

Research Article

Control of Chaos in Rate-Dependent Friction-Induced Vibration Using Adaptive Sliding Mode Control and Impulse Damper

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Two different control methods, namely, adaptive sliding mode control and impulse damper, are used to control the chaotic vibration of a block on a belt system due to the rate-dependent friction. In the first method, using the sliding mode control technique and based on the Lyapunov stability theory, a sliding surface is determined, and an adaptive control law is established which stabilizes the chaotic response of the system. In the second control method, the vibration of this system is controlled by an impulse damper. In this method, an impulsive force is applied to the system by expanding and contracting the PZT stack according to efficient control law. Numerical simulations demonstrate the effectiveness of both methods in controlling the chaotic vibration of the system. It is shown that the settling time of the controlled system using impulse damper is less than that one controlled by adaptive sliding mode control; however, it needs more control effort.

1. Introduction

There exist a lot of works on the theory of friction-driven oscillations in the literature, for example, influence of the belt speed on the system response [1], dynamics of three-block mechanical system with dry friction [2], investigation on the geometry of chaotic attractors for dry friction oscillators [3, 4], influence of parametric and external excitations on a dry friction oscillator dynamics [5], and the dynamic behavior of friction-driven oscillator with an impact damper [6].

The characteristics of the friction force between two surfaces are quite complex and depend on many parameters such as, normal pressure, slip velocity, surface, and material properties [7]. The friction-actuated oscillation is strongly nonlinear, and discontinuous and has nonsmooth process, which is a source of instabilities generating stick-slip, chatter, squeal, and chaos [8, 9]. LuGre friction law is one of the most widely used friction laws which models rate dependency of the friction force by one additional inner variable [10]. In [11], LuGre friction model is applied to the single degree of freedom friction-induced oscillator, and it is shown that the oscillations of this system turned out to be chaotic for most parameter combinations. Many active control methods have been presented for control of chaotic systems such as nonlinear feedback control [12], drive-response synchronization method [13], adaptive control method [14, 15], variable structure (or sliding mode) control method [16–19], back stepping control method [20, 21], fractional control [22], impulsive control [23], and adaptive sliding mode control [24].

Chatterjee presented a novel method for controlling the vibration, called impulse damper [25]. In this method, mass loaded PZT stack is attached to the system, and applying suitable voltage to the PZT stack leads to impulsive force which can control the system by setting control parameters correctly. This technique was used to control a nonlinear friction-driven oscillator [26] and to control the chaotic vibration of friction-driven oscillator with coulomb friction model [27, 28].

In the present study, two different control methods, namely, impulse damper and adaptive sliding mode control, are used to control the chaotic vibration of a single degree of freedom system subjected to the rate-dependent friction. To control the chaotic vibration of this system using adaptive sliding mode control, an adaptive control law is established which stabilizes the chaotic response of the system. Since chaotic systems are sensitive to the initial conditions, applying appropriate impulses at suitable positions causes the system to maintain its path within a specified bound. This was our motivation to investigate the effectiveness of impulse damper to suppress the chaotic behavior of the system. The effectiveness of both methods will be investigated in following sections.

2. Mathematical Model

In this section, the mathematical modeling of uncontrolled and controlled system, using adaptive sliding mode control and impulse damper is presented.

2.1. Friction Mathematical Model. Experimental investigations, especially at low sliding speeds, have shown that friction laws which give the friction force as an algebraic function of the underlying parameters (i.e., static friction laws) do not capture all of the observable frictional effects. Among the additional effects is the presliding displacement due to lateral contact elasticity, the increase of static friction with time due to diffusion processes on the interface, and frictional lag in sliding, which stands for the effect of the friction force lagging in time behind changes in relative velocity or normal load. To model the effects, the so-called dynamic friction models have been proposed. The following investigation is based on one of the most widespread models, the LuGre model [10], which allows a comparatively simple representation of the dynamic effects. Based on this model, the friction force is given by

$$F = \frac{N}{N_0} \left(\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r \right). \tag{1}$$

Governing equation of z is

$$\dot{z} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z, \qquad (2)$$

where z represents the average deflection of the bristles, v_r is the relative sliding velocity, N is the normal load with a reference value N_0 , and $g(v_r)$ is the friction force which is formulated as a function of the constant relative sliding velocity v_r . Moreover, σ_0 , σ_1 , and σ_2 parameterize the presliding displacement, an internal viscous frictional damping, and a viscous damping contribution due to the relative velocity [11], respectively. For steady sliding, σ_2 vanishes and the friction force reduces to

$$F = \frac{N}{N_0} g\left(v_r\right) \operatorname{sign}\left(v_r\right). \tag{3}$$

Function g(x) is given as follows:

$$g(\dot{x}) = F_c + \left(F_s - F_c\right) \exp\left[-\left(\frac{\dot{x}}{v_s}\right)^2\right],\tag{4}$$

where v_s is the speed of belt. In this research, sliding block on a rigid belt moving with a constant velocity as shown in Figure 1 is considered.



FIGURE 1: The structure of the block on a rigid belt.

Using LuGre friction, the equations of motion are

$$m\ddot{x} + c\dot{x} + kx = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \left(v_s - \dot{x} \right),$$

$$\dot{z} = v_s - \dot{x} - \frac{\sigma_0 \left| v_s - \dot{x} \right|}{q \left(v_s - \dot{x} \right)} z,$$
(5)

where the dot denotes differentiation with respect to *t*. Introducing the new parameter $\tau = \omega_n t$, where $\omega_n = \sqrt{k/m}$, the following relations will be concluded.

$$x'' + \frac{c}{\sqrt{km}}x' + x = \frac{\sigma_0}{k}z + \frac{\sigma_1}{\sqrt{km}}z' + \frac{\sigma_2}{k}\left(v_s - \omega_n x'\right), \quad (6)$$
$$z' = \frac{v_s}{\omega_n} - x' - \frac{1}{\omega_n}\frac{\sigma_0\left|v_s - \omega_n x'\right|}{g\left(v_s - \omega_n x'\right)}z. \quad (7)$$

It should be noted that the dimension of (6) and (7) is length. In these equations, the prime denotes differentiation with respect to τ , and the values of parameters are set as follows.

 $m = 1 \text{ kg}, k = 200 \text{ N/m}, \sigma_0 = 300 \text{ N/m}, \sigma_1 = 100 \text{ N/m}, F_c = 2 \text{ N}, F_s = 8 \text{ N}, v_s = 0.5 \text{ m/s}, D = 0.01.$

Here, a relative damping measure $D = (c + \sigma_2)/2\sqrt{km}$ has been introduced to combine the effects of linear damping from both structure and friction.

It is shown that for these values of parameters, the system has chaotic vibration [11].

2.2. Adaptive Sliding Mode Controller (ASMC) Design. In this section, the adaptive sliding mode control is used to control the chaotic vibration. The controlled chaotic system can be rewritten as follows:

$$x'' + \frac{c}{\sqrt{km}}x' + x = \frac{\sigma_0}{k}z + \frac{\sigma_1}{\sqrt{km}}z' + \frac{\sigma_2}{k}\left(v - \omega_n x'\right),$$

$$z' = \frac{v}{\omega_n} - x' - \frac{1}{\omega_n}\frac{\sigma_0\left|v - \omega_n x'\right|}{g\left(v - \omega_n x'\right)}z.$$
(8)

It is desired that *x* be adjusted to equilibrium state x_0 , where x_0 is a given constant. The error states are defined as

$$e_1 = x - x_0,$$

 $e_2 = e'_1 = x'.$
(9)

To guarantee the stability of the sliding mode, a switching surface s(t) in the error space is defined as follows:

$$s(t) = e_1(t) + e_2(t).$$
 (10)

As well known, when the system operates in the sliding mode, it satisfies the following [29]:

$$s(t) = s'(t) = 0.$$
 (11)

Therefore, the following sliding mode dynamics can be obtained as

$$e'_{1} = e_{2} = -e_{1},$$

$$e'_{2} = -\frac{c}{\sqrt{km}}e_{2} - (e_{1} + x_{0}) + \frac{\sigma_{0}}{k}z$$

$$+\frac{\sigma_{1}}{\sqrt{km}}z' + \frac{\sigma_{2}}{k}(v - \omega_{n}e_{2}) + u.$$
(12)

From (8), it can be seen that the internal parameter z will converge and be stabilized when $(e_1, e_2) \rightarrow (0, 0)$.

The following function is defined:

$$f(e_{1}, e_{2}, z, z') = -\frac{c}{\sqrt{km}}e_{2} - (e_{1} + x_{0}) + \frac{\sigma_{0}}{k}z + \frac{\sigma_{1}}{\sqrt{km}}z' + \frac{\sigma_{2}}{k}(v - \omega_{n}e_{2}).$$
(13)

Using (13), (12) can be rewritten as

$$e'_{2} = f(e_{1}, e_{2}, z, z') + u.$$
 (14)

The controller is designed as

$$u = -\gamma \eta \operatorname{sign} \left(s\left(t \right) \right), \tag{15}$$

where $\eta = |f(e_1, e_2, z, z')| + |e_2|$ and the adaptive law for parameter γ is proposed as

$$\gamma' = -\gamma - \xi, \quad \xi > 1. \tag{16}$$

The proposed adaptive control scheme will guarantee the global asymptotic stability of the error and is shown in the following.

2.3. Stability Analysis. In the following, we analyze the stability of the sliding mode dynamics based on the Lyapunov stability theory. The Lyapunov function is selected as $V(t) = (1/2)(s^2 + \gamma^2)$, which leads to

$$V' = ss' + \gamma\gamma' = s(e'_{1} + e'_{2}) + \gamma\gamma'$$

= $s(e_{2} + f(e_{1}, e_{2}, z, z') + u) + \gamma\gamma'$ (17)
 $\leq |s|(|e_{2}| + |f(e_{1}, e_{2}, z, z')|) + su + \gamma\gamma'.$

Solving (16) leads to $\gamma = e^{-\tau} + \xi$. Substituting (16) in (17), the following is obtained:

$$V' \le |s| \eta \left(1 - e^{-\tau} - \xi \right) + \left(-e^{-2\tau} - \xi e^{-\tau} \right) < 0.$$
 (18)

Now, if we define $w(x) = |s|\eta(1 - e^{-x} - \xi) + (-e^{-2x} - \xi e^{-x})$ and integrating the previous equation from zero to *t*, it yields

$$V(\tau) \leq V(0) - \int_{0}^{\tau} w(\lambda) d\lambda$$
$$\implies V(0) \geq V(\tau) + \int_{0}^{\tau} w(\lambda) d\lambda \qquad (19)$$
$$\int_{0}^{\tau} w(\lambda) d\lambda \leq V(0) < \infty.$$

Taking the limit as $\tau \rightarrow \infty$ on both sides of (19) yields

$$\lim_{\tau \to \infty} \int_0^\tau w(\lambda) \, d\lambda \le V(0) < \infty.$$
 (20)

Now, Barbalat's lemma is applied as shown in the following.

Lemma 1 (Barbalat's lemma [30]). If $w : R \to R$ is a uniformly continuous function for $t \ge 0$ and if $\lim_{t\to\infty} \int_0^t |w(t)| dt \le V(0) < \infty$ exists and is finite, then $\lim_{t\to\infty} w(t) \to 0$.

Thus, according to Barbalat's lemma, it is obtained that

$$\lim_{\tau \to \infty} w(t) = \lim_{\tau \to \infty} \left(|s| \eta \left(1 - e^{-\tau} - \xi \right) + \left(-e^{-2\tau} - \xi e^{-\tau} \right) \right) \longrightarrow 0.$$
(21)

Since $\lim_{t\to\infty} e^{-\tau} = 0$, $\eta > 0$, and $\xi > 1$, it is concluded that $s(t) \to 0$ as $\tau \to \infty$.

Thus, the system is in the sliding mode, and using (12), stability of e_1 and e_2 is surely guaranteed. Therefore, both $e_1(\tau)$ and $e_2(\tau)$ converge to zero.

2.4. Mathematical Model of Impulse Damper. In this section, for the purpose of controlling and suppressing vibration of the system, an impulse damper is used (Figure 2). The basic principle of the impulse damper is to generate suitable impulsive force by quickly expanding or contracting the PZT stack actuator. The stack is expanded by applying a suitable positive voltage at the instants of the displacement zero crossing with a positive velocity. In addition, a negative voltage is applied to contract the stack when the zero crossing takes place with a negative velocity. In fact, damping is generated by velocity feedback.

Applied force by impulse damper can be written as $F_a + C_p X_p$, where X_p is the elongation of PZT stack and F_a is the force produced by the actuator. F_a can be found from the electromechanical characteristics of the PZT actuator which is given by the following:

$$\begin{cases} Q_p \\ X_p \end{cases} = \begin{bmatrix} C_{\text{PZT}} & -nd_{33} \\ nd_{33} & \frac{-1}{K_p} \end{bmatrix} \begin{cases} V_p \\ F_a \end{cases} ,$$
 (22)

where Q_p , X_p , V_p , F_a , and C_{PZT} represent charge, elongation, voltage, force, and the effective capacitance of the PZT actuator, respectively. d_{33} is the piezoelectric constant of each



FIGURE 2: PZT stack attached to the system as controller.

wafer, and *n* is the total number of wafers in the actuator. k_p^{-1} is the compliance of the actuator in short circuit condition and is given by $K_p = EA/l$, where *E* is the elastic modulus, *A* is the cross-sectional area, and *l* is the length of the actuator. From (22), the following expression is obtained for the actuator force:

$$F_a = nd_{33}K_pV_p - K_pX_p.$$
 (23)

Employing the previous equation and adding it to the right side of (6) as an actuation force which is applied to the system by the impulse damper, one can conclude that

$$x'' + \frac{c}{\sqrt{km}}x' + x = \frac{\sigma_0}{k}z + \frac{\sigma_1}{\sqrt{km}}z' + \frac{\sigma_2}{k}\left(v - \omega_n x'\right) + \alpha y + h_e y' - \frac{\lambda^2}{1 - \lambda^2}\alpha V.$$
(24)

The equation of motion of the added mass can be written as [25]

$$r_m y'' + h_e y' + \alpha y = -\frac{\lambda^2}{1 - \lambda^2} \alpha V - r_m x'',$$
 (25)

where $x = x_1/x_0$, $y = x_p/x_0$, $\omega_n = \sqrt{k/m}$, $\xi = c/2\sqrt{km}$, $r_m = m_a/m$, $h_e = h_p + (\lambda^2/(1 - \lambda^2))\alpha\gamma$, $h_p = c_p/m\omega_n$, $V = V_p/V_{ref}$, $V_{ref} = \lambda^2/(1 - \lambda^2)nd_{33}$, $\alpha = K_p/K$, and $\lambda^2 = n^2 d_{33}^2 K_p/C_{PZT}$.

Depending upon the problem, the reference displacement x_0 can be suitably defined. The following values are chosen for the previous mentioned parameters in this study: $r_m = 0.1$, $\lambda = 0.7$, $h_p = 25$, $V_m = 4$, $\varepsilon = 0.003$, and $\alpha = 100$.

The proposed control law is mathematically recast as [25]

$$V = \begin{cases} V_m \operatorname{sgn} (X) & |X| \ge \frac{\varepsilon}{2}, \\ \frac{2V_m}{\varepsilon} X & |X| \le \frac{\varepsilon}{2}. \end{cases}$$
(26)

3. Results and Discussion

Equations of this system are solved numerically. For the numerical analysis, Runge-Kutta's integration procedure is employed where the initial condition of system is set as $(x(0), \dot{x}(0), z(0)) = (0, 0, 0)$. Figure 3 shows the time history of the mass block displacement, and Figure 4 shows the phase



FIGURE 3: Block displacement of the uncontrolled system.



FIGURE 4: Phase plane of the uncontrolled system.

plane of the uncontrolled system. Chaotic dynamics of the system can be clearly seen in these figures.

In the following, the numerical results are given to confirm the validity of the proposed adaptive sliding mode method. In the numerical simulations, ξ is selected as $\xi = 1.5$, and the desired response is set at $x_0 = 0.0383$ (equilibrium point). Synchronization error of the state variables is depicted in Figure 5.

Phase plane of the controlled system has been shown in Figure 6. As can be seen in this figure, the controlled system is not chaotic.

The time response of the switching function $s(\tau)$ and the control input $u(\tau)$ are shown in Figures 7 and 8, respectively.

For the purpose of indicating the efficacy of the proposed impulse damper controller, we have examined the system in two different situations. In the first situation, the controller is turned on when the system is at origin, and accordingly, the mass moves towards its equilibrium position at x = 0.0383. In the second situation, controller is off for $\tau < 0.0383$



FIGURE 5: Time response of error states using adaptive sliding mode method.



FIGURE 6: Phase plane of the controlled system.

100; therefore, the mass is vibrating chaotically around its equilibrium position at x = 0.0383. At this moment, the controller is turned on and the effect of the control input on the behavior of the system is evaluated. After that, to show the usefulness of the controller, the controller is turned off, and there is no suppressing signal to damp system vibrations. Figure 9 shows response of the system controlled by the impulse damper when the controller is turned on at $\tau = 0$. Figure 10 shows applied force by the impulse damper when the controller is 11 and 12 illustrate the response and control input for the system controlled by the impulse damper when the controller is turned on at $\tau = 100$ and is turned off at $\tau = 200$.



FIGURE 7: Time response of the switching function using adaptive sliding mode method.

In Figure 9, when the controller is turned on at $\tau = 0$, response of the system is absolutely fast. The mass reaches its equilibrium position in a very short time. Although previously mentioned quick response and short settling time demonstrate high performance of the controller, it is achieved at the expense of a high actuating force (shown in Figure 10) which is applied by the impulse damper. The achieved result is rational since the mass is at origin and, thus, is far from its equilibrium position. Therefore, impulse damper controller applies big impulses to the mass for the purpose of moving it towards the equilibrium position. In contrast, when the controller is off for $\tau < 100$ and thus the chaotic trajectory enters a small neighborhood of the fixed point x = 0.0383 (m)



FIGURE 8: Time response of the input control using adaptive sliding mode method.



FIGURE 9: Block displacement (controller is turned on at t = 0).



FIGURE 10: Applied force (controller is turned on at t = 0).



FIGURE 11: Block displacement (controller is turned on at t = 100 and is turned off at t = 200).



FIGURE 12: Applied force (controller is turned on at t = 100 and is turned off at t = 200).

(Figure 10), there is no need for such a high force as in the previous case and impulses with a short amplitude as shown in Figure 12 can control the system effectively. In this case, the distance between the current position of the mass and its equilibrium position is not as far as the first case. In other words, the farther (close) the system is to the goal dynamics, the bigger (small) the weight given to the control effort is. Feasibility of using impulse damper for controlling the system is obvious in these figures. As seen in these figures, impulse damper can damp the vibration of the system absolutely in a short time.

4. Conclusion

In this paper, chaotic vibration control of a single degree of freedom oscillator subjected to a LuGre type friction law was

Journal of Chaos

presented. Two different methods were used to control the chaos, ASMC, and impulse damper.

For adaptive control of the chaotic vibration of this system, a switching surface was adopted such that it becomes easy to ensure the stability of the error dynamics in the sliding mode. Then, an ASMC was derived to guarantee the occurrence of the sliding motion. The adaptive laws were derived in the Lyapunov sense to guarantee the stability of the controlled system. Furthermore, impulse damper was also used for controlling the system. The idea is that, since chaotic systems are exponentially sensitive to perturbations, careful choice of even small control perturbations can, after some time, have a large effect on the trajectory location and can be used to guide it. In this method, controlled impulses are generated by expanding and contracting a mass loaded PZT actuator used between the primary system and the secondary mass. Numerical results verified the effectiveness of the proposed methods in controlling the chaotic vibration of the rate-dependent friction-driven oscillator. It was shown that impulse damper decreases the settling time of the controlled system by applying higher control forces in comparison with ASMC.

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