Conference Paper

Controlling Nonlinear Behavior in Current Mode Controlled Boost Converter Based on the Monodromy Matrix

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Recently it has been observed that power electronic converters working under current mode control exhibit codimensional-2 bifurcations through the interaction of their slow-scale and fast-scale dynamics. In this paper, the authors further probe this phenomenon with the use of the saltation matrix instead of the Poincaré map. Using this method, the authors are able to study and analyze more exotic bifurcation phenomena that occur in cascade current mode controlled boost converter. Finally, we propose two control strategies that guarantee the stable period-one operation. Numerical and analytical results validate our analysis.

1. Introduction

Power electronic circuits are normally designed to operate in a periodic steady state. The region in the parameter space where this behaviour can be obtained is delimited by various instability conditions. The nature of these instabilities has been recently understood in terms of nonlinear dynamics. In this approach, the periodic orbit is sampled in synchronism with the clock signal (called the Poincaré section), thus obtaining a discrete-time model or a map [1, 2]. The fixed point of the map signifies the periodic orbit, and its stability is given by the eigenvalues of the Jacobian matrix, computed at the fixed point. There are two basic ways in which such a periodic orbit may lose stability.

- (1) When an eigenvalue becomes equal to -1 , the bifurcation is called a period-doubling bifurcation, which results in a period-2 orbit. This instability is not visible in an averaged model, and so it is also called a “fast-scale” instability [3–5].
- (2) When a pair of complex conjugate eigenvalues assume a magnitude of 1, this bifurcation is called a Neimark-Sacker bifurcation, which results in the onset of a slow sinusoidal oscillation in the state variables. The orbit

rests on the surface of a torus. This instability can be predicted using the averaged model, and so it is also called the “slow-scale” instability [6, 7].

In [8, 9], Chen, Tse, and others showed that dynamical behavior resulting from these two types of bifurcations can interact, giving rise to interesting dynamics. In our earlier papers [10, 11], we further investigated this phenomena using the technique developed in [12–15]. In these papers, we reported creation of a two-loop torus through a Neimark-Sacker bifurcation occurring on a period-2 orbit. There are complex interactions between periodic orbits, tori, and a saturation behavior, in which unstable tori play an important role. We have detected the unstable tori and have demonstrated that the sudden departure from stable torus to a saturation behavior is caused by a collision between a stable and an unstable torus. Such complex nonlinear phenomena and bifurcations need comprehensive efforts to capture their dynamical behaviour and analyse their stability in order to apply appropriate controllers to avoid such instabilities. Any control method is required to ensure a stable period-1 operation over a wide range of operating conditions and at the same time be relatively simple and easy to implement in practice. In the last two decades, a number of control methods

have been proposed for controlling chaos and bifurcations as described in [16, 17]. However, there are significant problems in applying such controllers in switching systems. The main reason for failures in these controllers is susceptibility to noise and sensitivity to measurement accuracy. In this paper a new control technique is developed based on the expression of the saltation matrix to control nonlinear behaviours in a cascade current-mode control boost converter to overcome a number of complex instabilities occurring in the system. This technique has been successfully applied in [12–15].

2. Model Description

The closed-loop current controlled boost converter system has been studied in [8]. The current controlled boost converter with cascade control is shown in Figure 1. The controller has a primary and a secondary control loops. Primary control consists of a PI controller to achieve the required output voltage. The output of the primary controller i_{ref} is added to the compensation ramp signal and used as the set point of the secondary controller. The output of the secondary controller i_M is given by the inductor current i_L multiplied by a gain M . The outputs of the two controllers are compared to generate the flip-flop reset signal. The parameters of the system are chosen in such a way that the system is operating in CCM. In the normal periodic operation, the switch and the diode are complementarily activated. The flip-flop latch is set periodically by the clock signal, turning ON the switch S_T and causing i_M to rise linearly. When i_M reaches the reference value i_{ref} , the output of the comparator resets the flip-flop turning the switch S_T OFF. Figure 2 shows the control signal for normal period-1 operation in continuous conduction mode. The converter itself is governed by two sets of linear differential equations related to the ON and OFF states of the converter. The state equations of the system can be written as two linear differential equations when the circuit operates in CCM (continuous conduction mode):

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{A}_1 \mathbf{X} + \mathbf{B}_1 V_{in} \quad S_T \text{ is ON and } S_D \text{ is OFF,} \\ \dot{\mathbf{X}} &= \mathbf{A}_2 \mathbf{X} + \mathbf{B}_2 V_{in} \quad S_T \text{ is OFF and } S_D \text{ is ON,} \end{aligned} \quad (1)$$

where $\mathbf{X} = [i_L \ v_c \ v_a]^T = [x_1 \ x_2 \ x_3]^T$; i_L , v_c , and v_a are inductor current, voltage across the capacitor C , and voltage across the compensation capacitor C_a , respectively.

The state matrices of the system are as described in [8]:

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{r_L + r_T}{L} & 0 & 0 \\ 0 & -\frac{1}{\tau_m(1+k_c)} & 0 \\ 0 & \frac{g}{\tau_a(1+k_c)} & 0 \end{bmatrix},$$

FIGURE 1: The closed-loop current controlled boost converter with PI controller.

FIGURE 2: The control signals and generated PWM waveform.

$$\mathbf{A}_2 = \begin{bmatrix} -\frac{r_L + r_D}{L} - \frac{r_c}{L(1+k_c)} & -\frac{1}{L(1+k_c)} & 0 \\ \frac{1}{C(1+k_c)} & -\frac{1}{\tau_m(1+k_c)} & 0 \\ \frac{g}{\tau_a(1+k_c)} & \frac{g}{\tau_a(1+k_c)} & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} \frac{1}{L} & 0 & \frac{g k_d V_{ref}}{\tau_a V_{in}} \end{bmatrix}^T, \quad (2)$$

where $k_c = r_c/R$, $k_d = (R_1 + R_2)/R_2$, $g = R_a/R_1$, $\tau_m = RC$, $\tau_a = R_a C_a$, V_{ref} is the reference voltage, and V_{in} is input voltage.

FIGURE 3: Bifurcation diagram with $R = 26 \Omega$ as V_{in} is varied. FIGURE 4: The stable period-1 ($V_{in} = 3.7$). FIGURE 5: Fast-scale instability ($V_{in} = 3.31$). FIGURE 6: The slow-scale instability ($V_{in} = 3.3056$ V).

3. Simulation Results

The circuit parameters were chosen from [8] as shown in Table 1. Figure 3 shows the bifurcation diagram of the system when $R = 26 \Omega$, and the value of V_{in} is varied taking into account the initial conditions. According to the bifurcation diagram, if the parameter V_{in} is reduced continuously, the behaviour of the system goes from period-1 to period-2 through a period-normal period-doubling bifurcation. The time domain period-1 and period-2 inductor current waveforms are shown in Figures 4 and 5, respectively. When V_{in} is reduced to 3.3058 V, the period-2 orbit loses its stability via a slow-scale bifurcation. In this case, there will be an interaction between the fast-scale (period doubling) and slow-scale instabilities as shown in Figure 6.

4. The Stability Analysis for the Boost Converter

In DC/DC converters, one is interested in the stability of a periodic orbit that starts at a specific state at a clock instant and returns to the same state at the end of the clock period. The stability of such a periodic orbit can be understood

in terms of the evolution of a perturbation. If the initial condition is perturbed and the solution converges back to the orbit, then the orbit is stable. For any DC/DC converter, the state evolves through a number of subsystems (ON state of the switch, OFF state of the switch, etc.). The boost converter studied in this paper operates in CCM; therefore in the steady state period-1 operation, there are two topologies. The solution for each circuit topology is linear; however, for a complete switching cycle, the system becomes piecewise linear and the solution is not defined at the switching instant. Filippov showed that in such a situation one has to additionally consider the evolution of the perturbation across the switching event [18]. Filippov derived the form of the state transition matrix S that relates the perturbation just after the switching event to that just before the switching event. The Monodromy matrix is the product of all these solutions, and the stability of the system can then be investigated by examining the eigenvalues of the Monodromy matrix as in [12–15].

TABLE 1: Parameters of the system.

$V_{in} = 2.9\text{--}15\text{ V}$	$R = 20\text{--}30\ \Omega$	$L = 165\ \mu\text{H}$	$r_L = 0.04\ \Omega$
$C = 1500\ \mu\text{F}$	$r_C = 0.03\ \Omega$	$r_T = 0.055\ \Omega$	$r_D = 0.01\ \Omega$
$T = 40\ \mu\text{s}$	$V_{ref} = 2.5\text{ V}$	$R_1 = 47\text{ K}\Omega$	$R_2 = 6.8\text{ K}\Omega$
$R_a = 10\text{ K}\Omega$	$C_a = 2.2\text{ nF}$	$V_p = 0.445\text{ V}$	$M = 0.3\text{ V/A}$

TABLE 2: The eigenvalues of the Monodromy matrix, $R = 26\ \Omega$.

V_{in}	Eigenvalues (λ_1, λ_2 and λ_3)			Abs (λ_2 or λ_3)	The stability of period-1
3.5000	-0.9700	$0.9970 \pm 0.0403i$		0.9979	Stable
3.4300	-0.9860	$0.9962 \pm 0.0403i$		0.9970	Stable
3.3105	-1.0000	$0.9983 \pm 0.0384i$		0.9990	Fast-scale
3.3100	-1.0001	$0.9984 \pm 0.0384i$		0.9991	Fast-scale
3.3058	-1.0008	$0.9985 \pm 0.0384i$		0.9992	Fast-scale
3.2700	-1.0063	$0.9987 \pm 0.0380i$		0.9994	Fast-scale

5. Derivation of the Monodromy Matrix for the Boost Converter

Figure 7 shows the period-1 steady state operation of the system. The system changes its topologies when the switching manifold $h(x(t)) = 0$, corresponding to the transition from the ON state to the OFF state. The switching manifold is defined by the equations

$$h(\mathbf{X}(t), t) = i_{ref} - i_M, \quad i_{ref} = K_0 + K_1 \mathbf{X}(dT) - V_p, \quad i_M = M i_L, \quad (3)$$

$$h(\mathbf{X}(dT), t) = K_0 + K_1 \mathbf{X}(dT) - V_p - M x_1(dT) = 0.$$

V_p is the peak value of the ramp.

During each switching interval, the system is governed by linear time invariant equations (1).

In Figure 7 Φ_1 is the state transition matrix for the ON period ($t_0, t_0 + dT$), where d is the duty ratio. And Φ_2 is the state transition matrix for the OFF period ($t_0 + dT, T$):

$$\Phi_1 = e^{A_1 dT}, \quad \Phi_2 = e^{A_2 (1-d)T}. \quad (4)$$

From [12–15, 18], the saltation matrix can be calculated as

$$S_1(dT) = \mathbf{I} + \frac{(\mathbf{f}_+ - \mathbf{f}_-) \mathbf{n}^T}{\mathbf{n}^T \mathbf{f}_- + \partial h / \partial t} \Big|_{t=dT}, \quad (5)$$

where $\mathbf{I}$ is identity matrix of the same order as the number of state variables, $\mathbf{n}$ is the vector normal to the switching surface and $\mathbf{n}^T$ is its transpose, $\mathbf{f}_-$ represents the right-hand side of the differential equations before the switching had occurred, and $\mathbf{f}_+$ represents the right-hand side of the differential equations after the switching:

$$\frac{\partial h}{\partial t} = -\frac{V_p}{T}. \quad (6)$$

The normal vector $\mathbf{n}^T$ is given by

$$\mathbf{n}^T = \nabla h(\mathbf{X}(dT), t) = \left[-M \quad \frac{-g}{(1+k_c)} \quad -1 \right]. \quad (7)$$

The two vectors field $\mathbf{f}_-$ before switching and $\mathbf{f}_+$ after switching can be calculated as:

$$\mathbf{f}_- = \begin{bmatrix} -\frac{r_L + r_T}{L} x_1 + \frac{1}{L} V_{in} \\ \frac{1}{\tau_m (1+k_c)} x_2 \\ \frac{g}{\tau_a (1+k_c)} x_2 - \frac{g k_d V_{ref}}{\tau_a} \end{bmatrix}, \quad \mathbf{f}_+ = \begin{bmatrix} \left(-\frac{r_L + r_D}{L} - \frac{r_c}{L(1+k_c)} \right) x_1 - \frac{1}{L(1+k_c)} + \frac{V_{in}}{L} \\ \frac{1}{C(1+k_c)} x_1 - \frac{1}{\tau_m (1+k_c)} x_2 \\ \frac{g r_c}{\tau_a (1+k_c)} x_1 + \frac{g}{\tau_a (1+k_c)} x_2 - \frac{g k_d V_{ref}}{\tau_a} \end{bmatrix}. \quad (8)$$

S_1 can be obtained by evaluating the previous terms at the switching time dT and substituting into (5). The second saltation matrix S_2 relates to the end of the cycle where the switching changes topology from the OFF state to the ON at T . The perturbation and original trajectories reach the end of the switching period T at the same instant. Therefore, $\partial h = V_p$ and $\partial t = 0$, so that the slope $\partial h / \partial t \rightarrow \infty$. Substituting this value in (5), S_2 becomes the identity matrix ($\mathbf{I}$). Hence, the Monodromy matrix over the complete one cycle is obtained as:

$$\mathbf{M}(0, T) = S_2 \times \Phi_2(T, dT) \times S_1 \times \Phi_1(dT, 0). \quad (9)$$

The stability of the system has been investigated by examining (9), and all the results are tabulated in Table 2. Table 2 shows the calculation of the eigenvalues of the period-1 orbit for $R = 26\ \Omega$ as the input voltage is varied corresponding to Figure 3. It is clear from the results shown in Table 2 that the period-1 orbit is always stable for higher values of input voltage where all the eigenvalues are located inside the unit circle. The period-1 orbit loses stability via a fast-scale bifurcation at $V_{in} = 3.3105\text{ V}$, when the real eigenvalue moves out of the unit circle leading to a stable period-2 fixed point.

6. Controlling the Fast-Scale Bifurcation in the Boost Converter

In general, the stability of the periodic orbit of dc/dc switching converters can be investigated by the Monodromy matrix. The Monodromy matrix is described as the state transition matrix over a full-clock period. It is composed of matrix exponentials and the saltation matrices. The expression of the saltation matrix shows that S depends on the two vector fields $\mathbf{f}_-$ and $\mathbf{f}_+$ (which cannot be manipulated), the rate of change $\partial h / \partial t$, and the normal vector $\mathbf{n}$ (which can both be manipulated). The normal vector $\mathbf{n}$ can be altered by changing the slope of the switching manifold h , and the rate of change $\partial h / \partial t$ can be altered by changing the slope of the compensation ramp for the system. By monitoring the bifurcation parameters, one can make these small changes in

FIGURE 7: The normal operation of the system.

FIGURE 8: Values of (a) to maintain stability as V_{in} is varied and $R = 26 \Omega$.

the saltation matrix and can therefore force the eigenvalues to remain inside the unit circle.

6.1. Ramp Slope Change. The first control method is proposed to influence the saltation matrix. This is based on the slope of the ramp voltage signal as the perturbed parameter and is achieved by changing the tip of V_p to $(V_p + aV_p)$. Hence, the switching manifold will be $h(X, t) = K_0 + K_1 X(t) - M(x_1(t)) - (V_p + aV_p)$. The value of (a) is calculated to keep the magnitude of the eigenvalues exactly the same as that of the stable period-1 orbit obtained for the nominal value of V_p . This can be obtained by solving equation $|\text{eig}(\mathbf{M}(T, 0))| - 0.99 = 0$. The results of this equation are shown in Figure 8.

FIGURE 9: Forced period-1 operation via the effect of the new controller; $a = 0.005$, $V_{in} = 3.080 \text{ V}$, and $R = 26 \Omega$. FIGURE 10: Values of (b) to maintain stability as V_{in} is varied and $R = 26 \Omega$.

It is clear that this system can be forced to operate in normal operation without overshooting as seen in Figure 9.

6.2. Adding a Small Component to the Inductor Current Signal. As the slope of the switching manifold is expressed by its normal vector, it is possible to stabilize the boost converter by adding a component to the feedback inductor current signal. This will force $\mathbf{n}$ to be non zero and hence change the slope of h . In this case, the switching manifold will be modified as:

$$h(\mathbf{X}, t) = K_0 + K_1 \mathbf{X}(t) - (1 + b) M(x_1(t)) - V_p = 0, \quad (10)$$

where $K_1 = [0 \quad (-g/(1 + k_c)) \quad -1]$.

The new normal vector is given by

$$\mathbf{n} = \frac{dh}{d\mathbf{X}} = \left[-(1 + b) M \quad -\frac{g}{(1 + k_c)} \quad -1 \right]^T. \quad (11)$$

FIGURE 11: Forced period-1 operation via the effect of the new controller; $b = -0.005$, $V_{in} = 3.080$ V, and $R = 26 \Omega$.

FIGURE 12: Forced period-1 operation via the effect of the new controller; $b = -0.05$, $V_{in} = 3.080$ V, and $R = 26 \Omega$.

Hence, in addition to the voltage feedback loop ($g = 1$), the current feedback loop changes the dynamics. To ensure that the system is stable (period-1 operation) for a wide range of V_{in} , the parameter (b) is calculated as the input voltage is varied from 3.32 V to 3.24 V. The real eigenvalues must be kept less than one based on the equation $|\text{eig}(\mathbf{M}(T, 0))| - 0.99 = 0$. Figure 10 shows the calculated values of (b) versus the values of input voltage. Figure 11 shows the response of system with the new controller for the minimum value of (b). It is obvious that the new controller makes the system stable with an overshooting. The reason for choosing the spectral radius of the Monodromy matrix at 0.99 is to limit the transient overshoot resulting from higher values of the variable (b) needed for a smaller value of the spectral radius. If the overshoot reaches a high enough value, it might hit the unstable boundary, and the response of the system will be saturated as described in [10, 11]. The behavior of the system is very sensitive to the initial conditions. Therefore, the stability of the system will be local and not global. As the value of

FIGURE 13: Forced period-1 operation via the effect of the new controller; $b = -0.09335$, $V_{in} = 3.080$ V, and $R = 26 \Omega$.

FIGURE 14: The behaviour of the system is saturated by the effect of the new controller; $b = -0.09337$, $V_{in} = 3.080$ V, and $R = 26 \Omega$.

parameter (b) is further increased, the overshoot becomes larger and the transient response of the system takes long time before it settles down to a steady state operation as shown in Figures 12 and 13. At a value of $b = -0.09337$, the overshoot hits the unstable torus and the response is saturated as shown in Figure 14.

7. Conclusions

In this paper, the Monodromy matrix has been derived for the dc-dc boost converter with PI controller. The Monodromy matrix which is the fundamental solution matrix over one full cycle offers a deeper insight of how and why these systems lose their stability. As a result, supervisory controllers have been developed to place the eigenvalues of the state transition matrix of the system over one complete switching cycle (the Floquet multipliers of the system) within the unit circle to ensure system stability for operation.

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