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Research Article

Eccentric Connectivity and Zagreb Coindices of the Generalized Hierarchical Product of Graphs

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Formulas for calculations of the eccentric connectivity index and Zagreb coindices of graphs under generalized hierarchical product are presented. As an application, explicit formulas for eccentric connectivity index and Zagreb coindices of some chemical graphs are obtained.

1. Introduction

All graphs considered here are undirected, simple, and connected. For two vertices u and v of a graph G, the distance $d(u, v)$ is equal to the length of a shortest path connecting u and v. Suppose that $V(G)$ and $E(G)$ are the set of vertices and edges of G, respectively. For every vertex $u \in V(G)$, the edge connecting u and v is denoted by uv and $\deg_G(u)$ ($\deg(u)$ for short) denotes the degree of u in G. The diameter of G, denoted by $\operatorname{diam}(G)$, is the maximum distance among all pairs of vertices in the graph.

The first and second Zagreb indices are defined as

$$M_1(G) = \sum_{u \in V(G)} \deg(u)^2, \qquad M_2(G) = \sum_{uv \in E(G)} \deg(u)^2 \deg(v), \tag{1}$$

respectively [1]. The applications of these graph invariants and their mathematical properties are reviewed in two important survey articles [2, 3]. When a topological index or a new graph operation is introduced, then the following mathematical questions are usually raised.

(1) What are the extremal properties of this new topological index?

(2) Is it possible to find exact formulas for this topological index under old and new graph operations?

We refer to [4–7] for such questions about Zagreb group indices.

The Zagreb indices can be viewed as the contributions of pairs of adjacent vertices to certain degree-weighted generalizations of Wiener polynomials. The first and second Zagreb coindices were first introduced by Došlić [8]. They are defined as follows:

$$\overline{M}_1(G) = \sum_{uv \notin E(G)} [\deg(u) + \deg(v)], \qquad \overline{M}_2(G) = \sum_{uv \notin E(G)} [\deg(u) \deg(v)]. \tag{2}$$

In [9], the authors computed exact formulas for these graph parameters under some graph operations.

Now we define a vertex version of Zagreb indices as follows:

$$\overline{M}_1^*(G) = \sum_{\{u,v\} \subseteq V(G)} [\deg(u) + \deg(v)], \qquad \overline{M}_2^*(G) = \sum_{\{u,v\} \subseteq V(G)} [\deg(u) \deg(v)]. \tag{3}$$

The graph invariants $\overline{M}_1^*(G)$ and $\overline{M}_2^*(G)$ are called the first and second vertex Zagreb indices of G.

The eccentricity $\varepsilon_G(u)$ is the largest distance between u and any other vertex v of G. The total connectivity index

$\zeta(G)$ of a graph G is defined as $\zeta(G) = \sum_{u\in V(G)} \varepsilon_G(u)$. Also, the eccentric connectivity index of G is defined as $\xi^c(G) = \sum_{u\in V(G)} \varepsilon_G(u)\deg_G(u)$ [10]. We refer to [11] for a good survey on this topological index.

A graph G with a specified vertex subset $U \subseteq V(G)$ is denoted by $G(U)$. Suppose G and H are graphs and $U \subseteq V(G)$. The generalized hierarchical product, denoted by $G(U) \sqcap H$, is the graph with vertex set $V(G) \times V(H)$ and two vertices (g,h) and (g',h') are adjacent if and only if $g = g' \in U$ and $hh' \in E(H)$ or $gg' \in E(G)$ and $h = h'$; see Figure 1. This graph operation was introduced recently by Barrière et al. [12, 13] and found some applications in computer science. We encourage the reader to consult [14–16] for the mathematical properties of the hierarchical product of graphs. The Cartesian product, $G\times H$, of graphs G and H has the vertex set $V(G\times H) = V(G)\times V(H)$ and $(u,x)(v,y)$ is an edge of $G\times H$ if $u = v$ and $xy \in E(H)$ or $uv \in E(G)$ and $x = y$ [17].

We denote by P_n and C_n the path and cycle with n vertices, respectively. Our notations are standard and can be taken from the standard books on graph theory.

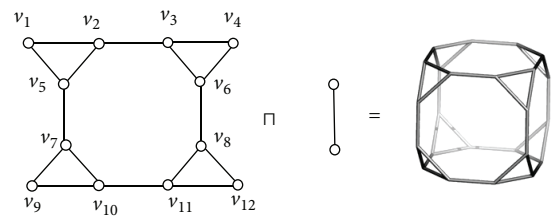

Figure 1: The molecular graph of truncated cube $H = G(U) \sqcap P_2$, where $U = \{v_1, v_4, v_9, v_{12}\}$.

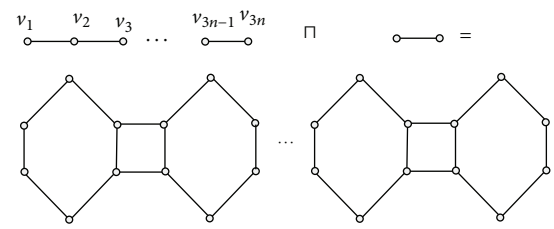

Figure 2: The linear phenylene $F_n = P_{3n}(U) \sqcap P_2$, where $U = \{v_{3k+1} \mid 0 \leqslant k \leqslant n-1\} \cup \{v_{3k} \mid 1 \leqslant k \leqslant n\}$.

2. Main Results

We start by introducing some notations that will be kept throughout this section. Let $G = (V,E)$ be a graph and $U \subseteq V$. Following Pattabiraman and Paulraja [18], an $u-v$ path through U in $G(U)$ is an $u-v$ path in G containing some vertices $w \in U$ (not necessarily distinct from u and v). Let $d_{G(U)}(u,v)$ denote the length of a shortest $u-v$ path through U in G. Notice that if one of the vertices u and v belongs to U, then $d_{G(U)}(u,v) = d_G(u,v)$. Furthermore, let $\varepsilon_{G(U)}(u) = \max\{d_{G(U)}(u,v) \mid v \in V(G(U))\}$; then $\zeta(G(U))$ and $\xi^c(G(U))$ can be defined as follows:

$$\begin{aligned} \zeta(G(U)) &= \sum_{u\in V(G(U))} \varepsilon_{G(U)}(u), \\ \xi^c(G(U)) &= \sum_{u\in V(G(U))} \varepsilon_{G(U)}(u)\deg_G(u). \end{aligned} \tag{4}$$

Lemma 1 (see [13]). *Let G and H be graphs with $U \subseteq V(G)$. Then one has the following:*

(a) *if $U = V(G)$, then the generalized hierarchical product $G(U) \sqcap H$ is the Cartesian product of G and H;*

(b) $|V(G(U) \sqcap H)| = |V(G)||V(H)|$; $|E(G(U) \sqcap H)| = |E(G)||V(H)| + |E(H)||U|$;

(c) *$G(U) \sqcap H$ is connected if and only if G and H are connected;*

(d) $d_{G(U)\sqcap H}((g,h),(g',h')) = \{d_{G(U)}(g,g') + d_H(h,h')$, *if $h \neq h'$; $d_G(g,g')$, if $h = h'$*$\}$.

Theorem 2 (see [19]). *Let G and H be two connected graphs and let U be a nonempty subset of $V(G)$. Then*

$$\begin{aligned} \xi^c(G(U) \sqcap H) = {} & |V(H)|\,\xi^c(G(U)) + |U|\,\xi^c(H) \\ & + 2|E(H)| \sum_{u\in U} \varepsilon_{G(U)}(u) \\ & + 2|E(G)|\,\zeta(H). \end{aligned} \tag{5}$$

Put $U = V(G)$. Then $G(U)\sqcap H = G\times H$, $\xi^c(G(U)) = \xi^c(G)$, and $\sum_{u\in U} \varepsilon_{G(U)}(u) = \zeta(G)$. Therefore,

Corollary 3 (see [19]). *Let G and H be two connected graphs. Then*

$$\begin{aligned} \xi^c(G\times H) = {} & |V(H)|\,\xi^c(G) + |V(G)|\,\xi^c(H) \\ & + 2|E(H)|\,\zeta(G) + 2|E(G)|\,\zeta(H). \end{aligned} \tag{6}$$

Example 4. Let H be the graph of truncated cube. Then $H = G(U)\sqcap P_2$, where $U = \{v_1, v_4, v_9, v_{12}\}$, as shown in Figure 1. It is not difficult to check that $\sum_{u\in U} \varepsilon_{G(U)}(u) = 20$, $\xi^c(G(U)) = 160$ and so, by Theorem 2, we have $\xi^c(H) = \xi^c(G(U) \sqcap P_2) = 2 \times 160 + 4 \times 2 + 2 \times 20 + 2 \times 16 \times 2 = 432$.

Example 5. Consider the linear phenylene F_n including $n > 1$ benzene rings; see Figure 2. The linear phenylene F_n is the graph $P_{3n}(U) \sqcap P_2$, where

$$U = \{v_{3k+1} \mid 0 \leqslant k \leqslant n-1\} \cup \{v_{3k} \mid 1 \leqslant k \leqslant n\}. \tag{7}$$

On the other hand, by tedious calculations, one can check that

$$|U| = 2n\sum_{u\in U}\varepsilon_{P_{3n}(U)}(u) = \begin{cases} \frac{9}{2}n^2 - n + \frac{1}{2} & 2\nmid n \\ \frac{9}{2}n^2 - n & 2\mid n, \end{cases} \tag{8}$$

$$\xi^c(P_{3n}(U)) = \begin{cases} \frac{27}{2}n^2 - 9n + \frac{3}{2} & 2\nmid n \\ \frac{27}{2}n^2 - 9n + 2 & 2\mid n. \end{cases}$$

So, by Theorem 2, we obtain

$$\xi^c(F_n) = \xi^c(P_{3n}(U)\sqcap P_2) = 36n^2 - 4n. \tag{9}$$

Theorem 6. *Let G and H be two connected graphs and let U be a nonempty subset of $V(G)$. Then,*

(1) $\overline{M}_1^*(G(U)\sqcap H) = 2(|V(G)||V(H)|-1)(|U||E(H)| + |V(H)||E(G)|)$.

(2) $\overline{M}_2^*(G(U)\sqcap H) = |V(H)|^2\overline{M}_2^*(G) + 2|E(H)|^2|U|(|U|-1) + (1/2)|V(H)|\ (|V(H)|-1)M_1(G) + |U|\overline{M}_2^*(H) + 4|V(H)||E(H)||U||E(G)| + (\overline{M}_1^*(H) - 2|V(H)||E(H)|)\sum_{u\in U}\deg_G(u)$.

Proof. Let G and H be two connected graphs and let U be a nonempty subset of $V(G)$. The part (1) of this theorem is clear. To prove the second part, we notice that by definitions of generalized hierarchical product and $\overline{M}_2^*$, we have

$$\begin{aligned}
&\overline{M}_2^*(G(U)\sqcap H) \\
&= \sum_{\{(g,h),(g',h')\}\subseteq V(G(U)\sqcap H)} \deg_{(G(U)\sqcap H)}((g,h)) \cdot \deg_{(G(U)\sqcap H)}((g',h')) \\
&= \sum_{h\in V(H)}\sum_{h'\in V(H)}\sum_{g\in U}\sum_{g'\in V(G)-U}(\deg_G(g)+\deg_H(h))\deg_G(g') \\
&\quad + \sum_{h\in V(H)}\sum_{h'\in V(H)}\sum_{\{g,g'\}\subseteq U}(\deg_G(g)+\deg_H(h)) \cdot (\deg_G(g')+\deg_H(h')) \\
&\quad + \sum_{h\in V(H)}\sum_{h'\in V(H)}\sum_{\{g,g'\}\subseteq V(G)-U}\deg_G(g)\deg_G(g') \\
&\quad + \sum_{\{h,h'\}\subseteq V(H)}\sum_{g\in U}(\deg_G(g)+\deg_H(h)) \cdot (\deg_G(g)+\deg_H(h')) \\
&\quad + \sum_{\{h,h'\}\subseteq V(H)}\sum_{g\in V(G)-U}\deg_G(g)^2 \\
&= |V(H)|^2\sum_{g\in U,g'\in V(G)-U}\deg_G(g)\deg_G(g') \\
&\quad + 2|V(H)||E(H)||U|\sum_{g'\in V(G)-U}\deg_G(g') \\
&\quad + |V(H)|^2\sum_{\{g,g'\}\subseteq U}\deg_G(g)\deg_G(g') \\
&\quad + 2|E(H)|^2|U|(|U|-1) \\
&\quad + \sum_{h\in V(H)}\sum_{h'\in V(H)}\sum_{\{g,g'\}\subseteq U}(\deg_G(g)\deg_H(h') + \deg_G(g')\deg_H(h)) \\
&\quad + |V(H)|^2\sum_{\{g,g'\}\subseteq V(G)-U}\deg_G(g)\deg_G(g') \\
&\quad + \frac{1}{2}|V(H)|(|V(H)|-1)\sum_{g\in U}\deg_G(g)^2 \\
&\quad + \overline{M}_1^*(H)\sum_{g\in U}\deg_G(g) + |U|\overline{M}_2^*(H) \\
&\quad + \frac{1}{2}|V(H)|(|V(H)|-1)\sum_{g\in V(G)-U}\deg_G(g)^2 \\
&= |V(H)|^2\overline{M}_2^*(G) + 2|E(H)|^2|U|(|U|-1) \\
&\quad + \frac{1}{2}|V(H)|(|V(H)|-1)M_1(G) \\
&\quad + |U|\overline{M}_2^*(H) + 4|V(H)||E(H)||U||E(G)| \\
&\quad + (\overline{M}_1^*(H) - 2|V(H)||E(H)|)\sum_{u\in U}\deg_G(u).
\end{aligned} \tag{10}$$

This completes the proof. □

Corollary 7 (see [15]). *Let G and H be two connected graphs and let U be a nonempty subset of $V(G)$. Then*

$$M_1(G(U)\sqcap H) = |V(H)|M_1(G) + |U|M_1(H) + 4|E(H)|\sum_{u\in U}\deg_G(u). \tag{11}$$

Proof. It follows from definition of $\overline{M}_2^*$ that $M_1(G(U)\sqcap H) = 4|E(G(U)\sqcap H)|^2 - 2\overline{M}_2^*(G(U)\sqcap H)$. On the other hand, $|E(G(U)\sqcap H)| = |V(H)||E(H)| + |U||E(G)|$. Therefore,

$$\begin{aligned} M_1&(G(U)\sqcap H)\\ &= 4|V(H)|^2|E(G)|^2 + 4|U|^2|E(H)|^2\\ &\quad - 2|V(H)|^2\overline{M}_2^*(G) - 4|E(H)|^2|U|(|U|-1)\\ &\quad - |V(H)|(|V(H)|-1)M_1(G) - 2|U|\overline{M}_2^*(H)\\ &\quad - 2\left(\overline{M}_1^*(H) - 2|V(H)||E(H)|\right)\sum_{u\in U}\deg_G(u). \end{aligned} \tag{12}$$

So, by replacing $\overline{M}_1^*(G)$ and $\overline{M}_2^*(G)$ with $2|E(G)|(|V(G)|-1)$ and $\overline{M}_2^*(G) = 2|E(G)|^2 - (1/2)M_1(G)$, respectively, in the above equation, the assertion follows immediately. □

Corollary 8. *Let G and H be two connected graphs and let U be a nonempty subset of V(G). Then*

$$\begin{aligned} \overline{M}_1(G(U)\sqcap H) &= 2(|V(G)||V(H)|-1)\\ &\quad\cdot(|U||E(H)| + |V(H)||E(G)|)\\ &\quad - |V(H)|M_1(G) - |U|M_1(H)\\ &\quad - 4|E(H)|\sum_{u\in U}\deg_G(u). \end{aligned} \tag{13}$$

Proof. The assertion follows from Theorem 6, Corollary 7, and the fact that $\overline{M}_1(G(U)\sqcap H) = \overline{M}_1^*(G(U)\sqcap H) - M_1(G(U)\sqcap H)$. □

Theorem 9 (see [15]). *Let G and H be two connected graphs and let U be a nonempty subset of V(G). Then*

$$\begin{aligned} M_2&(G(U)\sqcap H)\\ &= |V(H)|M_2(G) + |U|M_2(H)\\ &\quad + 2|E(H)|\sum_{u\in U}\sum_{x\in N[u]}\deg_G(x)\\ &\quad + M_1(H)\left|\{xy\in E(G)\mid x,y\in U\}\right|\\ &\quad + |E(H)|\sum_{u\in U}\deg_G(u)^2\\ &\quad + M_1(H)\sum_{u\in U}\deg_G(u), \end{aligned} \tag{14}$$

where $N[u]$ is the set of all adjacent vertices of u.

By Theorems 6 and 9 and this fact that $\overline{M}_2(G(U)\sqcap H) = \overline{M}_2^*(G(U)\sqcap H) - M_2(G(U)\sqcap H)$, we can write the following.

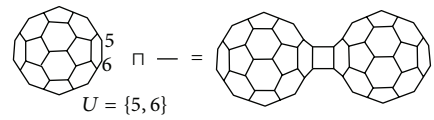

Figure 3: The dimer fullerene C_{60}.

Corollary 10. *Let G and H be two connected graphs and let U be a nonempty subset of V(G). Then*

$$\begin{aligned} \overline{M}_2&(G(U)\sqcap H)\\ &= |V(H)|^2\overline{M}_2^*(G) + 2|E(H)|^2|U|(|U|-1)\\ &\quad + \frac{1}{2}|V(H)|(|V(H)|-1)M_1(G) + |U|\overline{M}_2^*(H)\\ &\quad + 4|V(H)||E(H)||U||E(G)|\\ &\quad + \left(\overline{M}_1^*(H) - M_1(H) - 2|V(H)||E(H)|\right)\\ &\quad\cdot\sum_{u\in U}\deg_G(u) - |V(H)|M_2(G)\\ &\quad - |U|M_2(H) - 2|E(H)|\sum_{u\in U}\sum_{x\in N[u]}\deg_G(x) - M_1(H)\\ &\quad\cdot\left|\{xy\in E(G)\mid x,y\in U\}\right| - |E(H)|\sum_{u\in U}\deg_G(u)^2. \end{aligned} \tag{15}$$

Example 11. The molecular graph of dimer fullerene H is the graph $C_{60}(U)\sqcap P_2$, where $U = \{5,6\}$ (Figure 3). On the other hand, it is not difficult to check that $M_1(C_{60}) = 540$, $M_2(C_{60}) = 810$, $\sum_{u\in U}\deg_{C_{60}}(u) = 6$, $\sum_{u\in U}\deg_{C_{60}}(u)^2 = 18$, $\sum_{u\in U}\sum_{x\in N[u]}\deg_{C_{60}}(x) = 18$, $\overline{M}_2^*(C_{60}) = 15930$, and $|\{xy\in E(C_{60})\mid x,y\in U\}| = 1$. Thus, by the above results, we obtain

(1) $\overline{M}_1^*(C_{60}(U)\sqcap P_2) = 43316$ and $\overline{M}_2^*(C_{60}(U)\sqcap P_2) = 65694$.

(2) $\overline{M}_1(C_{60}(U)\sqcap P_2) = 42208$ and $\overline{M}_2(C_{60}(U)\sqcap P_2) = 64004$.

Example 12. Consider the linear phenylene F_n including n benzene rings (Figure 2). The linear phenylene F_n is the graph $P_{3n}(U)\sqcap P_2$, where $U = \{v_{3k+1}\mid 0\leqslant k\leqslant n-1\}\cup\{v_{3k}\mid 1\leqslant k\leqslant n\}$; see Figure 2. On the other hand, it is not difficult to check that $M_1(P_{3n}) = 12n-6$, $M_2(P_{3n}) = 12n-8$, $\sum_{u\in U}\deg_{P_{3n}}(u) = 4n-2$, $\sum_{u\in U}\deg_{P_{3n}}(u)^2 = 8n-6$, $\sum_{u\in U}\sum_{x\in N[u]}\deg_{P_{3n}}(x) = 8n-4$, $\overline{M}_2^*(P_{3n}) = 18n^2-18n+5$, and $|\{xy\in E(P_{3n})\mid x,y\in U\}| = n-1$. Hence, by previous result, we obtain the following:

(1) $\overline{M}_1^*(F_n) = \overline{M}_1^*(P_{3n}(U)\sqcap P_2) = 96n^2-40n+4$ and $\overline{M}_2^*(F_n) = \overline{M}_2^*(P_{3n}(U)\sqcap P_2) = 128n^2-86n+18$;

(2) $\overline{M}_1(F_n) = \overline{M}_1(P_{3n}(U)\sqcap P_2) = 96n^2-84n+24$ and $\overline{M}_2(F_n) = \overline{M}_2(P_{3n}(U)\sqcap P_2) = 128n^2-146n+54$.

Example 13. Let H be the graph of truncated cube. Then $H = G(U) \sqcap P_2$, where $U = \{v_1, v_4, v_9, v_{12}\}$, as shown in Figure 1. Therefore,

(1) $\overline{M}_1^*(G(U) \sqcap P_2) = 1656$ and $\overline{M}_2^*(P_{3n}(U) \sqcap P_2) = 2484$.

(2) $\overline{M}_1(P_{3n}(U) \sqcap P_2) = 1440$ and $\overline{M}_2(G(U) \sqcap P_2) = 2160$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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