

Research Article

Second Law Analysis of a Gas-Liquid Absorption Film

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This paper reports an analytical study of the second law in the case of gas absorption into a laminar falling viscous incompressible liquid film. Velocity, temperature, and concentration profiles are determined and used for the entropy generation calculation. Irreversibilities due to heat transfer, fluid friction, and coupling effects between heat and mass transfer are derived. The obtained results show that entropy generation is mainly due to coupling effects between heat and mass transfer near the gas-liquid interface. Total irreversibility is minimum at the diffusion film thickness. On approaching the liquid film thickness, entropy generation is mainly due to viscous irreversibility.

1. Introduction

Gas-liquid (and vapor-liquid) flows occur in a wide variety of situations such as: power generation, chemical processing, and energy production facilities. Some practical applications include condensers and reboilers, gas-liquid reactors (used in the production of pharmaceuticals and specialty chemicals), wetted wall absorbers, power systems, core cooling of nuclear power plant, and falling film reactors. In addition to normal gravity applications, gas-liquid (and vapor-liquid) flows also occur in many planned space operations such as: power cycles, active thermal control systems [1], propulsion devices, and storage and transfer of cryogenic fluids.

Gas-liquid flows can be grouped into a number of different flow patterns. These flow patterns are based on the spatial and temporal distribution of the gas and liquid phases. Gravity is the major reason for such two-phase flows to exhibit a wide range of flow patterns with different characteristics.

Gas-liquid absorption is one of the most important separation processes, the gas-liquid contactors in which absorption takes place are typically spray, trayed, or packed systems involving falling films. Falling film flow has found wide applications in modern equipments, because it is widely used in modern process industries such as vertical condensers, film evaporators, distillation columns, and packed and absorption towers.

Various works are elaborated in order to study the coupled heat and mass transfer processes in gas or vapor absorption into a liquid film. The theoretical analysis of combined heat and mass transfer processes in the case of gas or vapor absorption into a laminar and/or turbulent liquid film as well as the temperature evolution and the concentration profiles in the film are developed [2–5]. Conlisk and Mao [6] considered the condensation of a two-component vapor mixture into a two-component liquid film on a falling film absorber in an absorption heat pump. Analytical expressions for temperature and mass fraction in the falling film have been determined. It is important in this situation to recall the experimental works of Miller and Keyhani [7], in which they presented experimental data of the local heat and mass transfer processes in aqueous LiBr absorption cycle. They studied heat and mass transfer processes and presented measurements of the temperature profile and the bulk concentration along the absorber length of the used test tube. Islam et al. [8] developed a linearized coupled model for heat and mass transfer in falling film absorbers. In order to test its accuracy, the developed model was compared with a nonlinear model and a numerical simulation. The used model provides analytical expressions of heat and mass transfer coefficients from the experimental data concerning vertical and horizontal tube absorbers. Killion and Garimella [9] reported an interesting review about the main used models to

illustrate coupled heat and mass transfer phenomena during falling film absorption. Detailed expressions of the governing equations, the boundary conditions, and the methods of resolution used for modeling combined heat and mass transfer problems have been developed. Other extensive researches have been performed on gas absorption in liquid films with different flow regimes, geometries, and boundary conditions [10–12]. In many real cases, the heat effect is small and the absorption process may be considered isothermal [11, 13].

The improvement of industrial systems as well as the energy utilization in any fluid is one of the fundamental problems of the engineering processes, since improved systems will provide better material processing, energy conservation, and environmental effects. One of the used methods for predicting the performance of the engineering processes is the second law analysis. The second law of thermodynamics is applied to determine the irreversibilities in terms of the entropy generation. Entropy generation is the measure of the destruction of the available work of the system. In the present work, entropy generation analysis concerns gas absorption through a laminar falling liquid film. In the following paragraph, some researches about entropy generation analysis through a falling film will be presented.

The study of laminar falling viscous incompressible liquid film along an inclined porous heated plate has many significant applications in thermal engineering and industries, starting from petroleum drilling equipments to various industrial exchanger systems [16]. Adeyinka and Naterer [17] investigated the physical significance of entropy generation, and a resulting optimization correlation for a laminar film condensation on a flat plate was developed. Galović [18] studied entropy generation in evaporator/condenser types of heat exchangers. A model was developed to calculate the overall entropy generation of heat exchange for given values of heat transfer effectiveness and ratios of flow rates. Boulama et al. [19] derived an analytical expression of entropy generation rate in a fully developed, laminar, and time-independent simultaneous thermal and solute mixed convection for the case of a nonreacting binary gas mixture flowing upwards in a vertical parallel plate. Carrington and Sun [20] developed different expressions of entropy generation due to the coupling effects between heat and mass transfer in a multicomponent fluid. They identified restrictions on the applicability of expressions for entropy generation due to mass transfer and heat transfer. They used an ideal binary fluid to illustrate the analysis in the case of small pressure gradient in the diffusion flux direction. Magherbi et al. [21] reported a numerical study of entropy generation due to heat transfer, mass transfer, and fluid friction in steady state for laminar double diffusive convection in an inclined enclosure with heat and mass diffusive walls. They studied the influences of the inclination angle of the enclosure, the thermal Grashof number, and the buoyancy ratio on total entropy generation due to heat transfer, mass transfer, and fluid friction irreversibilities. Saouli and Aïboud-Saouli [22] investigated second law analysis of a laminar falling liquid film along an inclined heated plate. They considered the upper surface of the liquid film free and adiabatic and the lower wall fixed and impermeable. Results show

that entropy generation number transversely decreases and increases for all values of group parameters. They found that the irreversibility ratio decreases in the transverse direction and increases as the group parameter increases. Makinde and Osalusi [23] analytically investigated entropy generation of a laminar falling viscous incompressible liquid film along an inclined porous heated plate. Velocity and temperature profiles are obtained and used to calculate entropy generation number. They found that, near the inclined porous heated plate, heat transfer irreversibility dominates, while near the liquid-free surface, viscous dissipation irreversibility dominates. Gorla and Pratt [24] analytically studied second law of a non-Newtonian laminar falling liquid film along an inclined heated plate, where the upper surface of the liquid film is considered free and adiabatic. Velocity and temperature profiles are calculated and used to compute the entropy generation number (N_g), the irreversibility ratio (ϕ), and the Bejan number (Be), for different values of the viscous dissipation parameter ($Br\Omega^{-1}$), the viscosity index (n), and the dimensionless axial distance (X). Results show that heat transfer dominates the viscous irreversibility when $0 \leq \phi < 1$, and the fluid friction irreversibility dominates heat transfer irreversibility when $\phi > 1$. For $\phi = 1$, both heat transfer and fluid friction irreversibilities have equal contribution for entropy generation. Recently, Chermiti et al. [25] analytically investigated entropy generation of an isothermal absorption of carbon dioxide into a laminar viscous incompressible falling liquid film (H_2O). Entropy generation expression is developed, and the velocity profile as well as the absorbed gas concentration is determined and used for the calculation of the diffusive and the convective mass transfer fluxes, as well as the entropy generation. It was found that, from the gas-liquid interface towards the diffusion film thickness δ_a , entropy generation increases towards a maximum value due to diffusive mass transfer and then decreases towards a minimum value by convective mode. This minimum is obtained at the diffusion film thickness. For values of liquid film thickness greater than the diffusion film thickness (i.e., $x > \delta_a$), entropy generation linearly increases due to the fluid friction irreversibility. Near the diffusion film thickness, mass transfer irreversibility dominates. On approaching the liquid film thickness δ , viscous irreversibility dominates. The increase of the falling film length induces the decrease of entropy generation magnitude. Aïboud-Saouli et al. [26] presented the application of the second law to gravity-driven liquid film along an inclined heated plate in the presence of a transverse magnetic field and viscous dissipation effects. It was found that entropy generation increases with Hartman number, Brinkman number, and the dimensionless group. Irreversibility due to the magnetic field, the conduction heat in the transverse direction, and the fluid friction increases by the increase of Hartman number, Brinkman number, and the dimensionless group, respectively.

The main originality of the present work is the second law analysis for the case of gas absorption through a vertical laminar falling viscous incompressible liquid film, by the determination of different sources of irreversibility. The study is based on the analytical flow field solutions obtained for

the velocity, the temperature, and the concentration profiles. The velocity distribution was determined by solving the momentum equation subject to an appropriate set of boundary conditions. The concentration profile was determined by solving the species conservation equation with appropriate boundary conditions, and the temperature distribution was obtained by using the energy conservation equation. In many previous works, authors are interested to determine the concentration and the temperature profiles only. In the case of vapor absorption [2, 3], a linear relation between the temperature and the concentration in equilibrium with vapor at constant pressure is adopted and makes the resolution of the system equations. In this study, velocity, temperature, and concentration fields are analytically determined and directly used to evaluate the entropy generation.

In the presence of simultaneous heat and mass transfer, four sources of irreversibility have been identified, which are due to coupling effects between heat and mass transfer by diffusion, due to heat transfer, due to coupling effects between heat and mass transfer by convection and fluid friction.

2. Mathematical Formulation

2.1. Physical Considerations. Generally, the fluid density depends on pressure and temperature. Thus, the presence of temperature variation or chemical species (thermal and concentration gradients) may change the fluid density which can result in thermal or solute natural convection. The system under consideration is illustrated in Figure 1. It consists of a laminar falling viscous incompressible liquid film (designated by the letter B), flowing down on a vertical plate under gravity. The gas, designated by the letter A, is absorbed without chemical reactions. In the present study, the assumptions employed in the problem formulation are

- (i) physical absorption (no chemical reactions),
- (ii) two dimensional flow in steady state regime,
- (iii) falling laminar flow of a Newtonian incompressible viscous liquid film (water),
- (iv) ammonia as absorbed gas,
- (v) the diffusion film thickness is too small as compared to the falling film thickness (i.e., $\delta_d \ll \delta$), so that the thermophysical properties of the falling film are similar to those of the liquid,
- (vi) the Boussinesq approximation is valid for assuming a linear variation of the fluid density with temperature and concentration,
- (vii) the curvature of the liquid film is neglected, and the velocity only depends on the transverse direction (i.e., $v = v_z(x)$),
- (ix) the absorption process takes place at constant pressure (i.e., $(\partial P/\partial z) \approx 0$),
- (x) for the falling film (a thin film), the axial conduction terms are neglected as compared to the transverse terms (i.e., $(\partial^2 T/\partial z^2) \ll (\partial^2 T/\partial x^2)$). Further, for the thin film, small values of temperature difference ΔT are considered.

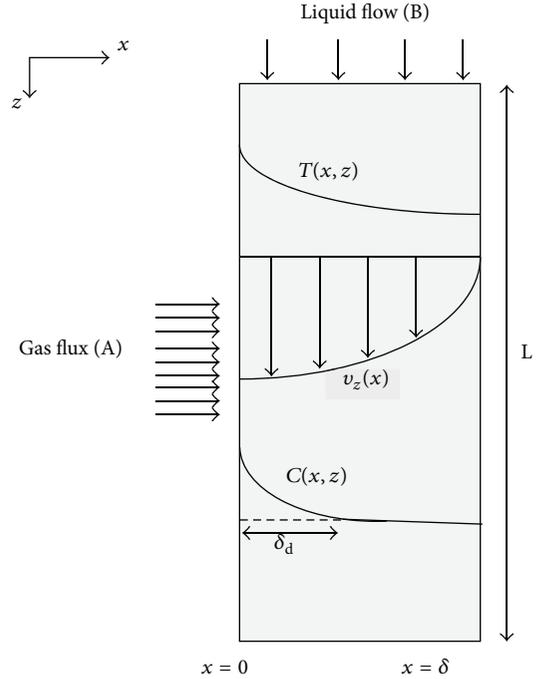


FIGURE 1: Description of gas absorption into a laminar falling liquid film: typical profiles of temperature, velocity, and concentration.

2.2. Velocity Profile. Under the Boussinesq approximation and for a small density variation, the fluid density linearly varies with temperature and species concentration such that

$$\rho = \rho_{\text{ref}} [1 - \beta_T (T - T_{\text{ref}}) - \beta_C (C - C_{\text{ref}})]. \quad (1)$$

ρ_{ref} is the reference density measured at reference temperature (T_{ref}) and concentration (C_{ref}).

Thermal and solute expansion coefficients β_T and β_C are, respectively, given by

$$\beta_T = -\frac{1}{\rho_{\text{ref}}} \left(\frac{\partial \rho}{\partial T} \right), \quad \beta_C = -\frac{1}{\rho_{\text{ref}}} \left(\frac{\partial \rho}{\partial C} \right). \quad (2)$$

The continuity and momentum equations are given by

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} &= 0, \\ \rho_{\text{ref}} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial z} \right) &= -\frac{\partial P}{\partial x} + \eta_L \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right), \\ \rho_{\text{ref}} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \eta_L \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right) + f. \end{aligned} \quad (3)$$

f describes the buoyancy forces due to thermal and concentration gradients. It is given by

$$f = g \int_{\rho_0}^{\rho_\delta} d\rho = g(\rho_\delta - \rho_0). \quad (4)$$

ρ_δ and ρ_0 are the fluid densities measured at the falling film thickness ($x = \delta$) and at the gas-liquid interface ($x = 0$), respectively. Since the flow takes place along a vertical plate, the velocity only depends on the diffusion film thickness (i.e., in transverse direction). That is, the transverse velocity $u \approx 0$ and $v = v_z(x)$ [27]. Further, the absorption process takes place at constant pressure. After rearrangement, the continuity and the momentum equations are, respectively, given by

$$\frac{\partial v}{\partial z} = 0, \quad (5)$$

$$\frac{\eta_L}{\rho_{\text{ref}}} \left(\frac{\partial^2 v}{\partial x^2} \right) + \frac{f}{\rho_{\text{ref}}} = 0. \quad (6)$$

By using (1) and (4), the buoyancy forces are given by

$$f = g\rho_{\text{ref}} [(\beta_T(T_0 - T_\delta) + \beta_C(C_0 - C_\delta))]. \quad (7)$$

T_0 and T_δ are the gas-liquid interface and the falling film thickness temperatures, respectively. Thus, (6) can be written as follows:

$$\eta_L \frac{\partial^2 v}{\partial x^2} = g\rho_{\text{ref}} [\beta_T(T_\delta - T_0) + \beta_C(C_\delta - C_0)]. \quad (8)$$

This differential equation is the liquid motion equation. The parabolic profile of the velocity enables us to give the appropriate boundary conditions as follows:

$$\begin{aligned} \text{at } x = 0, \quad v &= v_{\text{max}}, \\ \text{at } x = \delta, \quad v &= 0. \end{aligned} \quad (9)$$

The reference density is supposed to be equal to that of the liquid (i.e., $\rho_{\text{ref}} = \rho_L$). Thus, the solution of (8) is given by

$$\begin{aligned} v(x) &= v_{\text{max}} \left(1 - \left(\frac{x}{\delta} \right)^2 \right) \\ &= \frac{g\rho_L}{2\eta_L} \delta^2 [\beta_T(T_0 - T_\delta) + \beta_C(C_0 - C_\delta)] \left(1 - \left(\frac{x}{\delta} \right)^2 \right). \end{aligned} \quad (10)$$

For small density differences, thermal and solute expansion coefficients are given by [28, 29]

$$\beta_T = -\frac{1}{\rho_L} \left(\frac{\partial \rho}{\partial T} \right) \approx \frac{1}{\Delta T}, \quad \beta_C = -\frac{1}{\rho_L} \left(\frac{\partial \rho}{\partial C} \right) \approx \frac{1}{\Delta C}, \quad (11)$$

where

$$\Delta T = T_0 - T_\delta, \quad \Delta C = C_0 - C_\delta. \quad (12)$$

The maximum velocity is, therefore, given by

$$v_{\text{max}} = \frac{g\rho_L}{\eta_L} \delta^2. \quad (13)$$

2.3. Concentration Profile. For the gas-liquid absorption system under consideration, the diffusing species is the absorbed gas, which is designated by the letter A (see Figure 1). Thus, the species conservation equation is given by

$$u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial z} - D_A \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial z^2} \right) = 0. \quad (14)$$

As the Reynolds number of the system is smaller than 25 (laminar flow) [29], the above equation is, therefore, given by

$$v_{\text{max}} \left(1 - \left(\frac{x}{\delta} \right)^2 \right) \frac{\partial C_A}{\partial z} - D_A \frac{\partial^2 C_A}{\partial x^2} = 0. \quad (15)$$

For a small penetration distance of the gas into the falling liquid film and a short contact time (physical absorption), the film is supposed to move with a velocity equal to v_{max} [30, 31]. Finally, the required solution for the gas concentration through the falling film can be obtained by using the method of combination of variables (see details in [25]):

$$C_A = C_{A0} \left(1 - \text{erf} \left(\frac{x}{\sqrt{4(D_A/v_{\text{max}})z}} \right) \right). \quad (16)$$

The transverse and axial molar fluxes J_{Ax} and J_{Az} of the gas are, respectively, given by

$$\begin{aligned} J_{Ax} &= -D_A \frac{\partial C_A}{\partial x}, \\ J_{Az} &= C_A v(x). \end{aligned} \quad (17)$$

2.4. Temperature Profile. Due to the strong dependence of the gas solubility on temperature, it is important to determine the temperature distribution through the falling liquid film. Indeed, the gas absorption heat is the most heat influencing the temperature profile and leading to a rise in the temperature of the liquid phase [32]. Energy conservation equation throughout the falling liquid film is given by

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial z} = \frac{k_L}{\rho_L C_{p,L}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right). \quad (18)$$

By taking into consideration the above-mentioned assumptions and by introducing the thermal diffusivity of the fluid given by $\alpha = k_L/(\rho_L \cdot C_{p,L})$, the energy conservation equation is simply given by

$$\alpha \frac{\partial^2 T}{\partial x^2} = v_{\text{max}} \left(1 - \left(\frac{x}{\delta} \right)^2 \right) \frac{\partial T}{\partial z}. \quad (19)$$

The appropriate boundary conditions associated to this equation are

$$z = 0, \quad T(x, 0) = T_0, \quad (20)$$

$$x = 0, \quad q^* = -k_L \frac{\partial T(0, z)}{\partial x}, \quad (21)$$

$$x = \delta, \quad \frac{\partial T(\delta, z)}{\partial x} = 0. \quad (22)$$

q^* is the interfacial heat flux given by the heat of absorption.

Before proceeding with the solution of the temperature equation (19), it is convenient to rewrite the equations in a dimensionless form. Let us define the following dimensionless variables:

$$\theta(x, z) = \frac{T(x, z) - T_0}{\Delta T}, \quad X = \frac{x}{\delta}, \quad Z = \frac{\alpha z}{v_{\max} \delta^2}. \quad (23)$$

ΔT is the temperature difference given by $\Delta T = (q^* \delta)/k_L$.

Equation (19) with the new dimensionless variables becomes

$$\frac{\partial^2 \theta}{\partial X^2} = (1 - X^2) \frac{\partial \theta}{\partial Z}. \quad (24)$$

The dimensionless boundary conditions are given by

$$Z = 0, \quad \theta(X, 0) = 0, \quad (25)$$

$$X = 0, \quad \frac{\partial \theta(0, Z)}{\partial X} = -1, \quad (26)$$

$$X = 1, \quad \frac{\partial \theta(1, Z)}{\partial X} = 0.$$

To solve the dimensionless temperature equation (24), the best approach is to use the separation of variables method [33], so that the dimensionless temperature is given by

$$\theta(X, Z) = \theta_1(X) + \theta_2(Z). \quad (27)$$

The solution of (27) is the sum of two functions $\theta_1(X)$ and $\theta_2(Z)$ that depend only on X and Z , respectively. Consequently, the separated solutions into the energy equation give

$$\frac{\partial^2 \theta_1(X)}{\partial X^2} = (1 - X^2) \frac{\partial \theta_2(Z)}{\partial Z}. \quad (28)$$

The two sides of the above equation are equal for any chosen values of X and Z , so both sides are equal to the same constant. In other words, the above equation yields a pair of separated differential equations given by

$$\begin{aligned} \frac{\partial^2 \theta_1(X)}{\partial X^2} &= \lambda_1 (1 - X^2), \\ \frac{\partial \theta_2(Z)}{\partial Z} &= \lambda_1. \end{aligned} \quad (29)$$

The general solution of the dimensionless temperature is, therefore, given by

$$\theta(X, Z) = -\frac{X^4}{12} \lambda_1 + \frac{X^2}{2} \lambda_1 + X \lambda_2 + \lambda_1 Z + \lambda_3. \quad (30)$$

λ_1 , λ_2 , and λ_3 are constants of integration.

In order to determine the third constant λ_3 , the mean bulk temperature should be introduced. It is given by

$$\theta_b(Z) = \int_0^1 \theta(X, Z) dX. \quad (31)$$

The first boundary condition given by (25) gives the mean bulk temperature $\theta_b(0) = 0$. By using this condition and the two other boundary conditions, the temperature profile is finally given by

$$\begin{aligned} T(x, z) = T_0 + \Delta T \left[-\frac{1}{8} \left(\frac{x}{\delta} \right)^4 + \frac{3}{4} \left(\frac{x}{\delta} \right)^2 \right. \\ \left. - \left(\frac{x}{\delta} \right) + \frac{3}{2} \left(\frac{\alpha z}{v_{\max} \delta^2} \right) + \frac{11}{40} \right]. \end{aligned} \quad (32)$$

3. Entropy Generation

The existence of thermal, velocity, and concentration gradients along the absorbing falling film yields a nonequilibrium state, which causes entropy generation. Under these conditions, the volumetric rate of entropy generation in the flow field with both heat and mass transfer and without body force and chemical reaction effects is given by [34]

$$\sigma_c = \vec{q} \cdot \overrightarrow{\text{grad}} \left(\frac{1}{T} \right) + \frac{\bar{\tau} : \overrightarrow{\text{grad}}(\vec{V})}{T} - \frac{1}{T} \sum_k \vec{J}_k \cdot \overrightarrow{\text{grad}}(\mu_k). \quad (33)$$

The first term of the right hand side of (33) is the entropy generation due to heat transfer, the second is due to fluid friction, and the third is due to mass transfer.

The energy flux \vec{q} is the sum of the heat flux \vec{q}' , given by the Fourier law, and the enthalpy flux due to species diffusion. That is

$$\vec{q} = \vec{q}' - \sum_k h_k \vec{J}_k = -k_L \cdot \overrightarrow{\text{grad}}(T) - \sum_k h_k \vec{J}_k. \quad (34)$$

Rearrangement of (33) gives

$$\begin{aligned} \sigma_c = -\frac{1}{T^2} \left[\vec{q}' \cdot \overrightarrow{\text{grad}}(T) \right] + \frac{1}{T^2} \left[\left(\sum_k h_k \vec{J}_k \right) \cdot \overrightarrow{\text{grad}}(T) \right] \\ + \frac{\bar{\tau} : \overrightarrow{\text{grad}}(\vec{V})}{T} - \frac{1}{T} \sum_k \vec{J}_k \cdot \overrightarrow{\text{grad}}(\mu_k). \end{aligned} \quad (35)$$

The first term of the right-hand side of (35) is the irreversibility due to thermal gradients, the second is due to coupling effects between thermal gradients and mass

diffusion, the third is due to fluid friction, and the fourth is due to mass diffusion.

The specific chemical potential of the gas (A) is given by

$$\begin{aligned} \mu_A(T, P_o) = & C_{p,A}(T - T_o) - TC_{p,A} \ln\left(\frac{T}{T_o}\right) \\ & + h_{A_o}(T_o, P_o) - Ts_{A_o}(T_o, P_o). \end{aligned} \quad (36)$$

P_o and T_o are the reference pressure and temperature, respectively. h_{A_o} and s_{A_o} are the enthalpy and entropy of the gas, at T_o and P_o .

The shear stress is simply given by [28]

$$\tau_{ij} = \eta_L \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \quad (37)$$

Thus, the local entropy generation rate in a two-dimensional flow with a single diffusing species can, therefore, be written as follows:

$$\begin{aligned} \sigma_c = & \frac{k_L}{T^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right] + \frac{h_A}{T^2} \left[J_{Ax} \cdot \frac{\partial T}{\partial x} + J_{Az} \cdot \frac{\partial T}{\partial z} \right] \\ & + \frac{\eta_L}{T} \left(\frac{\partial v}{\partial x} \right)^2 - \frac{1}{T} \left(J_{Ax} \cdot \frac{\partial \mu_A}{\partial x} + J_{Az} \cdot \frac{\partial \mu_A}{\partial z} \right). \end{aligned} \quad (38)$$

Expressions of specific enthalpy and entropy of the absorbed gas are, respectively, given by

$$h_A = C_{pA}(T - T_o) + h_{A_o}, \quad s_A = C_{pA} \ln\left(\frac{T}{T_o}\right) + s_{A_o}. \quad (39)$$

By replacing the fluxes J_{Ax} and J_{Az} and the specific enthalpy and entropy given above by their expressions and by making the derivatives of the specific chemical potential, the expression of the local entropy generation rate for the case of gas absorption into a falling liquid film is, therefore, given by

$$\begin{aligned} \sigma_c = & \frac{k_L}{T^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right] - \left(\frac{D_A}{T^2} \right) \left(\frac{\partial C_A}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) \\ & \times \left[c_{pA}(T - T_o) + h_{A_o} + T \left(c_{pA} \ln\left(\frac{T}{T_o}\right) + s_{A_o} \right) \right] \\ & + \left(\frac{C_A v}{T^2} \right) \left(\frac{\partial T}{\partial z} \right) \\ & \times \left[c_{pA}(T - T_o) + h_{A_o} \right. \\ & \left. + T \left(c_{pA} \ln\left(\frac{T}{T_o}\right) + s_{A_o} \right) \right] + \frac{\eta_L}{T} \left(\frac{\partial v}{\partial x} \right)^2. \end{aligned} \quad (40)$$

The first term of the right-hand side of (40) is the irreversibility due to heat transfer $\sigma_{c,th}$, the second and the third terms are due to the coupling effects between heat and mass transfer $\sigma_{c,m-th}$ (by diffusion and by convection, resp.), and the last one is due to the fluid friction $\sigma_{c,v}$ (viscous dissipation irreversibility).

TABLE 1: Gas and liquid physical properties [14, 15].

	Property	
Liquid	Inlet temperature, T_o (K)	298
	Reference pressure, P_o (atm)	1
	Dynamic viscosity, η_L (Pa · s)	$0,911 \cdot 10^{-3}$
(Water)	Mass density, ρ_L ($\text{kg} \cdot \text{m}^{-3}$)	1000
	Thermal conductivity, k_L ($\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$)	0,607
Gas	Diffusion coefficient, D_A ($\text{m}^2 \cdot \text{s}^{-1}$)	$1,9 \cdot 10^{-9}$
	Solubility in water at T_o , C_A ($\text{g} \cdot \text{L}^{-1}$)	540
(Ammonia)	Reference molar entropy, s_{A_o} ($\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$)	192,77
	Specific heat, C_{pA} ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$)	2129,45

4. Results and Discussions

Velocity and concentration are calculated by using (10) and (16), respectively. Temperature Equation (32) is solved for a given value of temperature difference ΔT . Values of x are selected in order to ensure a laminar flow of the liquid film. The absorption of ammonia into a falling water film is considered. All the following results are calculated using the physical properties of water and ammonia given in Table 1.

Figures 2–5 show the general behavior of irreversibilities due to thermal gradients, coupling effects between heat and mass transfer, fluid friction, and total entropy generation versus the transverse direction (i.e., from the gas-liquid interface to the falling liquid film thickness δ), for known values of the liquid film length and the temperature difference ΔT . The choice of the temperature difference ranging between 1°C and 5°C is based on an experimental study elaborated by Mahmoud [35] about heat and mass transfer accompanying ammonia vapor absorption into a solution of water-ammonia. The study shows that for a null fraction of ammonia in the solution and a penetration distance equal to 4×10^{-4} m, the solution temperature increases from 22°C to 29°C. Using this argument, it is important to consider the above-mentioned temperature difference values to examine the evolution of different sources of irreversibility in the system.

As it can be seen, Figure 2 shows that irreversibility due to thermal gradients $\sigma_{c,th}$ is at its maximum value at the gas-liquid interface, where the gas absorption process begins. For any fixed value of the temperature difference ΔT , entropy generation due to thermal gradients exponentially decreases as x increases; since the temperature is a fractional polynomial function of the transverse coordinate x , its derivative gives another polynomial function with lower degree (i.e., the irreversibility due to thermal gradients). Consequently, thermal irreversibility mainly happens near the gas-liquid interface, then it decreases and tends towards a minimum constant value on approaching the falling film thickness δ , where the absorption process is practically absent.

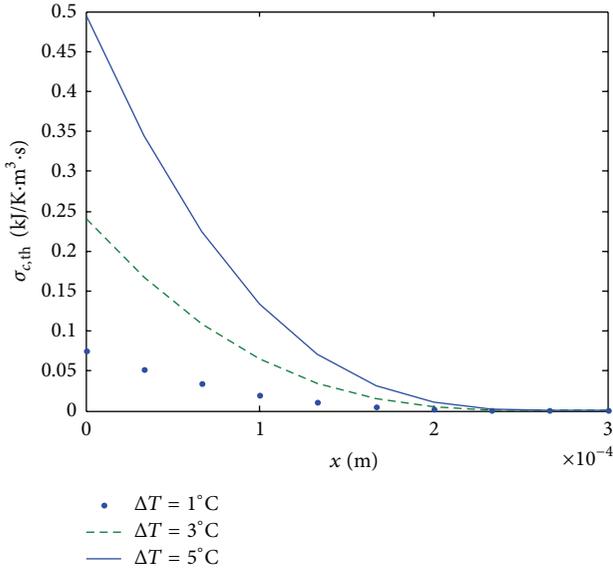


FIGURE 2: Entropy generation due to thermal gradients, $\sigma_{c,th}$ versus x and ΔT .

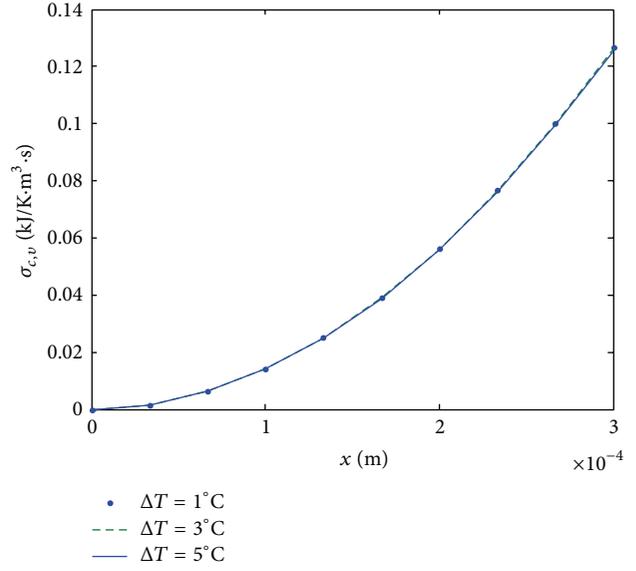


FIGURE 4: Entropy generation due to fluid friction, $\sigma_{c,v}$ versus x and ΔT .

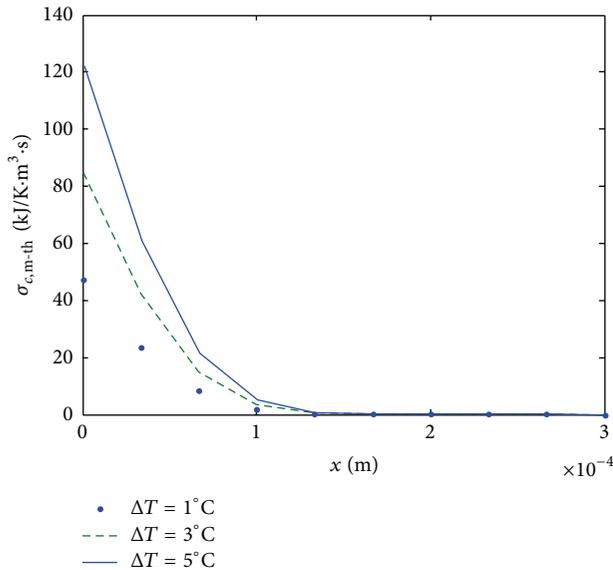


FIGURE 3: Entropy generation due to heat and mass transfer, $\sigma_{c,m-th}$ versus x and ΔT .

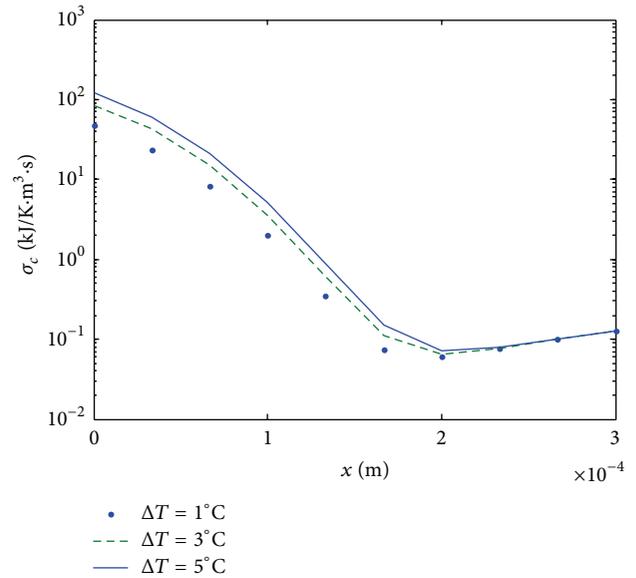


FIGURE 5: Total entropy generation, σ_c versus x and ΔT .

For a fixed value of the transverse distance x , it can be observed that as the temperature gradients increase, entropy generation due to thermal gradients also increases.

Figure 3 illustrates entropy generation due to the coupling effects between heat and mass transfer, $\sigma_{c,m-th}$. It can be seen that near the gas-liquid interface, entropy generation is at its maximum value, since the absorption process mainly takes place close to the gas-liquid interface, which is accompanied by the heat of absorption. Thus, entropy generation due to the coupling effects between heat and mass transfer is similar to the concentration profile, which is maximum at the gas-liquid interface (i.e., at $x = 0$) as found by Chermiti et al.

[25]. The increase of the transverse distance (i.e., $x > 0$) induces the decrease of $\sigma_{c,m-th}$, to become null near the falling film thickness (i.e., $x = \delta$), for all the studied values of ΔT . This is due to the fact that when the transverse distance increases, the absorption process and the fluid motion decrease. Consequently, irreversibilities due coupling effects between heat and mass transfer by diffusion and convection decrease. At the film thickness ($x = \delta$), the absorption process is insignificant (no concentration gradients).

It could be noticed that, at lower values of temperature gradient ΔT , the gas absorption proceeds at a faster rate due to the availability of larger driving potentials for mass and heat transfer.

Throughout the diffusion film thickness region, the decrease of the temperature difference induces the decrease of both irreversibilities due to heat transfer (Figure 2) and coupling effects between heat and mass transfer (Figure 3). This is attributed to the decrease of temperature and concentration as well as velocity gradients in this region.

Figure 4 shows entropy generation due to fluid friction, $\sigma_{c,v}$ along the liquid film. As shown, the analytically obtained viscous dissipation irreversibility is null near the gas-liquid interface, since velocity is at its maximum value at this region. As a consequence, the friction effects contribution is insignificant near the interface.

On approaching the film thickness, viscous dissipation irreversibility significantly increases and it is maximum at the falling film thickness ($x = \delta$). The fluid friction irreversibility exhibits a parabolic profile. This behavior is expected, since its expression includes the term (x^2) via the velocity expression (see (40)). The fluid friction irreversibility magnitude slightly increases with the decrease of the temperature difference ΔT as it is inversely proportional to the temperature.

As a consequence, the viscous dissipation irreversibility through the falling liquid film increases as the transverse distance increases and, inversely, as the temperature difference decreases.

Total entropy generation σ_c behavior is illustrated in Figure 5. As it can be seen, entropy generation is at its maximum value near the gas-liquid interface for all the studies values of ΔT . This is attributed, on one hand, to the dominant effect of coupled heat and mass transfer irreversibility induced by the absorption process and, on the other hand, to the thermal irreversibility. As mentioned, at the interface region, irreversibility due to fluid friction is insignificant (Figure 3). Further, the increase of the temperature difference induces the increase of total entropy generation magnitude.

On increasing the transverse distance, total entropy generation significantly decreases towards a minimum value for all the studied values of the temperature difference. Entropy generation is minimum at the diffusion film thickness δ_d . The decrease of entropy generation from the gas-liquid interface to the diffusion film thickness is due to the decrease of both convective and diffusive irreversibilities, since the gas diffusion occurs at the beginning of the falling film thickness and the mass transfer by convection decreases while approaching the diffusion film thickness. At this point, the influence of both thermal and coupled effects between heat and mass transfer (by diffusion and convection) irreversibilities on total entropy generation becomes insignificant. For $x > \delta_d$, total entropy generation increases towards an asymptotic value, which is mainly due to the viscous dissipation irreversibility. This result is consistent with the findings of Chermiti et al. [25]. For $x > \delta_d$, entropy generation slightly increases with the decrease of the temperature difference as explained above.

5. Conclusions

The second law of thermodynamics is analytically investigated for the case of gas absorption through a laminar

falling film. Under some physical assumptions, velocity, temperature, and gas concentration equations are derived and used for the entropy generation calculation concerning the absorption of ammonia in a water falling film. The analysis shows that entropy generation is due to heat transfer, coupling effects between heat and mass transfer (by convection and diffusion), and fluid friction. Entropy generation due to fluid friction is insignificant at the gas-liquid interface, where thermal and coupled effects between heat and mass transfer irreversibilities dominate. At the interface ($x = 0$), total irreversibility magnitude is maximum, whereas it is minimum at the diffusion film thickness ($x = \delta_d$). On approaching the liquid film thickness ($x = \delta_d$), entropy generation is mainly due to fluid friction effects. At the diffusion film region ($0 < x \leq \delta_d$), entropy generation magnitude increases with the increase of the temperature difference of the falling film; this behavior is reversed for values of the transverse distance greater than the diffusion film thickness ($\delta_d < x \leq \delta$).

Nomenclature

C_A :	Molar concentration of gas ($\text{mol}\cdot\text{m}^{-3}$)
C_{pA} :	Isobaric specific heat of gas ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
D_A :	Molecular diffusivity coefficient ($\text{m}^2\cdot\text{s}^{-1}$)
erf:	Error function
q :	Energy flux ($\text{J}\cdot\text{s}^{-1}\cdot\text{m}^2$)
J_A :	Mass transfer flux of gas ($\text{mol}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$)
h_A :	Specific enthalpy of gas (J/mol)
k_L :	Liquid thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$)
L :	Falling film length (m)
P :	Pressure (Pa)
s_A :	Specific entropy of gas ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
T :	Temperature (K)
u, v :	Falling film velocity components ($\text{m}\cdot\text{s}^{-1}$)
x, z :	Rectangular coordinates (m)
X, Z :	Dimensionless coordinates (—)

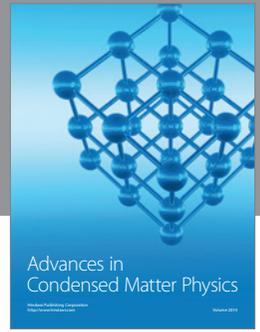
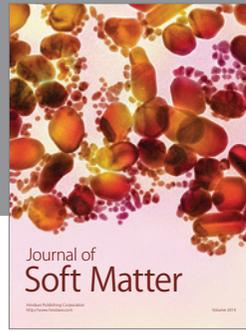
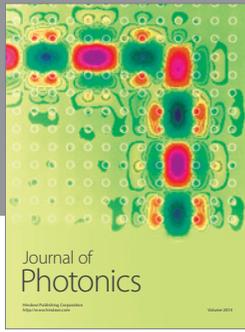
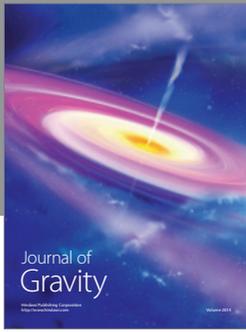
Greek Symbols

α :	Thermal diffusivity of liquid ($\text{m}^2\cdot\text{s}^{-1}$)
β_c :	Solute expansion coefficient ($\text{m}^3\cdot\text{mol}^{-1}$)
β_T :	Thermal expansion coefficient (K^{-1})
η_L :	Dynamic viscosity (Pa·s)
δ :	Falling film thickness (m)
δ_d :	Diffusion film thickness (m)
ΔT :	Temperature difference through the liquid film ($^{\circ}\text{C}$)
μ_A :	Specific chemical potential of gas ($\text{J}\cdot\text{mol}^{-1}$)
ρ :	Fluid mass density ($\text{kg}\cdot\text{m}^{-3}$)
τ :	Stress tensor ($\text{N}\cdot\text{m}^{-2}$)
θ :	Dimensionless temperature (—)
λ :	Integration constant (—)
σ_c :	Entropy generation rate per unit volume ($\text{J}\cdot\text{m}^{-3}\cdot\text{s}^{-1}\cdot\text{K}^{-1}$).

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