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Research Article

Higgs Field in Universe: Long-Term Oscillation and Deceleration/Acceleration Phases

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It is shown that the Einstein gravity and Higgs scalar field have (a) a long-term oscillation phase; (b) cosmological regular solutions with deceleration/acceleration phases. The first has a preceding contracting and subsequent expanding phases and between them there exists an oscillating phase with arbitrary time duration. The behavior of the second solution near to a flex point is in detail considered.

1. Introduction

The standard cosmological model (for review, see [1]) gives us an accurate description of the evolution of the Universe. In spite of its success, the standard cosmological model has a series of problems such as the initial singularity, the cosmological horizon, the flatness problem, the baryon asymmetry, and the nature of dark matter and dark energy.

Under the dynamical laws of general relativity, the standard FLRW cosmology becomes singular at the origin of Universe. The matter density and geometrical invariants diverge as the volume of the Universe goes to zero. The Big Bang singularity seems to be an unavoidable aspect of the currently established cosmological model [2] which probably only a full quantum theory of gravity could resolve. A bouncing Universe with an initial contraction to a nonvanishing minimal radius; then subsequently an expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology.

Bouncing cosmologies, in which the present era of expansion is preceded by a contracting phase, have been studied as potential alternatives to inflation in solving the problems of standard FRW cosmology. The first explicit semianalytic solution for a closed bouncing FRW model filled by a massive scalar field was found by Starobinskii [3]. Later explicit solutions for a bouncing geometry were obtained by Novello and Salim [4] and Melnikov and Orlov [5]. For the review of the cosmological bounce one can see review [6].

Supernova observations [7, 8] were the first to suggest that our Universe is currently accelerating. For this acceleration now it is believed that as much as 2/3 of the total density of the Universe is in a form which has large negative pressure and which is usually referred to as dark energy. A number of various models have been proposed aiming at the description of dark energy universe (for review, see [9–11]). It is evident that to have deceleration (where $\ddot{a} < 0$) and acceleration (where $\ddot{a} > 0$) phases it is necessary to have the moment with $\ddot{a} = 0$.

Here we would like to show that (a) a Universe bounce can be not only a short time event but also it can be a long-term oscillating process; (b) the gravitating Higgs scalar field may have cosmological solutions with such property. Such solution exists only with a single value of cosmological constant.

2. Long-Term Bouncing

2.1. The Statement of the Problem. In this section we will investigate a cosmological solution for a closed Universe filled

with a Higgs scalar field. Our goal is to find a regular solution with contracting, expanding, and oscillating phases.

We start with the Lagrangian

$$L = -\frac{R}{2} + \varkappa \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right], \tag{1}$$

where R is the 4D scalar curvature and ϕ is the scalar Higgs field with the potentials $V(\phi)$. The corresponding field equations are

$$\begin{gathered} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \varkappa T_{\mu\nu}, \\ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu} \right) = -\frac{dV(\phi)}{d\phi}. \end{gathered} \tag{2}$$

The potential $V(\phi)$ is the Mexican has potential

$$V(\phi) = \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0, \tag{3}$$

where m and V_0 are constants and V_0 can be considered as a cosmological constant. The energy-momentum tensor $T_{\mu\nu}$ for the scalar field is

$$T_{\mu\nu} = \left(\nabla_\mu \phi \right) \left(\nabla_\nu \phi \right) - g_{\mu\nu} L. \tag{4}$$

For the investigation of Universe having an oscillating phase between contraction and expanding phases, we consider the cosmological metric

$$ds^2 = dt^2 - a^2(t) \left[d\chi^2 + \sin^2 \chi \left(d\theta^2 + \sin^2 \theta d\varphi^2 \right) \right]. \tag{5}$$

After the substitution of the metric (5) into field equations (2), we have following equations set:

$$\frac{3\dot{a}^2}{a^2} + \frac{3}{a^2} = \varkappa \left[\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right], \tag{6}$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \varkappa \left[-\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right], \tag{7}$$

$$\ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} = \lambda \phi \left(m^2 - \phi^2 \right). \tag{8}$$

To begin with we would like to define the notion "bouncing-off of the Universe." We will say that the Universe undergoes bouncing-off if a long enough the stage of contraction is replaced by the stage of expansion. A long time interval Δt means that $\Delta t \gg t_{\text{Planck}}$ or $\Delta t \approx t_{\text{Universe}}$, where t_{Universe} is the life time of the Universe.

Why we need such definition? Below we will show that bouncing-off can be a complicated process lasting a long enough time which we will call as an oscillating stage.

The statement of the problem: *to find solutions having big enough contraction/expansion stages and between them a long enough oscillation stage.*

2.2. Solution with Oscillating Stage. We will investigate solutions with an oscillating stage having a point of inflection with the following conditions:

$$\ddot{a}(t) \begin{cases} < 0, & \text{if } t < t_0, \\ = 0, & \text{if } t = t_0, \\ > 0, & \text{if } t > t_0. \end{cases} \tag{9}$$

Here t_0 is the point of inflection. In the consequence of the existence of the inflection point the functions $a(t)$, $\phi(t)$ can be presented in the form

$$\begin{gathered} a(t) = a_0 + a_1 (t - t_0) + a_3 (t - t_0)^3 + \cdots, \\ \phi(t) = \phi_0 + \phi_1 (t - t_0) + \phi_2 (t - t_0)^2 + \phi_3 (t - t_0)^3 + \cdots. \end{gathered} \tag{10}$$

Using the conditions (9) and (6)–(8) one can find the following constraints on the initial conditions $a_0 = a(t_0)$, $a_1 = \dot{a}_0 = \dot{a}(t_0)$, $\phi_0 = \phi(0)$, and $\phi_1 = \dot{\phi}_0 = \dot{\phi}(0)$ and the cosmological constant V_0

$$\begin{gathered} \frac{2\dot{a}_0^2}{a_0^2} + \frac{2}{a_0^2} = \varkappa \left[\frac{\lambda}{4} \left(\phi_0^2 - m^2 \right)^2 - V_0 \right], \\ \frac{2\dot{a}_0^2}{a_0^2} + \frac{2}{a_0^2} = \varkappa \dot{\phi}_0^2. \end{gathered} \tag{11}$$

It means that the cosmological constant V_0 is defined uniquely:

$$\Lambda = \varkappa V_0 = \frac{\lambda}{4} \varkappa \left(\phi_0^2 - m^2 \right)^2 - 2 \left(\frac{\dot{a}_0^2}{a_0^2} + \frac{1}{a_0^2} \right). \tag{12}$$

For the numerical investigation we introduce the dimensionless time $x = t/\sqrt{\varkappa}$ and dimensionless functions $\phi\sqrt{\varkappa} \to \phi$, $a/\sqrt{\varkappa} \to a$. Then (6)–(8) become

$$\begin{gathered} \frac{3\dot{a}^2}{a^2} + \frac{3}{a^2} = \left[\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right], \\ \frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \left[-\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right], \\ \ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} = \lambda \phi \left(m^2 - \phi^2 \right) \end{gathered} \tag{13}$$

with the following initial conditions:

$$\begin{gathered} a(0) = a_0, \qquad \dot{a}(0) = a_0, \qquad \phi(0) = \phi_0, \\ \dot{\phi}_0 = -\frac{\sqrt{2\dot{a}_0^2 + 2}}{a_0}. \end{gathered} \tag{14}$$

The numerical solutions are presented in Figures 1, 2, 3, 4, and 5. In Figure 1 the profiles for different a_1 are presented. From this figure we see that there exists the bounce with

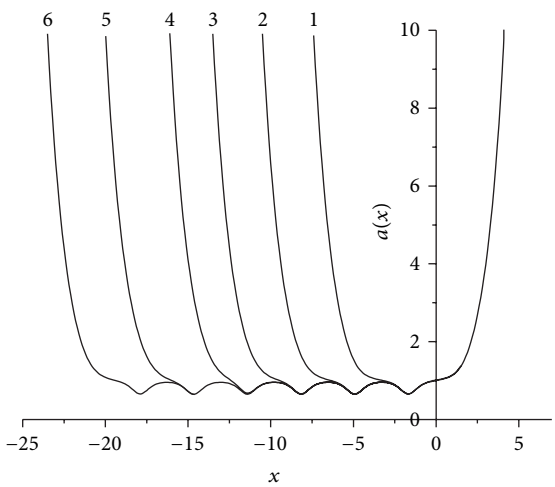

Figure 1: The profiles $a(t)$. The curve 1 for $a_1 = 0.08$, the curve 2 for $a_1 = 0.0805$, the curve 3 for $a_1 = 0.080502$, the curve 4 for $a_1 = 0.0805020498$, the curve 5 for $a_1 = 0.080502049893$, and the curve 6 for $a_1 = 0.08050204989312597$.

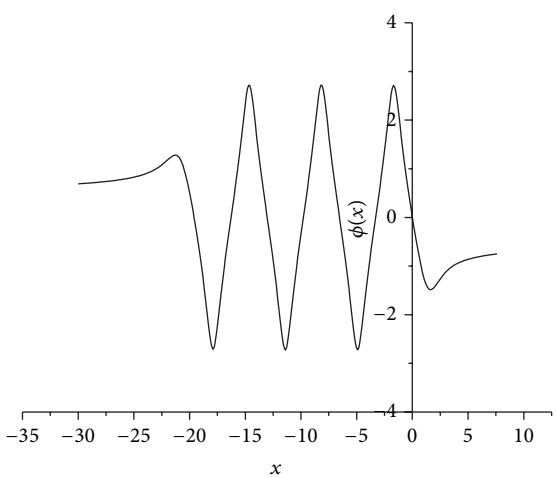

Figure 2: The profile $\phi(t)$ for $a_1 = 0.08050204989312597$.

different time durations. Every such solution can be enumerated with the number of minima or maxima. In Figures 1–4 the profiles of $\phi(t)$, Hubble parameter $H(t) = \dot{a}(t)/a(t)$, and the state equation $w(t)$ for $a_1 = 0.08050204989312597$ are presented. It is useful to present the profile of a state equation (see Figure 4) in the form

$$w = \frac{p}{\varepsilon} = \frac{T^{11}}{T^{00}} = \frac{\left(\dot{\phi}^2/2\right) - (\lambda/4)\left(\phi^2 - m^2\right) + V_0}{\left(\dot{\phi}^2/2\right) + (\lambda/4)\left(\phi^2 - m^2\right) - V_0}, \qquad (15)$$

where p is the pressure and ε is the energy density. We see that in the contracting and expanding stages $w \approx -1$ and additionally during the oscillation stage there are points where $w \approx -1$.

How long could such prolonged Universe oscillation be? For the investigation of this question we are addressing this in Figure 5. We see that for $a_1 = 0.080502049893$,

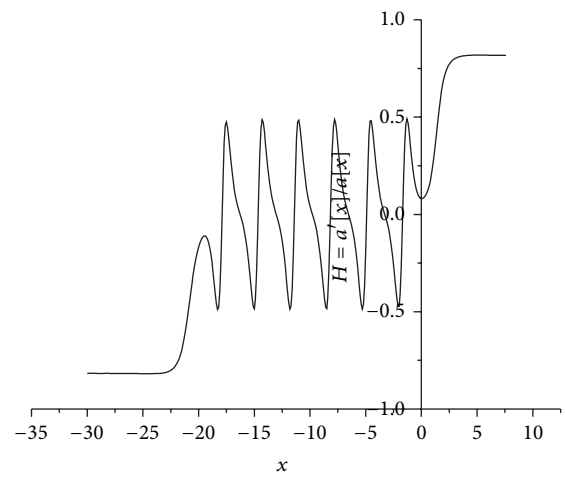

Figure 3: The profile of Hubble parameter $H(t) = \dot{a}(t)/a(t)$ for $a_1 = 0.08050204989312597$.

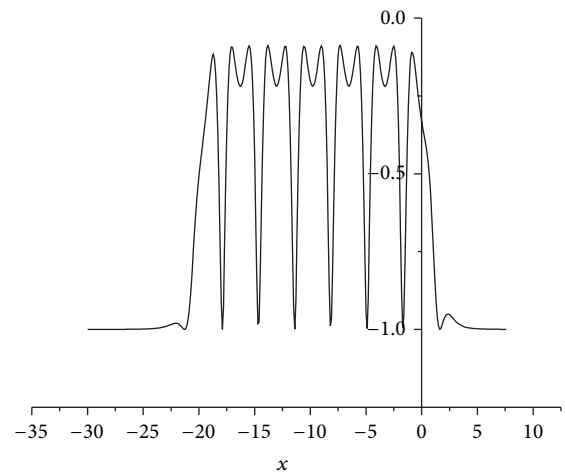

Figure 4: The profile of the equation state $w(t)$ for $a_1 = 0.08050204989312597$.

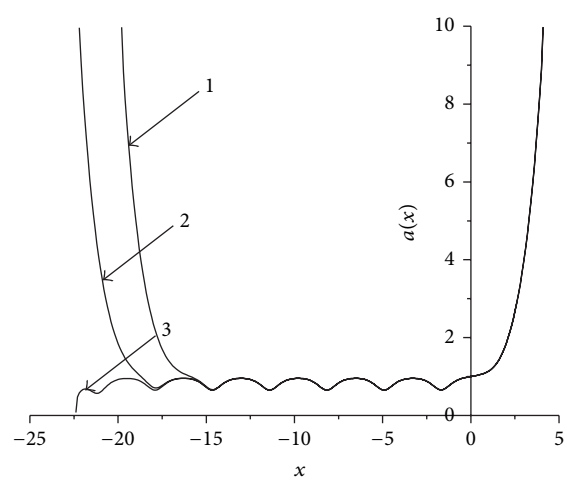

Figure 5: The profiles of regular and singular $a(t)$. The regular solutions: curve 1 for $a_1 = 0.080502049893$, curve 2 for $a_1 = 0.08050204989312597$. The singular solutions: curve 3 for $a_1 = 0.08050204989312599$.

0.08050204989312597 the solution has the oscillations, but for $a_1 = 0.08050204989312599$ the solution is singular one. It means that there exists a special solution with $a_1 = a_1^*$, where $0.08050204989312597 < a_1^* < 0.08050204989312599$. From Figures 1 and 5 we see that such solution should have an infinite long oscillation stage with a contracting phase at $t \to -\infty$ and an expansion phase at $t \to +\infty$. The most interesting is that by a_1 close enough to a_1^* we may have the stage with oscillations with any duration.

During the oscillations the size of Universe is $a \approx a_{Pl}$. It means that the existence time of Universe in such state can be somehow long with possible expansion at any time.

3. Deceleration/Acceleration Phases from Gravitating Higgs Field

3.1. Numerical Solution with Deceleration/Acceleration Phases. The aim of this section is to show that in the ordinary Einstein gravity interacting with the Higgs scalar field there exist solutions having the deceleration and acceleration phases. The corresponding Einstein and scalar field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \varkappa T_{\mu\nu},$$
$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu} \right) = -\frac{dV(\phi)}{d\phi}, \tag{16}$$

where $V(\phi)$ is

$$V(\phi) = \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0. \tag{17}$$

ϕ is the Higgs field and the Lagrangian for the scalar field is

$$L = \frac{1}{2} \left(\nabla_\mu \phi \right) \left(\nabla^\mu \phi \right) - V(\phi),$$
$$T_{\mu\nu} = \left(\nabla_\mu \phi \right) \left(\nabla_\nu \phi \right) - g_{\mu\nu} L. \tag{18}$$

The quantity V_0 in (17) is identical to a cosmological constant. Later we will see that V_0 is defined uniquely in a flex point t_0, where $\ddot{a}(t_0) = 0$.

We consider the cosmological metric

$$ds^2 = dt^2 - a^2(t) \left[d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right]. \tag{19}$$

The equations for $a(t)$ and $\phi(t)$ are

$$\frac{3\dot{a}^2}{a^2} + \frac{3}{a^2} = \varkappa \left[\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right], \tag{20}$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \varkappa \left[-\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right], \tag{21}$$

$$\ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} = \lambda\phi \left(m^2 - \phi^2 \right). \tag{22}$$

For the deceleration phase we have $\ddot{a}(t) < 0$, for the acceleration phase: $\ddot{a}(t) > 0$. Consequently there is a flex point t_0 where

$$\ddot{a}(t_0) = 0. \tag{23}$$

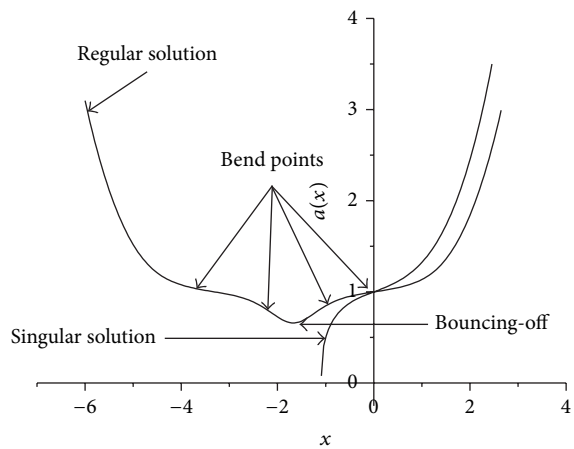

Figure 6: The profiles $a(t)$ for regular ($\dot{a}_0 = 0.08$) and singular ($\dot{a}_0 = 0.2$) solutions. The parameters $\lambda = 0.41$, $a_0 = 1.0$, $\phi_0 = -0.0028$; $\dot{\phi}_0 = -\sqrt{2\dot{a}_0^2 + 2}/a_0$, and $m = 0.603$ are the same for both solutions.

Using the conditions (23) and (20), (21) one can find the following constraints on the initial conditions and cosmological constant:

$$\frac{2\dot{a}_0^2}{a_0^2} + \frac{2}{a_0^2} = \varkappa \left[\frac{\lambda}{4} \left(\phi_0^2 - m^2 \right)^2 - V_0 \right],$$
$$\frac{2\dot{a}_0^2}{a_0^2} + \frac{2}{a_0^2} = \varkappa \dot{\phi}_0^2, \tag{24}$$

where $a_0 = a(t_0)$, $\dot{a}_0 = \dot{a}(t_0)$, $\phi_0 = \phi(0)$, and $\dot{\phi}_0 = \dot{\phi}(0)$. It means that the cosmological constant is defined uniquely:

$$\Lambda = \varkappa V_0 = \frac{\lambda}{4} \varkappa \left(\phi_0^2 - m^2 \right)^2 - 2 \left(\frac{\dot{a}_0^2}{a_0^2} + \frac{1}{a_0^2} \right). \tag{25}$$

For the numerical solution we introduce the dimensionless quantities $x = t/\sqrt{\varkappa}$, $\phi\sqrt{\varkappa} \to \phi$, and $a/\sqrt{\varkappa} \to a$. Then (20)–(22) are

$$\frac{3\dot{a}^2}{a^2} + \frac{3}{a^2} = \left[\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right],$$
$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \left[-\frac{\dot{\phi}^2}{2} + \frac{\lambda}{4} \left(\phi^2 - m^2 \right)^2 - V_0 \right], \tag{26}$$
$$\ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} = \lambda\phi \left(m^2 - \phi^2 \right)$$

with the following initial conditions:

$$a(0) = a_0, \qquad \dot{a}(0) = a_0, \qquad \phi(0) = \phi_0,$$
$$\dot{\phi}_0 = -\frac{\sqrt{2\dot{a}_0^2 + 2}}{a_0}. \tag{27}$$

The numerical solution is presented in Figures 6-7. We see that there are two type of solutions: regular and singular one.

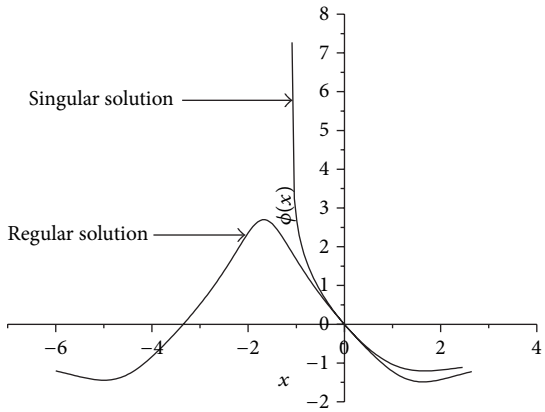

Figure 7: The profiles $\phi(t)$ for regular and singular solutions.

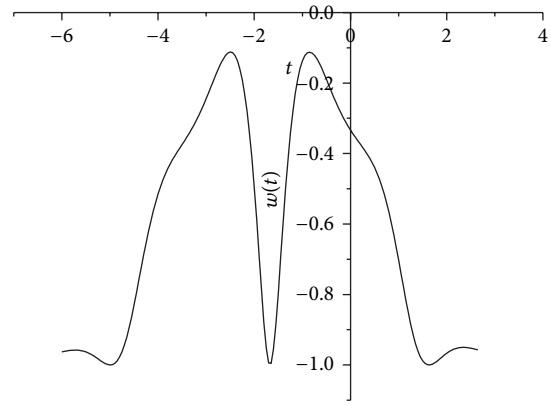

Figure 8: The profiles $w(t)$ for the regular solution.

The regular solution exists for $-\infty < t < +\infty$ and has bouncing-off point and four flex points. The singular solution has one flex point only. The presented solution is symmetrical one relative to the bouncing-off moment, but there exist nonsymmetrical solutions with the initial conditions different from (27).

The asymptotical behavior of the solution is

$$\begin{gathered} a(t) \approx a_\infty e^{t\sqrt{-\varkappa V_0/3}}, \quad t \longrightarrow +\infty, \\ a(t) \approx a_\infty e^{-t\sqrt{-\varkappa V_0/3}}, \quad t \longrightarrow -\infty, \\ \phi(t) \approx -m + \phi_\infty e^{-\alpha t}, \\ \alpha_{1,2} = \sqrt{\frac{-3\varkappa V_0}{4}} \pm \sqrt{\frac{-3\varkappa V_0}{4} - 2\lambda m^2}, \\ |t| \longrightarrow \infty, \end{gathered} \tag{28}$$

where a_∞, ϕ_∞ are constants. This solution can describe the inflation of Universe with the posterior standard decay of the scalar field.

It is useful to present the profile of a state equation (see Figure 8) in the form

$$w = \frac{p}{\varepsilon} = \frac{T^{11}}{T^{00}} = \frac{\left(\dot\phi^2/2\right) - (\lambda/4)\left(\phi^2 - m^2\right) + V_0}{\left(\dot\phi^2/2\right) + (\lambda/4)\left(\phi^2 - m^2\right) - V_0}, \tag{29}$$

where p is the pressure and ε is the energy density. Particularly interesting is the behavior of w in the region $x > x_0$, that is, in the acceleration region. We see there that $0 < w < -1$.

3.2. The Deceleration ⟶ Acceleration Transition. In the preceding section we have shown that the Universe filled with the Higgs field may have the deceleration/acceleration phases and have presented the solution with $a_0 \approx l_{Pl} \propto \sqrt{\varkappa}$. In this section we would like to investigate more carefully the solution near the flex point where the transition from the deceleration to acceleration phase has happened and with $a_0 \gg l_{Pl} \propto \sqrt{\varkappa}$. It is not too hard to find the solution of equations set (20)–(22) in the following form:

$$\begin{aligned} a(t) &= a_0 + \dot a_0 t \\ &\quad + \left[4\frac{\dot a_0\left(\dot a_0^2 + 1\right)}{a_0^2} \right. \\ &\quad \left. + \lambda\phi_0\sqrt{2\varkappa\left(\dot a_0^2 + 1\right)}\left(m^2 - \phi_0^2\right) \right] \frac{t^3}{6} + \cdots, \end{aligned} \tag{30}$$

$$\begin{aligned} \phi(t) &= \phi_0 - \frac{1}{a_0}\sqrt{2\frac{\dot a_0^2 + 1}{\varkappa}}\, t \\ &\quad + \left[3\frac{\dot a_0}{a_0^2}\sqrt{2\frac{\dot a_0^2 + 1}{\varkappa}} + \lambda\phi_0\left(m^2 - \phi_0^2\right) \right] \frac{t^2}{2} \\ &\quad - \left[12\frac{\dot a_0^2}{a_0^3}\sqrt{2\frac{\dot a_0^2 + 1}{\varkappa}} + 3\lambda\frac{\dot a_0\phi_0}{a_0}\left(m^2 - \phi_0^2\right) \right. \\ &\quad \left. + \frac{\lambda}{a_0}\sqrt{2\frac{\dot a_0^2 + 1}{\varkappa}}\left(m^2 - 3\phi_0^2\right) \right] \frac{t^3}{6} + \cdots. \end{aligned} \tag{31}$$

The deceleration parameter is

$$\begin{gathered} q(t) = -\frac{a\ddot a}{\dot a^2} \approx -\frac{a_0 a_3}{\dot a_0^2} t, \\ a_3 = 4\frac{\dot a_0\left(\dot a_0^2 + 1\right)}{a_0^2} + \lambda\phi_0\sqrt{2\varkappa\left(\dot a_0^2 + 1\right)}\left(m^2 - \phi_0^2\right). \end{gathered} \tag{32}$$

More convenient in this approach is the modified deceleration parameter

$$q(t) = -\frac{(a - a_0)\,\ddot a}{\dot a^2} \approx -\frac{a_3}{\dot a_0} t^2. \tag{33}$$

The Habble constant is

$$H(t) = \frac{\dot a}{a} \approx \frac{\dot a_0}{a_0} - \frac{\dot a_0^2}{a_0^2} t + \left(\frac{\dot a_0^3}{a_0^3} + \frac{a_3}{2}\right) t^2. \tag{34}$$

Let us remind that the time t is counted from the flex point moment t_0. Unfortunately it is not for a while yet unknown: is the solution with $a_0 \gg l_{Pl}$ regular or singular, that is, has the solution bouncing-off from a cosmological singularity or not.

4. Outlook

We have shown the following.

(i) The bouncing-off process of Universe is not a simple process. Implicitly it is supposed that bouncing-off happens at *a moment*. Here we have shown that it can be *a long-term process* taking place at the Planck region (oscillation). It allows us to consider a quantum birth of the Universe as the following process: the Universe exists in the state with $a_1 = a_1^*$ and in the consequence of quantum fluctuations of a_1 the Universe passes to the state with $a_1 \approx a_1^*$. After that the Universe goes into an inflation phase.

(ii) The Einstein-Higgs gravity has cosmological solutions with the deceleration/acceleration phases. Additionally these solutions may have bouncing-off from a cosmological singularity. The detailed investigation is made near the moment where the transition from the deceleration epoch to the acceleration one happens.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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