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Research Article

gsΛ Continuous Function in Topological Space

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We introduce the different notions of a new class of continuous functions called generalized semi Lambda (gsΛ) continuous function in topological spaces. Its properties and characterization are also discussed.

1. Introduction

In 1986, Maki [1] continued the work of Levine and Dunham on generalized closed sets and closure operators by introducing the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel (= saturated set), that is, to the intersection of all open supersets of A. Arenas et al. [2] introduced and investigated the notion of λ-closed sets and λ-open sets by involving Λ-sets.

In 2008 Caldas et al. [3] introduced Λ generalized closed sets (Λg, Λ-g, gΛ) and their properties. They also studied the concept of λ closed maps. In 2007, Caldas et al. [4] introduced the concept of λ irresolute maps.

In this paper, we establish a new class of maps called gsΛ continuous function and study its properties and characteristics.

Throughout this paper, (X,τ), (Y,σ), and (Z,η) (or simply X, Y, and Z will always denote topological spaces on which no separation axioms are assumed unless explicitly stated. $\mathrm{Int}(A)$, $\mathrm{Cl}(A)$, $\mathrm{Int}_\lambda A$, $\mathrm{Cl}_\lambda A$, gsΛCl(A), and gsΛInt(A) denote the interior of A, closure of A, lambda interior of A, lambda closure of A, gs Lambda closure of A and gs Lambda interior of A, respectively.

2. Preliminary Definitions

Definition 1. A subset A of a space (X,τ) is called

(1) a semiopen set [5] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,

(2) a preopen set [6] if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,

(3) a regular open set [6] if $A = \mathrm{Int}(\mathrm{Cl}(A))$.

The complement sets of semi open (resp., preopen and regular open) are called semi closed sets (resp., preclosed and regular closed). The semiclosure (resp., preclosure) of a subset A of X denoted by $\mathrm{sCl}(A)$, $(\mathrm{pCl}(A))$ is the intersection of all semi closed sets (pre closed sets) containing A.

A topological space (X,τ) is said to be

(1) a generalized closed [7] if $\mathrm{Cl}(A) \subset U$, whenever $A \subset U$ and U is open in (X,τ),

(2) a g^* closed [8] if $\mathrm{Cl}(A) \subset U$, whenever $A \subset U$ and U is g-open in (X,τ),

(3) semigeneralized closed (denoted by sg-closed) [9] if $\mathrm{sCl}(A) \subset U$, whenever $A \subset U$ and U is semi open in (X,τ),

(4) generalized semiclosed (denoted by gs-closed) [9] if $\mathrm{sCl}(A) \subset U$, whenever $A \subset U$ and U is open in (X,τ),

(5) a subset A of a space (X,τ) is called λ-closed [2] if $A = B \cap C$, where B is a Λ-set and C is a closed set,

(6) a subset A of (X,τ) is said to be a Λg closed set [3] if $\mathrm{Cl}(A) \subset U$ whenever $A \subset U$, where U is λ open in (X,τ),

(7) a subset A of (X,τ) is said to be a Λ-g closed set [3] if $\mathrm{Cl}_\lambda(A) \subset U$ whenever $A \subset U$, where U is λ open in (X,τ),

(8) a subset A of (X,τ) is said to be a gΛ closed set [3] if $\mathrm{Cl}_\lambda(A) \subset U$ whenever $A \subset U$, where U is open in (X,τ),

(9) a subset A of (X,τ) is said to be a gsΛ closed set [10] if $\mathrm{Cl}_\lambda(A) \subset U$ whenever $A \subset U$, where U is semi open in X.

The complement of the above closed sets are called its respective open sets.

Definition 2. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called

(1) semicontinuous [5] if $f^{-1}(V)$ is semi open in (X,τ) for every open set V in (Y,σ),

(2) semi open (semiclosed) [9] if $f(F)$ is semi open (semi closed) in (Y,σ) for any open (closed) set F in (X,τ),

(3) pre-semiopen (pre-semiclosed) [9] if $f(F)$ is semi open (semi closed) in (Y,σ) for every semi open (semi closed) set in (X,τ),

(4) irresolute [9] if for any semi open set S of (Y,σ), $f^{-1}(S)$ is semi open in (X,τ),

(5) g continuous [11] if $f^{-1}(V)$ is g open in (X,τ) for every open set V in (Y,σ),

(6) $\widehat{g}$ continuous [12] if $f^{-1}(V)$ is $\widehat{g}$ open in (X,τ) for every open set V in (Y,σ),

(7) contracontinuous [13] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ),

(8) λ continuous [2, 14] if $f^{-1}(V)$ is λ open (λ closed) in (X,τ) for every open (closed) set V in (Y,σ),

(9) λ closed [3] if $f(F)$ is λ closed in (Y,σ) for every λ closed set F of (X,τ),

(10) λ irresolute [4] if the inverse image of λ open sets in Y are λ open in (X,τ),

(11) sg continuous [15] if $f^{-1}(V)$ is sg open (sg closed) in (X,τ) for every open (closed) set in (Y,σ),

(12) gsΛ closed map (gsΛ open map) [16] if the image of each closed (open) set in X is gsΛ closed (gsΛ open) in Y.

(13) M.gsΛ closed map (gsΛ open map) [16] if the image of each gsΛ closed (gsΛ open) set in X is (gsΛ open) gsΛ closed in Y.

Lemma 3 (see [7]). *If $f : X \longrightarrow Y$ is continuous and closed and if B is g closed (or g open) subset of Y, then $f^{-1}(B)$ is g closed (or g open) in X.*

Definition 4. A space (X,τ) is called

(i) [7] a $T_{1/2}$ space if every g closed subset of X is closed in X,

(ii) [12] a $T\widehat{g}$ space if every $\widehat{g}$ closed subset of X is closed in X,

(iii) [12] a T_b space if every gs closed subset of X is closed in X.

Proposition 5 (see [10]). *In a topological space (X,τ), the following properties hold.*

(1) *Every closed set is gsΛ closed.*

(2) *Every open set is gsΛ closed.*

(3) *Every λ closed set is gsΛ closed.*

(4) *In T_1 space every gsΛ closed set is λ closed.*

(5) *In T_1 space every Λg closed set is gsΛ closed.*

(6) *In partition space every gsΛ closed set is g closed and $\widehat{g}$ closed.*

(7) *In a door space every subset is gsΛ closed.*

(8) *In $T_{1/2}$ space every subset is gsΛ closed.*

Definition 6. Let (X,τ) be the topological space and $A \subseteq X$. We define gsΛ closure of A (briefly gsΛCl(A)) to be the intersection of all gsΛ closed sets containing A, gsΛ interior of A (briefly gsΛInt(A)) to be the union of all gsΛ open sets contained in A.

Lemma 7 (see [10]). *Let A and B be subsets of a topological space (X,τ). The following properties hold.*

(1) *gsΛCl(A) is the smallest gsΛ closed set containing A.*

(2) *If A is gsΛ closed then A = gsΛCl(A). Converse need not be true.*

(3) $A \subset gs\Lambda Cl(A) \subset Cl_\lambda(A) \subset Cl(A)$.

(4) *If $A \subset B$, then* $gs\Lambda Cl(A) \subset gs\Lambda Cl(B)$.

(5) $gs\Lambda Cl(gs\Lambda Cl(A)) = gs\Lambda Cl(A)$.

(6) gsΛInt(A) *is the largest gsΛ open set contained in A.*

(7) *If A is gsΛ open then A =* gsΛint(A). *Converse need not be true.*

Proofs are obvious from the definition and properties of gsΛ closed sets and gsΛ open sets.

3. gsΛ Continuous Function

Definition 8. A map $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called gsΛ continuous function if the inverse image $f^{-1}(V)$ of each closed set V in (Y,σ) is gsΛ closed in (X,τ).

Theorem 9. *Every continuous function is gsΛ continuous function.*

Proof. Let F be a closed set in (Y,σ) and a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a continuous function. Hence, $f^{-1}(F)$ is closed in (X,τ). As every closed set is gsΛ closed set by Proposition 5, we have $f^{-1}(F)$ is gsΛ closed in X. Thus, f is a gsΛ continuous function. □

Converse need not be true as seen from the following example.

Example 10. Let $X = Y = \{a,b,c,d,e\}$ and $(X,\tau) = \{\emptyset, X, \{a\}, \{b,c,d\}, \{a,b,c,d\}\}$, $(Y,\sigma) = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}\}$. The identity function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ continuous function but not continuous function as $A = \{a,d,e\}$ is closed in (Y,σ) but $f^{-1}(A) = \{a,d,e\}$ is not closed in (X,τ).

Theorem 11. *Every λ continuous function is gsΛ continuous function.*

Proof. Let F be a closed set in (Y,σ) and a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a λ continuous function. Hence, $f^{-1}(F)$ is λ closed in (X,τ). As every λ closed set is gsΛ closed set by Proposition 5, we have $f^{-1}(F)$ is gsΛ closed in X. Thus, f is a gsΛ continuous function. □

Converse need not be true as seen from the following example.

Example 12. Let $X = Y = \{a,b,c,d,e\}$ and $(X,\tau) = \{\emptyset, X, \{a\}, \{b,c,d\}, \{a,b,c,d\}\}$, $(Y,\sigma) = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}\}$. The identity function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ continuous function but not λ continuous function as $A = \{a,d,e\}$ is closed in (Y,σ) but $f^{-1}(A) = \{a,d,e\}$ is not λ closed in (X,τ).

Theorem 13. *If (X,τ) is a T_1 space then every gsΛ continuous function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is λ continuous function.*

Proof. The proof is simple to the readers as in T_1 space every gsΛ closed set is λ closed. □

Theorem 14. *Every $\widehat{g}$ continuous function is gsΛ continuous function.*

Proof. As every $\widehat{g}$ closed set is gsΛ closed set [10] f is a gsΛ continuous function. □

Converse need not be true as seen from the following example.

Example 15. Let $X = Y = \{a,b,c,d,e\}$ and $(X,\tau) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}, \{a,b,c,e\}, \{a,b,d,e\}, \{a,b,d\}, \{a,b,e\}\}$, $(Y,\sigma) = \{\emptyset, Y, \{a\}, \{b,c,d\}, \{a,b,c,d\}\}$. The identity function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ continuous function but not $\widehat{g}$ continuous function as $A = \{a,e\}$ is closed in (Y,σ) but $f^{-1}(A) = \{a,e\}$ is not $\widehat{g}$ closed in (X,τ).

Theorem 16. *If (X,τ) is a partition space, then every gsΛ continuous function $f : (X,\tau) \rightarrow (Y,\sigma)$ is $\widehat{g}$ continuous function.*

Proof. The proof is simple as in a partition space every gsΛ closed set is $\widehat{g}$ closed [10]. □

Theorem 17. *Every contra continuous function is gsΛ continuous function.*

Proof. Let F be a closed set in (Y,σ) and a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a contra continuous function. Hence, $f^{-1}(F)$ is open in (X,τ). As every open set is gsΛ closed set by Proposition 5, we have $f^{-1}(F)$ is gsΛ closed in X. Thus, f is a gsΛ continuous function. □

Converse need not be true as seen from the following example.

Example 18. Let $X = Y = \{a,b,c,d,e\}$ and $(X,\tau) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}, \{a,b,c,e\}, \{a,b,d,e\}, \{a,b,d\}, \{a,b,e\}\}$, $(Y,\sigma) = \{\emptyset, Y, \{a\}, \{b,c,d\}, \{a,b,c,d\}\}$. The identity function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ continuous function but not contra continuous function as $A = \{e\}$ is closed in (Y,σ) but $f^{-1}(A) = \{e\}$ is not open in (X,τ).

Theorem 19. *Every λ irresolute function is gsΛ continuous function.*

Proof. Let F be a closed set in (Y,σ). As every closed set is λ closed set, F is a λ closed set in (Y,σ). Since $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a λ irresolute function, $f^{-1}(F)$ is λ closed in (X,τ). As every λ closed set is gsΛ closed set by Proposition 5, we have $f^{-1}(F)$ is gsΛ closed in (X,τ). Thus, f is a gsΛ continuous function. □

Converse need not be true as seen from the following example.

Example 20. Let $X = Y = \{a,b,c,d,e\}$ and $(X,\tau) = \{\emptyset, X, \{a\}, \{a,b\}, \{b,e\}, \{c,d\}, \{a,c,d\}, \{a,b,e\}, \{a,b,c,d\}, \{b,c,d,e\}\}$, $(Y,\sigma) = \{\emptyset, Y, \{a\}, \{b,c\}, \{a,b,c\}\}$. The identity function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ continuous function but not λ irresolute function as $A = \{d,e\}$ is λ closed in (Y,σ) but $f^{-1}(A) = \{d,e\}$ is not λ closed in (X,τ).

Theorem 21. *If (X,τ) is a partition space, then every gsΛ continuous function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is g continuous function.*

Proof. Let F be a closed set in (Y,σ). Since a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ continuous function $f^{-1}(F)$ is gsΛ closed in (X,τ). Since (X,τ) is a partition space by Proposition 5, we have $f^{-1}(F)$ is g closed in X. Thus, f is a g continuous function. □

Theorem 22. *Any function $f : (X,\tau) \longrightarrow (Y,\sigma)$, where the domain (X,τ) is a $T_{1/2}$ (door) space is a gsΛ continuous function.*

Proof. Since in $T_{1/2}$ (door) space every subset is gsΛ closed set [10], the proof is clear to the readers. □

Theorem 23. *Let a bijective map $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a λ irresolute and pre-semiopen function. If F is gsΛ closed in (Y,σ) then $f^{-1}(F)$ is gsΛ closed in (X,τ).*

Proof. Let A be a gsΛ closed set of (Y,σ). Let U be a semi open set of (X,τ) containing $f^{-1}(A)$. Then $A \subseteq f(U)$, where $f(U)$ is semi open in (Y,σ), as f is a pre semi open function. Since A is a gsΛ closed set of (Y,σ), we have $\mathrm{Cl}_\lambda(A) \subseteq f(U)$. Thus, we have $f^{-1}(\mathrm{Cl}_\lambda(A)) \subseteq U$, where $f^{-1}(\mathrm{Cl}_\lambda(A))$ is λ closed in (X,τ). Thus, we have $\mathrm{Cl}_\lambda(f^{-1}(A)) \subseteq \mathrm{Cl}_\lambda(f^{-1}(\mathrm{Cl}_\lambda(A)) = f^{-1}(\mathrm{Cl}_\lambda(A)) \subseteq U$. Hence, $f^{-1}(A)$ is gsΛ closed in (X,τ). □

Lemma 24 (see [10]). *If A is semi open and gsΛ closed then A is λ closed.*

Lemma 25 (see [17]). *A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is contra irresolute if and only if the inverse image of each semi closed set in Y is semi open in X.*

Theorem 26. *If a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ continuous function and contra irresolute function then f is a λ continuous function.*

Proof. Let V be a closed set of Y. As every closed set is semi closed, V is a semi closed set in Y. Since f is a gsΛ continuous function and contra irresolute function, $f^{-1}(V)$ is gsΛ closed and semi open in (Y,σ). Now, by Lemma 24 the proof is very clear. □

Lemma 27 (see [10]). *Let $F \subseteq A \subseteq X$, where A is open in X. If F is gsΛ closed in X, then F is gsΛ closed in A.*

Theorem 28. *If a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a λ continuous function and A is an open subset of X, then the restriction $f_A : A \longrightarrow Y$ is also gsΛ continuous.*

Proof. Let V be a closed set of Y and A be an open subset of X. As every open set is λ closed, A is λ closed in X and since f is λ continuous function $f^{-1}(V)$ is λ closed in X. Hence, we have $f^{-1}(V) \cap A$ is λ closed in X, which is also gsΛ closed in X. Since $f^{-1}(V) \cap A \subseteq A \subseteq X$ where A is an open subset of X, by Lemma 27 $f^{-1}(V) \cap A = ({f_A}^{-1}(V))$ is gsΛ closed in A. Thus, the restriction $f_A : A \longrightarrow Y$ is also gsΛ continuous. □

Definition 29. (i) Let x be a point of (X,τ) and V be a subset of X. Then V is called a gsΛ neighbourhood of x in (X,τ), if there exist a gsΛ open set U of (X,τ), such that $x \in U \subseteq V$.

(ii) The intersection of all gsΛ closed sets each containing a set A in a topological space X is called the gsΛ closure of A and is denoted by gsΛCl(A).

Theorem 30. *Let A be a subset of (X,τ). Then $x \in gs\Lambda Cl(A)$ if and only if for any gsΛ neighbourhood N_x of x in (X,τ), $A \cap N_x \neq \phi$.*

Proof. Necessity. Assume that $x \in$ gsΛCl(A). Suppose that there exist a gsΛ neighbourhood N_x of x such that $A \cap N_x = \phi$. Since N_x is a gsΛ neighbourhood of x in (X,τ) by definition, there exists a gsΛ open set V such that $x \in V \subseteq N_x$. Since N_x is such that $A \cap N_x = \phi$, we have $A \cap V = \phi$ we have $A \subseteq (V)^C$. Since $(V)^C$ is a gsΛ closed set containing A, we have gsΛCl(A) $\subseteq (V)^C$ and therefore $x \notin$ gsΛ Cl(A), which is a contradiction.

Sufficiency. Assume that for each gsΛ neighbourhood N_x of x in (X,τ), $A \cap N_x \neq \phi$. Suppose $x \notin$ gsΛCl(A). Then there exist a gsΛ closed set F of (X,τ), such that $A \subseteq F$ and $x \notin F$. Thus, $x \in F^C$ and F^C is a gsΛ open set in (X,τ). But $A \cap F^C = \phi$ which is a contradiction. Thus, $x \in$ gsΛCl(A). □

Theorem 31. *Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a function.Then the following are equivalent.*

(i) *The function is gsΛ continuous.*

(ii) *The inverse of each open set in (Y,σ) is gsΛ open in (X,τ).*

(iii) *For each x in (X,τ), the inverse of every neighbourhood of $f(x)$ is a gsΛ neighbourhood of x.*

(iv) *For each x in (X,τ), and each neighbourhood N of $f(x)$, there is a gsΛ neighbourhood W of x, such that $f(W) \subseteq N$.*

(v) *For each subset A of (X,τ), $f(gs\Lambda Cl(A)) \subseteq Cl(f(A))$.*

(vi) *For each subset B of (Y,σ), $gs\Lambda Cl(f^{-1}(B)) \subseteq f^{-1}Cl(B)$.*

Proof. (i)⇔(ii). Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a gsΛ continuous function and U be an open set in (Y,σ). Then U^C is closed set in Y. Hence, by the definition of gsΛ continuous function $f^{-1}(U^C) = (f^{-1}(U))^C$ is gsΛ closed set in X. Thus, $f^{-1}(U)$ is a gsΛ open set in X. The converse is analogous.

(ii)⇔(iii). For $x \in (X,\tau)$, let N be the neighbourhood of $f(x)$. Then there exist an open set V in (Y,σ) such that $f(x) \in V \subseteq N$. Consequently, $f^{-1}(V)$ is gsΛ open set in (X,τ) and $x \in f^{-1}(V) \subseteq f^{-1}(N)$. Thus, $f^{-1}(N)$ is a gsΛ neighbourhood in (X,τ). Converse is very simple to the reader.

(iii)⇔(iv). Let $x \in X$ and N be the neighbourhood of $f(x)$. Then by assumption $W = f^{-1}(N)$ is a gsΛ neighbourhood of x and $f(W) = f(f^{-1}(N)) \subseteq N$. Converse is clear.

(iv)⇔(v). Suppose that (iv) holds and let $y \in f($gsΛCl$(A))$ and let N be the neighbourhood of y. Then there exist $x \in X$ and a gsΛ neighbourhood W of x, such that $f(x) = y, x \in W$, $x \in$ gsΛCl(A) and $f(W) \subseteq N$. Since $x \in$ gsΛCl(A), by Theorem 30 $W \cap A \neq \phi$ and hence $f(A) \cap N \neq \phi$. Hence, $y = f(x) \in \mathrm{Cl}(f(A))$.

Conversely suppose that (v) holds and let $x \in X$, N be the neighbourhood of $f(x)$. Let $A = f^{-1}(Y \setminus N)$. Since $f($gsΛCl$(A)) \subseteq \mathrm{Cl}(f(A)) \subseteq Y \setminus N$, we have gs$\Lambda$Cl$(A) = A$. Since $x \notin$ gsΛ Cl(A), there exist a gsΛ neighbourhood W of x such that $W \cap A = \phi$ and hence $f(W) \subseteq f(X \setminus A) \subseteq N$.

(v)⇔(vi). Suppose that (v) holds and B be any subset of (Y,σ). Then replacing A by $f^{-1}(B)$ in (v), we obtain $f($gsΛCl$(f^{-1}(B))) \subseteq \mathrm{Cl}(f(f^{-1}(B)) \subseteq \mathrm{Cl}(B)$. That is gs$\Lambda$Cl$(f^{-1}(B))) \subseteq f^{-1}(\mathrm{Cl}(B)$.

Conversely, suppose that (vi) holds, and let $B = f(A)$, where A is the subset of (X,τ). Then we have gsΛCl(A) $\subseteq$ gsΛCl$(f^{-1}(B)) \subseteq f^{-1}\mathrm{Cl}(f(A))$ and so $f($gsΛCl$(A)) \subseteq \mathrm{Cl}(f(A))$.

This completes the proof of the theorem. □

Theorem 32. *Let $\{X_i/i \in I\}$ be any family of topological spaces. If $f : X \longrightarrow \Pi X_j$ is a gsΛ continuous function, then P_{r_i} of $: X \longrightarrow X_i$ is gsΛ continuous for each $i \in I$, where P_{r_i} is the projection of ΠX_j on X_i.*

Proof. We will consider a fixed $i \in I$. Suppose U_i is an arbitrary open set in X_i. Then ${P_{r_i}}^{-1}(U_i)$ is open in ΠX_j. Since

f is a gsΛ continuous function, we have $f^{-1}(P_{r_i}{}^{-1}(U_i)) = (P_{r_i} \circ f)^{-1}(U_i)$ is gsΛ open in X. Hence, P_{r_i} of is a gsΛ continuous function. □

Definition 33. Let A be a subset of X. A mapping $r : (X, \tau) \longrightarrow A$ is called a gsΛ continuous retraction if r is gsΛ continuous and the restriction r_A is the identity mapping on A.

Definition 34. A topological space (X, τ) is called a gsΛ Hausdroff if for each pair x, y of distinct points of X, there exist gsΛ neighbourhoods U_1 and U_2 of x and y, respectively, that are disjoint.

Theorem 35. *Let A be a subset of X and $r : X \longrightarrow A$ be a gsΛ continuous retraction. If X is gsΛ Hausdroff, then A is a gsΛ closed set of X.*

Proof. Suppose that A is not gsΛ closed. Then there exist a point x in X such that $x \in \text{gs}\Lambda\text{Cl}(A)$ but $x \notin A$. It follows that $r(x) \neq x$ because r is gsΛ continuous retraction. Since X is gsΛ Hausdroff there exist disjoint gsΛ open sets U_1 and U_2 in X such that $x \in U_1$ and $r(x) \in U_2$. Now let W be an arbitrary gsΛ neighbourhood of x. Then $W \cap U_1$ is a gsΛ neighbourhood of x. Since $x \in \text{gs}\Lambda\text{Cl}(A)$, by Theorem 30, we have $(W \cap U_1) \cap A \neq \emptyset$. Therefore, there exist a point $y \in W \cap U_1 \cap A$. Since $y \in A$, we have $r(y) = y \in U_1$ and hence $r(y) \notin U_2$. This implies that $r(W) \nsubseteq U_2$ because $y \in W$. This is a contrary to gsΛ continuity of r. Consequently, A is a gsΛ closed set of X. □

4. On Composite Functions

Theorem 36. *Composition of continuous functions is a gsΛ continuous function.*

Proof. Since every closed set is a gsΛ closed set, the proof is simple to prove. □

Theorem 37. *Composition of λ irresolute functions is a gsΛ continuous function.*

Proof. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \varsigma)$ are λ irresolute functions and let F be a closed set of (Z, ς), which is also λ closed set by [2]. Then $g^{-1}(F)$ is a λ closed set in (Y, σ) and $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is a λ closed set in (X, τ). Since every λ closed set is gsΛ closed set by Proposition 5, we get $g \circ f$ is a gsΛ continuous function. □

Theorem 38. *Composition of contra continuous functions is a gsΛ continuous function.*

Proof. Proof is clear by definition. □

Remark 39. But composition of gsΛ continuous function is not a gsΛ continuous function.

Example 40. Let $X = Y = Z = \{a, b, c, d, e\}$ and $(X, \tau) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,b,c,d\}, \{b,c,d,e\}\}$, $(Y, \sigma) = \{\emptyset, Y, \{a\}, \{b,c,d\}, \{a,b,c,d\}\}$, and $(Z, \varsigma) = \{\emptyset, Z, \{a\}, \{b\}, \{a,b\}, \{b,e\}, \{c,d\}, \{b,c,d\}, \{a,b,e\}, \{a,c,d\}, \{a,b,c,d\}, \{b,c,d,e\}\}$. The identity function $f : (X, \tau) \longrightarrow (Y, \sigma)$ and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous functions but $(g \circ f)$ is not gsΛ continuous function as $A = \{b, e\}$ is closed in (Z, ς) but $(g \circ f)^{-1}(A) = \{b, e\}$ is not gsΛ closed in (X, τ).

Theorem 41. *(i) If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a gsΛ continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function, then $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.*

(ii) If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a λ continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function, then $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(iii) If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a λ irresolute function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a λ continuous function, then $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(iv) If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a $\widehat{g}$ continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function, then $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(v) If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a contra continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function, then $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

Proof. (i) Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a gsΛ continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function and let F be a closed set of (Z, ς). Then $g^{-1}(F)$ is a closed set in (Y, σ) and $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is a gsΛ closed set in (X, τ) as $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a gsΛ continuous function. Thus $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(ii) It is clear that $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a λ continuous function and hence gsΛ continuous function by Theorem 11.

(iii) Since every closed set is λ closed and every λ closed set is gsΛ closed set (Proposition 5), $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(iv) Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a $\widehat{g}$ continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function and let F be a closed set of (Z, ς). Then $g^{-1}(F)$ is a closed set in (Y, σ) and $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is a $\widehat{g}$ closed set in (X, τ) as $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a $\widehat{g}$ continuous function. Since every $\widehat{g}$ continuous function is gsΛ continuous function by Theorem 14, we get $g \circ f$ is a gsΛ continuous function.

(v) As every closed set is gsΛ open by Proposition 5 the proof is very clear. □

Theorem 42. *(i) Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is a gsΛ continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function, then $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a g continuous ($\widehat{g}$ continuous) function if (X, τ) is a partition space.*

(ii) Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a gsΛ continuous function and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a continuous function, then $g \circ f : (X, \tau) \rightarrow (Z, \varsigma)$ is a λ continuous function if (X, τ) is a T_1 space.

Proof. (i) In partition space every gsΛ closed set is g closed ($\widehat{g}$ closed) set by Proposition 5, the theorem is proved.

(ii) The proof is clear as in a T_1 space every gsΛ closed set is λ closed by Proposition 5. □

Lemma 43 (see [10]). *If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is irresolute and a λ closed map then for every gsΛ closed set B of (X, τ), $f(B)$ is gsΛ closed set in (Y, σ).*

Theorem 44. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ are bijective.*

(i) If $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a continuous function and $f : (X, \tau) \longrightarrow (Y, \sigma)$ is gsΛ closed map then $g : (Y, \sigma) \rightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(ii) If $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a λ continuous function and $f : (X, \tau) \longrightarrow (Y, \sigma)$ is λ closed map then $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(iii) If $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a λ irresolute function and $f : (X, \tau) \longrightarrow (Y, \sigma)$ is λ closed map then $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(iv) If $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a continuous function and $f : (X, \tau) \longrightarrow (Y, \sigma)$ is λ closed map then $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(v) If $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function and $f : (X, \tau) \longrightarrow (Y, \sigma)$ is irresolute and a λ closed map then $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(vi) If $g \circ f : (X, \tau) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function and $f : (X, \tau) \longrightarrow (Y, \sigma)$ is M.gsΛ closed map then $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

Proof. (i) Let F be a closed set in (Z, ς). Since $g \circ f$ is a continuous function $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is closed in (X, τ). Now since f is a gsΛ closed map $f(f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is gsΛ closed set in (Y, σ). Thus $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(ii) Let F be a closed set in (Z, ς). Since $g \circ f$ is a λ continuous function $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is λ closed in (X, τ). Now since f is a λ closed map $f(f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is λ closed which is also gsΛ closed set in (Y, σ) [10]. Thus, $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(iii) Let F be a closed set in (Z, ς) which is also λ closed in Z [10]. Since $g \circ f$ is a λ irresolute function $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is λ closed in (X, τ). Now, since f is a λ closed map $f(f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is λ closed which is also gsΛ closed set in (Y, σ) [10]. Thus, $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(iv) Let F be a closed set in (Z, ς). Since $g \circ f$ is a continuous function $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is closed in (X, τ), which is also λ closed in X [10]. Now, since f is a λ closed map $f(f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is λ closed which is also gsΛ closed set in (Y, σ) [10]. Thus, $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(v) Let F be a closed set in (Z, ς). Since $g \circ f$ is a gsΛ continuous function $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is gsΛ closed in (X, τ). By Lemma 43 $f(f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is gsΛ closed set in (Y, σ). Thus $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function.

(vi) Let F be a closed set in (Z, ς). Since $g \circ f$ is a gsΛ continuous function $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is gsΛ closed in (X, τ). Now, since $f$is a M.gsΛ closed map $f(f^{-1}(g^{-1}(F)) = g^{-1}(F)$ is gsΛ closed. Thus, $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is a gsΛ continuous function. □

Theorem 45. *For any bijection $f : (X, \tau) \longrightarrow (Y, \sigma)$, the following are equivalent:*

(i) $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ *is gsΛ continuous,*

(ii) $f : (X, \tau) \rightarrow (Y, \sigma)$ *is gsΛ open,*

(iii) $f : (X, \tau) \rightarrow (Y, \sigma)$ *is gsΛ closed.*

Proof. (i)⟶(ii) Let U be an open set in (X, τ). By assumption $(f^{-1})^{-1}(U)) = f(U)$ is gsΛ open in (Y, σ) and hence f is gsΛ open map.

(ii)⟶(iii) Let F be a closed set in (X, τ). Then F^C is open in (X, τ). By assumption $f(F^C) = (f(F))^C$ is gsΛ open in (Y, σ) and therefore $f(F)$ is gsΛ closed in (Y, σ). Hence, f is gsΛ closed.

(iii)⟶(i) Let F be a closed set in (X, τ). By assumption$f(F)$ is gsΛ closed in (Y, σ). But $f(F) = (f^{-1})^{-1}(F))$ and therefore f^{-1} is is gsΛ continuous. □

References

[1] H. Maki, "Generalised-sets and the associated closure operator," *The special issue in Commemoration of Professor Kazusada IKEDS Retirement*, pp. 139–146, 1986.

[2] F. G. Arenas, J. Dontchev, and M. Ganster, "On λ-sets and the dual of generalized continuity," *Questions and Answers in General Topology*, vol. 15, pp. 3–13, 1997.

[3] M. Caldas, S. Jafari, and T. Noiri, "On Λ-generalized closed sets in topological spaces," *International Journal Of General Topology*, vol. 118, no. 4, pp. 337–343, 2008.

[4] M. Caldas, S. Jafari, and T. Navalgi, "More on λ-closed sets in topological spaces," *Revista Colombina de Mathematics*, vol. 41, pp. 355–369, 2007.

[5] N. Levine, "Semi-open sets ans semi continuity in topological spaces," *The American Mathematical Monthly*, vol. 70, no. 1, pp. 36–41, 1963.

[6] I. Arockiarani, K. Balachandran, and J. Dontchev, "Some Characterizations of gp-irresolute and gp-continuous maps between topological spaces," *Memoirs of the Faculty of Science Kochi University Series A*, vol. 20, pp. 93–104, 1999.

[7] N. Levine, "Generalized closed sets in topology," *Rendiconti del Circolo Matematico di Palermo*, vol. 19, no. 1, pp. 89–96, 1970.

[8] M. K. R. S. VeeraKumar, "gs-Closed sets in topological space," *Memoirs of the Faculty of Science Kochi University Series A*, vol. 21, pp. 1–19, 2000.

[9] R. Devi, H. Maki, and K. Balachandran, "semi generalized closed maps and generalized semi closed maps," *Memoirs of the Faculty of Science Kochi University Series A*, vol. 14, pp. 41–54, 1993.

[10] S. P. Missier and V. H. Raj, "gsΛ closed sets in topological spaces," *International Journal of General Topology*, vol. 5, pp. 33–44, 2012.

[11] K. Balachandran, P. Sundaram, and H. Maki, "On Generalised Continuous Maps in topological spaces," *Memoirs of the Faculty of Science Kochi University Series A*, vol. 12, pp. 5–13, 1991.

[12] M. K. R. S. Veera Kumar, "On $\hat{g}$ closed sets in topological spaces," *Bulletin of the Allahabad Mathematical Society*, vol. 18, pp. 99–112, 2003.

[13] J. Dontchev, "Contra continuous functions and strongly S-closed spaces," *International Journal of Mathematics Sciences*, vol. 19, no. 2, pp. 303–310, 1996.

[14] M. Caldas and S. Jafari, "On some low separation axioms via λ open and λ closure operator," *Rendiconti del Circolo Matematico di Palermo*, vol. 54, no. 2, pp. 195–208, 2005.

[15] P. Sundaram, H. Maki, and K. Balachandran, "Semi-Generalised Continuous Maps and Semi-$T^{1/2}$ spaces," *Bulletine of Fukuoka University of Education*, vol. 40, no. 3, pp. 33–40, 1991.

[16] V. H. Raj and S. P. Missier, "gsΛ closed and open functions in topological space," *South Asian Journal of Mathematics*, vol. 2, no. 5, pp. 527–539, 2012.

[17] M. C. Cueva, "Weak and Strong forms of irresolute maps," *International Journal of Mathematics and Mathematical Sciences*, vol. 23, no. 4, pp. 253–259, 2000.


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