Research Article

Numerical Approximation for Nonlinear Gas Dynamic Equation

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Laplace transform and new homotopy perturbation methods are adopted to study gas dynamic equation analytically. The solutions introduced in this study can be used to obtain the closed form of the solutions if they are required. The combined method needs less work in comparison with the other homotopy perturbation methods and decreases volume of calculations considerably. Results show that the new method is more effective and convenient to use, and is high accuracy evident.

1. Introduction

Many phenomena in engineering, physics, chemistry, and other sciences can be described very successfully by nonlinear models. Except in a limited number of these problems, we have difficulty in finding their exact analytical solutions. Therefore, there have been attempts to develop new techniques for obtaining analytical solutions which reasonably approximate the exact solutions [1]. In recent decades, numerical calculation methods were good means of analyzing the nonlinear equations, but as the numerical calculation methods improved, analytical methods did, too. Most scientists believe that the combination of numerical and analytical methods can also end with useful results. In recent years, several of such techniques have drawn special attention, such as homotopy perturbation method. In the recent years, an increasing interest of scientists and engineers has been devoted to the analytical asymptotic techniques for solving nonlinear problems. Many new numerical techniques have been widely applied to the nonlinear problems. Based on homotopy, which is a basic concept in topology, general analytical method, namely, the homotopy perturbation method (HPM) is established by He [2–8] in 1998 to obtain series solutions of nonlinear differential equations. The He's HPM has been already used to solve various functional equations. In this method, the nonlinear problem is transferred to an infinite number of subproblems, and then the solution is

approximated by the sum of the solutions of the first several subproblems. This simple method has been applied to solve linear and nonlinear equations of heat transfer [9–11], fluid mechanics [12], nonlinear Schrodinger equations [13], integral equations [14], boundary value problems [15], fractional KdV-Burgers equation [16], nonlinear system of second-order boundary value problems [17], and delay differential equations [18].

The equations of gas dynamics are mathematical expressions based on the physical laws of conservation, namely, the laws of conservation of mass, conservation of momentum, conservation of energy, and so forth. The nonlinear equations of ideal gas dynamics are applicable for three types of nonlinear waves like shock fronts, rare factions, and contact discontinuities. Different types of gas dynamics equations in physics have been solved by Evans and Bulut [19], Polyanin and Zaitsev [20], Elizarova [21], Jafari et al. [22], and Ames [23], by applying various kinds of analytical and numerical methods. In 2003, Rasulov and Karaguler [24] applied the difference scheme for solving nonlinear system of equations of gas dynamic problems for a class of discontinuous functions.

In this work, we construct the solution of gas dynamic equation by using a powerful method. We obtain an analytical approximation to the solution of the nonlinear gas dynamic equation using combination of Laplace transform and new homotopy perturbation methods (LTNHPM). The

gas dynamic equation as a nonlinear model is as follows [19, 22]:

$$u_t + uu_x - u(1 - u) = 0, \quad (1)$$

where $0 \leq x \leq 1$ and $t > 0$. The results obtained via LTNHPM confirm validity of the proposed method. The rest of this paper is organized as follows.

In Section 2, basic ideas of LTNHPM and the homotopy perturbation method are presented. In Section 3, the application of LTNHPM for solving nonlinear gas dynamic equation is presented. Conclusion will appear in the last section.

2. Basic Ideas of the LTNHPM

In this section, we propose LTNHPM, a new hybrid of Laplace transform and new homotopy perturbation methods (NHPM) for solving nonlinear gas dynamic equation. This method is simple and finds exact solution of the equations analytically using the initial condition only. This method unlike the most numerical techniques provides a closed form of the solution. The new method developed in the current paper was tested on several equations [25–27]. To illustrate the basic ideas of this method, let us consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega, \quad (2)$$

with the following initial conditions:

$$u(0) = \alpha_0, \quad u'(0) = \alpha_1, \dots, u^{(n-1)}(0) = \alpha_{n-1}, \quad (3)$$

where A is a general differential operator and $f(r)$ is a known analytical function. The operator A can be divided into two parts, L and N , where L is a linear and N is a nonlinear operator. Therefore, (2) can be rewritten as

$$L(u) + N(u) - f(r) = 0. \quad (4)$$

Based on NHPM [28], we construct a homotopy $U(r, p) : \Omega \times [0, 1] \rightarrow \mathbb{R}$, which satisfies

$$\begin{aligned} H(U, p) &= (1 - p)[L(U) - u_0] \\ &\quad + p[A(U) - f(r)] \\ &= 0, \quad p \in [0, 1], \quad r \in \Omega, \end{aligned} \quad (5)$$

or equivalently,

$$H(U, p) = L(U) - u_0 + pu_0 + p[N(U) - f(r)] = 0, \quad (6)$$

where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation for the solution of (2). Clearly, (5) and (6) give

$$\begin{aligned} H(U, 0) &= L(U) - u_0 = 0, \\ H(U(x), 1) &= A(U) - f(r) = 0. \end{aligned} \quad (7)$$

Applying Laplace transform to both sides of (6), we arrive at

$$L\{L(U) - u_0 + pu_0 + p[N(U) - f(r)]\} = 0. \quad (8)$$

Using the differential property of Laplace transform we have

$$\begin{aligned} s^n L\{U\} - s^{n-1}U(0) - s^{n-2}U'(0) - \dots - U^{(n-1)}(0) \\ = L\{u_0 - pu_0 + p[N(U) - f(r)]\} \end{aligned} \quad (9)$$

or

$$\begin{aligned} L\{U\} &= \frac{1}{s^n} \{s^{n-1}U(0) + s^{n-2}U'(0) + \dots + U^{(n-1)}(0) \\ &\quad + L\{u_0 - pu_0 + p[N(U) - f(r)]\}\}. \end{aligned} \quad (10)$$

Finally, applying the inverse Laplace transform to both sides of (10), one can successfully reach the following:

$$\begin{aligned} U &= L^{-1} \left\{ \frac{1}{s^n} \left\{ s^{n-1}U(0) + s^{n-2}U'(0) + \dots + U^{(n-1)}(0) \right. \right. \\ &\quad \left. \left. + L\{u_0 - pu_0 + p[N(U) - f(r)]\} \right\} \right\}. \end{aligned} \quad (11)$$

According to the HPM, we can first use the embedding parameter p as a small parameter and assume that the solutions of (11) can be represented as a power series in p as

$$U(x) = \sum_{n=0}^{\infty} p^n U_n. \quad (12)$$

Now let us rewrite (11) using (12) as

$$\begin{aligned} &\sum_{n=0}^{\infty} p^n U_n \\ &= L^{-1} \left\{ \frac{1}{s^n} \left\{ s^{n-1}U(0) + s^{n-2}U'(0) + \dots + U^{(n-1)}(0) \right. \right. \\ &\quad \left. \left. + L \left\{ u_0 - pu_0 + p \left[N \left(\sum_{n=0}^{\infty} p^n U_n \right) - f(r) \right] \right\} \right\} \right\}. \end{aligned} \quad (13)$$

Therefore, equating the coefficients of p with the same power leads to

$$\begin{aligned} p^0 : U_0 &= L^{-1} \left\{ \frac{1}{s^n} \left(s^{n-1} U(0) + s^{n-2} U'(0) \right. \right. \\ &\quad \left. \left. + \dots + U^{(n-1)}(0) + L \{u_0\} \right) \right\}, \\ p^1 : U_1 &= L^{-1} \left\{ \frac{1}{s^n} \left(L \{N(U_0) - u_0 - f(r)\} \right) \right\}, \\ p^2 : U_2 &= L^{-1} \left\{ \frac{1}{s^n} \left(L \{N(U_0, U_1)\} \right) \right\}, \\ p^3 : U_3 &= L^{-1} \left\{ \frac{1}{s^n} \left(L \{N(U_0, U_1, U_2)\} \right) \right\}, \\ &\vdots \\ p^j : U_j &= L^{-1} \left\{ \frac{1}{s^n} \left(L \{N(U_0, U_1, U_2, \dots, U_{j-1})\} \right) \right\}, \\ &\vdots \end{aligned} \quad (14)$$

Suppose that the initial approximation has the form $U(0) = u_0 = \alpha_0$, $U'(0) = \alpha_1, \dots, U^{(n-1)}(0) = \alpha_{n-1}$, therefore the exact solution may be obtained as following

$$u = \lim_{p \rightarrow 1} U = U_0 + U_1 + U_2 + \dots \quad (15)$$

To show the capability of the method, we apply the NHPM to some examples in Section 3.

3. Analysis of the Method

Example 1. Consider the nonlinear gas dynamic equation (1)

$$u_t + uu_x - u(1-u) = 0 \quad (16)$$

subject to a constant initial condition

$$u(x, 0) = ae^{-x}. \quad (17)$$

For solving this equation by applying the new homotopy perturbation method, we construct the following homotopy:

$$H(U, p) = U_t - u_0 + p[u_0 + UU_x - U(1-U)] = 0, \quad (18)$$

where $p \in [0, 1]$ is an embedding parameter, u_0 is an initial approximation of solution of equation. Clearly, we have from (18)

$$\begin{aligned} H(U, 0) &= U_t - u_0 = 0, \\ H(U, 1) &= U_t + UU_x - U(1-U) = 0. \end{aligned} \quad (19)$$

By applying Laplace transform on both sides of (18), we have

$$L\{H(U, p)\} = L\{U_t - u_0 + p[u_0 + UU_x - U(1-U)]\}. \quad (20)$$

Using the differential property of Laplace transform we have:

$$\begin{aligned} sL\{U(x, t)\} - U(x, 0) \\ = L\{u_0 - p[u_0 + UU_x - U(1-U)]\} \end{aligned} \quad (21)$$

or

$$\begin{aligned} L\{U(x, t)\} \\ = \frac{1}{s} (U(x, 0) + L\{u_0 - p[u_0 + UU_x - U(1-U)]\}). \end{aligned} \quad (22)$$

By applying inverse Laplace transform on both sides of (22), we have

$$\begin{aligned} U(x, t) \\ = L^{-1} \left\{ \frac{1}{s} (U(x, 0) + L\{u_0 - p[u_0 + UU_x - U(1-U)]\}) \right\}. \end{aligned} \quad (23)$$

According to the HPM, we use the embedding parameter p as a small parameter, and assume that the solutions of (23) can be represented as a power series in p as

$$U(x, t) = \sum_{n=0}^{\infty} p^n U_n(x, t). \quad (24)$$

Substituting (24) into (23), and equating the terms with the identical powers of p , leads to calculate $U_j(x, t)$, $j = 0, 1, 2, \dots$

$$\begin{aligned} p^0 : U_0(x, t) &= L^{-1} \left\{ \frac{1}{s} (U(x, 0) + L\{u_0(x, t)\}) \right\} \\ p^1 : U_1(x, t) \\ &= L^{-1} \left\{ \frac{-1}{s} L\{u_0(x, t) + U_0(U_0)_x - U_0 + U_0^2\} \right\} \\ p^2 : U_2(x, t) \\ &= L^{-1} \left\{ \frac{-1}{s} L\{U_0(U_1)_x + U_1(U_0)_x - U_1 + 2U_0U_1\} \right\} \\ &\vdots \\ p^j : U_j(x, t) \\ &= L^{-1} \left\{ \frac{-1}{s} L \left\{ \sum_{k=0}^{j-1} U_k (U_{j-k-1})_x - U_{j-1} \right. \right. \\ &\quad \left. \left. + \sum_{k=0}^{j-1} U_k U_{j-k-1} \right\} \right\}. \end{aligned} \quad (25)$$

Suppose that the initial approximation has the form $U(x, 0) = u_0(x, t) = ae^{-x}$, therefore the exact solution may be obtained as following

$$\begin{aligned} u(x, t) &= \lim_{p \rightarrow 1} U(x, t) \\ &= U_0(x, t) + U_1(x, t) + U_2(x, t) + \dots \end{aligned} \quad (26)$$

Solving (25) for $U_j(x, t)$, $j = 0, 1, \dots$ leads to the result

$$\begin{aligned} U_0(x, t) &= e^{-x} (1 + t), \\ U_1(x, t) &= \frac{1}{2} a e^{-x} t^2, \\ U_2(x, t) &= \frac{1}{6} a e^{-x} t^3, \\ U_3(x, t) &= \frac{1}{24} a e^{-x} t^4, \\ U_4(x, t) &= \frac{1}{120} a e^{-x} t^5, \\ &\vdots \end{aligned} \quad (27)$$

Therefore, we gain the solution of (27) as

$$\begin{aligned} u(x, t) &= U_0(x, t) + U_1(x, t) + U_2(x, t) \\ &\quad + U_3(x, t) + \dots \\ &= a e^{-x} + a e^{-x} t + a e^{-x} \frac{t^2}{2!} + a e^{-x} \frac{t^3}{3!} \\ &\quad + a e^{-x} \frac{t^4}{4!} + a e^{-x} \frac{t^5}{5!} + \dots = a e^{t-x}, \end{aligned} \quad (28)$$

which is the exact solution of the problem. Figure 1 shows the plot of $u(x, t) = a e^{t-x}$ with $a = 1$ and $t > 0$, $0 \leq x \leq 1$.

Example 2. Consider the following nonhomogeneous nonlinear gas dynamic equation [19]:

$$u_t + uu_x - u(1 - u) = -e^{t-x}, \quad (29)$$

with the initial condition

$$u(x, 0) = 1 - e^{-x}. \quad (30)$$

For solving this equation by applying the new homotopy perturbation method, we construct the following homotopy:

$$\begin{aligned} H(U, p) &= U_t - u_0 + p [u_0 + UU_x - U(1 - U) + e^{t-x}] \\ &= 0, \end{aligned} \quad (31)$$

where $p \in [0, 1]$ is an embedding parameter, u_0 is an initial approximation of solution of equation. Clearly, we have from (31)

$$H(U, 0) = U_t - u_0 = 0, \quad (32)$$

$$H(U, 1) = U_t + UU_x - U(1 - U) + e^{t-x} = 0.$$

By applying Laplace transform on both sides of (31), we have

$$\begin{aligned} L\{H(U, p)\} &= L\{U_t - u_0 + p [u_0 + UU_x - U(1 - U) + e^{t-x}]\}. \end{aligned} \quad (33)$$

Using the differential property of Laplace transform we have

$$\begin{aligned} sL\{U(x, t)\} - U(x, 0) &= L\{u_0 - p [u_0 + UU_x - U(1 - U) + e^{t-x}]\} \end{aligned} \quad (34)$$

or

$$\begin{aligned} L\{U(x, t)\} &= \frac{1}{s} (U(x, 0) \\ &\quad + L\{u_0 - p [u_0 + UU_x - U(1 - U) + e^{t-x}]\}). \end{aligned} \quad (35)$$

By applying inverse Laplace transform on both sides of (35), we have

$$\begin{aligned} U(x, t) &= L^{-1} \left\{ \frac{1}{s} (U(x, 0) \right. \\ &\quad \left. + L\{u_0 - p [u_0 + UU_x - U(1 - U) + e^{t-x}]\}) \right\}. \end{aligned} \quad (36)$$

According to the HPM, we use the embedding parameter p as a small parameter and assume that the solutions of (36) can be represented as a power series in p as

$$U(x, t) = \sum_{n=0}^{\infty} p^n U_n(x, t). \quad (37)$$

Substituting (37) into (36) and equating the terms with the identical powers of p lead to calculating $U_j(x, t)$, $j = 0, 1, 2, \dots$

$$\begin{aligned} p^0 : U_0(x, t) &= L^{-1} \left\{ \frac{1}{s} (U(x, 0) + L\{u_0(x, t)\}) \right\}, \\ p^1 : U_1(x, t) &= L^{-1} \left\{ \frac{-1}{s} L\{u_0(x, t) + U_0(U_0)_x - U_0 + U_0^2 + e^{t-x}\} \right\}, \\ p^2 : U_2(x, t) &= L^{-1} \left\{ \frac{-1}{s} L\{U_0(U_1)_x + U_1(U_0)_x - U_1 + 2U_0U_1\} \right\}, \\ &\vdots \\ p^j : U_j(x, t) &= L^{-1} \left\{ \frac{-1}{s} L \left\{ \sum_{k=0}^{j-1} U_k (U_{j-k-1})_x - U_{j-1} + \sum_{k=0}^{j-1} U_k U_{j-k-1} \right\} \right\}. \end{aligned} \quad (38)$$

Suppose that the initial approximation has the form $U(x, 0) = u_0(x, t) = 1 - e^{-x}$; therefore, the exact solution may be obtained as follows

$$\begin{aligned} u(x, t) &= \lim_{p \rightarrow 1} U(x, t) \\ &= U_0(x, t) + U_1(x, t) + U_2(x, t) + \dots \end{aligned} \quad (39)$$

FIGURE 1: Plot of $u(x, t) = ae^{t-x}$ with $a = 1$ and $t > 0$, $0 \leq x \leq 1$.

Solving (38) for $U_j(x, t)$, $j = 0, 1, 2, \dots$ leads to the result

$$U_0(x, t) = 1 + t - e^{-x}(1 + t),$$

$$U_1(x, t) = -t - \frac{t^2}{2} - \frac{t^3}{3} - e^{t-x} + \frac{1}{6}e^{-x}(2t^3 + 3t^2 + 6t + 6),$$

$$U_2(x, t) = \frac{t^2}{2} + \frac{5}{6}t^3 + \frac{t^4}{3} + \frac{2}{15}t^5 - \frac{1}{30}e^{-x}(10t^4 + 4t^5 - 30 + 30t^2 + 25t^3) + e^{t-x}(t - 1),$$

$$U_3(x, t) = -\frac{t^3}{2} - \frac{17}{90}t^6 - \frac{17}{24}t^4 - \frac{7}{12}t^5 - \frac{17}{315}t^7 - \frac{1}{6}e^{t-x}(2t^3 + 3t^2 - 6t + 6) + \frac{1}{1260}e^{-x}(68t^7 + 1155t^4 + 238t^6 + 735t^5 + 1260 + 840t^3),$$

$$U_4(x, t) = \frac{31}{315}t^8 + \frac{23}{36}t^6 + \frac{113}{315}t^7 + \frac{3}{8}t^4 + \frac{31}{40}t^5 + \frac{62}{2835}t^9 - \frac{1}{45360}e^{-x}(16272t^7 + 4464t^8 + 28350t^4 + 32823t^6 + 41958t^5 + 992t^9 - 45360)$$

$$+ \frac{1}{30}e^{t-x}(-15t^2 + 5t^3 + 4t^5 + 30t + 10t^4 - 30),$$

$$U_5(x, t) = -\frac{578}{2835}t^9 - \frac{148}{315}t^8 - \frac{57}{80}t^6 - \frac{31}{40}t^7 - \frac{691}{14175}t^{10} - \frac{13}{40}t^5 - \frac{1382}{155925}t^{11} + \frac{1}{19958400}e^{-x}(17364600t^7 + 10062855t^8 + 176896t^{11} + 18905040t^6 + 9646560t^5 + 4069120t^9 + 19958400 + 972928t^{10}) - \frac{1}{2520}e^{t-x}(105t^4 - 420t^3 + 798t^5 - 2520t + 476t^6 + 1260t^2 + 136t^7 + 2520),$$

⋮

(40)

Therefore, we gain the solution of (40) as follows:

$$u(x, t) \cong U_0(x, t) + U_1(x, t) + U_2(x, t) + U_3(x, t) + U_4(x, t) + U_5(x, t) = 1 + 4e^{-x} - 5e^{t-x} - \frac{13}{35}t^8 - \frac{172}{945}t^9 - \frac{11}{60}e^{t-x}t^5 + \frac{7}{24}e^{t-x}t^4 + \frac{16361}{40320}e^{-x}t^8 + \frac{172}{945}e^{-x}t^9 - \frac{17}{90}e^{t-x}t^6 - \frac{17}{315}e^{t-x}t^7$$

FIGURE 2: Plot of $u(x, t) = 1 - e^{t-x}$ with $t > 0$, $0 \leq x \leq 1$, and the 5-term approximation of $u(x, t)$.

$$\begin{aligned}
 & + \frac{1382}{155925}e^{-x}t^{11} + \frac{691}{14175}e^{-x}t^{10} \\
 & - \frac{21}{80}t^6 - \frac{79}{168}t^7 - \frac{3}{2}e^{t-x}t^2 \\
 & + 4e^{t-x}t + \frac{407}{720}e^{-x}t^7 - \frac{1}{24}e^{-x}t^4 \\
 & + \frac{33}{80}e^{-x}t^6 + \frac{1}{120}e^{-x}t^5 + \frac{1}{6}e^{-x}t^3 \\
 & - \frac{691}{14175}t^{10} - \frac{1382}{155925}t^{11} - \frac{1}{2}e^{-x}t^2
 \end{aligned} \tag{41}$$

which is the exact solution of the problem. Figure 2 shows the plots of $u(x, t) = 1 - e^{t-x}$ with $t > 0$, $0 \leq x \leq 1$, and the 5-term approximation of $u(x, t)$.

4. Summary and Conclusion

In the present work, we proposed a combination of Laplace transform method and homotopy perturbation method to solve nonlinear gas dynamic equation. This method finds an exact solution of the equation using the initial condition only. This method unlike the most numerical techniques provides a closed form of the solution. The obtained results show that the new method developed in the current paper can solve the problem effectively.

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