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Research Article

Pedagogical Visualization of a Nonideal Carnot Engine

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We have implemented a visualization tool for the demonstration of a nonideal Carnot engine, operating at finite time. The cycle time can be varied using a slide bar and the pressure-volume, temperature-entropy, power-time, and efficiency-time diagrams change interactively and are shown on one screen. We have evaluated the visualization tool among engineering students at university level during an introductory course on thermodynamics and we review and discuss the outcome of the evaluation.

1. Introduction

In 1975, Curzon and Ahlborn made the observation that most undergraduate textbooks on thermodynamics do not treat the time aspects of thermodynamic cycles; thus lacking the explanation of how power is generated from, for example, heat engines. In their investigation, Curzon and Ahlborn considered a Carnot cycle operating at finite time by modeling the time-dependent energy losses in the isotherms [1]. This approach enabled them to derive a general expression for the efficiency at maximum power, depending only on the temperatures of the reservoirs, just as the Carnot efficiency. Curzon and Ahlborn's motivation was purely pedagogical and at the time they were probably not aware that the same expression had been published already in 1957 by Novikov [2] (the original paper in Russian was published in 1957) and by Chambadal [3]. Nevertheless, this efficiency is most commonly referred to as the Curzon-Ahlborn efficiency and their work paved the way for finite time thermodynamics [4].

The purpose of this investigation is to construct an interactive visualization tool in the form of a computer program illustrating the time dependence of a nonideal Carnot engine and to evaluate this demonstration tool among university students during an introductory thermodynamics course. The reason for doing this is the lack of time and power aspects of thermodynamic cycles in undergraduate textbooks. The visualization tool may thus fill a gap in thermodynamics courses since it increases the awareness of these important engineering aspects. Illustrating the concepts by an interactive visualization may enable a more holistic understanding, which, in turn, will allow for deep learning.

2. Theoretical Model

We base our nonideal Carnot engine on Curzon-Ahlborn's model [1]; that is, the engine spends finite times in the isotherms so that the high temperature of the working medium (which we will implicitly consider to be an ideal gas), T_{hw}, will not reach the temperature of the hot reservoir, T_h. Correspondingly, the low temperature of the working medium, T_{cw}, will not reach the temperature of the cold reservoir, T_c, and we have $T_{hw} \leq T_h$ and $T_{cw} \geq T_c$. Apart from these heat losses, the engine is an ideal Carnot engine and it is schematically depicted in Figure 1.

The heat input, Q_h, and heat output, Q_c, are given by the times spent in the isotherms multiplied by the heat fluxes through the vessel containing the working medium, such that $Q_h = F_h t_h$ and $Q_c = F_c t_c$, where F_h and t_h (F_c and t_c) are the heat flux and the time spent in the hot (cold) isotherm. The heat fluxes are modeled as being proportional to the temperature differences between the working medium (an ideal gas) and the reservoirs, $F_h = \alpha(T_h - T_{hw})$ and $F_c = \beta(T_{cw} - T_c)$, where α and β are heat conduction coefficients, which depend on the material and the geometry of the vessel. If the material of the vessel is uniform, its thickness is constant, and the areas exposed to the reservoirs are the same for the hot and the cold reservoir, then $\alpha = \beta$.

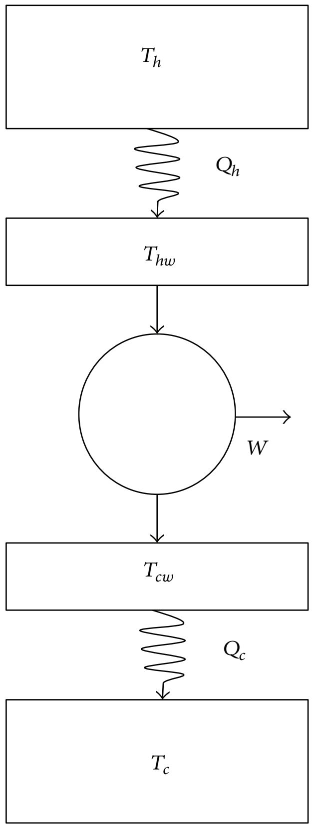

Figure 1: Schematic illustration of the Curzon-Ahlborn engine. The symbols are explained in the text. The zigzag arrows indicate the heat losses in the isotherms. The process operating between T_{hw} and T_{cw} is an ideal Carnot engine.

In our following analysis we take this to be the case and α is given by $\alpha = \kappa S/d$, where κ is the thermal conductivity of the vessel material, S is the area of the vessel that is exposed to any of the reservoirs, and d is the thickness of the vessel.

Due to the reversibility of the adiabatic stages, we note that it must hold that

$$\frac{Q_h}{T_{hw}} = \frac{Q_c}{T_{cw}} = \lambda, \tag{1}$$

where λ is a constant. Based on (1), the high and the low temperatures of the working medium can be expressed as

$$T_{hw} = \frac{\alpha t_h}{\alpha t_h + \lambda} T_h, \tag{2}$$

$$T_{cw} = \frac{\alpha t_c}{\alpha t_c - \lambda} T_c, \tag{3}$$

respectively. In the following analysis we let the engine spend equal amounts of time in the hot and the cold reservoir; that is, $t_h = t_c = t$. As $t \to \infty$, we see that $T_{hw} \to T_h$ and $T_{cw} \to T_c$. By taking this limit in (1), Q_h approaches its ideal value $nRT_h \ln(V_2/V_1)$ and we can identify $\lambda = nR\ln(V_2/V_1)$.

Power is defined as work per time and the output power of the engine can be calculated as work per cycle, $Q_h - Q_c$, divided by the cycle time. If we assume that the times spent in the adiabats are considerably shorter than the times spent in the isotherms, t for each isotherm, the cycle time is approximately given by $2t$ and the power can be expressed as

$$P = \frac{Q_h - Q_c}{2t} = \frac{\lambda\alpha}{2}\left(\frac{T_h}{\alpha t + \lambda} - \frac{T_c}{\alpha t - \lambda}\right). \tag{4}$$

The efficiency is given by

$$\eta = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{T_{cw}}{T_{hw}} = 1 - \frac{\alpha t + \lambda}{\alpha t - \lambda}\,\frac{T_c}{T_h} \tag{5}$$

and we immediately see that η approaches the Carnot efficiency, $\eta_C = 1 - T_c/T_h$, for long cycle times ($t \to \infty$). By differentiating (4) with respect to t, we see that P has optima at the times given by

$$\frac{\alpha t - \lambda}{\alpha t + \lambda} = \pm\sqrt{\frac{T_c}{T_h}}. \tag{6}$$

The equation with negative right hand side in (6) can be discarded, since it is nonphysical. Its solution is a time shorter than the minimum time, which in turn is a solution to $P = 0$. By instead inserting the time corresponding to the positive right hand side in (6) into (5), we get the efficiency at maximum power, the so-called Curzon-Ahlborn efficiency,

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}. \tag{7}$$

3. Visualization Tool

With our visualization tool we show the time dependence of four interesting properties of the nonideal Carnot engine simultaneously on one screen: the pressure-volume, the temperature-entropy, the power-time, and the efficiency-time diagrams. The reason that we restrict the tool to show precisely these four properties is that they are the most essential for the understanding of the nonideal, finite time Carnot engine. The pressure-volume and the temperature-entropy diagrams are well covered in introductory thermodynamics courses; therefore, it is instructive to see how these diagrams change as an effect of a finite cycle time. The power-time diagram shows the output power as a function of time. It should also be instructive to see that this curve has a maximum and also at what efficiency this maximum power is reached, which is shown in the efficiency-time diagram. In these two time-dependent diagrams it is also obvious that the engine cannot generate any power when operated at Carnot efficiency.

The time the engine spends in one of the isotherms can be interactively varied. We have implemented the tool as a cdf (computational document format) document [5]. This is an interactive document format that is programmed using the software Mathematica. cdf documents can be used and viewed with the free cdf player that can be downloaded at [5]. This format can be considered as a candidate for innovative,

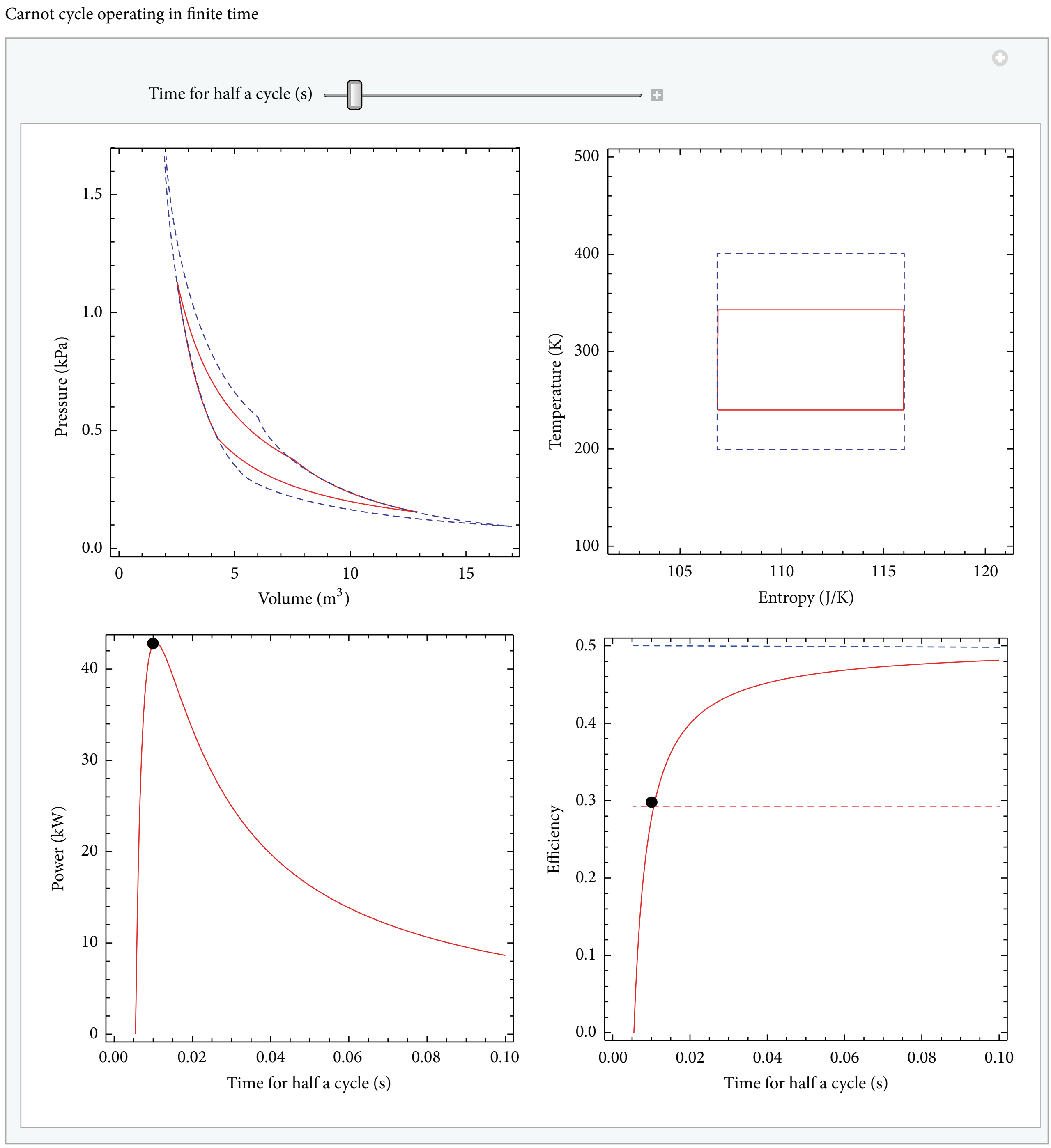

Figure 2: Snapshot of the visualization tool. The time spent in one of the isotherms (time for half a cycle) can be interactively changed. The blue dashed curves in the pressure-volume and temperature-entropy diagrams represent the curves for the ideal Carnot engine (infinite cycle time), while the red curves correspond to the current value of the time for half a cycle. The black dots on the power-time and efficiency-time curves display the current, time-dependent values of power and efficiency. The blue and red dashed lines in the efficiency-time diagram represent the Carnot and Curzon-Ahlborn efficiencies, respectively.

interactive, and electronic textbooks [6] as well as for online course materials.

In Figure 2 we display a snapshot of our visualization tool. In the upper left corner the p-V (pressure-volume) diagram is shown and in the upper right corner the T-S (temperature-entropy) diagram is shown. The dashed blue curves represent the ideal Carnot cycles, while the solid red curves represent the finite time Carnot cycles. The lower left graph shows the power as a function of time and the lower right graph shows the efficiency as a function of time (red solid curve). The black solid circles in the lower graphs indicate the instantaneous time, the blue dashed line in the lower right graph indicates the Carnot efficiency, and the red dashed line indicates the Curzon-Ahlborn efficiency. The time, or more precisely, the time spent in one of the isotherms can be adjusted with the slide bar, entitled "time for half a cycle," above the graphs. The red solid cycles in the upper diagrams and the black solid circles in the lower diagrams change interactively and continuously on the screen, as the slide bar is moved. At long times, the red cycles approach the blue dashed cycles in the p-V and T-S diagrams, while $P \rightarrow 0$ and $\eta \rightarrow \eta_C$. As the time is decreased, the area of the red solid cycles decreases and the power increases, while the efficiency decreases. After a while, the power passes through a maximum, corresponding to $\eta = \eta_{CA}$, and for even shorter times, the power decreases.

The curves making up the p-V cycle were calculated according to the ideal gas law, where the pressure as function of volume is given by $p = nRT/V$ for the isotherms and by $p = nRT_1V_1^{\gamma-1}/V^{\gamma}$ for the adiabats, where (T_1, V_1) is a reference state along the adiabat and γ is the adiabatic index. The entropies in the T-S cycle were modeled so that the entropy change during the isotherms is given by $S_2 - S_1 = nR\ln(V_2/V_1)$. For further details, see [7], where two versions of the cdf file are available.

The visualization tool has a very simple and user-friendly interface, since everything is contained on the same screen view; see Figure 2. Thus no switching between screens is necessary. Moreover, the only way a user can interact with the tool is to change the time, that is, to change the setting of the slide bar. If, however, an advanced user wants to add more functionality, this can easily be done, since the cdf file is publicly available for download [7].

4. Evaluation Method

We evaluated the visualization tool in the setting of an introductory course on thermodynamics for first year students at a university level engineering program. Prior to our demonstration of the visualization tool, the students had been taught about thermodynamic cycles, so this moment came timely in their course.

The evaluation was conducted on the entire class but set up during two consecutive, half-class problem solving occasions (12 students in group A and 16 students in group B). We started with a short (approximately 10 min) discussion about the Carnot cycle, how to calculate the work as the area enclosed in the p-V diagram, Carnot efficiency, and how power is defined and calculated as work divided by time. In addition, we showed the students data on temperatures and efficiencies of some real power plants (from Curzon and Ahlborn's paper [1]). After this, each student was given a piece of paper with the question (translated from Swedish):

(1) *Why do most power plants and engines have an efficiency that is so far from the Carnot efficiency?*

After all students had written their answers, the papers were collected and the visualization tool was demonstrated. Care was taken to go through the process slowly and systematically and point out the simultaneous changes in all four diagrams. Special emphasis was put on the interconnection time-power efficiency. That is, for long cycle time, the power approaches zero, while the efficiency approaches its theoretical maximum, the Carnot efficiency. For shorter cycle times, the power passes through a maximum, and the efficiency at maximum power is known as the Curzon-Ahlborn efficiency.

Following the demonstration, we again looked at the power plant data and we explained to the students that the observed efficiencies match very well the Curzon-Ahlborn efficiencies and that no power plant will operate at the Carnot efficiency since the output power in this case will approach zero. Instead, the Curzon-Ahlborn efficiency is in many cases a better measure of the working efficiency of power plants and engines. Effectively, this reasoning provided the answer to question 1 above. After this, each student was given the remaining evaluation questions (translated from Swedish):

(2) *Did you find the interactive demonstration helpful for your understanding? If so, in which way?*

(3) *Was there anything difficult to understand? In that case, what?*

(4) *Which alternative do you think you would prefer (mark your choice):*

   (a) *An instructor shows the demonstration and discusses*

   (b) *Work individually with the demonstration together with written facts and exercises*

   (c) *First a demonstration and then possibility to work individually*

   (d) *Other, namely...*

(5) *Would you like to see more interactive demonstrations of this kind in your education?*

(6) *If you answered yes to question 5, what do you think is important for the lecturer to keep in mind when showing interactive demonstrations or visualizations?*

In addition there was one note where we asked the students to send an email to us if they were interested to work with the visualization themselves. If so, we would reply and send them the visualization tool (the cdf) together with the download link for the free cdf reader [5].

5. Results and Discussion

Since the preparation of the two groups was identical and we did not find any systematic differences among the answers, we do not distinguish between the groups in our analysis but rather treat the two groups as one. Still, the number of students is quite small (28), which prevents any quantitative conclusions. Instead we discuss the results qualitatively.

The purpose of question 1 was to assess the prior knowledge of the students. While analyzing the answers, it soon became apparent that they could be divided into the following categories: heat losses—unspecified, heat losses—frictional, ideality aspects, timing aspects, and others. The number of answers belonging to each category is found in Table 1. In a few cases the answers contain parts from more than one category (e.g., mentioning both heat losses due to friction and timing aspects). In such cases, the answer will also contribute to more than one category.

Not very surprising, the category with most of the answers is heat losses—unspecified with 14 answers. It is interesting to note that five of the answers from the heat losses categories also include ideality aspects; see Table 1. Many of these students appear to believe that the reason that real power plants do not operate at Carnot efficiency is that the Carnot process is ideal and thus not possible to reach in practice, because of heat losses. This is in part true but it is clear from the answers that the students do not understand why there are heat losses. Their answers are also phrased in a way that one would think that they believe that the designer of the power plant strived for Carnot efficiency (instead of high power output). Besides, a few of the answers in the ideality aspects category show a vague understanding of the ideality concept. One such answer reads (translated from Swedish): *Because I think that the Carnot process refers to an ideal process, the reality is not always like that*.

Five of the answers in the heat loss categories also contain timing aspects. It is clear that these students have reached a higher level of understanding. They seem to understand that Carnot efficiency can in principle be reached, but that it would require a very long cycle time and a minimum of heat losses, which is not practically feasible. Only one student mentions power in the answer (translated from Swedish): *The isothermal reaction does not proceed slowly enough, which gives rise to energy losses. This decreases the efficiency. However, it cannot proceed infinitely slowly because of the power one would like to extract*.

It is interesting to note that one student in the timing aspects category gives a confused answer: *Because the time in the isotherms cannot be allowed to approach zero in reality*. This student knows that there is a timing issue but somehow misunderstands and believes that the time in the isotherms should approach zero instead of infinity in order to reach Carnot efficiency.

The two answers in the other category are not relevant. One of these specifically mentions the regenerator, which is not part of the Carnot but the Stirling engine. The other one reads: *To get a really high efficiency, a very low minimum temperature is required. This is difficult to achieve in a power plant and as we see on the chart, the low temperatures are high; that is, T_{cold}/T_{warm} is still a fairly high ratio*. This answers a different question, for example, What would you do to get a high Carnot efficiency?

The evaluation of the prior knowledge shows that some students believe that the Carnot process is an idealization that one cannot reach because of energy losses, while some students realize that there is a tradeoff between cycle time and efficiency. In any case, the prior knowledge of all the students seemed to be on the right level for further evaluation of the visualization to be meaningful.

All the responses to question 2 are positive with one exception. Some selected positive answers read (translated from Swedish and separated by semicolons): *Yes, it was interesting to see how the time directly influenced the efficiency of the cycle*; *Yes, a little bit about understanding the connection between efficiency, power, and so forth*; *Yes, it was good to see what happened with all the graphs as the time was changed*; *Yes, it was very good to see the different kinds of graphs at the same time*; *Yes, to a certain degree. It is easier to grasp when you see a visual demonstration like this*; *Definitely. To see how power and efficiency changed in relation to the ideal Carnot process with respect to time was very pedagogical*. The one negative answer here reads: *Not so much, take it slower and comment more on each separate graph*. From the collected answers to question 2, we conclude that the students in general appreciate the visualization since it offers a complementary way to understand the Carnot process.

Table 1: Categorization of answers to question 1. In the middle column, the contributions of the individual students are shown. The students are labeled as A1–A12 and B1–B16.

Category	Students	Number of answers
Heat losses—unspecified	A1, A3, A9, A12 B2, B3, B5, B7, B8, B9, B11, B12, B15, B16	14
Heat losses—frictional	A2, A8, A10 B5	4
Ideality aspects	A3, A5, A6, A7 B3, B4, B6, B7, B9	9
Timing aspects	A4, A9, A10, A11, A12 B13, B14, B15, B16	9
Other	B1, B10	2

While the answers to question 2 are very much aligned, the answers to question 3 are quite diverse. 10 of the 28 students give negative answers; that is, there was nothing that was difficult to understand. The other answers are so scattered, so no pattern is discernible. One student answers that *I don't think I fully understood the graphs* and another one writes: *The figure where the Carnot process changed its size*. At least two students seemed to need more time. They answer: *It is hard to get the time to think things through* and *The different graphs could be explained a little bit more*. This is in contrast to two other answers: *No, the demonstration was almost on a too low level* and *No, it was calm and methodical*.

The answers to question 4, which addresses how the students would like to use visualizations, are consistent. All students except one would prefer that an instructor demonstrates the visualization. After this demonstration half of the students would like to have a discussion, led by the instructor. The other half would like to have the possibility to work individually. Only one student would prefer the second option, that is, to work individually together with written facts and exercises.

The responses to question 5 are overwhelmingly positive. All students would, to a varying degree, like to see more of this kind of interactive demonstrations in their education. About half (12) of the students simply answer *Yes*, while some of the students specify or motivate their answer. Some examples of this are as follows: *Yes, I like practical examples and demonstrations provided that they are at the right level* and *It would certainly be helpful at some moments*, as well as *Yes, especially on topics that are difficult to grasp intuitively the first time one encounters them*. Only one of the students is more reserved and answers *Maybe*.

The responses to question 6 are quite diversified and most students have clear ideas on what the instructor needs to think about. However, not all students agree in their answers. The pace of the presentation seems to be the factor that most students have opinions about and here we find opposing views. One student writes: *Fast, clear, consice* [*sic*], while another one answers: *Calm and methodical*. Yet another student notes that the pace should be adapted to the group of students. One student thinks that it is important that the understanding of the theory precedes the visualization, while another one wants the visualization to be on the same difficulty level as the one they are currently studying. Moreover, according to the students it is important that the visualization makes the understanding easier and does not complicate matter. There should be some degree of discussion and feedback. A holistic view and a sense of reality are also desirable. Only one of the students requested the visualization tool as cdf file for individual study.

Most students welcome more demonstrations of this kind in the education but they think the lecturer needs to be careful so that the demonstrations or visualizations do not complicate or confuse things. It is also important that the visualization is at the right level and that the pace of the demonstration is appropriate. However, among the students there is a lack of consensus about the right level and the appropriate pace. Some students want the visualization to be aligned with what they are currently studying, while others want the theory to precede the visualization. Concerning the pace, some students want a calm and methodical demonstration, while others want it to be fast and concise. Vavra et al. [8] have proposed a set of recommendations for the use of visualizations in science teaching and it is interesting to see how well aligned some of the student's answers are with these recommendations.

In one of the recommendations it is pointed out that "students require a repertoire of knowledge and skills to use visualization objects effectively" [8] and partly due to this there is a risk that only the best students benefit from the visualization, which has been pointed out by Gelaan [9]. Those students can use the visualization to test and enhance the understanding of concepts that they already have grasped, while the less able students are likely to misunderstand significant parts of the visualization [9]. Since the prior knowledge and understanding can vary a lot among students, this was probably the case in the current investigation, where the prior knowledge was diverse (see Table 1). Some of the answers to question 2 in the survey confirm this.

Even if it is unfeasible to require all students to have the same prior knowledge, the lecturer should at least make sure that all students have prior knowledge enough to make predictions of the outcome of the visualization, no matter if the predictions are correct or not. It has been shown that if students can make predictions, they are more likely to understand the demonstration in a correct way [10].

All students were positive about the visualization and would like to see more of this in their education. We are, however, cautious of drawing any conclusions about the changes in the students' understanding, as we have not quantified this. It has previously been reported that students find learning with visualization engaging and enjoyable [11]. We believe that this could facilitate a deep approach to learning [12].

It is hardly surprising that only one of the students wanted to work individually with the visualization. Many engineering students tend to choose the easiest path, in this case that an instructor demonstrates the visualization, even if this might not be optimal for their learning. Nevertheless, to actively work with the visualization is the most efficient way to use the visualization tool [13].

6. Conclusion

We have constructed an interactive visualization of a nonideal Carnot engine operating at finite time. When the cycle time is changed, the pressure-volume, temperature-entropy, power-time, and efficiency-time diagrams, which are displayed in parallel on one screen, change interactively. We have implemented the visualization in Wolfram's computational document format (cdf).

Our evaluation of the visualization tool in the setting of an introductory course on thermodynamics for engineering students who had just been taught about thermodynamic cycles showed that their prior knowledge about which aspects that make the Carnot cycle ideal is vague. According to the survey, all students found the interactive demonstration helpful in their understanding. A general remark we extracted from several of the students' answers is that the relationships between efficiency, power, and cycle time became clearer. Several students also conclude that seeing the different diagrams being varied simultaneously gives increased holistic understanding.

Since students are individuals and at more or less different levels of understanding, care must be taken when designing the visualization tool and planning the pace of the demonstration. From the lack of consensus concerning what the students think that the lecturer should keep in mind, we conclude that it will be extremely difficult to make a visualization that pleases and facilitates learning for all students in a class.

Conflict of Interests

The author declares that there is no conflict of interests in this paper.

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