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Research Article

Is Einstein-Cartan Theory Coupled to Light Fermions Asymptotically Safe?

Eckehard W. Mielke

Departamento de Física, Universidad Autónoma Metropolitana-Iztapalapa, Av. San Rafael Atlixco No. 186, Col. Vicentina, 09340 México, DF, Mexico

Correspondence should be addressed to Eckehard W. Mielke; ekke@xanum.uam.mx

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The difference between Einstein's general relativity and its Cartan extension is analyzed within the scenario of *asymptotic safety* of quantum gravity. In particular, we focus on the four-fermion interaction which distinguishes the Einstein-Cartan theory from its Riemannian limit.

1. Introduction

In the coupling of gravity to Dirac type spinor fields [1], it is at times surmised that the Einstein-Cartan (EC) theory [2] is superior to standard General Relativity (GR), inasmuch as the involved torsion tensor of Cartan [3, 4] can accommodate the spin of fundamental Fermions of electrons and quarks in gravity.

However, classically,the effects of spin and torsion cannot be detected by Lageos or Gravity Probe B [5] and would be significant only at densities of matter that are very high but nevertheless smaller than the Planck density at which quantum gravitational effects are believed to dominate. It was even claimed [6] that EC theory may avert the problem of singularities in cosmology, but for a coupling to Dirac fields, the opposite happens [7–9].

Recently, it has been stressed by Weinberg [10–12] that the Riemann-Cartan (RC) connection $\Gamma = \Gamma^{\{\}} - K$, a one-form, is just a *deformation* of the Christoffel connection $\Gamma^{\{\}}$ by the *(con-)tortion* tensor-valued one-form $K = iK^{\alpha\beta}\sigma_{\alpha\beta}/4$, at least from the field theoretical point of view. Although algebraically complying with [13], this argument has been refuted [14] on the basis of the special geometrical interpretation [15, 16] of Cartan's torsion.

It is well-known [17, 18] that EC theory coupled to the Dirac field is effectively GR with an additional *four-fermion* (FF) interaction. However, such *contact* interactions are perturbatively nonrenormalizable in $D > 2$ without Chern-Simons (CS) terms [19], which was one of the reasons for giving up Fermi's theory of the beta decay.

Since GR with a cosmological constant Λ appears to be asymptotically safe, in the scenario [20] first devised by Weinberg [21], one may ask [22] what the situation in EC theory is, where Cartan's algebraic equation relates torsion to spin, that is, to the axial current j_5^i in the case of Dirac fields, on dimensional grounds coupled with gravitational strength.

2. Dirac Fields in Riemann-Cartan Spacetime

In our notation [13, 23–25], a Dirac field is a bispinor-valued zero-form ψ for which $\overline{\psi} := \psi^{\dagger}\gamma_0$ denotes the Dirac adjoint and $D\psi := d\psi + \Gamma \wedge \psi$ is the exterior covariant derivative with respect to the RC connection one-form $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha}$, providing a minimal gravitational coupling.

In the manifestly *Hermitian* formulation, the Dirac Lagrangian is given by the four-form

$$\begin{aligned} L_D &= L(\gamma, \psi, D\psi) \\ &= \frac{i}{2}\left\{\overline{\psi}\, {}^*\gamma \wedge D\psi + \overline{D\psi} \wedge {}^*\gamma\, \psi\right\} + m\overline{\psi}\psi\eta, \end{aligned} \tag{1}$$

where $\gamma := \gamma_\alpha \vartheta^\alpha$ is the Clifford algebra-valued coframe, obeying $D\gamma = [\gamma, K] = \gamma_\alpha T^\alpha$, and

$$T^\alpha := D\vartheta^\alpha = d\vartheta^\alpha + \Gamma^\alpha_\beta \wedge \vartheta^\beta = \frac{1}{2} T^\alpha_{ij} dx^i \wedge dx^j \tag{2}$$

is the torsion two-form.

Since $L_D = \overline{L}_D = L_D^\dagger$ even in a nonholonomic frame, the minimal coupling provides us automatically with the *Hermitian* charge current and standard axial current three-forms

$$j := \overline{\psi}\,{}^*\gamma\,\psi = j^\mu \eta_\mu, \qquad j_5 := \overline{\psi}\,{}^*\gamma\,\gamma_5 \psi = j_5^\mu \eta_\mu, \tag{3}$$

respectively, which are familiar with quantum electrodynamics (QED) in curved spacetime.

Let us now separate the purely Riemannian part from spin-contortion pieces:

$$\begin{aligned} L_D &= L\left(\gamma, \psi, D^{\{\}}\psi\right) - \frac{i}{2}\overline{\psi}\left({}^*\gamma \wedge K - K \wedge {}^*\gamma\right)\psi \\ &= L\left(\gamma, \psi, D^{\{\}}\psi\right) + \frac{1}{4}\mathscr{A} \wedge j_5. \end{aligned} \tag{4}$$

Hence, in an RC spacetime, a massive Dirac spinor only feels the *axial torsion* one-form

$$\begin{aligned} \mathscr{A} &:= \frac{1}{4}\,{}^*\mathrm{Tr}\left(\gamma \wedge D\gamma\right) = {}^*\left(\vartheta^\alpha \wedge T_\alpha\right) \\ &= \frac{1}{2} T^{[\alpha\beta\gamma]} \eta_{\alpha\beta\gamma} = \mathscr{A}_i dx^i. \end{aligned} \tag{5}$$

The *spin current* of the Dirac field is given by the Hermitian three-form

$$\begin{aligned} \tau_{\alpha\beta} &:= \frac{\partial L_D}{\partial \Gamma^{\alpha\beta}} = \frac{1}{8}\overline{\Psi}\left({}^*\gamma\,\sigma_{\alpha\beta} + \sigma_{\alpha\beta}\,{}^*\gamma\right)\Psi \\ &= \frac{1}{4}\eta_{\alpha\beta\gamma\delta}\overline{\Psi}\gamma^\delta\gamma_5\Psi\eta^\gamma = \tau_{\alpha\beta\gamma}\eta^\gamma, \end{aligned} \tag{6}$$

with *totally antisymmetric* components $\tau_{\alpha\beta\gamma} = \tau_{[\alpha\beta\gamma]}$. Equivalently, torsion merely couples to the *spin-energy potential* $\mu_\alpha = \vartheta_\alpha \wedge {}^*j_5/4$, that is; to a two-form that is proportional to the axial current j_5 compare [25] for more details.

2.1. Axial Anomaly in Riemann-Cartan Spacetime. In quantum field theory (QFT), however, the axial current is not conserved, rather there arises in RC spacetime the *axial anomaly*

$$\langle dj_5 \rangle = 2im\left\langle \overline{\psi}\gamma_5\psi \right\rangle \eta - \frac{1}{96\pi^2}\left[2R^{\{\}}_{\alpha\beta} \wedge R^{\{\}\alpha\beta} + d\mathscr{A} \wedge d\mathscr{A}\right], \tag{7}$$

for its vacuum expectation value, which involves the topological Pontrjagin term quadratic in the curvature. This result [26], which can easily be transferred to the chiral current $j_\pm$, is based on the Pauli-Villars regularization schem; compare also [27].

Since the axial torsion $\mathscr{A}$ is *not* a gauge field, it is legitimate to absorb [28] its contribution to the anomaly (7) into the redefined current

$$\widehat{j}_5 := j_5 + \frac{1}{96\pi^2}\mathscr{A} \wedge d\mathscr{A}, \tag{8}$$

such that

$$\langle d\widehat{j}_5 \rangle = 2im\langle\overline{\psi}\gamma_5\psi\rangle\eta + \frac{1}{48\pi^2}R^{\{\}}_{\alpha\beta} \wedge R^{\{\}\alpha\beta} \tag{9}$$

is the same result as in the Riemannian spacetime of GR.

One way to avoid such anomalies is to employ curvature constraints like $R_{\alpha\beta} \equiv 0$ typical for teleparallel models [29]. Another approach, inspired by the BF schemes [30, 31] of Topological Quantum Field Theory (TQFT), is to start from a minimalists $SL(5, R)$ gauge model which includes only a "bare" Pontrjagin type four-form as its own counterterm. However, then a tiny symmetry breaking is mandatory, in order to recover the classical metrical background of GR.

3. Effective Einstein-Cartan Theory

The Einstein-Cartan Lagrangian

$$L_{\mathrm{EC}} := -\frac{1}{2\kappa}R^{\alpha\beta} \wedge \eta_{\alpha\beta} = L_{\mathrm{HE}} + \frac{1}{12\kappa}\mathscr{A} \wedge {}^*\mathscr{A}, \tag{10}$$

where $\kappa = 8\pi G_N$ is the gravitational constant in natural units, generalizes the metrical Hilbert-Einstein Lagrangian L_{HE} to an RC spacetime with torsion, where only the axial torsion $\mathscr{A}$ enters algebraically. (Adding torsion squared terms [32, 33] is not an unambiguous procedure, since the particular combination $T^\alpha \wedge {}^*({}^{(1)}T_\alpha - 2\,{}^{(2)}T_\alpha - (1/2)\,{}^{(3)}T_\alpha)$ of irreducible pieces is related to a fourth boundary term derived from the dual CS term $C_{TT^*} := \vartheta^\alpha \wedge {}^*T_\alpha$; cf. [34]. In the space of gravity theories, the nontopological boundary term dC_{TT^*} is interrelating GR with its teleparallelism equivalent [35]. Exactly the previous teleparallel "nucleus" leaves its traces in the controversies [36, 37] about the well posedness of the classical Cauchy problem and the particle content of the (broken) Poincaré gauge theory.)

The Einstein-Cartan equation [2]

$$G_\alpha := \frac{1}{2}R^{\beta\gamma} \wedge \eta_{\alpha\beta\gamma} = \kappa\Sigma_\alpha, \tag{11}$$

coupled to the canonical energy-momentum current Σ_α of matter, is obtained by varying for the coframe ϑ^α. Likewise, the EC three-form

$$\begin{aligned} G_\alpha &:= \frac{1}{2}R^{\beta\gamma} \wedge \eta_{\alpha\beta\gamma} \\ &= G^{\{\}}_\alpha + \frac{(-1)^s}{12}\left(e_\alpha \rfloor \mathscr{A} \wedge {}^*\mathscr{A} - \frac{1}{3}\mathscr{A} \wedge e_\alpha \rfloor {}^*\mathscr{A}\right) \\ &\quad + \frac{(-1)^s}{6}\vartheta_\alpha \wedge d\mathscr{A} \end{aligned} \tag{12}$$

can be decomposed into the Einstein three-form $G^{\{\}}_\alpha = G^\beta_\alpha \eta_\beta$ with respect to the Riemannian connection $\Gamma^{\{\}}$ and additional axial torsion pieces [17, 18]. It satisfies the first Noether identity

$$\widehat{D}\, G_\alpha \equiv \frac{1}{2}\left(e_\alpha \rfloor R^{\beta\gamma}\right) \wedge \eta_{\beta\gamma\mu} \wedge T^\mu \tag{13}$$

with respect to the transposed connection $\widehat{\Gamma}_\alpha^{\beta} := \Gamma_\alpha^\beta + e_\alpha \rfloor T^\beta$; compare equation (5.4.13) of [13]. (Observe that the three-form (12) is not covariantly conserved in RC spacetime. Only for vanishing torsion, it reduces to the conservation law $D^{\{\}} G_\alpha^{\{\}} \equiv 0$ familiar with GR as a consequence of the contracted second Bianchi identity.)

By varying with respect to the linear connection $\Gamma^{\alpha\beta}$, we obtain the second field equation of EC theory, that is, Cartan's algebraic relation:

$$\eta_{\alpha\beta\gamma} \wedge T^\gamma = 2\kappa \tau_{\alpha\beta} \tag{14}$$

between torsion and the canonical spin of matter. Due to (15), in the case of Dirac fields, this is equivalent to

$${}^*\mathscr{A} = \frac{\kappa}{2} j_5, \tag{15}$$

coupled via the "bare" fundamental length $\ell = \sqrt{\kappa}$. Then, "on shell," EC theory deviates from GR merely via

$$\Delta\widetilde{L} := \kappa\left(L_{\mathrm{EC}} - L_{\mathrm{HE}}\right) \simeq \frac{\kappa^2}{48} j_5 \wedge {}^* j_5 = 4 f^2 j_5 \wedge {}^* j_5 \,. \tag{16}$$

4. Asymptotic Safety of EC Theory

For the Hilbert-Einstein Lagrangian

$$L_{\mathrm{HE}\Lambda} := L_{\mathrm{HE}} + \frac{\Lambda}{\kappa}\eta \tag{17}$$

with cosmological term, one can define the *dimensionless running* coupling constants

$$g_N := \kappa k^2, \qquad \lambda := \frac{\Lambda}{k^2}, \tag{18}$$

where k is the renormalization group (RG) scale in momentum space and Λ the cosmological constant related to dark energy (DE) of density ρ_Λ; see also [38].

Asymptotic safety amounts to the requirement that dimensionless coupling constants remain bounded in the ultraviolet limit $k \to \infty$. In 4D, this is controlled by the *renormalization group equations*

$$\begin{aligned} k\frac{\partial}{\partial k} g_N &= \beta_1\left(g_N, \lambda\right) = \left(2 + d_N\right) g_N, \\ k\frac{\partial}{\partial k}\lambda &= \beta_2\left(g_N, \lambda\right), \end{aligned} \tag{19}$$

where d_N is the anomalous dimension of the running Newton coupling g_N. According to the *Asymptotic Safety* (AS) scenario [20], they run into some nontrivial fixed points g_{N*} and λ_*, depending on the specific truncation of the effective Lagrangian to the celebrated Hilbert-Einstein one (10) without torsion. This can be extended [39] to high-order polynomials R^n of the Ricci scalar similarly as in the classically bifurcating $f(R)$ models [40], but then the issue of physical ghosts or nonunitarity known [41, 42] from Stelle-type higher-derivative models needs to be seen.

Quite generally, the dimensionless product

$$\mu = \frac{4}{3}\kappa\Lambda = \frac{1}{3}(2\kappa)^2 \rho_\Lambda \le \frac{4}{3} g_{N*}\lambda_* \simeq 0.2. \tag{20}$$

exhibits a universal bound independent of the particular truncation.

4.1. The Issue of the Four-Fermion Interaction. Interesting enough, the EC induced FF interaction (16) with its tiny "bare" coupling constant

$$f^2 = \frac{1}{192}\kappa^2 = 2^{-10}\left(\frac{\mu}{\Lambda}\right)^2 = 2^{-8}\frac{\mu}{\rho_\Lambda} \tag{21}$$

also scales with the gravitational constant κ but is inversely compared to the Hilbert-Einstein and cosmological terms.

If the renormalization flow starts to the right from the non-Gaussian fixed point, the coupling actually diverges [43] at a finite RG scale. When the contact- or point-like truncation breaks down, a boson-like description of fermion bilinears is mandatory, including the $1/k^4$ dependence in the functional integral. Then, the FF interaction becomes nonlocal [44], and the corresponding dimensionless renormalized running coupling f_*^2 becomes asymptotically safe or even free. In a nonlinear σ model [45], nonrenormalizable FF interactions may be instrumental for restoring asymptotic safety.

In view of these problems, the EC theory has been amended [32, 46] by the pseudocurvature scalar term of Hojman et al. [47] (the infamous "Holst" term, cf. [34]), or even nonminimally coupled Dirac fields [48]. Unfortunately, many of these extensions [49–51] are ignoring a possible running of the gravitational couplings and therefore appear not to be conclusive.

Apparently, the search for a Quantum Theory of Gravity (QG) which is free of anomalies and is leaving Einstein's GR as a well-established macroscopic sign post has produced rather contradictory partial results, to some extent resembling a Babylonian confusion; compare [52].

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