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Research Article

Compression of Meanders

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This paper refers to the algorithmic transformation of a meander to its uniquely defined compression. We obtain this directly from meandric permutations, thus creating representations of large classes of meanders of different orders. We prove basic properties, give arithmetic results, and produce generating procedures.

1. Introduction

A closed meander of order n is a closed self-avoiding curve crossing an infinite horizontal line $2n$ times [1]. In this paper, we obtain the compression as the determination of a unique simple meander, directly from its permutation. The meanders as planar permutations were introduced by Rosenstiehl [2] and they have been studied with nested sets [3, 4].

More specifically, in Section 2, we define the flow of a meander consisted by its traces and corresponding blocks. In Section 3, we create a specific form of meanders: the simple ones, we study the properties of their numbers of cuttings and cutting degree and we use them in order to introduce the compression. In Section 4, we determine the flow of the meandric permutations and we achieve also numerical results for the classification of the meanders of the compressions according to their order. Finally, in Section 5, we establish the compression of meanders directly from their meandric permutations divided in suitable blocks. Thus, we change their interpretation and produce a simplified procedure for generating the compressions.

The following definitions and notation are necessary for the rest of the paper [3].

A set S of disjoint pairs of $[2n]$ such that $\bigcup_{\{a,b\}\in S}\{a,b\} = [2n]$ and for any $\{a,b\},\{c,d\} \in S$ we never have $a < c < b < d$ is called *nested set* of pairs on $[2n]$. Each pair of a nested set consists of an odd and an even number. We denote the set of all nested sets of pairs on $[2n]$ by N_{2n}. Two nested sets $S_1, S_2 \in N_{2n}$ define a permutation σ on $[2n]$, such that $\sigma(2i-1) = j$ iff $\{2i-1, j\} \in S_1$ and $\sigma(2i) = j$ iff $\{2i, j\} \in S_2$, for every $i \in [n]$. The sets S_1, S_2 are *k-matching* if and only if σ has k cycles. In the case where $k = 1$, S_1, S_2 are simply called *matching*. This definition is equivalent to the one given in [3].

We call *short pair* of S any pair of consecutive numbers that belongs to S, and *outer pair* of S any pair $\{a,b\} \in S$ such that there is no pair $\{c,d\} \in S$ with $c < a < b < d$. Each nested set of pairs contains at least one outer and one short pair.

2. Meanders

A meander of order n is equivalently defined [3] as a cyclic permutation

$$\mu = \mu(1)\,\mu(2)\cdots\mu(2n) \tag{1}$$

on $[2n]$, for which the following properties hold true: $\mu(1) = 1$, and the sets

$$\begin{gathered} U = \{\{\mu(i), \mu(i+1)\} : i = 1, 3, \ldots, 2n-1\}, \\ L = \{\{\mu(i), \mu(i+1)\} : i = 2, 4, \ldots, 2n\} \end{gathered} \tag{2}$$

are both nested and matching.

We take all numbers mod $2n$. It is clear that $\mu(i)$ is odd if and only if i is odd. In the corresponding geometrical representation, the nested arcs correspond to nested pairs. A pair of nested sets U, L should be matching, in order to

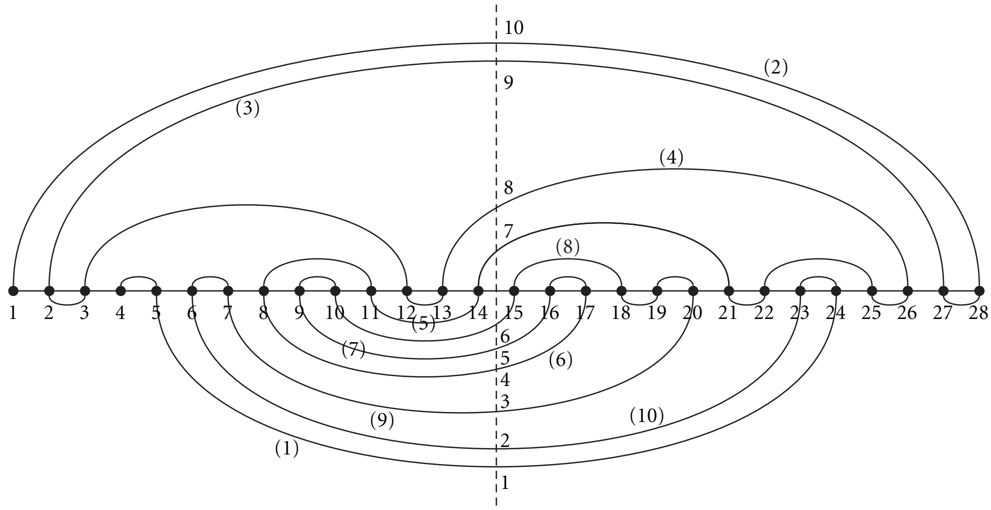

Figure 1: A meander of order 14.

generate a meander. For example, the meander of Figure 1 is of order 14, with

$$\begin{aligned} \mu &= 1\ 28\ 27\ 2\ 3\ 12\ 13\ 26\ 25\ 22\ 21\ 14\ 11 \\ &\quad 8\ 17\ 16\ 9\ 10\ 15\ 18\ 19\ 20\ 7\ 6\ 23\ 24\ 5\ 4, \\ U &= \{\{1,28\},\{2,27\},\{3,12\},\{4,5\},\{6,7\},\{8,11\},\{9,10\}, \\ &\quad \{13,26\},\{14,21\},\{15,18\},\{16,17\},\{19,20\},\{22,25\}, \\ &\quad \{23,24\}\}, \\ L &= \{\{1,4\},\{2,3\},\{5,24\},\{6,23\},\{7,20\},\{8,17\},\{9,16\}, \\ &\quad \{10,15\},\{11,14\},\{12,13\},\{18,19\},\{21,22\},\{25,26\}, \\ &\quad \{27,28\}\}. \end{aligned} \tag{3}$$

The set of all the meanders of order n is denoted by $\mathscr{M}_{2n}$. Let $\mu \in \mathscr{M}_{2n}$ be a meander crossing a horizontal line. Following [5], for any $i \in [2n-1]$ we consider the vertical line, which shall be called the *i-line*, passing through the middle point of the segment $(i, i+1)$ of the horizontal line. The numbers of those arcs of the meandric curve which are intersected by the i-line and lie above and beneath the horizontal line of μ, are called the *numbers of cuttings* $\theta(i)$ and $\theta'(i)$, respectively [6].

The sum $\varepsilon(i) = \theta(i) + \theta'(i)$ of the number of those arcs of the meandric curve which are intersected by the i-line is called the *cutting degree* of the meander at i. We notice that $\theta(i)$ and $\theta'(i)$ are of the same parity; hence, $\varepsilon(i)$ is always even [5].

The meandric curve always has points of intersection with the i-line, which we call *traces*. Obviously, the number of the traces is equal to $\varepsilon(i)$. Starting below the horizontal line, we label the traces with the numbers $1, 2, 3, \ldots, \varepsilon(i)$, knowing that $\theta(i)$ (resp., $\theta'(i)$) of them are lying above (resp., beneath) the horizontal line. From now on, we will consider that the traces are identical to their corresponding labels. For the meander of Figure 1 and for $i = 14$, we have $\theta(14) = 4$, $\theta'(14) = 6$, and $\varepsilon(14) = 10$.

Beginning from trace 1 and moving clockwise upon the meandric curve, following its "natural flow," we obtain a shuffle of the permutation of the traces and the meandric permutation, see Figure 2 where the circled elements are the traces.

In the general case, we have the shuffle

$$t_i = \tau_i(1) B_1^i \tau_i(2) B_2^i \cdots \tau_i(\varepsilon(i)) B_{\varepsilon(i)}^i \tag{4}$$

with $\tau_i(1) = 1, \tau_i(2), \ldots, \tau_i(\varepsilon(i))$ being the traces of the meander at i and $B_1^i, B_2^i, \ldots, B_{\varepsilon(i)}^i$ the parts of consecutive elements of the meandric permutation μ, lying between consecutive traces, called *blocks* of the meander; that is, the block B_k^i, $k \in [\varepsilon(i)]$, is the set of the consecutive elements of the permutation μ, which are lying between the traces $\tau_i(k)$ and $\tau_i(k+1)$. These two traces are called the "entrance" trace and the "exit" trace of the block, respectively, while the shuffle t_i is called *flow* of the meander from the trace (1) of the i-line, or for simplicity *i-flow*.

For every $k \in [\varepsilon(i)]$, $\tau_i(k) B_k^i \tau_i(k+1)$ corresponds to the part of the meandric curve starting from the trace $\tau_i(k)$ and ending at the trace $\tau_i(k+1)$, which we denote by c_k^i. If k is odd (resp., even), then this curve lies on the left (resp., right) of the i-line. In Figure 1, we denote the curve c_k^i by (k).

3. Simple Meanders and Compression

Let $\mu \in \mathscr{M}_{2n}$ and its flow $t_i = \tau_i(1) B_1^i \tau_i(2) B_2^i \cdots \tau_i(\varepsilon(i)) B_{\varepsilon(i)}^i$. To each block B_k^i, $k \in [\varepsilon(i)]$, we correspond a number b_k^i such that

$$b_k^i = \begin{cases} 1, & \text{if } \left|B_k^i\right| \text{ is odd}, \\ 2, & \text{if } \left|B_k^i\right| \text{ is even}. \end{cases} \tag{5}$$

$t =$ ① 5 4 1 ⑩ 28 27 ⑨ 2 3 12 13 ⑧ 26 25 22 21 ⑦ 14 11 8 ④ 17 16 ⑤ 9 10 ⑥ 15 18 19 20 ③ 7 6 ② 23 24

Figure 2

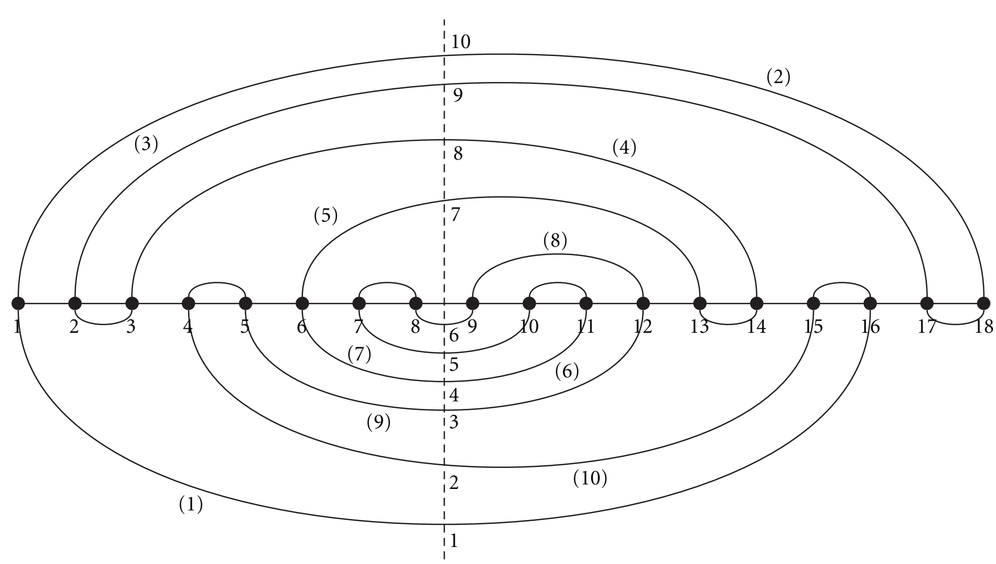

Figure 3: A meander of order 9, simple at $i = 8$.

The set $B(i) = \{B_1^i, B_1^i, \ldots, B_{\varepsilon(i)}^i\}$ can be partitioned into three classes $B_1(i)$, $B_2(i)$, and $B_2^{'}(i)$, where the set $B_1(i)$ consists of the blocks B_k^i with $b_k^i = 1$ and the set $B_2(i)$ consists of the blocks B_k^i with $b_k^i = 2$ and $\theta^{'}(i) < \tau_i(k), \tau_i(k+1)$, while the set $B_2^{'}(i)$ consists of the blocks B_k^i with $b_k^i = 2$ and $\tau_i(k), \tau_i(k+1) < \theta^{'}(i)$.

A meander $\mu \in \mathscr{M}_{2n}$ is called *simple* at i (i.e., simple referring to the i-line) if $|B_k^i| \leq 2$, $k \in [\varepsilon(i)]$. Hence, every block of the sets $B_1(i)$ (resp., $B_2(i)$, $B_2^{'}(i)$) has exactly one element (resp., two elements). We notice that the following necessary and sufficient condition holds true.

A meander is simple at i if and only if every triple of consecutive terms of its permutation contains elements from both of the sets $\{1, 2, \ldots, i\}$ and $\{i+1, i+2, \ldots, 2n\}$.

For example, the meander of Figure 3 is simple at $i = 8$.

We note that a meander can be simple at more than one point. For example, the meander $\mu = 1\ 4\ 3\ 2\ 5\ 6$ is simple at $i = 2$ and $i = 3$.

Let $\mu \in \mathscr{M}_{2n}$ be a meander that is not simple at n. Further on, we will study every meander according to its n-line, so for simplicity we omit the index n from the notation. Let $B(n) = \{B_1, B_2, \ldots, B_{2\nu}\}$ be its already defined set of blocks, where $2\nu = \varepsilon(n)$ and the blocks B_k, for $k \in I_1 = \{1, 3, \ldots, 2\nu - 1\}$ (resp., $k \in I_2 = \{2, 4, \ldots, 2\nu\}$), are the ones placed at the left (resp., right) of the n-line. Each block $B_k \in B(n)$ lies between the trace $\tau(k)$ and the trace $\tau(k+1)$ of the flow t.

We denote by γ_k, $k \in [2\nu]$, the closed interval of $[2\nu]$ with ends the traces $\tau(k)$, $\tau(k+1)$. Given a pair of blocks $B_k, B_\lambda \in B(n)$, $k, \lambda \in I_1$ or $k, \lambda \in I_2$, with $\gamma_\lambda \subset \gamma_k$, then the block B_λ is called *internal* of the block B_k, and the block B_k is called *external* to the block B_λ. We can easily deduce that the number of the internal blocks of a block $B_k \in B(n)$ is equal to $(1/2)(|\tau(k) - \tau(k+1)| - 1)$.

If we replace the blocks B_k, $k \in [2\nu]$, of the meander μ by blocks having one element (resp., two elements) whenever $b_k = 1$ (resp., $b_k = 2$), then we obtain a meander $\overline{\mu}$ of order $\overline{n} = (1/2)\sum_{k\in[2\nu]} b_k$ (since its blocks have one or two elements, corresponding to crossing points with the horizontal line) and simple at $u = \sum_{k\in I_1} b_k$ (counting the points of intersection with the horizontal line to the left of the n-line).

The result of the above replacement is the set of blocks $\overline{B}(u) = \{\overline{B}_1, \overline{B}_2, \ldots, \overline{B}_{2\nu}\}$, where $\overline{B}_k = \overline{B}_k^u$ for simplicity. The set $\overline{B}(u)$ is partitioned into the classes $\overline{B}_1(u)$, $\overline{B}_2(u)$, and $\overline{B}_2^{'}(u)$ corresponding to the classes $B_1(n)$, $B_2(n)$, and $B_2^{'}(n)$ of the set $B(n)$.

When we put a dash upon any existing notation, we refer to the elements of the deduced simple meander $\overline{\mu}$. We easily obtain that

(i) u, n are of the same parity,

(ii) $\overline{\varepsilon}(u) = 2\nu$, $\overline{\theta}(u) = \theta(n)$, $\overline{\theta}^{'}(u) = \theta^{'}(n)$, $\overline{b}_k = b_k$,

(iii) if $B_k \in B_2(n)$ (resp. $B_k \in B_2^{'}(n)$) and $|\tau(k) - \tau(k+1)| = 1$, then its corresponding block $\overline{B}_k \in \overline{B}_2(u)$ (resp., $\overline{B}_k \in \overline{B}_2^{'}(u)$) contains one short pair of $\overline{L}$ (resp., $\overline{U}$). If $B_k \in B_1(n)$, then its corresponding block $\overline{B}_k \in \overline{B}_1(u)$ is the pair $\{\tau(k), \tau(k+1)\}$.

The pair $(\overline{\mu}, u)$ is called *central compression* or simply *compression* of the meander μ. The simple at u meander $\overline{\mu}$

μ = 1 28 27 2 3 12 13 26 25 22 21 14 11 8 17 16 9 10 15 18 19 20 7 6 23 24 5 4

1 | 28 27 | 2 3 12 13 | 26 25 22 21 | 14 11 8 | 17 16 | 9 10 | 15 18 19 20 | 7 6 | 23 24 | 5 4

5 4 1 $\overset{(10)}{|}$ 28 27 $\overset{(9)}{|}$ 2 3 12 13 $\overset{(8)}{|}$ 26 25 22 21 $\overset{(7)}{|}$ 14 11 8 $\underset{(4)}{|}$ 17 16 $\underset{(5)}{|}$ 9 10 $\underset{(6)}{|}$ 15 18 19 20 $\underset{(3)}{|}$ 7 6 $\underset{(2)}{|}$ 23 24 $\underset{(1)}{|}$

Figure 4

has as same invariants with the meander μ the traces and the flow of curves. For example, the compression of the meander μ of Figure 1 is the pair $(\overline{\mu}, u)$, where $\overline{\mu}$ is the meander of Figure 3 and $u = 1 + 2 + 2 + 1 + 2 = 8$.

4. The Flow t

The traces and the blocks of the flow t of a meander μ can be found from its permutation with the help of the subsets $U(n) \subseteq U$ and $L(n) \subseteq L$, containing the pairs $\{\mu(i), \mu(i+1)\}$ satisfying the relation

$$\min\{\mu(i), \mu(i+1)\} \leq n < \max\{\mu(i), \mu(i+1)\}. \qquad (6)$$

These pairs have one element belonging to the set $\{1, 2, \ldots, n\}$ and the other belonging to the set $\{n+1, n+2, \ldots, 2n\}$, with $|U(n)| = \theta(n)$ and $|L(n)| = \theta'(n)$.

According to the absolute value $|\mu(i) - \mu(i+1)|$, we place the elements of $L(n)$ in decreasing order, while of $U(n)$ in increasing order. If $U(n), L(n) \neq \varnothing$, then we correspond the numbers $\theta'(n)+1, \ldots, 2\nu-1, 2\nu$ to the ordered pairs of $U(n)$, and the numbers $1, 2, \ldots, \theta'(n)$ to the ordered pairs of $L(n)$. If $L(n) = \varnothing$ (resp., $U(n) = \varnothing$), then we correspond to the ordered pairs of $U(n)$ (resp., $L(n)$) the numbers $1, 2, \ldots, 2\nu$. It is obvious that the previous numbers coincide with their corresponding traces.

In order to start the n-flow t from the first trace, we choose the pair $\{\mu(z), \mu(z+1)\}$ which corresponds to the trace $\tau(1) = 1$. If $L(n) \neq \varnothing$, then this pair is the outer pair of $L(n)$ with $\mu(z+1)$ odd and less than $\mu(z)$. If $L(n) = \varnothing$, then this is the pair of $U(n)$ with the smallest value of $|\mu(i) - \mu(i+1)|$.

For example, for the meander of Figure 1, we have that

$$U(14) = \{\{1, 28\}, \{2, 27\}, \{13, 26\}, \{14, 21\}\},$$
$$L(14) = \{\{5, 24\}, \{6, 23\}, \{7, 20\}, \{8, 17\}, \{9, 16\}, \{10, 15\}\}. \qquad (7)$$

Hence, we choose the pair $\{5, 24\}$ of $L(14)$ to correspond to the trace $\tau(1) = 1$.

Since the permutation μ is cyclic, we do not change the notation, that is,

$$\mu = \mu(z+1) \cdots \mu(1)\,\mu(2) \cdots \mu(z), \qquad (8)$$

which also defines a partition of $[2n]$ with classes including its consecutive elements, which belong, respectively, to the sets $\{1, 2, \ldots, n\}$ and $\{n+1, n+2, \ldots, 2n\}$. This partition gives the 2ν classes of the set $B(n) = \{B_1, B_2, \ldots, B_{2\nu}\}$.

Practically, at first we partition the permutation of the meander into classes including the consecutive terms of μ, which are less or equal to (resp., greater than) n. Thus, we have the partition of μ into blocks, putting at the beginning the last remaining elements and marking the traces.

For example, for the meander of Figure 1 we have Figure 4.

The placement of the traces follows the change of parity of the elements of the permutation, if we have an odd (resp., even) element followed by an even (resp., odd) element, then their intermediate trace is lying above (resp., beneath) the horizontal line. From the above partition, we obtain the flow: see Figure 2.

The meanders of the compression of the set $\mathscr{M}_{2n}$ are partitioned into classes, with elements belonging to the sets $\mathscr{M}_2, \mathscr{M}_4, \ldots, \mathscr{M}_{2n}$.

In the methods of generating planar permutations [7], we can also include the way to find the blocks of meanders, their corresponding numbers b_k, and consequently the orders of the meanders of their compressions.

These meanders can be used as generators for the reverse problem of "decompression." We can use them to extend a meander $\overline{\mu} \in \mathscr{M}_{2\overline{n}}$ simple at u to all possible meanders $\mu \in \mathscr{M}_{2n}$, with $n > \overline{n}$.

Table 1 presents the cardinalities of those classes for $n = 2, 3, \ldots, 10$, without taking into account the corresponding u-line.

The zeros of the first (resp., second) column verify the fact that if the order n of the meander is even (resp., odd), then there do not exist meanders of order n with compression of order 1 (resp., 2). We can easily prove that the values of the first column express that there exist $|\mathscr{M}_{(n+1)/2}|^2$ meanders of order n with compression of order 1. For meanders of larger order, we should try to calculate the number of different blocks of given orders.

5. Determining the Compression

We shall find the compression of a meander μ with the help of its n-flow $\overline{t} = \tau(1)\overline{B}_1\tau(2)\overline{B}_2 \cdots \tau(2\nu)\overline{B}_{2\nu}$. We recall that each block $\overline{B}_k \in \overline{B}(u)$ consists of one or two elements. Obviously, the elements of $\overline{B}_k$ belong to the sets $\{1, 2, \ldots, u\}$ (resp., $\{u+1, u+2, \ldots, 2\overline{n}\}$), when $k \in I_1$ (resp., $k \in I_2$). The relative position of these points defines a relation of preceding for the blocks of the set $\overline{B}(u)$; hence, the block $\overline{B}_p$ precedes the block $\overline{B}_q$ ($\overline{B}_p \prec \overline{B}_q$), iff $\min \overline{B}_p < \min \overline{B}_q$.

Table 1: The partition of the set $\mathscr{M}_{2n}$ into the classes $\mathscr{M}_{2\overline{n}}$.

$2n \setminus 2\overline{n}$	2	4	6	8	10	12	14	16	18	20	Total
4	0	2									2
6	4	0	4								8
8	0	18	16	8							42
10	64	0	144	24	30						262
12	0	392	616	480	268	72					1828
14	1764	0	6084	1760	3218	712	282				13820
16	0	13122	28000	27412	25040	12340	4220	820			110954
18	68644	0	304704	115200	261380	99664	66224	14512	3130		933458
20	0	578888	1491968	1684384	1980960	1315572	737944	284028	68892	10224	8152860

k	1	2	3	4	5	6	7	8	9	10
t	1 5 4 1	10 28 27	9 2 3 12 13	8 26 25 22 21	7 14 11 8	4 17 16	5 9 10	6 15 18 19 20	3 7 6	2 23 24
r^{-1}	1	10	2	8	4	7	5	6	3	9
$\overline{t}$	1 1	10 18 17	9 2 3	8 14 13	7 6	4 11 10	5 7 8	6 9 12	3 5 4	2 15 16

Figure 5

This relation is defined by the following conditions.

(a) If $B_p \in B_1(n)$, $B_q \in B(n)$, with $\gamma_q \subset \gamma_p$ and $p, q \in I_1$ (resp., $p, q \in I_2$), then $\overline{B}_p \prec \overline{B}_q$ (resp., $\overline{B}_q \prec \overline{B}_p$), since the block $\overline{B}_q$ is internal of the block $\overline{B}_p$.

(b) If $B_p \in B_2(n) \cup B_2'(n)$, $B_q \in B_1(n)$, with $\gamma_q \cap \gamma_p = \varnothing$ and $p, q \in I_1$ (resp., $p, q \in I_2$), then $\overline{B}_p \prec \overline{B}_q$ (resp., $\overline{B}_q \prec \overline{B}_p$), since this is imposed by the nature of the meander.

(c) If $B_p, B_q \in B_2(n) \cup B_2'(n)$, with $\gamma_q \subset \gamma_p$ and $p, q \in I_1$ or $p, q \in I_2$, then $\overline{B}_p \prec \overline{B}_q$, since the block $\overline{B}_q$ is internal of the block $\overline{B}_p$.

(d) If $B_p, B_q \in B_2(n)$, with $\gamma_q \cap \gamma_p = \varnothing$, $\min \gamma_q < \min \gamma_p$ and $p, q \in I_1$ (resp., $p, q \in I_2$), then $\overline{B}_p \prec \overline{B}_q$ (resp., $\overline{B}_q \prec \overline{B}_p$), since this is imposed by the nature of the meander. The case where $B_p, B_q \in B_2'(n)$ is similar.

Obviously, the above results do not cover the cases of two blocks, the one belonging to the set $B_2(n)$, and the other to the set $B_2'(n)$, where none of them is internal to the other. In order to obtain a unique solution, we have to make the following choice.

(e) If $B_p \in B_2(n)$, $B_q \in B_2'(n)$, with $\min \widetilde{B}_p < \min \widetilde{B}_q$ and $p, q \in I_1$ or $p, q \in I_2$, then $\overline{B}_p \prec \overline{B}_q$ (resp., $\overline{B}_q \prec \overline{B}_p$), where $\widetilde{B}_k = b_{k1} b_{k\omega}$, b_{k1} is the first element of B_k, $b_{k\omega}$ is the last element of B_k and $\omega = |B_k|$.

From the above conditions (a)–(e), we obtain an ordering for the blocks of the set $\overline{B}(u)$, concerning their relation of preceding, that is, $\overline{B}_{r(1)}, \overline{B}_{r(2)}, \ldots, \overline{B}_{r(2v)}$.

For example, applying the above conditions for the meander of Figure 1, we obtain that $\overline{B}_1 \prec \overline{B}_3 \prec \overline{B}_9 \prec \overline{B}_5 \prec \overline{B}_7$ and $\overline{B}_8 \prec \overline{B}_6 \prec \overline{B}_4 \prec \overline{B}_{10} \prec \overline{B}_2$. Indeed, $\overline{B}_1 \prec \overline{B}_3, \overline{B}_5, \overline{B}_7, \overline{B}_9$ due to the condition (a), $\overline{B}_3 \prec \overline{B}_5$ due to the condition (b), $\overline{B}_3 \prec \overline{B}_9$ due to the condition (e), $\overline{B}_5 \prec \overline{B}_7$ due to the condition (a), and finally $\overline{B}_9 \prec \overline{B}_5$ due to the condition (b). Similarly, we obtain the ordering for the rest of the blocks. Hence, $r = 1\ 3\ 9\ 5\ 7\ 8\ 6\ 4\ 10\ 2$.

In the general case, the ordering is deduced from a Hamiltonian path of the directed graphs with vertices the elements of the set I_1 (resp., I_2) and arcs the pairs $(p, q) \in I_1^2$ (resp., $(p, q) \in I_2^2$) such that $\overline{B}_p \prec \overline{B}_q$.

We note that $\overline{B}_p \prec \overline{B}_q$ iff $r^{-1}(p) < r^{-1}(q)$, given that $r^{-1}(k)$ defines the position of the block $\overline{B}_k$ at the total order of $\overline{B}(u)$. For our example, we have that $r^{-1} = 1\ 10\ 2\ 8\ 4\ 7\ 5\ 6\ 2\ 9$.

Practically, the whole procedure of finding the compression can be presented in a table, where the second row refers to the flow t, which includes all the elements necessary for the conditions (a)–(e), while from their application we deduce in the third row the total order of the set $\overline{B}(u)$. The last row refers to the flow $\overline{t}$ by assigning the numbers of the set $[2\overline{n}]$ to their corresponding blocks of $\overline{B}(u)$, according to the following remarks for the blocks $\overline{B}_k = \overline{b}_{k1} \overline{b}_{k2}$.

(i) Their elements have the same ordering (ascending or descending) with those of B_k.

(ii) When their elements are not consecutive, then $|\overline{b}_{k1} - \overline{b}_{k2}| = |\tau(k) - \tau(k+1)|$.

For example, for the meander of Figure 1, we have Figure 5. Hence, $\overline{\mu} = 1\ 18\ 17\ 2\ 3\ 16\ 15\ 6\ 11\ 10\ 7\ 8\ 9\ 12\ 5\ 4\ 13\ 14$, with $u = 1 + 2 + 2 + 1 + 2 = 8$.

6. Conclusions

We have introduced the compression of a meander, directly with the use of blocks of its permutation. The uniqueness of the compression is established by the ordering of the blocks, which is deduced according to its flow.

Various open questions can arise by the above meanders, when they are used as representatives of large classes of meanders as shown in Table 1. Yet, the main open problem is the reverse procedure of compression. The decompression of a meander to others of larger order having the same traces, number of cuttings, and flow seems to be the final step for integrating the procedures of cutting and compressing meanders, and in parallel being very promising for enumeration results and applications in physical phenomena.

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