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Research Article

Cooperative Object Manipulation by a Space Robot with Flexible Appendages

P. Zarafshan and S. Ali A. Moosavian

Center of Excellence in Robotics and Control, Advanced Robotics and Automated Systems Laboratory, Department of Mechanical Engineering, K. N. Toosi University of Technology, P.O. Box 19395-1999, Tehran, Iran

Correspondence should be addressed to P. Zarafshan; payam.zarafshan@gmail.com

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Modelling and control of rigid-flexible multibody systems is studied in this paper. As a specified application, a space robotic system with flexible appendages during a cooperative object manipulation task is considered. This robotic system necessitates delicate force exertion by several end-effectors to move an object along a desired path. During such maneuvers, flexible appendages like solar panels may get stimulated and vibrate. This vibrating motion will cause some oscillatory disturbing forces on the spacecraft, which in turn produces error in the motion of the end-effectors of the cooperative manipulating arms. In addition, vibration control of these flexible members to protect them from fracture is another challenging problem in an object manipulation task for the stated systems. Therefore, the multiple impedance control algorithm is extended to perform an object manipulation task by such complicated rigid-flexible multibody systems. This extension in the control algorithm considers the modification term which compensates the disturbing forces due to vibrating motion of flexible appendages. Finally, a space free-flying robotic system which contains two 2-DOF planar cooperative manipulators, appended with two highly flexible solar panels, is simulated. Obtained results reveal the merits of the developed model-based controller which will be discussed.

1. Introduction

Robotic manipulators are widely used in unsafe, costly, and repetitive boring tasks. Most available robotic manipulators are designed such that they can provide essential stiffness for end-effector to reach its desired position without flexible deformations [1]. This stiffness is usually attained by massive links. Consequently, design and use of weighty rigid manipulators may be deficient in energy consumption and the speed of operation. In particular, for space and on-orbit applications minimum weight design does not allow using such stiff manipulators. On the other hand, even assuming rigid manipulators, existence of flexible components on space robotic systems such as solar panels, necessitates considering their effect. The required settling time for vibration of such parts may delay the operation and so conflicts with increasing time efficiency of the system. This conflict of high speed and high accuracy during any operation makes these robots a disputative research problem [2–5].

Robotic systems with flexible components include continuous dynamic systems that are simplified by using a finite number of rigid degrees of freedom and a limited number of modes. This leads to a set of ordinary and partial differential equations that are usually nonlinear and coupled. Precise solution of these systems in most cases is almost impossible [6]. In studying these systems, if we ignore the flexibility effects in mathematic model, two types of error will be produced. The first one is related to the actuator torques, and the second one corresponds to the position of end-effectors. The position/orientation of end-effectors for precise tasks should not experience any vibration even with small amplitude. Therefore, to achieve high accuracy, we must begin with more precise mathematic models [7–11]. To study the dynamics of a rigid-flexible multibody space system, an inertia frame is used as a universal reference frame. Moreover, an intermediate reference frame is attached to each flexible or rigid body which is usually called floating frame. The motion relative to this intermediate frame for flexible parts occurs because of the body deformation only. This selection simplifies the calculations of internal forces since the magnitude of the stress and strain does not vary under the rigid body motion. To develop dynamics model of such systems, various approaches have been used, including the Lagrange method [9, 12], the Hamilton principle [13], the Newton-Euler equations [14], the virtual work principle [15], and Kane method [16].

Controller design for multibody systems with flexible members requires the development of proper dynamics model of such systems. Such models are also required to be as concise as possible for implementation of model-based control algorithms. In most researches on dynamics analysis, the modelling approach introduces an accumulation in the dynamics of rigid-flexible multibody systems, [17, 18]. This is done while the modelling approach does not affect their non-model based controller design, whereas, using an accumulated dynamic model to control these complicated systems by a model-based control algorithm will become as a challenging problem. On the other hand, control of flexible multibody systems is currently an attractive research subject because of its application in flexible manipulators and the articulated space structures [19–22]. This depends on determining the actuator torques that can produce the desired motion of such a complicated multibody system. In other words, the inverse dynamics is part of controller design, though control can be directly applied on a physical system without using a numerical model [23, 24]. In fact, operational problems with robotic manipulators in space due to structural flexibility lead to subsequent difficulties with their position control and have been widely studied [25, 26]. However, force interaction with the environment makes a more challenging problem than position control of such systems which is the main focus of this paper. The object manipulation operation by the rigid-flexible multibody systems is a problem that was less proceeded in the researches. Of course, various control algorithms were presented for the object manipulation task in the rigid robotic systems in which each of them has the advantages. These algorithms can be cited including multiple impedance control (MIC) [27], augmented object control (AOM) [28], and non-model-based impedance control (NMIC) [29]. Although, each of these algorithms were shown their capabilities for rigid systems, but their performance must be studied for rigid-flexible multibody systems.

As stated above, in this paper, a space robotic system with flexible appendages is considered to perform cooperative object manipulation task. It necessitates delicate force exertion by two or more end-effectors to move the object along a desired path. First, we must extract the simpler sets of dynamics equations which can be used for model-based controllers. To this end, the system dynamics is virtually partitioned into two rigid and flexible portions, and a convenient model is developed for control purposes of rigid-flexible multibody systems. Next, based on the genetic algorithm approach using MATLAB/GATOOL, appropriate trajectories are designed to study the stimulation effects of the flexible appendages. Then, an object manipulation operation is studied by extended multiple impedance control algorithm to suppress the vibration of the flexible members. Finally, using a comprehensive simulation routine, obtained results of the implementations of this controller on the rigid-flexible multibody system for the designed minimum time trajectory (CASE-I) and a circular path (CASE-II) will be discussed.

2. Dynamics Modelling

2.1. Modelling of Rigid Components. Space free-flying robots (SFFRs) are space systems that include an actuated spacecraft equipped with few manipulators. Distinct from fixed-based manipulators, the spacecraft (base) of an SFFR responds to dynamic reaction forces due to the arms motion. Unlike long-reach space manipulators, SFFRs are suggested to be comparable to a human body and an astronaut so are usually investigated under the assumption of rigid elements. The motion equations of a space robot with rigid components which were described by [30] can be extended to consider flexible elements as

$$\begin{aligned}&\mathbf{H}\left(\boldsymbol{\beta}_0,\boldsymbol{\theta}\right)\ddot{\mathbf{q}}+\mathbf{C}_1\left(\boldsymbol{\beta}_0,\dot{\boldsymbol{\beta}}_0,\boldsymbol{\theta},\dot{\boldsymbol{\theta}}\right)\dot{\mathbf{q}}+\mathbf{C}_2\left(\boldsymbol{\beta}_0,\dot{\boldsymbol{\beta}}_0,\boldsymbol{\theta},\dot{\boldsymbol{\theta}}\right)\\&\quad=\mathbf{Q}\left(\boldsymbol{\beta}_0,\boldsymbol{\theta}\right)+\mathbf{Q}_{\text{flex.}}\left(\boldsymbol{\beta}_0\right),\end{aligned}\tag{1}$$

where $\mathbf{Q}_{\text{flex.}}$ and $\mathbf{Q}$ are the resultant forces/torques applied on the main body of the space robot due to vibrating motion of the flexible solar panels and the generalized forces, respectively. Also, $\mathbf{C}_1$ and $\mathbf{C}_2$ are the vector of centrifugal and the Coriolis terms, and the mass matrixes $\mathbf{H}$ can be partitioned as

$$\begin{aligned}&\mathbf{H}_{ij}\\&\quad=M_{\text{sys}}\frac{\partial\mathbf{R}_{C_0}}{\partial q_i}\cdot\frac{\partial\mathbf{R}_{C_0}}{\partial q_j}+\frac{{}^0\partial\omega_0}{\partial\dot{q}_i}\cdot\mathbf{I}_0\cdot\frac{{}^0\partial\omega_0}{\partial\dot{q}_j}\\&\qquad+\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial\mathbf{r}_{C_k}^{(m)}}{\partial q_i}\cdot\frac{\partial\mathbf{r}_{C_k}^{(m)}}{\partial q_j}\right.\\&\qquad\qquad\left.+\frac{{}^k\partial\omega_k^{(m)}}{\partial\dot{q}_i}\cdot\mathbf{I}_k^{(m)}\cdot\frac{{}^k\partial\omega_k^{(m)}}{\partial\dot{q}_j}\right)\\&\qquad+\left(\sum_{m=1}^{n}\sum_{k=1}^{N_m}m_k^{(m)}\frac{\partial\mathbf{r}_{C_k}^{(m)}}{\partial q_i}\right)\cdot\frac{\partial\mathbf{R}_{C_0}}{\partial q_j}\\&\qquad+\left(\sum_{m=1}^{n}\sum_{k=1}^{N_m}m_k^{(m)}\frac{\partial\mathbf{r}_{C_k}^{(m)}}{\partial q_j}\right)\cdot\frac{\partial\mathbf{R}_{C_0}}{\partial q_i},\end{aligned}\tag{2a}$$

$$\begin{aligned}&\mathbf{C}_{1ij}\\&\quad=M_{\text{sys}}\frac{\partial\mathbf{R}_{C_0}}{\partial q_i}\cdot\left(\sum_{s=1}^{N}\frac{\partial^2\mathbf{R}_{C_0}}{\partial q_s\partial q_j}\dot{q}_s\right)\end{aligned}$$

$$
\begin{aligned}
&+\frac{{}^{0}\partial\omega_0}{\partial\dot{q}_i}\mathbf{I}_0\frac{{}^{0}\partial\omega_0}{\partial q_j}+\omega_0\mathbf{I}_0\frac{{}^{0}\partial^2\omega_0}{\partial\dot{q}_i\partial q_j}\\
&+\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial\mathbf{r}_{C_k}^{(m)}}{\partial q_i}\cdot\left(\sum_{s=1}^{N}\frac{\partial^2\mathbf{r}_{C_k}^{(m)}}{\partial q_s\partial q_j}\dot{q}_s\right)\right.\\
&\qquad\left.+\frac{{}^{k}\partial\omega_k^{(m)}}{\partial\dot{q}_i}\mathbf{I}_k^{(m)}\frac{{}^{k}\partial\omega_k^{(m)}}{\partial q_j}+\omega_k^{(m)}\mathbf{I}_k^{(m)}\frac{{}^{k}\partial^2\omega_k^{(m)}}{\partial\dot{q}_i\partial q_j}\right)\\
&+\left(\sum_{s=1}^{N}\frac{\partial^2\mathbf{R}_{C_0}}{\partial q_s\partial q_i}\dot{q}_s\right)\cdot\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial\mathbf{r}_{C_k}^{(m)}}{\partial q_j}\right)\\
&+\frac{\partial\mathbf{R}_{C_0}}{\partial q_i}\cdot\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\sum_{s=1}^{N}\frac{\partial^2\mathbf{r}_{C_k}^{(m)}}{\partial q_s\partial q_j}\dot{q}_s\right),
\end{aligned}
\tag{2b}
$$

$$
\mathbf{C}_{2i}=-\left(\omega_0\mathbf{I}_0\frac{{}^{0}\partial\omega_0}{\partial q_i}+\sum_{m=1}^{n}\sum_{k=1}^{N_m}\omega_k^{(m)}\mathbf{I}_k^{(m)}\frac{{}^{k}\partial\omega_k^{(m)}}{\partial q_i}\right),\tag{2c}
$$

where ω_0 and $\mathbf{I}_0$ are angular velocity and moment of inertia matrix of the base, M_{sys} is the total mass of the rigid subsystem, $\omega_k^{(m)}$ is the angular velocity of the kth link of the (m)th manipulator, $\mathbf{r}_{C_k}^{(m)}$ is the position vector of mass center of this link with mass of $m_k^{(m)}$ and inertia matrix of $\mathbf{I}_k^{(m)}$, and $\mathbf{R}_{C_0}$ is the inertial position vector of mass center of the base. Array $\mathbf{q}$ describes the rigid subsystem variable state that is defined as

$$
\mathbf{q}=\left\{\mathbf{R}_{C_0}^{T},\boldsymbol{\beta}_0^{T},\boldsymbol{\theta}^{T}\right\}^{T},\tag{3}
$$

where $\boldsymbol{\beta}_0$ is a set of Euler angles that determine the orientation of the base and $\boldsymbol{\theta}$ describes joint angle of the links that is defined as

$$
\boldsymbol{\theta}^{T}=\left\{\theta_1^{(1)},\ldots,\theta_{N_1}^{(1)},\ldots,\theta_1^{(m)},\ldots,\theta_{N_m}^{(m)},\ldots,\theta_1^{(n)},\ldots,\theta_{N_n}^{(n)}\right\}^{T},\tag{4}
$$

where n is the number of manipulators, N_m is the number of links of the (m)th manipulator, and $\theta_{N_m}^{(m)}$ is the joint angle of the (m)th of manipulator N_m. Also, $\mathbf{Q}$ is vector of generalized forces and torques that are fully described by [31] with determining all dynamics and kinematics parameters. Thus, (1) presents the motion's equation of a rigid robot in space, that is, a microgravity environment.

2.2. Modelling of Flexible Components. As mentioned before, based on real systems, it is assumed that a SFFR with rigid components is appended by flexible appendages, for example, the solar panels to supply the required electrical energy. In this section, dynamics of the flexible panels is developed by using a floating frame reference. It will be shown that the equations of motion of such systems can be written in terms of a set of inertia shape integrals in addition to the mass of the body, the inertia matrix, the generalized forces and the stiffness matrix. These inertia shape integrals depend on the assumed displacement field that appear in the nonlinear terms. They signify the inertia coupling between the reference motion and the elastic deformation of the body. It will be shown that the deformable body inertia matrix depends on the elastic deformation of the body, and hence, it is an implicit function of time. Here, the configuration of each flexible body in the multibody system is recognized by using two sets of coordinates, that is, reference and elastic coordinates. Reference coordinates describe the location and orientation of a selected body reference. On the other hand, elastic coordinates explain the body deformation with respect to the body reference. In order to avoid the computational difficulties of infinite-dimensional spaces, these coordinates are introduced by using classical approximation techniques such as Rayleigh-Ritz methods [31]. Thus, the global position of an arbitrary point on the flexible body is defined by using a coupled set of reference and elastic coordinates. Also, the kinetic energy of the flexible body is developed, and the inertia coupling between the reference motion and the elastic deformation is recognized. The kinetic energy as the virtual work of the forces acting on the body is written in terms of the coupled sets of reference and elastic coordinates [31]. Then, the motion's equations of the flexible members can be obtained as [4, 5]

$$
\mathbf{M}_f^{\{i\}}\ddot{\overline{\mathbf{q}}}^{\{i\}}+\mathbf{K}^{\{i\}}\overline{\mathbf{q}}^{\{i\}}=\mathbf{Q}_e^{\{i\}}+\mathbf{Q}_v^{\{i\}},\quad \{i\}=\{1,2,\ldots,n_b\},\tag{5}
$$

where n_b is the total number of the flexible bodies in the multibody system. Also, $\mathbf{Q}_v^{\{i\}}$ and $\mathbf{Q}_e^{\{i\}}$ are correspondingly a quadratic velocity vector which contains all gyroscopic and the Coriolis components and the vector of generalized forces associated with the $\{i\}$th body. Moreover, $\mathbf{M}_f^{\{i\}}$ and $\mathbf{K}^{\{i\}}$ are, respectively, recognized as the symmetric mass matrix and the symmetric positive definite stiffness matrix of the body $\{i\}$. It is recommended that $\overline{\mathbf{q}}^{\{i\}}$ is the vector of reference and elastic coordinates of the flexible body. This equation can be written in a partitioned matrix form as

$$
\begin{aligned}
&\begin{bmatrix}\mathbf{m}_{rr}^{\{i\}} & \mathbf{m}_{rf}^{\{i\}}\\ \mathbf{m}_{fr}^{\{i\}} & \mathbf{m}_{ff}^{\{i\}}\end{bmatrix}\begin{bmatrix}\ddot{\overline{\mathbf{q}}}_r^{\{i\}}\\ \ddot{\overline{\mathbf{q}}}_f^{\{i\}}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \mathbf{k}_{ff}^{\{i\}}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{q}}_r^{\{i\}}\\ \overline{\mathbf{q}}_f^{\{i\}}\end{bmatrix}\\
&\quad=\begin{bmatrix}\left(\mathbf{Q}_e^{\{i\}}\right)_r\\ \left(\mathbf{Q}_e^{\{i\}}\right)_f\end{bmatrix}+\begin{bmatrix}\left(\mathbf{Q}_v^{\{i\}}\right)_r\\ \left(\mathbf{Q}_v^{\{i\}}\right)_f\end{bmatrix},
\end{aligned}
\tag{6}
$$

where "r" and "f", respectively, refer to rigid and flexible coordinates of the flexible members. These dynamics parameters are fully described by [4, 5].

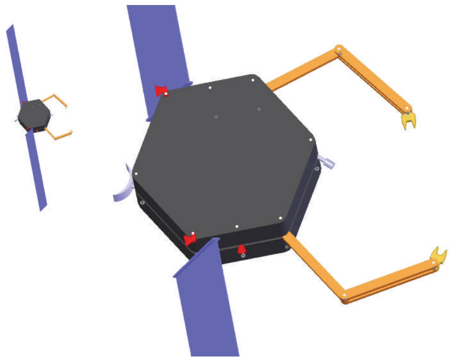

Figure 1: The considered space robotic system.

2.3. Complete Dynamics Equations of Motion. It should be noted that, in the developed modelling approach, the equations of motion for the $\{i\}$th flexible body are separated from those equations of the main rigid body system of the assumed space robot in Figure 1. This can be desirably used in designing the controller through a simulation study. In fact, after we solve the equations of motion for the rigid body system at each time step, the acceleration terms are considered as inputs for those equations of the flexible part. Then, we solve the equations of motion for the flexible subsystem considering these inputs. The obtained constraint forces are attained as the outputs which will be exerted on the mobile base of the space robot as estimated disturbance forces, [4, 5].

Considering (6), the generalized forces due to the stimulation of the flexible members which are applied on the rigid subsystem as the modification term or the constraint force or $\mathbf{Q}_{\text{flex.}}(\boldsymbol{\beta}_0)$ can be achieved as

$$\mathbf{Q}_{\text{flex.}}(\boldsymbol{\beta}_0) = \sum_{\{i\}=\{1\}}^{\{n_b\}} \mathbf{J}_f^{\{i\}^T} \left(\mathbf{Q}_e^{\{i\}}\right)_r, \tag{7}$$

where $\mathbf{J}_f^{\{i\}}$ is the Jacobian matrix of the floating frame of each flexible body related to the inertial frame of the main body. As detailed previously, this dynamics modelling approach combines the Lagrange and the Newton-Euler methods. To use this approach, the computation procedure at each time step includes the following calculations. First, the motion's equations of the rigid subsystem or (1) are solved. Then, the acceleration, velocity, and position terms of the rigid subsystem, that is, $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$, $\mathbf{q}$ in the previous formulation, are calculated. Then, the rigid components of the acceleration, velocity, and position terms of each flexile body, that is, $\ddot{\overline{\mathbf{q}}}_r^{\{i\}}$, $\dot{\overline{\mathbf{q}}}_r^{\{i\}}$, $\overline{\mathbf{q}}_r^{\{i\}}$ in the previous formulation, are computed. After that, they were inserted into the motion equations of the flexible members as input terms. The relationship between these two sets of variables is established by the kinematics constraints between the origin of the floating frame which is attached to the flexible member and the reference frame of the rigid subsystem. Considering these inputs and by solving the second row of (6), the flexible components of the acceleration, velocity, and position terms of each flexile body, that is, $\ddot{\overline{\mathbf{q}}}_f^{\{i\}}$, $\dot{\overline{\mathbf{q}}}_f^{\{i\}}$, $\overline{\mathbf{q}}_f^{\{i\}}$ in the previous formulation, are calculated. Using these values and substituting into the first row of (6), the constraint forces or $\mathbf{Q}_e^{\{i\}}$ are computed. Also, these results are applied to the equations of the rigid subsystem as the produced forces from the incitement of the flexible member, that is, $\mathbf{Q}_{\text{flex.}}(\boldsymbol{\beta}_0)$ by using (7). As being clear, this dynamics modelling approach of a rigid multibody system with flexible members increases the computations of dynamics analysis, and it is useful in model-based control algorithms. For instance, the inverse of a mass matrix of the accumulated rigid-flexible multibody system requires much more calculations than inversing two matrices of the rigid and flexible members of the multibody system separately. Next, we study the designated path which it is applied in the simulations of the object manipulation control algorithm.

3. Time Optimal Trajectory Planning

The minimum time trajectory (MTT) can be considered as a useful strategy for mobile robotic systems to move on a given path between the two points during a minimum time. This task is performed by maximum available force/torque capacity of the actuators. This causes the fast dynamics of the multibody system to be stimulated, and its effects can be studied. So, we consider an object manipulation operation along a straight path, and then, the various scenarios for MTT would be presented to perform this task. Considering the mobile base and a desired path length, several solutions exist. The first scenario is that the two manipulators of the robot perform this operation by the base movement. The second one is that the base remains stationary and performs the task using its manipulators whereas the desired path to follow is within the fixed work space of the manipulators. The third scenario is achieved by combining the two, which this one is considered in all of simulations [32].

Designing procedure of the MTT includes expressing the considered path using a path variable, then computation of the velocity and acceleration of the system variables in terms of the path variable and its derivatives [33]. So, replacing these in the dynamic equations and after some simplifications, the dynamic equations are obtained in terms of the path variable. Therefore, considering the actuators bound, the equation of the desired MTT is obtained. In this procedure, it is assumed that the path of each end-effector in its work space is specified. Thus, the path parameter "s" is selected on the given path. Then, relation between the joint variables $\mathbf{q}$ and the path parameter can be stated. By differentiating this equation and solving that, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$ are obtained according to the path parameter. Then, replacing these parameters in the motion's equations yields

$$\overline{\mathbf{C}}_1(s)\,\ddot{s} + \overline{\mathbf{C}}_2(s,\dot{s}) = \boldsymbol{\tau}, \tag{8}$$

where $\overline{\mathbf{C}}_1(s)$ and $\overline{\mathbf{C}}_2(s,\dot{s})$ are the mass matrix and the quadratic velocity vector of the robotic system which are explained as the path parameter, respectively. We should note that for the movement on the given path in minimum time, the acceleration $\ddot{s}$ is determined by considering the extreme bounds of all actuators as

$$\boldsymbol{\tau}_{\min} \leq \overline{\mathbf{C}}_1\ddot{s} + \overline{\mathbf{C}}_2 \leq \boldsymbol{\tau}_{\max}. \tag{9}$$

Thus

$$f_i(s,\dot{s}) \leq \ddot{s} \leq g_i(s,\dot{s}), \tag{10}$$

where $f_i(s,\dot{s})$ is the lower acceleration and $g_i(s,\dot{s})$ is the higher acceleration. We should not that $\ddot{s}$ is a scalar quantity and should be determined by considering the capacity of the weakest actuators. Therefore,

$$\begin{gathered} \ddot{s} = f = \max_{i=1,2} f_i(s,\dot{s}), \\ \ddot{s} = g = \min_{i=1,2} g_i(s,\dot{s}). \end{gathered} \tag{11}$$

Considering the limitation of actuators of the assumed space robot in the following case studies as [27],

$$|\tau_i| \leq 7\,\mathrm{N\cdot m}, \quad \text{for } i = 1,2,3,4. \tag{12}$$

Using genetic algorithm based on GATOOL toolbox of MATLAB for one manipulator [24], the $\overline{\mathbf{C}}_1(s)$ and $\overline{\mathbf{C}}_2(s,\dot{s})$ functions are optimized based on the assumed limitations or (12). Therefore, the acceleration is obtained. As shown in Figure 2, the designed trajectory and its velocity are achieved according to the obtained acceleration. It should be noted that the movement amplitude is 10 m and the optimum time in this maneuver is 5.8026 sec for this designed trajectory.

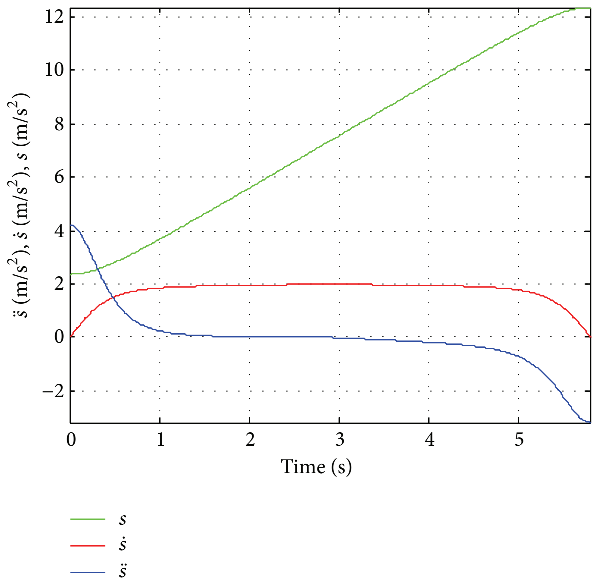

Figure 2: The position, velocity, and acceleration of the path parameter or s for the designed MTT.

4. Controller Design

There are several main issues that make the control problem of a rigid-flexible multibody system more complicated than control of a rigid system [34, 35]. First, the number of degrees of freedom is much larger than the number of actuators. A flexible body has an infinite number of degrees of freedom. As an example, the body can be discretized into a finite number of degree of freedom using various techniques such as finite element method or modal analysis, but the number of actuators is still generally much less than number of degrees of freedom, which may make the controller incapable to achieve an exact performance. At best, the controller can follow a trajectory that minimizes the error between the desired and the actual trajectories. The second issue is related to wave propagation delays. An action at one end of a flexible beam takes time to propagate to its tip. The third one is reversal action. This effect can be observed in a rotating flexible beam [36]. When a torque is applied to the beam in one direction, its tip position initially moves in the opposite direction. On the other hand, there are two main requirements for a rigid-flexible multibody system controller. These are fast and precise responses in following the desired trajectory. These two requirements are usually in conflict. The faster controller is the less accurate and vice versa. Several types of control laws have been proposed each offering benefits under some conditions. So, often more than one type of control law is used in these systems in order to improve the performance.

As studied in some researches, the multiple impedance control (MIC) law has the better performance rather than other object manipulation control algorithm [27]. So, the MIC law is introduced here. On the other hand, the rigid system assumption is considered for the MIC algorithm, whereas this assumption must be changed to control severe vibrations of flexible members in a rigid-flexible system and also to attain a successful object manipulation operation. Consequently, the MIC law is extended to compensate the disturbing forces due to the vibration of the flexible appendages. The modification term is placed on the control forces where the flexible members are installed on the affiliated part. In this assumed space robot, this means that the compensator term must be added on the control force of the robot, because the flexible appendages are located on this part. Also, to perform an object manipulation task in a planar maneuver, we should consider the following constraint in grasping the object:

$$(X_A - X_B)^2 + (Y_A - Y_B)^2 = l^2, \tag{13}$$

where X_A and Y_A are the end-effectors position of the left manipulator, X_B and Y_B are the end-effectors position of the right manipulator, and l is the object length.

Considering the formulation of MIC law, the end-effectors forces are

$$\mathbf{F}_e = \{\mathbf{F}_A^x \;\; \mathbf{F}_A^y \;\; \mathbf{F}_B^x \;\; \mathbf{F}_B^y\}^T. \tag{14}$$

Therefore, the equations of motion for the object are

$$\begin{aligned} F_A^x + F_B^x &= m\ddot{X}_o, \\ F_A^y + F_B^y &= m\ddot{Y}_o, \\ \frac{(F_B^x - F_A^x)\, l}{2C_0} + \frac{(F_B^y - F_A^y)\, l}{2S_0} &= I\ddot{\theta}_o, \end{aligned} \tag{15}$$

where S_0 and C_0 stand for the SIN and COS functions of the object orientation or $\boldsymbol{\theta}_o$, respectively. Thus, the grasp matrix is obtained as

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\frac{l}{2C_0} & -\frac{l}{2S_0} & \frac{l}{2C_0} & \frac{l}{2S_0} \end{bmatrix}. \tag{16}$$

For the object motion, we have

$$\mathbf{M}\ddot{\mathbf{X}}_o = \mathbf{G}\mathbf{F}_e + \mathbf{F}_o + \mathbf{F}_c, \tag{17}$$

where $\mathbf{F}_c$ is the force applied on the object due to contact with environment, $\mathbf{F}_o$ is the vector of other external forces applied on the object, and $\mathbf{M}$ is the mass matrix of the object. Choosing the impedance law for the object motion as

$$\mathbf{M}_{\text{des}}\ddot{\mathbf{e}}_o + \mathbf{K}_d\dot{\mathbf{e}}_o + \mathbf{K}_p\mathbf{e}_o = -\mathbf{F}_c, \tag{18}$$

where $\mathbf{e}_o = \mathbf{X}_{o_{\text{des}}} - \mathbf{X}_o$ is the tracking error of object variables and $\mathbf{M}_{\text{des}}$, $\mathbf{K}_d$, $\mathbf{K}_p$ are the gain matrices for the proposed controller. Then, the desired exerted forces from end-effectors to move the object are obtained as:

$$\begin{aligned} &\mathbf{F}_{e_{\text{req}}} \\ &= \mathbf{G}^{\#}\left\{\mathbf{M}\mathbf{M}_{\text{des}}^{-1}\left(\mathbf{M}_{\text{des}}\ddot{\mathbf{X}}_{o_{\text{des}}} + \mathbf{K}_d\dot{\mathbf{e}}_o + \mathbf{K}_p\mathbf{e}_o + \mathbf{F}_c\right) + \mathbf{F}_w + \left(\mathbf{F}_c + \mathbf{F}_o\right)\right\}, \end{aligned} \tag{19}$$

where $\mathbf{G}^{\#}$ is the pseudoinverse of $\mathbf{G}$ as

$$\mathbf{G}^{\#} = \mathbf{W}^{-1}\mathbf{G}^T\left(\mathbf{G}\mathbf{W}^{-1}\mathbf{G}^T\right)^{-1}, \tag{20}$$

where it is weighted by a task weighting matrix $\mathbf{W}$ [27]. Thus, the force that is applied on the object by the (i)th end-effectors is directly obtained from $\mathbf{F}_{e_{\text{req}}}$ as

$$\widetilde{\mathbf{Q}}_f^{(i)} = \mathbf{F}_{e_{\text{req}}}. \tag{21}$$

Next, to compute the required force for motion control, if the equations of motion of the space robotic system or (1) can be written in the task space as

$$\widetilde{\mathbf{H}}^{(i)}\left(\mathbf{q}^{(i)}\right)\ddot{\widetilde{\mathbf{X}}}^{(i)} + \widetilde{\mathbf{C}}^{(i)}\left(\mathbf{q}^{(i)}, \dot{\mathbf{q}}^{(i)}\right) = \widetilde{\mathbf{Q}}^{(i)} + \widetilde{\mathbf{Q}}_{\text{flex.}}^{(i)}, \tag{22}$$

where (i) indicates the (i)th manipulator and $\widetilde{\mathbf{X}}^{(i)}$ is the output coordinate as:

$$\widetilde{\mathbf{X}} = \left[X_{C_0} \;\; Y_{C_0} \;\; \delta_{C_0} \;\; X_{E,A} \;\; Y_{E,A} \;\; X_{E,B} \;\; Y_{E,B} \;\; \delta_{\text{Ant}} \;\; \delta_{\text{Cam}}\right]^T, \tag{23a}$$

$$\begin{aligned} \widetilde{\mathbf{H}}^{(i)} &= \mathbf{J}_c^{(i)^{-T}}\mathbf{H}^{(i)}\mathbf{J}_c^{(i)^{-1}}, \\ \widetilde{\mathbf{C}}^{(i)} &= \mathbf{J}_c^{(i)^{-T}}\left[\mathbf{C}_1^{(i)} + \mathbf{C}_2^{(i)}\right] - \widetilde{\mathbf{H}}^{(i)}\dot{\mathbf{J}}_c^{(i)}\dot{\mathbf{q}}^{(i)}, \\ \widetilde{\mathbf{Q}}^{(i)} &= \mathbf{J}_c^{(i)^{-T}}\mathbf{Q}^{(i)}, \\ \widetilde{\mathbf{Q}}_{\text{flex.}}^{(i)} &= \mathbf{J}_c^{(i)^{-T}}\mathbf{Q}_{\text{flex.}}^{(i)}, \end{aligned} \tag{23b}$$

where $\mathbf{J}_c^{(i)}$ is the Jacobian matrix for the (i)th manipulator and $\widetilde{\mathbf{X}}$ can be considered as the generalized workspace variables of the assumed robot. We should note that $\widetilde{\mathbf{Q}}_{\text{flex.}}^{(i)}$ is the disturbance force resulting from the solar panel vibrations in the object manipulation maneuver. Also, $\widetilde{\mathbf{Q}}^{(i)}$ is the vector of generalized forces in the work space. Similarly, choosing the impedance law for each end-effector as:

$$\widetilde{\mathbf{M}}_{\text{des}}\ddot{\widetilde{\mathbf{e}}}^{(i)} + \widetilde{\mathbf{K}}_d\dot{\widetilde{\mathbf{e}}}^{(i)} + \widetilde{\mathbf{K}}_p\widetilde{\mathbf{e}}^{(i)} = -\mathbf{F}_c, \tag{24}$$

where $\widetilde{\mathbf{e}}^{(i)} = \widetilde{\mathbf{X}}_{\text{des}}^{(i)} - \widetilde{\mathbf{X}}^{(i)}$ is the system tracking error for each manipulator and $\widetilde{\mathbf{M}}_{\text{des}}$, $\widetilde{\mathbf{K}}_d$, $\widetilde{\mathbf{K}}_p$ are the gain matrices for the proposed controller of the robotic system. Thus, the required force for motion control of the end-effectors by using the MIC law is expressed as

$$\begin{aligned} &\widetilde{\mathbf{Q}}_m^{(i)} \\ &\quad = \widetilde{\mathbf{H}}^{(i)}\widetilde{\mathbf{M}}_{\text{des}}^{-1}\left[\widetilde{\mathbf{M}}_{\text{des}}\ddot{\widetilde{\mathbf{X}}}_{\text{des}}^{(i)} + \widetilde{\mathbf{K}}_d\dot{\widetilde{\mathbf{e}}}^{(i)} + \widetilde{\mathbf{K}}_p\widetilde{\mathbf{e}}^{(i)} + \mathbf{F}_c\right] \\ &\quad + \widetilde{\mathbf{C}}^{(i)} - \widehat{\widetilde{\mathbf{Q}}}_{\text{flex.}}^{(i)}, \end{aligned} \tag{25}$$

in which $\widehat{\widetilde{\mathbf{Q}}}_{\text{flex.}}^{(i)}$ is a model-based term to compensate the vibration of solar panels. Considering the stated approach in the dynamics modelling, this modification term seems more necessary. In fact, the MIC law is extended and it can be applied to the rigid-flexible multibody systems in addition to the rigid systems by this modification term. Also, it has been recommended that the same impedance characteristics for the manipulated object and end-effectors can be chosen [37]:

$$\mathbf{M}_{\text{des}} = \widetilde{\mathbf{M}}_{\text{des}}, \qquad \mathbf{K}_d = \widetilde{\mathbf{K}}_d, \qquad \mathbf{K}_p = \widetilde{\mathbf{K}}_p, \tag{26}$$

Finally, the required forces for object manipulation to be supplied by actuators are

$$\widetilde{\mathbf{Q}}^{(i)} = \widetilde{\mathbf{Q}}^{(i)}_{\text{app}} + \widetilde{\mathbf{Q}}^{(i)}_{\text{react}} = \widetilde{\mathbf{Q}}^{(i)}_{m} + \widetilde{\mathbf{Q}}^{(i)}_{f} + \widetilde{\mathbf{Q}}^{(i)}_{\text{react}}, \tag{27}$$

where $\widetilde{\mathbf{Q}}^{(i)}_{m}$ is the control forces for end-effector motion and $\widetilde{\mathbf{Q}}^{(i)}_{\text{react}}$ is the reaction load on the end-effectors and virtually cancelled $\widetilde{\mathbf{Q}}^{(i)}_{f}$ as

$$\widetilde{\mathbf{Q}}^{(i)}_{\text{react}} = -\mathbf{F}^{(i)}_{e}, \tag{28}$$

where $\mathbf{F}^{(i)}_{e}$ is the exerted forces from end-effectors. Next, substituting (28), (25), and (21) into (27) and then the results into (22) yields

$$\begin{aligned}\widetilde{\mathbf{H}}^{(i)}\left(\mathbf{q}^{(i)}\right)\left\{\ddot{\widetilde{\mathbf{X}}}^{(i)} - \mathbf{M}^{-1}_{\text{des}}\left[\mathbf{M}_{\text{des}}\ddot{\widetilde{\mathbf{X}}}^{(i)}_{d} + \mathbf{K}_d\dot{\widetilde{\mathbf{e}}}^{(i)} + \mathbf{K}_p\widetilde{\mathbf{e}}^{(i)}\right]\right\} \\ + \mathbf{G}^{\#}\mathbf{M}\left\{\ddot{\mathbf{X}}_o - \mathbf{M}^{-1}_{\text{des}}\left[\mathbf{M}_{\text{des}}\ddot{\mathbf{X}}_{o_{\text{des}}} + \mathbf{K}_d\dot{\mathbf{e}}_o + \mathbf{K}_p\mathbf{e}_o\right]\right\} = \mathbf{0}.\end{aligned} \tag{29}$$

Because (29) must hold for any $\widetilde{\mathbf{H}}^{(i)}$ and $\mathbf{M}$, it can be concluded that:

$$\begin{aligned}\widetilde{\mathbf{H}}^{(i)}\left(\mathbf{q}^{(i)}\right)\left\{\ddot{\widetilde{\mathbf{X}}}^{(i)} - \mathbf{M}^{-1}_{\text{des}}\left[\mathbf{M}_{\text{des}}\ddot{\widetilde{\mathbf{X}}}^{(i)}_{d} + \mathbf{K}_d\dot{\widetilde{\mathbf{e}}}^{(i)} + \mathbf{K}_p\widetilde{\mathbf{e}}^{(i)}\right]\right\} = \mathbf{0}, \\ \mathbf{M}\left\{\ddot{\mathbf{X}}_o - \mathbf{M}^{-1}_{\text{des}}\left[\mathbf{M}_{\text{des}}\ddot{\mathbf{X}}_{o_{\text{des}}} + \mathbf{K}_d\dot{\mathbf{e}}_o + \mathbf{K}_p\mathbf{e}_o\right]\right\} = \mathbf{0}.\end{aligned} \tag{30}$$

By notating the fact that $\widetilde{\mathbf{H}}^{(i)}$ and $\mathbf{M}$ are positive definite mass matrices, it can result in

$$\begin{aligned}\mathbf{M}_{\text{des}}\ddot{\widetilde{\mathbf{e}}}^{(i)} + \mathbf{K}_d\dot{\widetilde{\mathbf{e}}}^{(i)} + \mathbf{K}_p\widetilde{\mathbf{e}}^{(i)} = \mathbf{0}, \\ \mathbf{M}_{\text{des}}\ddot{\mathbf{e}}_o + \mathbf{K}_d\dot{\mathbf{e}}_o + \mathbf{K}_p\mathbf{e}_o = \mathbf{0}\end{aligned} \tag{31}$$

which means all participating manipulators and the manipulated object exhibit the same designated impedance behaviour. Next, we study the extended multiple impedance control for the assumed space robot to perform an object manipulation task on the designated trajectory. Also, the simulations are done for the original MIC and the extended one to show the advantage of the new control algorithm.

Table 1: The geometric and mass parameters of the assumed robotic system.

i, j: $i = 1$, j	$i = 2$	m_{ij} (kg)	l (m)	$I_{Z_{ij}}$ (kg·m^2)
0		50	1	10
1		4	1	0.5
2		3	1	0.5

Table 2: The specifications parameters of the flexible members [38].

$m = 9$ (Kg)	$\rho = 1.4$ (g/cm^3)
$L_b = 4$ (m)	$E = 60$ (GPa)
$a = 16$ (cm^2)	$EI = 20$ (N·m^2)

5. Simulation Results and Discussions

Considering the same geometric and mass parameters for the main rigid system as given in Table 1 as they are applied by [27], two solar panels each 4 m long whose specification parameters are included in Table 2, are appended to the base. The assumed space robot with its geometric parameters and the defined frames is represented in Figure 3. Also, MATLAB/SIMULINK program is used to simulate and implement the stated control algorithm on the assumed space robot. So, the initial condition of the simulation study is stated in Table 3 which is the same for the two cases. It should be noted that the extracted dynamics model of space robot was verified by [4, 5]. As mentioned before, two cases of implementation of the extended MIC law are studied in simulations based on the designed trajectories, that is, CASE-I for the designed MTT and CASE-II for the circular path which has the radius of 15 m. These maneuvers are important cases since flexible modes are stimulated due to on-off nature of actuating forces in the first one, and centripetal accelerations in the second one. In each case, the simulations are done for the space robot with flexible solar panels (FSP) and without those, while the controller gain matrices remain the same. It should be noted that if the robotic system does not have FSP, then the extended MIC law changes to the original form of the MIC law.

First, to compare the MIC algorithm with the extended MIC, the MIC algorithm is applied on the assumed systems without FSP, and the operation is studied. Thus, considering the rigid system assumption (without FSP) for CASE-I, the MIC law can perfectly control the object on the designed path (Figures 4 and 5), which is absolutely misleading. Therefore, in a realistic analysis for practical implementations, the flexible effects should be considered, while those severe vibrations may even lead to fracture of flexible solar panels. However, as shown in Figures 6 and 7 for CASE-I and using the same controller gains, the effects of the disturbance forces due to vibrations of the flexible solar panels could create some errors in the position and the velocity, whereas the disturbance forces that generate these errors are shown in Figure 8 and

TABLE 3: The initial condition of the simulation study.

$\mathbf{q}(0) = [0\ 0\ 0\ 0.6\ 4.2\ 5.6\ 2.1\ \mu/7\ \pi/36]^T$	$\dot{\mathbf{q}}(0) = [0.85\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$
$\bar{\mathbf{q}}_f(0) = [0\ \ 0\ \ 0]^T$	$\dot{\bar{\mathbf{q}}}_f(0) = [0\ \ 0\ \ 0]^T$
$\mathbf{K}_p = \widetilde{\mathbf{K}}_p = 900$	$\mathbf{K}_d = \widetilde{\mathbf{K}}_d = 300$

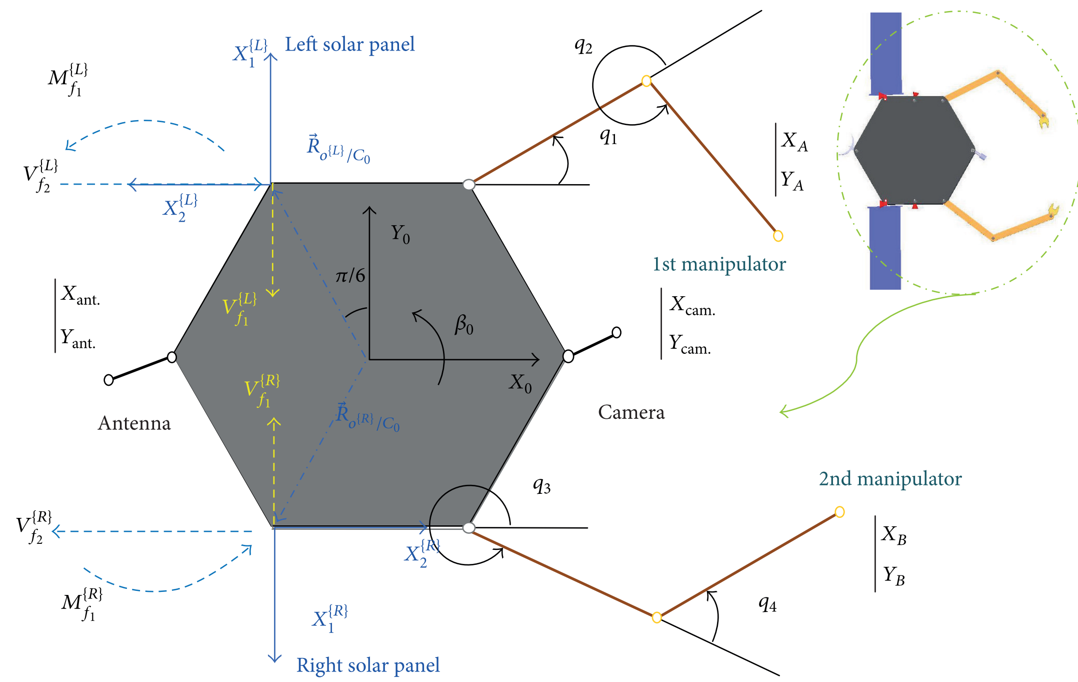

FIGURE 3: Schematic representation of assumed space robot system and its parameters and frames.

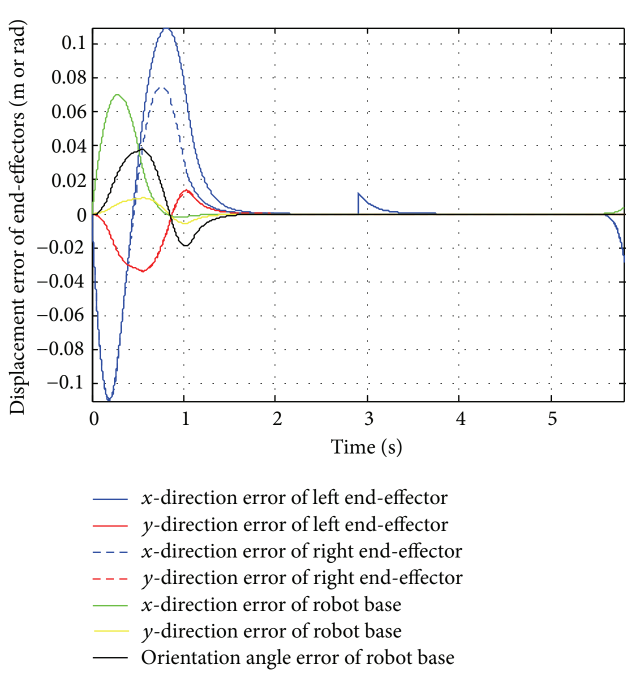

FIGURE 4: Error of the work space variables during object manipulation in CASE-I without FSP.

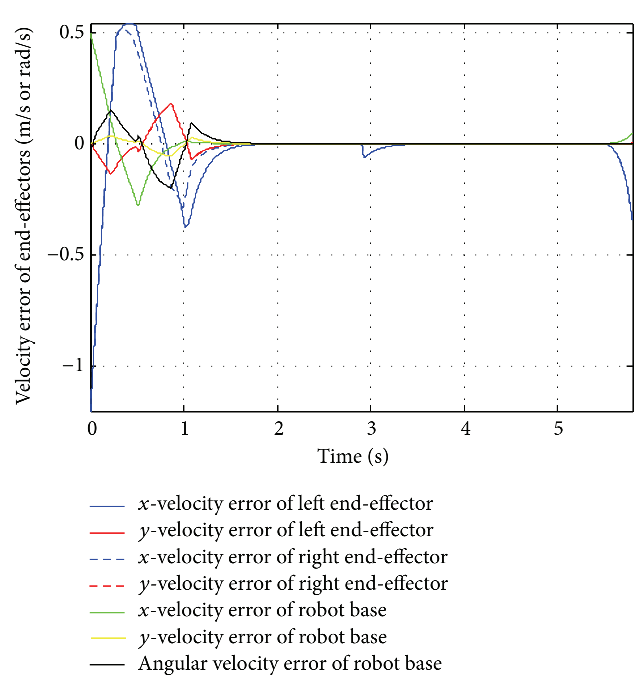

FIGURE 5: Error of the variable rates during object manipulation in CASE-I without FSP.

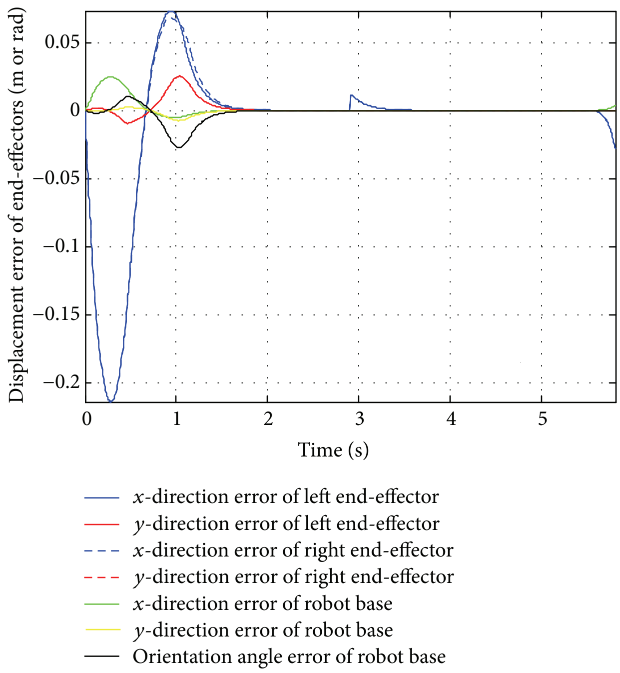

Figure 6: Error of the work space variables during object manipulation in CASE-I.

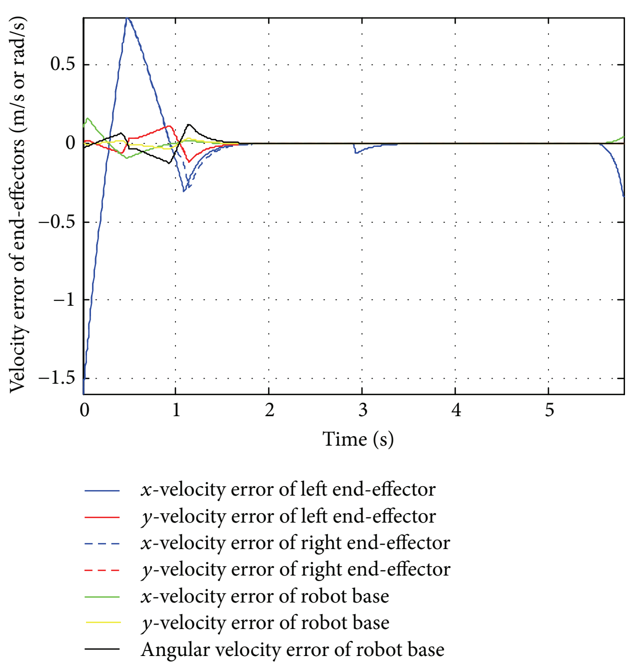

Figure 7: Error of the variable rates during object manipulation in CASE-I.

they could disturb the operation. These disturbing forces are caused by the deflection of the left and the right solar panels as shown in Figure 9. Although this is certainly undesirable, it should be controlled by this efficient strategy which is the property of the extended MIC controller than the original MIC algorithm. For instance, noting to some considerations

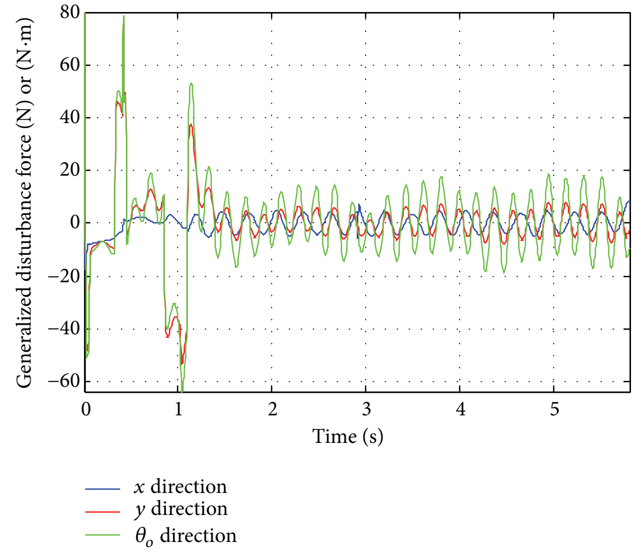

Figure 8: Generalized disturbance forces during object manipulation in CASE-I.

in path planning such as a constant speed for the robot base, or at least not inducing on-off impact disturbances, the extended MIC controller can successfully perform the object manipulation task. If the defined generalized variable of the robot base and the End-Effectors (the generalized workspace variables of the robot or $\widetilde{\mathbf{X}}$) are entirely controlled, the object would be on the desired path in which this is the basis of the MIC algorithm. It means that the object manipulation task is well done, while the flexible solar panels just stay true. This will be studied in the second case too. Also, an animated view of the system performing the object manipulation during the designed MTT of CASE-I is shown in Figure 10.

The implementation of the extended MIC law to move the object and accordingly the whole system on a circular path (CASE-II) is shown in Figures 11–15. This operation was done by the defined generalized variable of the robot base which is shown in Figure 3. Assuming a rigid system (without FSP), the original form of the MIC law can successfully complete the object manipulation task based on the planned path with negligible errors [4, 5]. Although the object manipulation task is successfully done in this case with FSP too, it would not be succeeded if the considerations in path designing such as a constant speed for the robot base are not taken into account, because the simulation was stopped after the flexible solar panels had been fractured. By using such considerations as discussed in CASE-I, the proposed extended MIC controller can effectively perform the task as shown in Figures 11 and 12. Deflection of solar panels is shown in Figure 13. Also, the disturbance forces of Figure 14 are produced due to these deflections along the designed circular path. An animated view of the system performing the task in CASE-II is shown in Figure 15. It should be noted that we have assumed that the space free-flying robot with the flexible solar panels possesses an initial velocity in all of these simulations. Also,

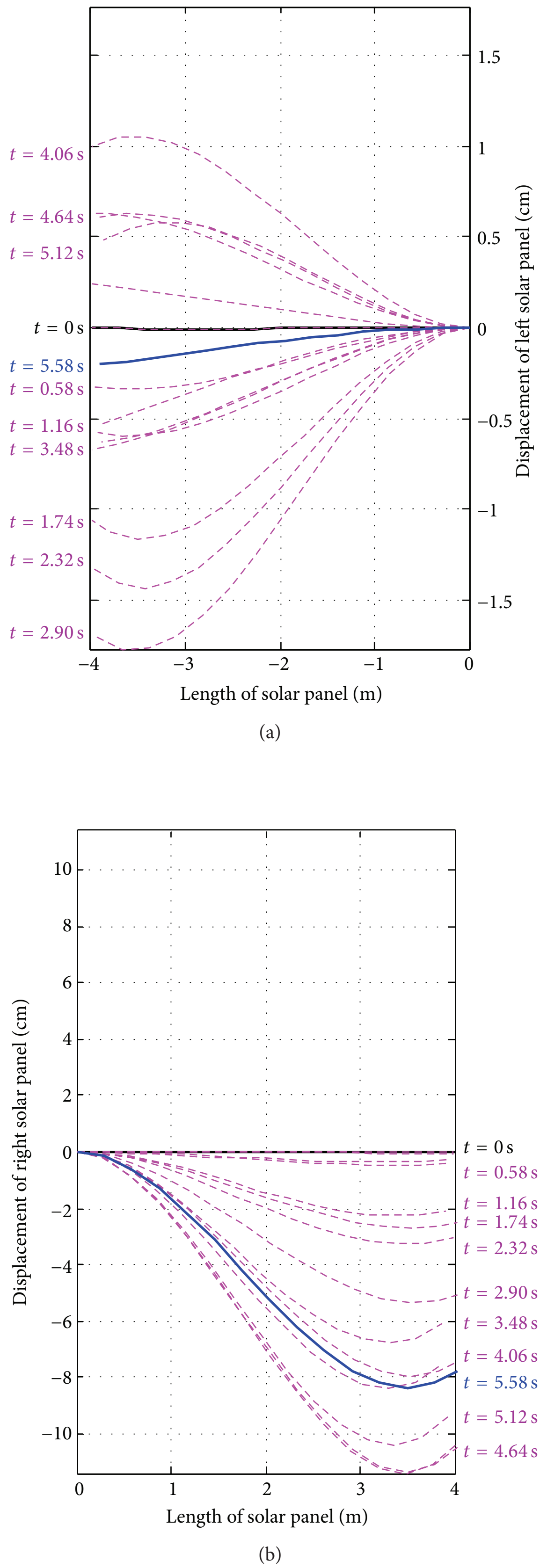

FIGURE 9: Deflection time history of the left and right flexible solar panels in CASE-I.

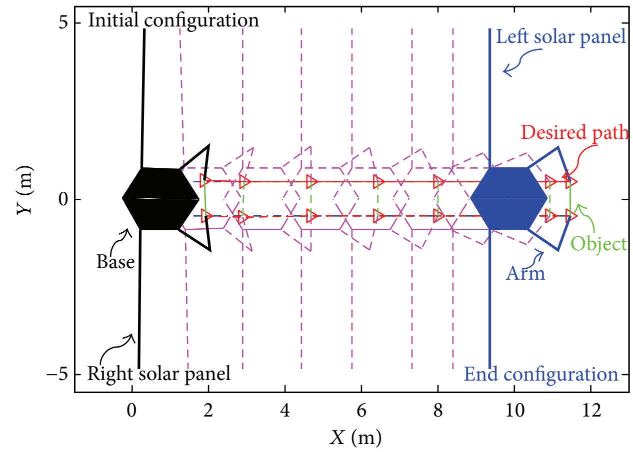

FIGURE 10: An animated view of the system during the object manipulation in CASE-I.

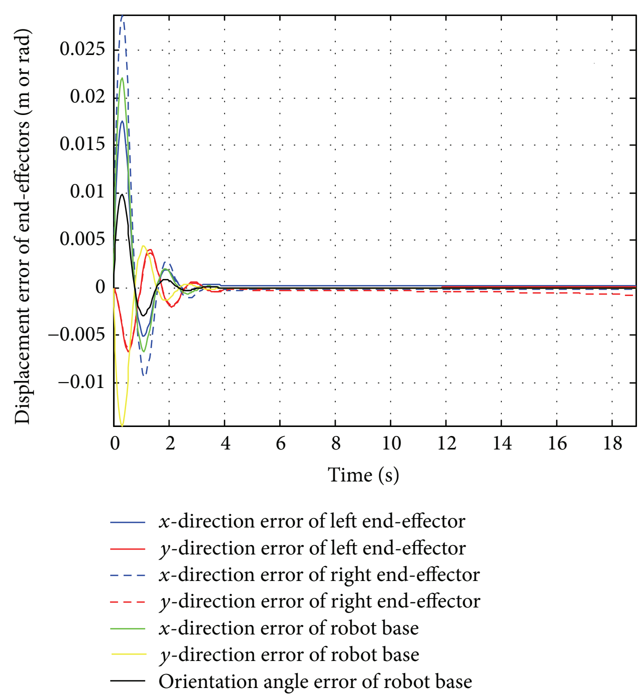

FIGURE 11: Error of the work space variables during object manipulation in CASE-II.

the flexible solar panels are at rest at the beginning of each operation as shown in Figures 9 and 14. It should be noted that by implementing the MIC algorithm on these two case studies, the simulation is broken since the flexible solar panels have more than the allowed displacement. This results in the fracture of these flexible appendages, and therefore, the object manipulation operation would not be perfectly done.

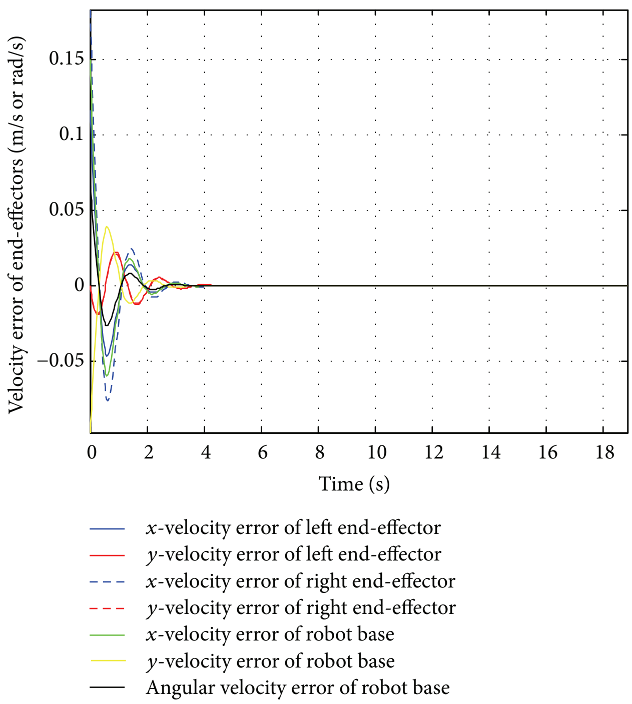

FIGURE 12: Error of the variable rates during object manipulation in CASE-II.

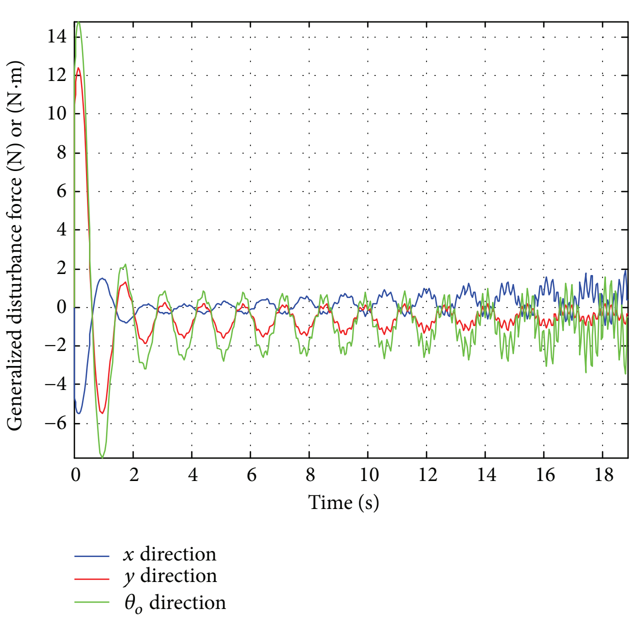

FIGURE 13: Generalized disturbance forces during object manipulation in CASE-II.

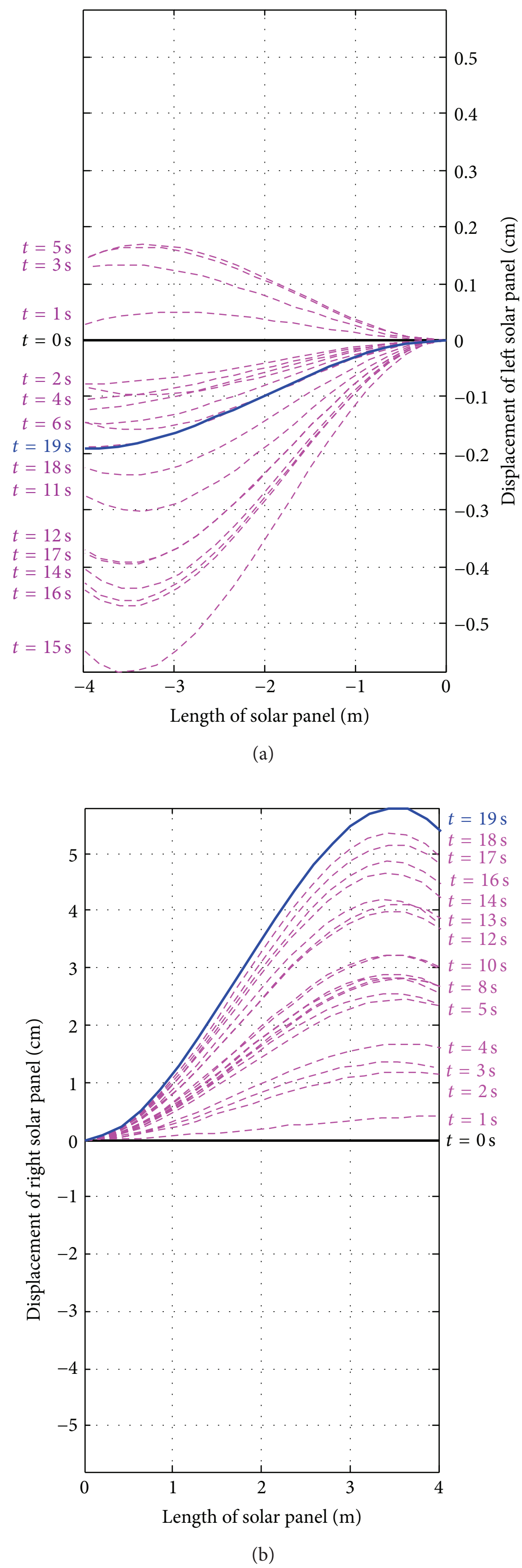

FIGURE 14: Deflection time history of the left and right flexible solar panels in CASE-II.

6. Conclusions

In this paper, dynamics modelling and control of a space robotic system with flexible appendages during a cooperative

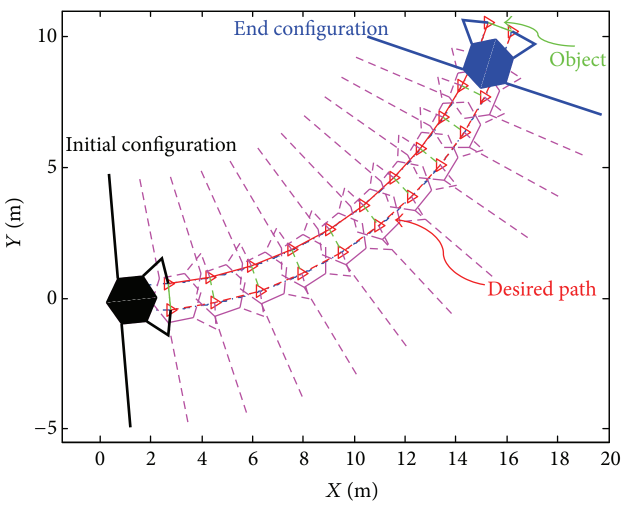

FIGURE 15: An animated view of the system during the object manipulation in CASE-II.

object manipulation task was discussed. This rigid-flexible multibody system necessitated delicate force exertion by numerous end-effectors to move the object along a specified path. After expressing the dynamic equations of a rigid space robot, the equations of the deformable bodies of this flexible multibody system were extracted. After that, the relationship between these two sets of equations was studied to obtain a practical precise dynamic model for designing the controller. So, the achieved dynamics model of a rigid-flexible multibody system decreased the computations of dynamics analysis, and it was useful in model-based control algorithms. Next, based on genetic algorithm approach using MATLAB/GATOOL, the trajectory was designed to move along a straight line based on a minimum time trajectory. This that the fast dynamics of the multibody system to be stimulated, and its effects could be studied. Then, an extended multiple impedance control was proposed and implemented on this system for object manipulation. As detailed, the multiple impedance control law was extended to compensate the disturbing forces due to vibrating motion of flexible appendages. Finally, by a comprehensive simulation routine, the obtained results were studied for the designed minimum time trajectory that complete the straight operation (CASE-I) and for the circular path (CASE-II). It was shown that vibration of the flexible solar panels results in generalized disturbance forces that are applied on the mobile base of the robot. These disturbance forces lead to undesirable errors of the end-effectors which were eliminated by the extended MIC controller. Moreover, these severe vibrations may even lead to fracture of flexible solar panels by applying the multiple impedance control. So, in this realistic analysis, the flexible effects could be considered and controlled by the stated efficient strategy. Furthermore, by noting some considerations in path planning to prevent on-off impact disturbances, the extended MIC controller could successfully perform the object manipulation task for these complicated rigid-flexible systems even if the straight path was designed based on a minimum time trajectory.

Nomenclature

$\mathbf{C}$: Vector of quadratic nonlinear terms of velocity where $\widetilde{\mathbf{C}}$ defines in task space
$\mathbf{G}$: Grasp matrix
$\mathbf{H}$: Positive definition mass matrix of system where $\widetilde{\mathbf{H}}$ defines in task space
$\mathbf{I}$: Second moment of area
$\mathbf{J}_c$: The Jacobian matrix for the manipulators
$\mathbf{K}$: Stiffness matrix of flexible member
$\mathbf{K}_p, \mathbf{K}_d, \mathbf{M}_{\text{des}}$: Gain matrix of controller for object
$\widetilde{\mathbf{K}}_p, \widetilde{\mathbf{K}}_d, \widetilde{\mathbf{M}}_{\text{des}}$: Gain matrix of controller for system in task space
$\mathbf{M}_f$: Positive definition mass matrix of flexible member
M_{sys}: Total mass of the rigid subsystem
$\mathbf{q}$: Entity vector of generalized coordinate of rigid system
$\overline{\mathbf{q}}$: Entity vector of generalized coordinate of flexible body
$\overline{\mathbf{q}}_f$: Vector of elastic generalized coordinate of flexible body
$\overline{\mathbf{q}}_r$: Vector of reference or rigid generalized coordinate of flexible body
$\mathbf{Q}$: Vector of generalized forces
$\widetilde{\mathbf{Q}}_{\text{app}}$: Vector of applied control forces
$\mathbf{Q}_e$: Vector of generalized external forces of the flexible members
$\mathbf{Q}_{\text{flex.}}$: Vector of generalized forces due to stimulation of the flexible members
$\widetilde{\mathbf{Q}}_m$: Vector of control forces for end-effector motion
$\widetilde{\mathbf{Q}}_{\text{react}}$: Vector of forces in task space that is exerted from object to end-effectors
$\mathbf{Q}_v$: Quadratic velocity vector of flexible member
$\mathbf{R}_{C_0}, \dot{\mathbf{R}}_{C_0}, \ddot{\mathbf{R}}_{C_0}$: Vector of position, velocity, and acceleration of robot bases in inertial frame
s: Path parameter
$\mathbf{W}$: Task weighting matrix
$\mathbf{X}_E^{(m)}, \dot{\mathbf{X}}_E^{(m)}$: Vector of position and velocity of (m)th end-effectors
$\boldsymbol{\beta}_0$: Generalized Euler angles variables of the robot base
$\boldsymbol{\theta}$: Generalized variables of the robot joints
$\boldsymbol{\theta}_o$: Object orientation angle
ω: Angular velocity.

Superscript

$\{i\}$: Counter of flexible member
(i): Counter of rigid member of manipulators.

Subscript

f: Showing flexibility in a member
r: Showing rigidly in a member
0: Index of the base.

Conflict of Interests

The authors declare that they do not have any conflict of interests with others.

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