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Research Article

Vibration, Oscillation, and Escape of the Fiber-Optic Signal under Two-Frequency Perturbations

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Chaos occurs easily in the nonlinear Schrödinger equation with external perturbations owing to the absence of damping. For the process of information transmission, the perturbation will cause distortion. If we add a suitable controller, it is easy to discover that chaos still appears in the process of propagation of fiber-optic signal when the strength of controller is weak. With the strength of controller increasing, the propagation of fiber-optic signal will arrive at the stable state. As the strength exceeds a certain degree, the propagation of fiber-optic signal system would tend toward the unstable state. Moreover, we consider the parameters' sensitivity to be controlled. The result demonstrates that the nonlinear term parameter and the two quite different frequencies have less effect on the propagation of fiber-optic signal. Meanwhile, the phenomena of vibration, oscillation, and escape occur in some regions.

1. Introduction

The cubic quintic nonlinear Schrödinger (NLS) equation

$$iq_t + aq_{xx} + b\left(|q|^2 + |q|^4\right)q = 0 \tag{1}$$

is extensively used in various fields, especially for the process of the fiber-optic signal propagation [1]. Here x represents the nondimensional distance along the fiber-optic signal, t represents time in a dimensionless form, and a and b are real valued constants. The dependent variable function $q(x,t)$ is a complex valued function that represents the signal wave profile. In a general way, (1) is a nonlinear partial differential equation, which is completely integrable on the infinite line or periodic boundary conditions in one dimension.

As a matter of fact, the propagation of fiber-optic signal must be perturbed with external environment. Extensive systems with external perturbations have been widely investigated by using analytic methods and numerical simulations. A mass of bifurcation sets, the routes to chaos, and Lyapunov exponents are given in [2–4]. More attention has been paid to the interaction of external perturbations. The analysis of complex dynamics in three-well forcing or other systems with two external forcings are shown in [5–7].

Although these researches have played a certain role in chaos control, there are rarely researches on the fiber-optic signal models with two-frequency perturbations. However, two-frequency perturbations can model the fiber-optic signal under complex external conditions better. Hence, we consider the fiber-optic signal system (1) under two-frequency perturbations

$$\begin{aligned} &iq_t + aq_{xx} + b\left(|q|^2 + |q|^4\right)q \\ &\quad = \gamma_1 \cos(\omega_1 x)\exp(-\sigma i t) + \gamma_2 \cos(\omega_2 x)\exp(-\sigma i t), \end{aligned} \tag{2}$$

where γ_i $(i = 1, 2)$ are the amplitudes of the perturbations and w_j $(j = 1, 2)$ are the frequencies; σ is the velocity of a certain signal.

The research purposes of this paper are the two following vital points. Firstly, it seems that chaos may be unavoidable under perturbations and has been observed in many practical applications such as engineering, biology, industry, and production [8–14]. Will the system (2) appear chaotic? If chaos appears, how do we design a controller to suppress chaos owing to the complex nonlinear term of system (2)? It is observed that there is no damping in the system. Once perturbed under external forcings, the system may tend

toward chaotic state easier. Therefore, we will select the controller which has the same function of the damping.

The second interesting problem is to analyze the parameter regions for fiber-optic signals stable propagation of the controlled system; that is, we will discuss the parameters' sensitivity to be controlled. The research is very important in practice. Through analysis for the parameters sensitivity of controlled system, we can get a set of reasonable parameters to guarantee the propagation of fiber-optic signal smoothly. For instance, the characters of media for fiber-optic signal relate to the system parameters. According to the analysis for the parameters' sensitivity to be controlled, we can obtain parameters in the controlling regions or design suitable media for signal propagation as to extremely reduce the impact from perturbations.

In this paper, we investigate the system (2) in detail. Via the four-order Runge-Kutta method, we also study the chaotic behaviors due to their high precision. Lyapunov exponents and bifurcation diagrams are used to show the behaviors of the fiber-optic signal system in some certain parametric regions. The plan of the paper is as follows: in Section 2 we analyze the chaotic behaviors and control of the fiber-optic signal system with two periodically perturbations. Numerical results concerning the fiber-optic signal propagation system are given for different parameters in Section 3. Last section is the conclusion.

2. Chaotic Behavior and Control

In this section, we consider the following model of the fiber-optic signal propagation system with two-frequency perturbations:

$$\begin{aligned} a\phi''(x) + \sigma\phi(x) + b\left(\phi^3(x) + \phi^5(x)\right) \\ = \gamma_1 \cos(w_1 x) + \gamma_2 \cos(w_2 x), \end{aligned} \tag{3}$$

by substituting the traveling wave solution $q(t,x) = \phi(x)\exp(-\sigma i t)$ into (2), where a, b, σ, γ_i $(i = 1,2)$ and w_j $(j = 1,2)$ are real parameters, respectively. Physically, γ_i $(i = 1,2)$ can be regarded as the amplitudes of the perturbations and w_j $(j = 1,2)$ as the frequencies; For this reason, $w_j \geq 0$ $(j = 1,2)$, $\gamma_i \geq 0$ $(i = 1,2)$, σ is the strength of linear, and b is the strength of nonlinear.

As in the general case, we assume that $a = 1$ and take the transformation

$$\varphi(x) \longrightarrow x_1, \qquad \varphi'(x) \longrightarrow x_2 \tag{4}$$

then we can obtain

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= -\sigma x_1 - b * \left(x_1^3 + x_1^5\right) + \gamma_1 \cos(\omega_1 x) + \gamma_2 \cos(\omega_2 x). \end{aligned} \tag{5}$$

Next we consider chaotic behaviors in system (5) by setting $\sigma = -0.08$, $b = 0.25$, $\gamma_2 = 0.6$, $w_1 = 1$, and $w_2 = 1.2$ with the initial conditions $x_1 = 1$, $x_2 = -1$. From the results given by bifurcation diagram and maximum Lyapunov exponent (see Figure 1), we can arrive at the conclusion that the system (5) is chaotic with a positive Lyapunov exponent within γ_1 in [0,150].

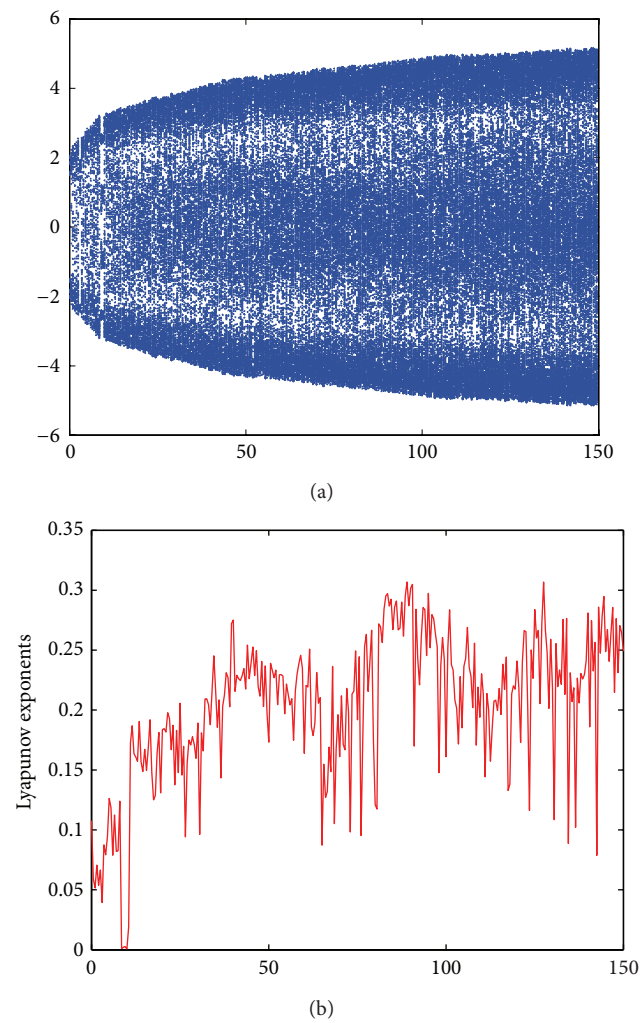

Figure 1: (a) Bifurcation diagram of system (5) in (γ_1, x) plane. (b) Maximum Lyapunov exponent corresponding to (a).

The system (5) is very sensitive to its parameter and chaos often causes irregular behavior, so chaos is undesirable. Thus, a suitable controller is desired for the fiber-optic propagation to suppress the chaos. It is not difficult to find that the system (5) is similar to the duffing system except the absence of damping in the system (5). Once perturbed with external forcing, the system may be in chaotic state. Therefore, we will select a controller that has the same function with the damping to control the chaos. The system (5) can be considered as follows:

$$\begin{aligned} iq_t + aq_{xx} + b\left(|q|^2 + |q|^4\right)q \\ = \gamma_1 \cos(\omega_1 x)\exp(-\sigma i t) + \gamma_2 \cos(\omega_2 x)\exp(-\sigma i t) \\ + \varepsilon\left(k_1 q + k_2 q_x\right). \end{aligned} \tag{6}$$

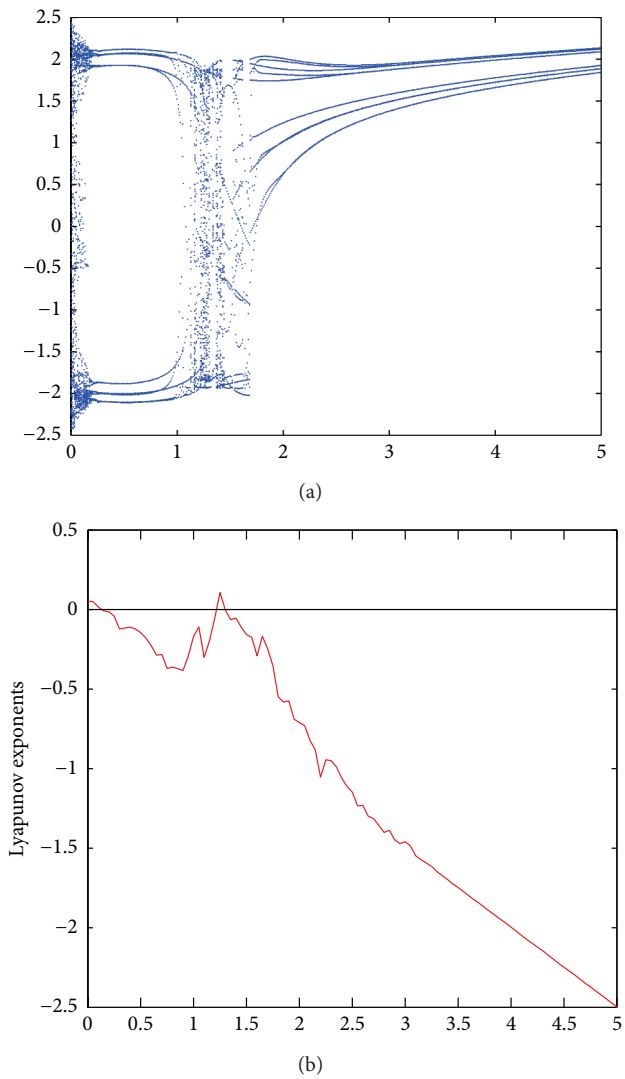

FIGURE 2: (a) Bifurcation diagram of system (8) in (ε, x) plane. (b) Maximum lyapunov exponent corresponding to (a).

We take the transformation $q(t,x) = \phi(x)\exp(-\sigma it)$, then (6) takes the form as follows:

$$\begin{aligned} a\varphi''(x) + \sigma\varphi(x) + b\left(\varphi^3(x) + \varphi^5(x)\right) \\ = \gamma_1\cos(\omega_1 x) + \gamma_2\cos(\omega_2 x) + \varepsilon\left(k_1x_1 + k_2x_2\right). \end{aligned} \tag{7}$$

Via the above transformation, we can get the following form:

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= -\sigma x_1 - b * \left(x_1^3 + x_1^5\right) + \gamma_1\cos(\omega_1 x) \\ &\quad + \gamma_2\cos(\omega_2 x) + \varepsilon\left(k_1x_1 + k_2x_2\right), \end{aligned} \tag{8}$$

where ε denote the strength of the controller, and k_1 as well as k_2 are real coefficients.

Now we consider the behavior of system (6) by setting $\sigma = -0.08$, $b = 0.25$, $\gamma_1 = 2$, $\gamma_2 = 0.6$, $w_1 = 1$, $w_2 = 1.2$, $k_1 = 1$, and $k_2 = -1$. Figure 2 presents the bifurcation diagram

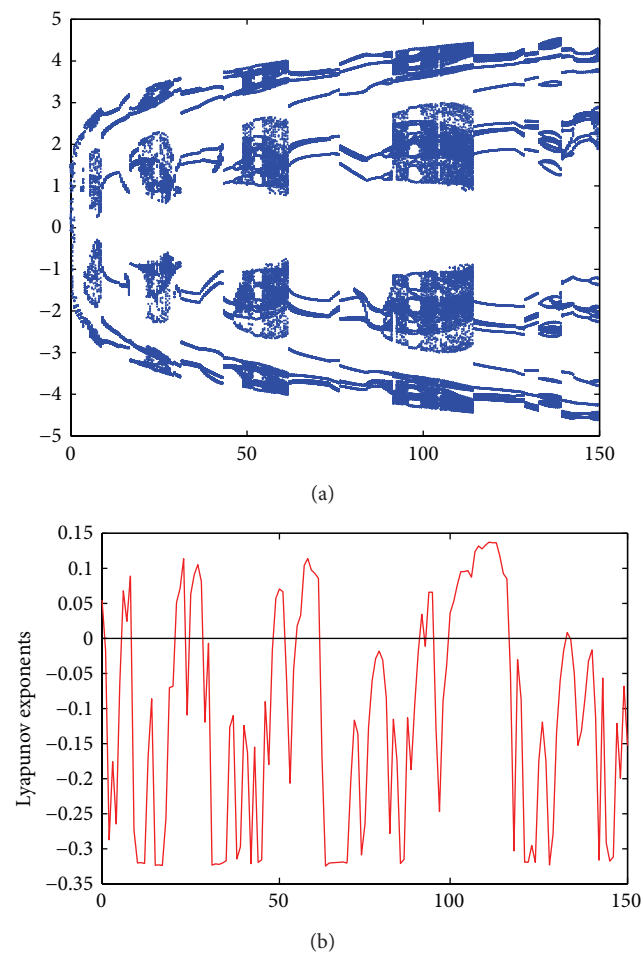

FIGURE 3: (a) Bifurcation diagram of system (8) in (γ_1, x) plane. (b) Maximum Lyapunov exponent corresponding to (a).

and maximum Lyapunov exponent of system (6) with the setting parameters. When the parameter of controller ε is varying, one can find that when ε in [0,0.02], the parameter of controller is so small that the chaotic state cannot be controlled. As the parameter ε continuously increased, the state of chaos in system (6) can be controlled within the region ε in [0.02,1.1]. The process for reverse periodic or chaotic within the region ε in [1.1,1.4]. Until when $\varepsilon > 1.4$, the fiber-optic signal will leak from the media, which is called escape.

Remark 1. From the above analysis, as the strength of controller sufficiently increases, the system will be more stable, so the selection of controller's parameter should be proper. As ε gets more and more close to 1.2, the signal propagation system (8) is more likely to be chaotic. Whenever ε is too small or too big, the chaos cannot be well controlled. So we should avoid selecting the parameter ε that is close to the frequencies for the signal system (8).

3. Parameters' Sensitivity to Be Controlled

The controlled fiber-optic signal propagation system (8) has a series of parameters. Certainly, every parameter of the system

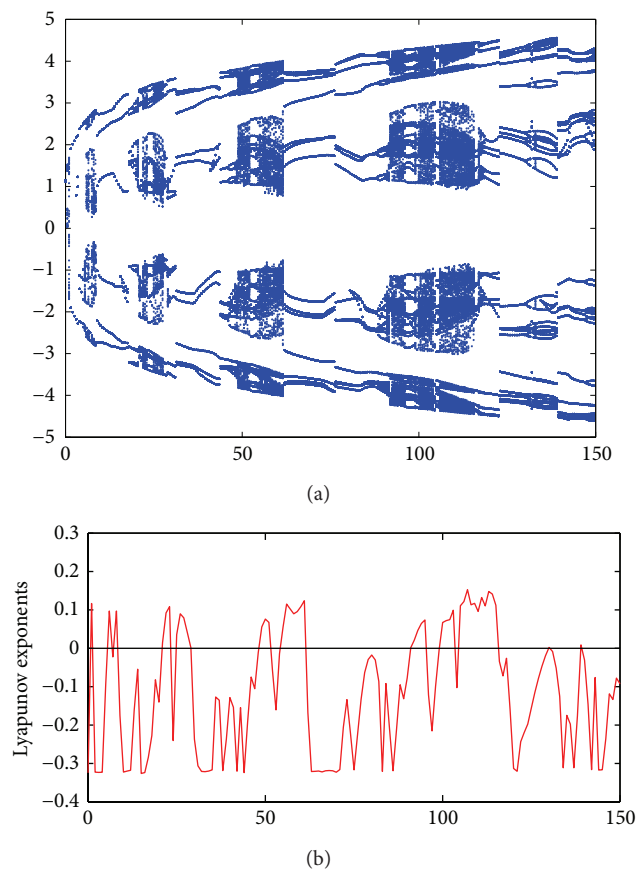

FIGURE 4: (a) Bifurcation diagram of system (8) in (γ_1, x) plane. (b) Maximum Lyapunov exponent corresponding to (a).

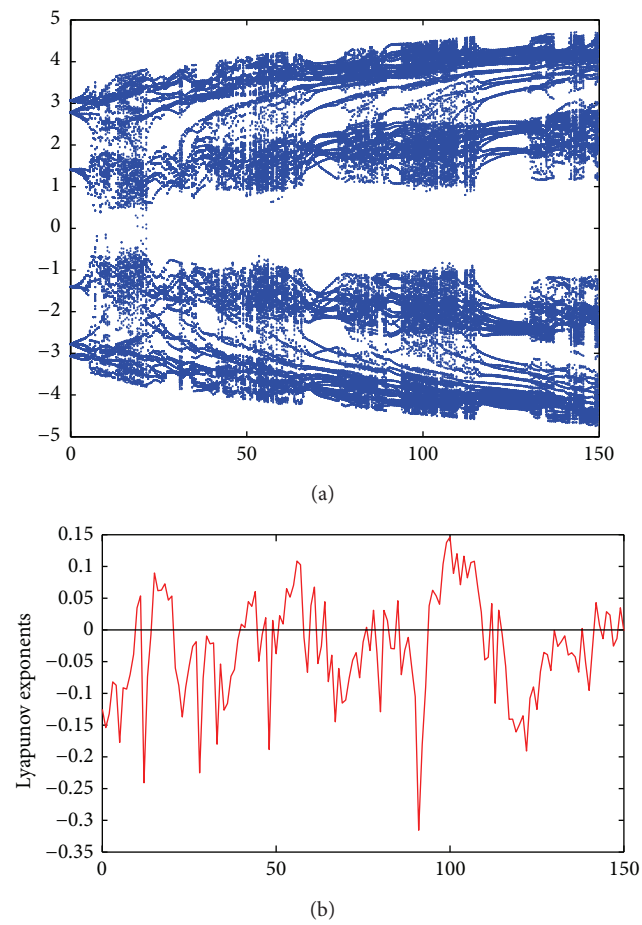

FIGURE 5: (a) Bifurcation diagram of system (8) in (γ_1, x) plane. (b) Maximum Lyapunov exponent corresponding to (a).

(8) plays an important role in the process for the signal propagation. In this section, we will analyze the parameter regions for fiber-optic signal propagation of controlled system (8) with the initials $x_1 = 1$, $x_2 = -1$.

Case 1. Analysis of parameters of perturbations varying γ_1 in range $0 < \gamma_1 < 150$:

(1) setting $\sigma = -0.08$, $b = 0.25$, $\gamma_2 = 0.6$, $w_1 = 1$, $w_2 = 1.2$, $k_1 = 1$, $k_2 = -1$, $\varepsilon = 0.65$;

(2) setting $\sigma = -0.08$, $b = 0.25$, $\gamma_2 = 0.6$, $w_1 = 1$, $w_2 = 12$, $k_1 = 1$, $k_2 = -1$, $\varepsilon = 0.65$;

(3) setting $\sigma = -0.08$, $b = 0.25$, $\gamma_2 = 20$, $w_1 = 1$, $w_2 = 1.2$, $k_1 = 1$, $k_2 = -1$, $\varepsilon = 0.65$;

(4) setting $\sigma = -0.08$, $b = 0.25$, $\gamma_2 = 20$, $w_1 = 1$, $w_2 = 12$, $k_1 = 1$, $k_2 = -1$, $\varepsilon = 0.65$.

Bifurcation diagrams and Lyapunov exponents of system (8) are given as follows:

For situation (1), the bifurcation diagram of system (8) in (γ_1, x) plane is given in Figure 3(a) and Figure 3(b) is the maximum Lyapunov exponent corresponding to Figure 3(a). The neighborhood of Figure 3(a) is from 0 to 150. Especially, when the parameter γ_1 is chosen from the locals of Figure 3(a) such as γ_1 in [0,0.21], [0.35,0.75], [5.25,8.77], [24.23,28.37], [48.15,51.98], [99.15,104.31], and [104.5,110.9] with two small frequencies w_i $(i = 1, 2)$, the system may easily tend to be chaotic.

For the situation (2), the bifurcation diagrams of system (8) in (γ_1, x) plane are given in Figure 4(a), and Figure 4(b) is the maximum Lyapunov exponent corresponding to Figure 4(a). The region of Figure 3(a) is from 0 to 150. Especially, if the parameter γ_1 is chosen from the intervals of Figure 4(a) for γ_1 in [0.5,0.725] and [5.21,8.77], the system (8) appears chaotic. With the increasing of amplitude, when $\gamma_1 > 20$, the system may easily tend to be chaotic.

For the situation (3), the bifurcation diagrams of system (8) in (γ_1, x) plane are given in Figure 5(a), and Figure 5(b) is the maximum Lyapunov exponent corresponding to Figure 5(a). The neighborhood of Figure 5(a) is from 0 to 150. Especially, If the parameter is chosen from these locals of Figure 5(a) in [5.38,5.46], the system may be easily lead to chaotic state. With increasing of amplitude, when $\gamma_1 > 7$, the system may easily tend to be chaotic.

For the situation (4), the bifurcation diagrams of system (8) in (γ_1, x) plane are given in Figure 6(a), and Figure 6(b) is the maximum Lyapunov exponent corresponding to Figure 6(a). If one fixes the parameter γ_1 in the regions of

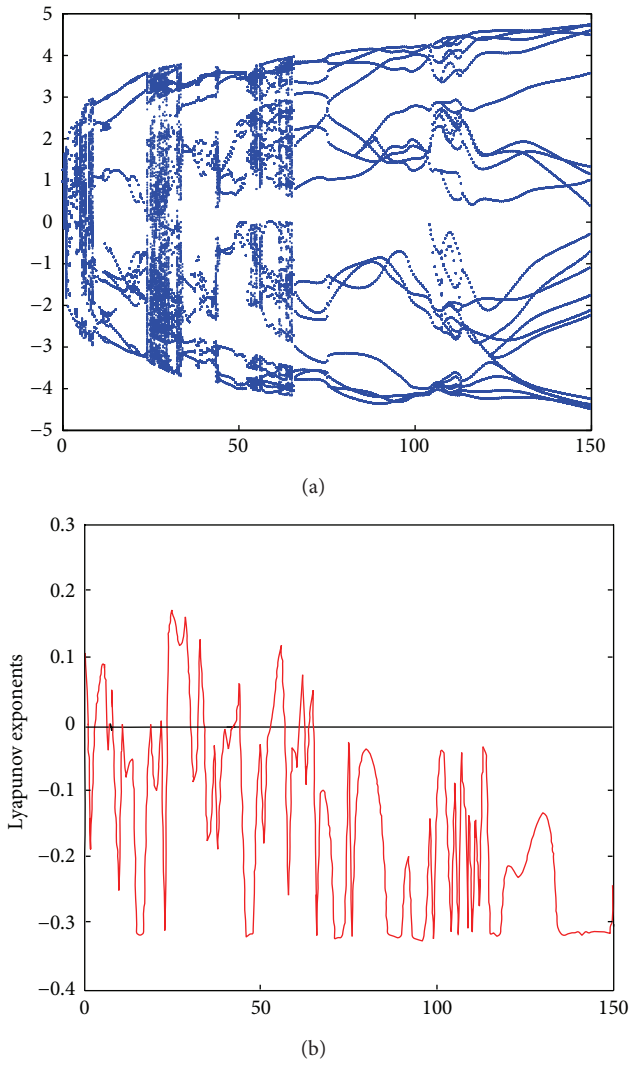

Figure 6: (a) Bifurcation diagram of system (8) in (γ_1, x) plane. (b) Maximum Lyapunov exponent corresponding to (a).

Figure 3(a) for γ_1 in [0.51,0.91], one will observe that the system may easily tend to be chaotic. With the increasing of amplitude, when $\gamma_1 > 5$, the system may more easily tend to be of chaotic state.

Remark 2. It can be observed that if one of the frequencies of the perturbations is close to another, the system will more easily tend toward resonanance. On the contrary, if one of the frequencies of the perturbations is far away from another, the influence for fiber-optic signal is smaller.

Case 2. Analysis of linear and nonlinear parameter b for controlled system (8) under perturbations. For the system (8), varying the linear parameter σ in a range (0,20) and fixing $b = 0.25$, $\gamma_1 = 2$, $\gamma_2 = 20$, $w_1 = 1$, $w_2 = 12$, $k_1 = 1$, $k_2 = -1$, $\varepsilon = 0.65$. For Case 2, the bifurcation diagrams of system (8) in (σ, x) plane are given in Figure 7(a), and Figure 7(b) is the Maximum Lyapunov exponent corresponding to Figure 7(a). When σ is in [1.07,1.74], the fiber-optic signal will be slightly vibrating. The phenomena of vibration, oscillation, and

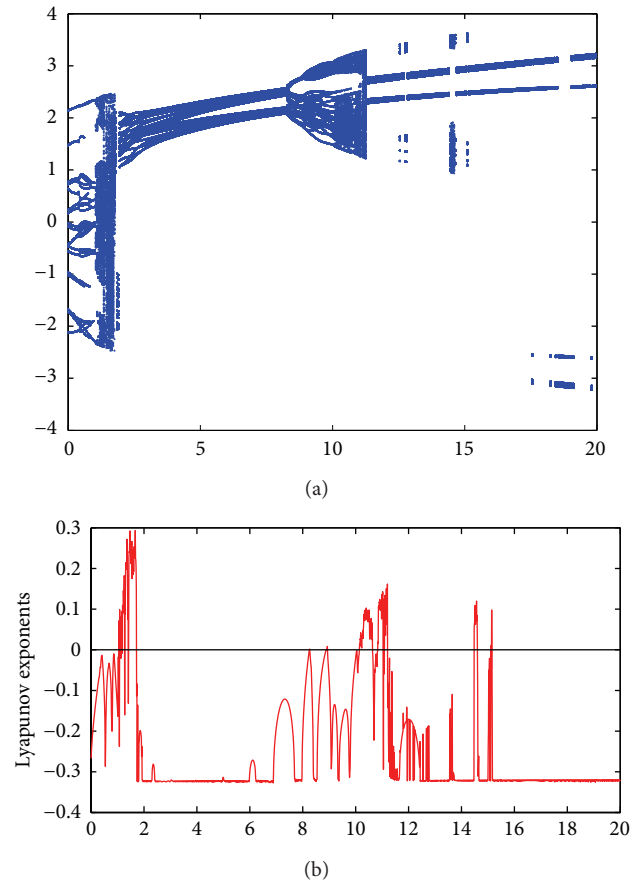

Figure 7: (a) Bifurcation diagram of system (8) in (σ, x) plane. (b) Maximum Lyapunov exponent corresponding to (a).

escape occur within the region σ in [10.15,14.57]. As the parameter σ is continuously increased, when $\sigma > 14.57$, the chaotic state of this system can be controlled.

For the system (8), varying the nonlinear parameter b in a range (0,20), and fixing $\sigma = -0.08$, $\gamma_1 = 2$, $\gamma_2 = 0.6$, $w_1 = 1$, $w_2 = 12$, $k_1 = 1$, $k_2 = -1$, $\varepsilon = 0.65$. For case 3, the bifurcation diagrams of system (6) in (b, x) plane are given in Figure 8(a), and Figure 8(b) is the maximum Lyapunov exponent corresponding to Figure 8(a). As the parameter b is continuously increased, when b is in [0,2.76], the chaotic state of this system can be controlled. The process for reverse period or chaos can be found within the regions b in [4.65,5.74], [6.71,7.43], and [15.05,18.04]. When $b > 18.04$, the amplitude of fiber-optic signal will fiercely vibrate within the region and gradually tend to be stable.

Remark 3. It can be observed that the change of parameter of nonlinear term is more easily lead to chaotic phenomena than the parameter of linear term. Hence, the linear term has less effect on the system of fiber-optic signal propagation.

4. Conclusions

Motivated by studying the duffing system with external excitations, we notice that the absence of damping in reduced

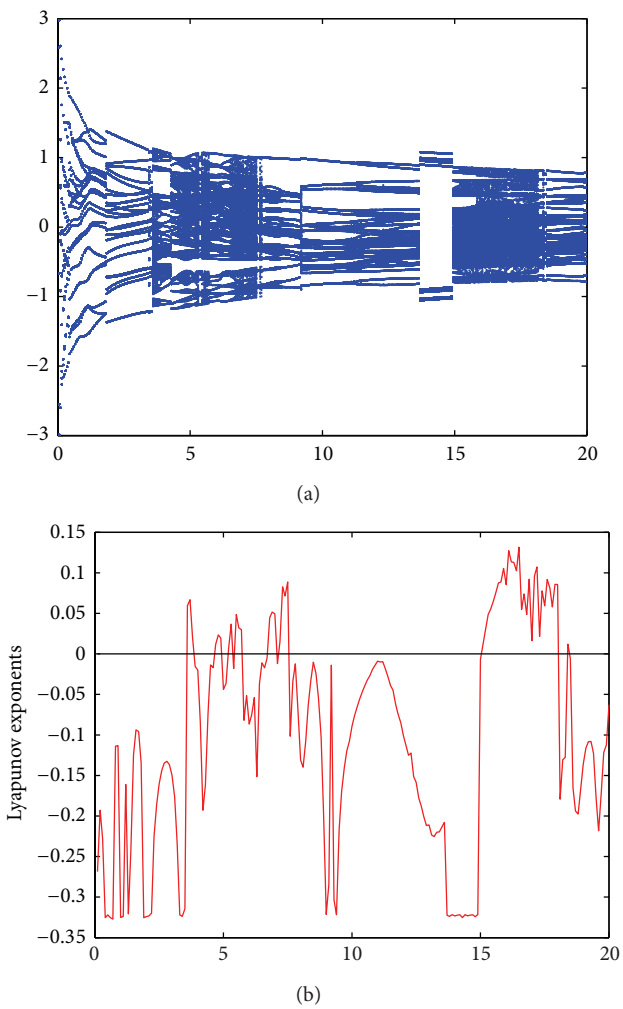

Figure 8: (a) Bifurcation diagram of system (8) in (b, x) plane. (b) Maximum Lyapunov exponent corresponding to (a).

system (5) for the fiber-optic propagation is more easily lead to chaotic state. We modified the signal system (5) into a more practical one by adding a controller, which possesses the properties of dumping. We analyze the system with two frequency perturbations and parameters' sensitivity to be controlled. Based on the above study, it may be concluded that the method is useful and efficient to suppress chaotic state. It can be extensively applied to other fiber-optic signal propagation system. Our study may be useful to further understand the effect of chaos control.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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