and $D = \sqrt{2/\pi}P$.

Case 2 ($\alpha = 1$ ($\Delta_s(r) \neq 0$)). Equation (16) reduces to

$$\frac{d^2 A_0}{dx^2} - \frac{2}{x} \frac{dA_0}{dx} + \frac{2}{x^2} A_0 = 0, \quad (20)$$

whose solution is given by

$$A_0(x) = Ax + Bx^2, \quad (21)$$

where A and B are integration constants [64]. Unfortunately, the above solution can not be regained from the general solution (17) due to the properties of Bessel functions and should be treated separately.

We thus have a model for an initially static stellar configuration which could either be isotropic or anisotropic in nature. After loss of equilibrium, the initially static star starts collapsing, and to generate a solution of the subsequent evolving system, we need to determine $f(t)$. This will be taken up in the following section.

4. Exterior Space-Time and Junction Conditions

In our construction, evolution of the collapsing object is governed by the function $f(t)$ which can be determined from the boundary conditions across the boundary surface joining the interior space-time and the exterior space-time described by the Vaidya [4] metric

$$ds_+^2 = -\left(1 - \frac{2m(v)}{\bar{r}}\right)dv^2 - 2dvd\bar{r} + \bar{r}^2 d(\theta^2 + \sin^2\theta d\phi^2). \quad (22)$$

In (22), v denotes the retarded time and $m(v)$ represents the total mass of the collapsing star. The junction conditions are determined by assuming a time-like 3-surface Σ which separates the interior and the exterior manifolds [5]. Continuity of the metric space times ($(ds^2)_\Sigma = (ds_+^2)_\Sigma = ds_\Sigma^2$) and the extrinsic curvatures ($K_{ij}^- = K_{ij}^+$) across the surface Σ then yield the following matching conditions linking smoothly the interior ($r \leq r_\Sigma$) and the exterior ($r \geq r_\Sigma$) space-times across the boundary:

$$A_0(r_\Sigma) dt = \left(1 - \frac{2m}{\bar{r}} + 2\frac{d\bar{r}}{dv}\right)_\Sigma^{1/2} dv, \quad (23)$$

$$r_\Sigma f(t) = \bar{r}_\Sigma(v), \quad (24)$$

$$m(v) = \left(\frac{rf(t)}{2} \left[\frac{r^2}{R^2 + r^2} + \left(\frac{r\dot{R}}{A_0}\right)^2 \right]\right)_\Sigma, \quad (25)$$

$$[p_r]_\Sigma = [qf(t)B_0]_\Sigma, \quad (26)$$

$$m(r, t) \stackrel{\Sigma}{=} m(v). \quad (27)$$

From (25) and (27), the mass enclosed within the boundary surface at any instant t within a boundary surface $r \leq r_\Sigma$ can be written as

$$m(r, t) = \frac{rf(t)}{2} \left[\frac{r^2}{R^2 + r^2} + \left(\frac{r\dot{f}}{A_0}\right)^2 \right]. \quad (28)$$

Combining (6), (9), and (26), together within the condition $(p_r)_s(r = r_\Sigma) = 0$, we deduce the surface equation governing the collapsing matter in the form

$$2\ddot{f}f + \dot{f}^2 - 2n\dot{f} = 0, \quad (29)$$

where we have defined

$$n = \left[\frac{A'_0}{B_0} \right]_\Sigma. \quad (30)$$

Note that for a given initial static configuration n appears as a constant in (29). Following Bonnor et al. [6], we write (29) as a first order differential equation

$$\dot{f} = -\frac{2n}{\sqrt{f}} \left(1 - \sqrt{f}\right), \quad (31)$$

which admits a solution

$$t = \frac{1}{n} \left[\frac{f}{2} + \sqrt{f} + \ln(1 - \sqrt{f}) \right]. \quad (32)$$

Note that at $t \rightarrow -\infty$, that is, at the onset of collapse, $f = 1$ and $f(t) \rightarrow 0$ as $t \rightarrow 0$.

We, therefore, have a complete description of the interior and exterior space-times of the collapsing body. In the following section, we shall analyze the impact of anisotropy by making use of the solutions thus obtained.

5. Physical Analysis

To understand the nature of collapse, we assume that a star starts its collapse at $t = -\infty$ (i.e., $f(t) = 1$) and the initial static star is described by the parameters A_0 , B_0 , and Δ_s . We assume that the collapsing object has an initial radius $r_\Sigma(t \rightarrow -\infty) = r_s$ and mass $m(r_s, -\infty) = m_s$ satisfying the condition $2m_s/r_s < 1$. If for some reason, instability develops inside the star, it begins to collapse. The comoving boundary surface $(rf)_\Sigma = r_s f(t)$ then starts shrinking until it reaches its Schwarzschild horizon value $[rf(t_{bh})]_\Sigma = r_s f(t_{bh}) = 2m(v)$, where $t = t_{bh}$ denotes the time of formation of the black hole corresponding to the value of $f(t) = f_{bh}$.

From (28), the mass of the evolving star at any instant t within the boundary radius r_Σ may be written as

$$m(r_\Sigma, t) = \left[m_s f + \frac{2n^2 r^3}{A_0^2} (1 - \sqrt{f})^2 \right]_\Sigma, \quad (33)$$

where mass of the initial static star has the form

$$m_s = \int_0^{r_s} 4\pi r^2 \rho_s dr = \frac{r_s^3}{2(r_s^2 + R^2)}. \quad (34)$$

Consequently, the condition $(rf_{bh})_\Sigma = 2m(v)$ yields

$$f_{bh} = \left[\frac{2nr/A_0}{(2nr/A_0) + \sqrt{1 - (2m_s/r)}} \right]_\Sigma^2, \quad (35)$$

and the time of black hole formation is obtained as

$$t_{bh} = \frac{1}{n} \left[\frac{f_{bh}}{2} + \sqrt{f_{bh}} + \ln(1 - \sqrt{f_{bh}}) \right]. \quad (36)$$

The model parameters (namely, R , P , and Q) involving the initial static star are determined using the following boundary conditions:

$$A_0(r_s) = \left(1 - \frac{2m_s}{r_s}\right)^{1/2}, \quad (37)$$

$$B_0(r_s) = \left(1 - \frac{2m_s}{r_s}\right)^{-1/2}, \quad (38)$$

$$(p_r)_s(r_s) = 0, \quad (39)$$

where we have matched the static interior space-time to the Schwarzschild exterior and imposed the condition that the radial pressure $((p_r)_s)$ must vanish at the boundary.

TABLE 1: Data for collapsing systems having different values of the anisotropic parameter α . All the configurations start collapsing with initial masses $m_s = 3.25 M_\odot$ and radii $r_s = 20$ km.

α	P	Q	R (km)	n	f_{bh}	t_{bh}	r_{bh} (km)	m_{bh} (M_\odot)
0.9	0.2848	-0.6341	20.8427	0.0244	0.2298	-2.3914	4.596	2.2980
0.6	0.3800	-0.4994	20.8427	0.0163	0.2298	-3.5816	4.596	2.2980
0.3	0.4065	-0.4619	20.8427	0.0142	0.2298	-4.1074	4.596	2.2980
0	0.42067	-0.4418	20.8427	0.0131	0.2298	-4.4578	4.596	2.2980
-0.3	0.4300	-0.4287	20.8427	0.0124	0.2298	-4.7239	4.596	2.2980
-0.6	0.4367	-0.4192	20.8427	0.0118	0.2298	-4.9401	4.596	2.2980
-0.9	0.4418	-0.4120	20.8427	0.0114	0.2298	-5.1230	4.596	2.2980
-1.5	0.4493	-0.4012	20.8427	0.0108	0.2298	-5.4231	4.596	2.2980

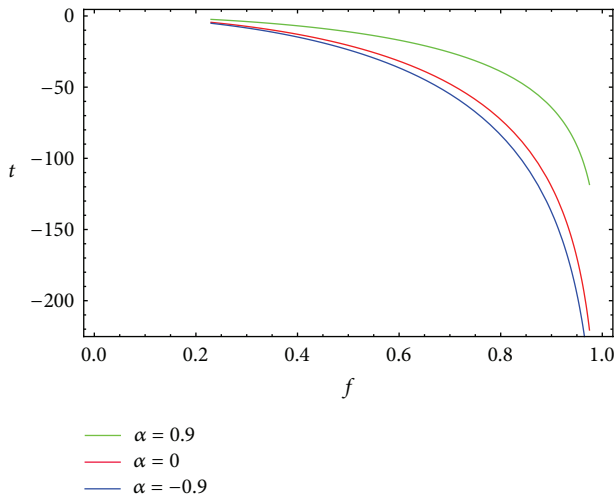


FIGURE 1: Variation of $f(t)$ for different anisotropic parameters.

Now, to get an insight about the effects of anisotropy, we consider different initial static configurations, both isotropic ($\alpha = 0$) and anisotropic ($\alpha \neq 0$). We consider different initially static stellar configurations of identical initial masses and radii (we have assumed $m_s = 3.25 M_\odot$ and radius $r_s = 20$ km) with different values of the anisotropic parameter α . Using numerical procedures, we have calculated the corresponding model parameters and also evaluated the time of formation of black holes t_{bh} and radius ($r_{\text{bh}} = r_s f_{\text{bh}}$) and mass (m_{bh}) of the black hole formed. Our results have been compiled in Table 1. Variations of $f(t)$ for different choices of the anisotropic parameter α have been shown in Figure 1. We have also calculated the rate of collapse in our model for different anisotropic parameters. The collapse rate, in our model, is obtained as

$$\Theta = u^\beta_{;\beta} = \frac{3\dot{f}}{A_0 f} = \frac{6n(\sqrt{f}-1)}{A_0 f \sqrt{f}}. \quad (40)$$

The collapse rate turns out to be $\Theta = -0.101464, -0.102096, -0.100749$ for $\alpha = 0, 0.9, -0.9$, respectively. Since, the collapsing object contracts in size as time evolves Θ , in our

construction, turns out to be negative. However, from the absolute values of Θ , we note that for a positive anisotropy ($p_r > p_t$) the collapse rate increases as compared to an isotropic star while for a negative value of α ($p_t > p_r$), the rate decreases. From Table 1, we note the following.

Case 1. When $\alpha > 0$ (implying $p_r > p_t$), the horizon is formed at a faster rate as compared to $\alpha = 0$, that is, isotropic case.

Case 2. When $\alpha < 0$ (implying $p_t > p_r$), the horizon is formed at a slower rate as compared to $\alpha = 0$, that is, isotropic case.

Similar observations may be found in [12]. However, we note that mass and radius of the collapsed configuration do not depend on anisotropy in our formulation.

6. Thermal Behaviour

To analyze the impact of anisotropy on the evolution of temperature of the collapsing system, we use the relativistic Maxwell-Cattaneo relation for temperature governing the heat transport [60, 67, 68] given by

$$\begin{aligned} \tau (g^{\alpha\beta} + u^\alpha u^\beta) u^\delta q_{\beta;\delta} + q^\alpha \\ = -\kappa (g^{\alpha\beta} + u^\alpha u^\beta) [T_{;\beta} + T \dot{u}_\beta], \end{aligned} \quad (41)$$

where $\kappa (\geq 0)$ is the thermal conductivity and $\tau (\geq 0)$ is the relaxation time. For the line element (1), (41) reduces to

$$\tau \frac{d}{dt} (q f B_0) + q^1 f A_0 B_0 = -\kappa \frac{1}{f B_0} \frac{d}{dr} (A_0 T). \quad (42)$$

Following an earlier work [51], we write the relativistic Fourier heat transport equation by setting $\tau = 0$ in (42). For $\tau = 0$, combining (6) and (42), we get

$$8\pi\kappa(A_0 T)' = \frac{2A_0' \dot{f}}{A_0 f}. \quad (43)$$

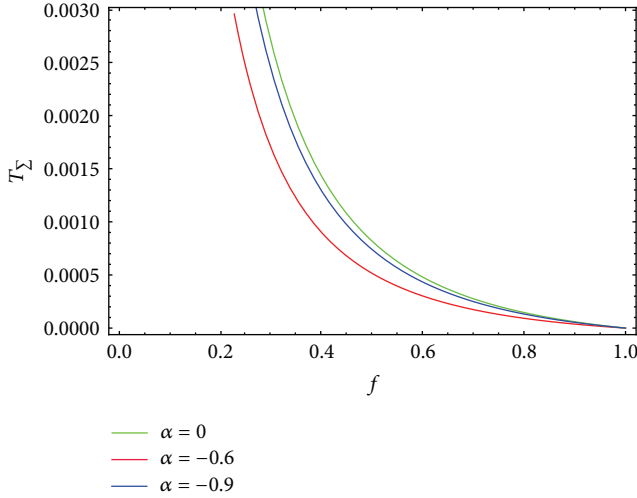


FIGURE 2: Evolution of surface temperature for different anisotropic parameters.

Let us now assume that the thermal conductivity varies as $\kappa = \gamma T^\omega$, where γ and ω are constants. Equation (43) then yields

$$8\pi\gamma(A_0 T)' = \frac{2A_0' T^{-\omega}}{A_0} \left[\frac{2n(\sqrt{f} - 1)}{f\sqrt{f}} \right], \quad (44)$$

where we have used (31). Integrating the above equation, we get

$$T^{\omega+1} = \frac{n(\sqrt{f} - 1)}{2\pi\gamma f\sqrt{f}} \left(\frac{\ln A_0}{A_0} \right) + T_0(t), \quad (45)$$

To get a simple estimate of temperature evolution, we set $\omega = 0$, $\gamma = 1$, and $T_0(t) = 0$ and evaluate the surface temperature at any instant as

$$T(r_\Sigma, t) = \frac{n(\sqrt{f} - 1)}{2\pi f\sqrt{f}} \left(\frac{\ln A_0}{A_0} \right)_\Sigma. \quad (46)$$

Time evolution of the surface temperature for different anisotropic parameter α has been shown in Figure 2, where we have used the data from Table 1.

7. Discussions

We have developed a minimalistic and basic framework of a gravitationally collapsing system which has allowed us to examine the impact of pressure anisotropy explicitly. In our construction, we have ignored the effects of various other factors relevant to collapsing systems such as shear, viscosity, and charge. Moreover, the line element describing the interior space-time has been assumed to be spherically symmetric where the metric functions have been chosen to be separable in its variables. Though formulation of a more general framework is always preferred to describe a realistic situation, the simple model developed here provides a mechanism to

capture the impact of anisotropy on gravitational collapse successfully. Though, as pointed out in [53], effects of factors like shear, charge, and so forth on self-bound systems can be absorbed by considering the system to be anisotropic, in general, we intend to formulate a more general framework so as to examine the combined impacts of various factor relevant to gravitationally collapsing systems. These issues will be taken up elsewhere.

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