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Research Article

Acceptance Sampling Plans Based on Truncated Life Tests for Gompertz Distribution

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An acceptance sampling plan for Gompertz distribution under a truncated life test is developed. For different acceptance numbers, consumer's confidence levels and values of the ratio of the experimental time to the specified mean lifetime, the minimum sample sizes required to ensure the specified mean lifetime are obtained. The operating characteristic function values and the associated producer's risks are also presented. An example is provided to illustrate the acceptance sampling plan.

1. Introduction

Gompertz [1] introduced the Gompertz distribution to describe human mortality and establish actuarial tables. It is a well-known lifetime model and has many applications such as biology [2], gerontology [3], and marketing science [4].

Recently, Gompertz distribution has attracted much attention. Some characteristics of this distribution are obtained by Pollard and Valkovics [5], Wu and Lee [6], Read [7], and Kunimura [8]. Saraçoğlu et al. [9] investigated the statistical inference for reliability and stress strength for Gompertz distribution. Jaheen [10] conducted a Bayesian analysis of record statistics from the Gompertz model. Lenart [11] derived exact formulas for its moment-generating function and central moments using the generalised integroexponential function.

The probability density function of the Gompertz distribution is given by

$$f(t;\theta,\sigma)=\frac{\theta}{\sigma}e^{t/\sigma}\exp\left[-\theta\left(e^{t/\sigma}-1\right)\right],\quad t>0, \tag{1}$$

where $\theta > 0$ is the shape parameter, $\sigma > 0$ is the scale parameter, and t is the life time. We denote it by writing $T \sim \text{Gompertz}(\theta,\sigma)$. If $0<\theta<1$, the density function is increasing and then decreasing with mode $-\sigma\ln\theta$. If $\theta\geq 1$, then the density function is decreasing with mode 0.

The corresponding cumulative distribution function of T is

$$F(t;\theta,\sigma)=1-\exp\left[-\theta\left(e^{t/\sigma}-1\right)\right],\quad t>0. \tag{2}$$

Gompertz distribution has an exponentially increasing failure rate function and it is given by $\theta e^{t/\sigma}/\sigma$.

The mean of the Gompertz distribution is given by

$$\mu=E(T)=e^{\theta}\Gamma(0,\theta)\sigma, \tag{3}$$

where $\Gamma(s,x)=\int_x^\infty t^{s-1}e^{-t}dt$ is known as the upper incomplete gamma function. Its mean is positively proportional to the scale parameter σ when the shape parameter θ is fixed. The mean acts as quality level for the lifetime distribution under consideration.

On the other hand, acceptance sampling plan is an important tool for ensuring quality in the field of statistical quality control. It is widely used when the testing is destructive and the cost of complete and thorough inspection is very high and/or it takes too long. A random sample is selected from the lot and on the basis of information yielded by the sample a decision is made regarding the accepting or rejecting the lot.

Since the lifetime of a product is expected to be very high and it might be time consuming to wait until all the products fail, it is usual to terminate a life test by a preassigned time t_0

for saving money and time. One object of these tests is to set a confidence limit on the mean life and to establish a specified mean life, μ_0, with a probability of at least P^* which is the consumer's confidence level.

The decision is to accept the lot if and only if the observed number of failures does not exceed a given acceptance number c. If the number of failures exceeds this number, one can stop the test at the time t_0 and reject the lot. The problem considered is that of finding the minimum sample size n necessary to ensure a certain mean lifetime based on the truncated life test. A lot is considered good if the true mean life of items, μ, is not less than the specified value μ_0. A lot is considered bad if $\mu < \mu_0$. For a given acceptance sampling plan, the consumer's risk and the producer's risk are the probabilities that a bad lot is accepted and a good lot is rejected, respectively.

Truncated life tests of this type have been developed by many authors. Sobel and Tischendrof [12] studied acceptance sampling plans for exponential distribution. These results are extended for the Weibull distribution by Goode and Kao [13]. Gupta and Groll [14] and Gupta [15] studied the sampling plans for the lifetime under the gamma and log-normal distributions, respectively. More recently, Aslam et al. [16], Balakrishnan et al. [17], Tsai and Wu [18], Rosaiah and Kantam [19], Kantam et al. [20], and Al-Nasser and Al-Omari [21] developed the acceptance sampling plans for generalized exponential distribution, generalized Birnbaum-Saunders distribution, generalized Rayleigh distribution, inverse Rayleigh distribution, log-logistic distribution, and exponentiated Fréchet distribution, respectively.

The acceptance sampling plans based on truncated life tests for the well-known Gom-pertz distribution have not been studied. This problem will be discussed in the paper. The rest of this paper is organized as follows. in Section 2, the proposed acceptance sampling plans are developed and the operating characteristic values and the producer's risk are analyzed. The numerical results and illustrative examples are presented in Section 3. The work is concluded in Section 4.

2. Design of the Sampling Plan

Suppose that life time of products follows a Gompertz distribution defined by (1). The life test terminates at a preassigned time t_0 and the number of failures during this time interval $[0, t]$ is recorded. The decision to accept the lot occurs if and only if the recorded number of failures at the end of the time point t_0 is less than or equal to the acceptance number c.

The lot size is assumed to be infinitely large so that the theory of binomial distribution can be applied. The acceptance or rejection of the lot is equivalent to the acceptance or rejection of the hypothesis $H_0 : \mu \geq \mu_0$. From (3), it is clear that, for fixed θ, $\mu \geq \mu_0 \Leftrightarrow \sigma \geq \sigma_0$, where

$$\sigma_0 = \frac{\mu_0}{e^{\theta}\Gamma(0,\theta)}. \tag{4}$$

Note that σ_0 also depends on the shape parameter θ. We assume that θ is known in this paper.

For the sake of convenience, set the termination time as a multiple of the specified mean lifetime; that is, $t_0 = a\mu_0$, where a is a positive constant. The proposed acceptance sampling plan is characterized by (n, c, a), which consists of

(1) the number of items n to be drawn from the lot,

(2) the acceptance number c,

(3) the ratio $a = t_0/\mu_0$, where μ_0 corresponds to the specified mean lifetime and t_0 is the preassigned testing time.

Let P^* be the consumer's confidence level in the sense that the chance of rejecting a bad lot (having mean lifetime $\mu < \mu_0$) is at least P^*. The consumer's risk, the probability of accepting a bad lot, is fixed not to exceed $1 - P^*$.

The probability of accepting a lot is obtained by $\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i}$, where $p = F(t_0;\theta,\sigma)$ = probability of a failure before time t_0 and is given by

$$\begin{aligned} p &= 1 - \exp\left[-\theta\left(e^{t_0/\sigma} - 1\right)\right] \\ &= 1 - \exp\left[-\theta\left(e^{ae^{\theta}\Gamma(0,\theta)/(\mu/\mu_0)} - 1\right)\right]. \end{aligned} \tag{5}$$

One wants to find the minimum sample size (n) satisfying the inequality

$$\sum_{i=0}^{c}\binom{n}{i}p_0^i(1-p_0)^{n-i} \leq 1 - P^*, \tag{6}$$

where $p_0 = F(t_0;\theta,\sigma_0)$ = probability of a failure in time t_0 if true mean life is equal to μ_0 and is given by

$$p_0 = 1 - \exp\left[-\theta\left(e^{ae^{\theta}\Gamma(0,\theta)} - 1\right)\right]. \tag{7}$$

It depends only on the ratio $a = t_0/\mu_0$ and is a monotonically increasing function of the ratio. Hence, the experiment needs to specify only this ratio.

If the number of observed failures is at most c, from (6), we can establish that $F(t;\theta,\sigma) \leq F(t;\theta,\sigma_0)$ with probability P^*, which implies $\sigma \geq \sigma_0$ (or $\mu \geq \mu_0$). Thus, the mean lifetime of the items can be assured to be at least equal to their specified value with probability P^*.

The minimum values of n satisfying inequality (6) have been obtained for $P^* = 0.75, 0.90, 0.95, 0.99$ and $a = t_0/\mu_0 = 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0$. These are presented in Tables 1 and 2 for $\theta = 1, 2$, respectively. This choice of a is consistent with the corresponding tables of Tsai and Wu [18], Gupta and Groll [14], Kantam et al. [20], Aslam et al. [16], and Al-Nasser and Al-Omari [21].

The operating characteristic function is associated with acceptance sampling plans. It measures the efficiency of a statistical hypothesis test designed to accept or reject a lot. The operating characteristic function of the proposed sampling plan $(n, c, a = t_0/\mu_0)$ provides the probability of accepting the lot and is given by

$$L(p) = \sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i}, \tag{8}$$

Table 1: Minimum sample sizes n to be tested for a time t_0 necessary to assert the mean life μ to exceed a given value, μ_0, with probability P^* and the corresponding acceptance number, c, using binomial probabilities ($\theta = 1$).

P^*	c	$a = t_0/m_0$							
		0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0
	0	6	4	3	2	1	1	1	1
	1	11	7	5	4	3	2	2	2
	2	16	11	8	6	4	4	3	3
	3	21	14	11	8	6	5	4	4
	4	26	17	13	10	7	6	5	5
0.75	5	31	20	15	12	9	7	6	6
	6	35	24	18	14	10	8	7	7
	7	40	27	20	16	11	9	8	8
	8	45	30	23	18	13	11	10	9
	9	49	33	25	20	14	12	11	10
	10	54	36	27	22	16	13	12	11
	0	9	6	4	3	2	2	1	1
	1	15	10	7	6	4	3	2	2
	2	21	14	10	8	5	4	3	3
	3	27	18	13	10	7	5	5	4
	4	32	21	16	13	8	7	6	5
0.90	5	38	25	18	15	10	8	7	6
	6	43	28	21	17	11	9	8	7
	7	48	32	24	19	13	10	9	8
	8	53	35	26	21	14	11	10	9
	9	58	38	29	23	16	13	11	10
	10	63	42	31	25	17	14	12	11
	0	12	7	5	4	3	2	1	1
	1	19	12	9	7	4	3	3	2
	2	25	16	12	9	6	5	4	3
	3	31	20	15	12	8	6	5	4
	4	37	24	18	14	9	7	6	5
0.95	5	42	28	20	16	11	8	7	6
	6	48	31	23	18	12	10	8	7
	7	53	35	26	21	14	11	9	9
	8	58	38	28	23	15	12	10	10
	9	63	42	31	25	17	13	11	11
	10	69	45	34	27	18	14	13	12
	0	18	11	8	6	4	3	2	1
	1	26	16	12	9	6	4	3	3
	2	33	21	15	12	7	5	4	4
	3	39	25	19	14	9	7	5	5
	4	46	30	22	17	11	8	7	6
0.99	5	52	34	25	19	13	9	8	7
	6	58	38	28	22	14	11	9	8
	7	64	41	30	24	16	12	10	9
	8	69	45	33	26	17	13	11	10
	9	75	49	36	29	19	14	12	11
	10	80	53	39	31	20	16	13	12

Table 2: Minimum sample sizes n to be tested for a time t_0 necessary to assert the mean life μ to exceed a given value, μ_0, with probability P^* and the corresponding acceptance number, c, using binomial probabilities ($\theta = 2$).

P^*	c	$a = t_0/m_0$							
		0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0
	0	5	3	3	2	1	1	1	1
	1	10	7	5	4	3	2	2	2
	2	14	10	7	6	4	4	3	3
	3	19	13	10	8	6	5	4	4
	4	23	16	12	10	7	6	5	5
0.75	5	27	19	14	12	9	7	7	6
	6	31	21	17	14	10	8	8	7
	7	35	24	19	16	12	10	9	8
	8	39	27	21	17	13	11	10	9
	9	44	30	23	19	14	12	11	10
	10	48	33	25	21	16	13	12	11
	0	8	5	4	3	2	2	1	1
	1	14	9	7	6	4	3	3	2
	2	19	13	10	8	5	4	4	3
	3	24	16	12	10	7	6	5	4
	4	28	19	15	12	8	7	6	5
0.90	5	33	22	17	14	10	8	7	7
	6	38	25	20	16	11	9	8	8
	7	42	29	22	18	13	11	9	9
	8	46	32	24	20	14	12	10	10
	9	51	35	27	22	16	13	12	11
	10	55	38	29	24	17	14	13	12
	0	10	7	5	4	3	2	2	1
	1	16	11	8	7	4	3	3	2
	2	22	15	11	9	6	5	4	4
	3	27	18	14	11	8	6	5	5
	4	32	22	16	13	9	7	6	6
0.95	5	37	25	19	15	11	9	7	7
	6	42	28	21	17	12	10	9	8
	7	46	31	24	20	14	11	10	9
	8	51	35	26	22	15	12	11	10
	9	56	38	29	24	17	14	12	11
	10	60	41	31	26	18	15	13	12
	0	15	10	7	6	4	3	2	2
	1	22	15	11	9	6	4	3	3
	2	29	19	14	11	8	6	5	4
	3	34	23	17	14	9	7	6	5
	4	40	27	20	16	11	8	7	6
0.99	5	45	30	23	18	13	10	8	7
	6	50	34	25	21	14	11	9	9
	7	56	37	28	23	16	12	11	10
	8	61	41	31	25	17	14	12	11
	9	66	44	33	27	19	15	13	12
	10	70	47	36	29	20	16	14	13

where $p = F(t_0; \theta, \sigma)$. Notice that $L(p)$ is a decreasing function of p and p is a decreasing function of σ (or the mean μ); thus the operating characteristic function is an increasing function of σ (or μ). For given P^*, $a = t_0/\mu_0$, the choice of c

Table 3: Operating characteristic values of the sampling plan (n, c, a) for a given P^*, under Gompertz distribution ($\theta = 1$).

P^*	n	c	$a = t_0/\mu_0$	μ/μ_0 2	4	6	8	10	12
0.75	16	2	0.4	0.705	0.934	0.976	0.989	0.994	0.996
	11	2	0.6	0.690	0.932	0.976	0.989	0.994	0.996
	8	2	0.8	0.713	0.941	0.979	0.991	0.995	0.997
	6	2	1.0	0.756	0.954	0.984	0.993	0.996	0.998
	4	2	1.5	0.783	0.964	0.988	0.995	0.997	0.998
	4	2	2.0	0.597	0.921	0.974	0.988	0.994	0.996
	3	2	2.5	0.700	0.952	0.985	0.994	0.997	0.998
	3	2	3.0	0.553	0.920	0.975	0.989	0.994	0.997
0.90	21	2	0.4	0.536	0.873	0.951	0.976	0.987	0.992
	14	2	0.6	0.534	0.877	0.953	0.978	0.988	0.992
	10	2	0.8	0.567	0.893	0.961	0.981	0.990	0.994
	8	2	1.0	0.568	0.897	0.963	0.983	0.991	0.994
	5	2	1.5	0.628	0.924	0.974	0.988	0.994	0.996
	4	2	2.0	0.597	0.921	0.974	0.988	0.994	0.996
	3	2	2.5	0.700	0.952	0.985	0.994	0.997	0.998
	3	2	3.0	0.553	0.920	0.975	0.989	0.994	0.997
0.95	25	2	0.4	0.415	0.815	0.925	0.963	0.979	0.987
	16	2	0.6	0.439	0.833	0.934	0.968	0.982	0.989
	12	2	0.8	0.435	0.836	0.936	0.969	0.983	0.989
	9	2	1.0	0.480	0.862	0.948	0.975	0.986	0.992
	6	2	1.5	0.480	0.873	0.954	0.979	0.988	0.993
	5	2	2.0	0.393	0.845	0.944	0.974	0.986	0.992
	4	2	2.5	0.402	0.860	0.952	0.978	0.988	0.993
	3	2	3.0	0.553	0.920	0.975	0.989	0.994	0.997
0.99	33	2	0.4	0.231	0.686	0.858	0.925	0.956	0.973
	21	2	0.6	0.251	0.712	0.873	0.935	0.962	0.976
	15	2	0.8	0.276	0.739	0.889	0.944	0.968	0.980
	12	2	1.0	0.269	0.741	0.891	0.945	0.969	0.981
	7	2	1.5	0.354	0.813	0.928	0.966	0.981	0.988
	5	2	2.0	0.393	0.845	0.944	0.974	0.986	0.992
	4	2	2.5	0.402	0.860	0.952	0.978	0.988	0.993
	4	2	3.0	0.237	0.783	0.921	0.964	0.980	0.988

Table 4: Operating characteristic values of the sampling plan (n, c, a) for a given P^*, under Gompertz distribution ($\theta = 2$).

P^*	n	c	$a = t_0/\mu_0$	μ/μ_0 2	4	6	8	10	12
0.75	14	2	0.4	0.692	0.928	0.974	0.988	0.993	0.996
	10	2	0.6	0.664	0.920	0.971	0.986	0.992	0.995
	7	2	0.8	0.720	0.939	0.978	0.990	0.995	0.997
	6	2	1.0	0.693	0.932	0.976	0.989	0.994	0.996
	4	2	1.5	0.740	0.948	0.982	0.992	0.996	0.997
	4	2	2.0	0.557	0.894	0.962	0.982	0.990	0.994
	3	2	2.5	0.685	0.937	0.979	0.990	0.995	0.997
	3	2	3.0	0.556	0.9	0.965	0.984	0.991	0.995
0.90	19	2	0.4	0.495	0.852	0.941	0.971	0.984	0.990
	13	2	0.6	0.485	0.850	0.940	0.971	0.984	0.990
	10	2	0.8	0.474	0.847	0.940	0.971	0.984	0.990
	8	2	1.0	0.484	0.854	0.943	0.973	0.985	0.991
	5	2	1.5	0.568	0.894	0.961	0.982	0.990	0.994
	4	2	2.0	0.557	0.894	0.962	0.982	0.990	0.994
	4	2	2.5	0.382	0.824	0.932	0.968	0.982	0.989
	3	2	3.0	0.556	0.9	0.965	0.984	0.991	0.995
0.95	22	2	0.4	0.392	0.798	0.915	0.957	0.976	0.985
	15	2	0.6	0.381	0.795	0.914	0.957	0.976	0.985
	11	2	0.8	0.403	0.811	0.923	0.962	0.978	0.987
	9	2	1.0	0.393	0.808	0.922	0.962	0.978	0.987
	6	2	1.5	0.415	0.828	0.932	0.967	0.982	0.989
	5	2	2.0	0.350	0.799	0.920	0.961	0.978	0.987
	4	2	2.5	0.382	0.824	0.932	0.968	0.982	0.989
	4	2	3.0	0.240	0.740	0.894	0.948	0.971	0.982
0.99	29	2	0.4	0.211	0.661	0.842	0.916	0.950	0.968
	19	2	0.6	0.222	0.676	0.852	0.922	0.954	0.971
	14	2	0.8	0.234	0.692	0.862	0.928	0.958	0.974
	11	2	1.0	0.248	0.709	0.872	0.934	0.962	0.976
	8	2	1.5	0.198	0.674	0.854	0.925	0.956	0.973
	6	2	2.0	0.206	0.693	0.867	0.932	0.961	0.976
	5	2	2.5	0.189	0.687	0.865	0.932	0.961	0.976
	4	2	3.0	0.240	0.740	0.894	0.948	0.971	0.982

and n is made on the basis of operating characteristics. Values of operating characteristics as a function of μ/μ_0 for several sampling plans are given in Tables 3 and 4 for $c = 2$ and $\theta = 1, 2$, respectively.

The producer's risk is the probability of rejection of the lot when it is good ($\mu > \mu_0$, or equivalently $\sigma > \sigma_0$). For the proposed sampling plan and a given value for the producer's risk γ, one is interested in knowing the value of μ/μ_0 that will ensure the producer's risk to be at most γ. Notice that p is a function of μ/μ_0 as shown in (5); then μ/μ_0 is the smallest positive number for which p satisfies the inequality

$$\sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \le \gamma. \tag{9}$$

For a given acceptance sampling plan (n, c, a), at a specified confidence level P^*, the minimum values of μ/μ_0 satisfying (9) are presented in Tables 5 and 6 for $\theta = 1, 2$, respectively. The tables can be generated for any other values of the shape parameter $\theta > 0$.

Figures 1 and 2 are the plots of the required minimum sample size versus $a = t_0/\mu_0$ for some selected values of the confidence level P^* and the shape parameter θ, respectively. Figure 1 shows that the required minimum sample size decreases as a increases, and the required minimum sample sizes get close for all selected confidence levels when a is large. Figure 2 shows that the required minimum sample size increases as the value of θ decreases when the value of a is small, and the required minimum sample sizes for different values of θ get close when the value of a is large.

3. Illustration of the Tables

Assume that the lifetime distribution of the test items follows Gompertz distribution with shape parameter $\theta = 1$ and we

Table 5: Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of $\gamma = 0.05$ ($\theta = 1$).

P^*	c	$a = t_0/\mu_0$							
		0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0
	0	28.03	28.09	28.15	23.55	17.89	23.85	29.81	35.77
	1	7.16	6.71	6.24	6.10	6.59	5.29	6.61	7.93
	2	4.49	4.54	4.29	3.88	3.56	4.75	3.95	4.74
	3	3.52	3.44	3.53	3.08	3.26	3.41	3.02	3.62
	4	3.02	2.89	2.87	2.66	2.57	2.76	2.55	3.06
0.75	5	2.72	2.56	2.49	2.41	2.54	2.38	2.27	2.72
	6	2.44	2.45	2.39	2.24	2.21	2.13	2.07	2.49
	7	2.30	2.28	2.18	2.12	1.98	1.95	1.94	2.32
	8	2.20	2.15	2.14	2.02	2.02	2.13	2.27	2.20
	9	2.07	2.04	2.01	1.95	1.87	1.99	2.14	2.10
	10	2.01	1.96	1.91	1.89	1.91	1.88	2.04	2.02
	0	41.98	42.04	37.45	35.18	35.33	47.10	29.81	35.77
	1	9.85	9.73	8.94	9.49	9.15	8.79	6.61	7.93
	2	5.95	5.86	5.46	5.36	4.70	4.75	3.95	4.74
	3	4.57	4.49	4.23	3.96	3.94	3.41	4.26	3.62
	4	3.75	3.62	3.61	3.59	3.05	3.42	3.45	3.06
0.90	5	3.36	3.25	3.04	3.11	2.90	2.89	2.97	2.72
	6	3.02	2.89	2.83	2.80	2.50	2.55	2.66	2.49
	7	2.78	2.73	2.67	2.58	2.47	2.31	2.44	2.32
	8	2.61	2.53	2.45	2.41	2.23	2.13	2.27	2.20
	9	2.47	2.38	2.37	2.29	2.23	2.24	2.14	2.10
	10	2.36	2.31	2.22	2.19	2.06	2.11	2.04	2.02
	0	55.93	49.01	46.75	46.81	52.77	47.10	29.81	35.77
	1	12.54	11.75	11.63	11.17	9.15	8.79	10.98	7.93
	2	7.12	6.73	6.64	6.10	5.82	6.27	5.94	4.74
	3	5.27	5.02	4.93	4.85	4.61	4.34	4.26	3.62
	4	4.36	4.17	4.09	3.89	3.52	3.42	3.45	3.06
0.95	5	3.72	3.66	3.41	3.34	3.26	2.89	2.97	2.72
	6	3.39	3.22	3.12	2.98	2.79	2.95	2.66	2.49
	7	3.08	3	2.91	2.88	2.70	2.64	2.44	2.92
	8	2.86	2.76	2.65	2.67	2.43	2.42	2.27	2.72
	9	2.69	2.64	2.55	2.51	2.40	2.24	2.14	2.57
	10	2.59	2.49	2.46	2.38	2.22	2.11	2.34	2.44
	0	83.83	76.92	74.65	70.06	70.21	70.36	58.88	35.77
	1	17.24	15.78	15.66	14.54	14.23	12.20	10.98	13.18
	2	9.46	8.93	8.39	8.30	6.93	6.27	5.94	7.12
	3	6.67	6.33	6.34	5.73	5.28	5.25	4.26	5.11
	4	5.45	5.26	5.07	4.81	4.46	4.07	4.28	4.14
0.99	5	4.64	4.49	4.33	4.03	3.96	3.39	3.61	3.57
	6	4.11	3.98	3.85	3.72	3.36	3.34	3.18	3.19
	7	3.74	3.54	3.40	3.34	3.17	2.97	2.88	2.92
	8	3.42	3.30	3.17	3.06	2.83	2.70	2.66	2.72
	9	3.22	3.11	2.99	2.96	2.75	2.49	2.48	2.57
	10	3.02	2.96	2.85	2.78	2.53	2.54	2.34	2.44

Table 6: Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of $\gamma = 0.05$ ($\theta = 2$).

P^*	c	$a = t_0/\mu_0$							
		0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0
	0	28.25	25.47	33.96	28.36	21.41	28.54	35.68	42.81
	1	7.79	8.02	7.42	7.22	7.73	6.07	7.59	9.10
	2	4.66	4.86	4.34	4.53	4.06	5.41	4.37	5.25
	3	3.77	3.74	3.70	3.55	3.69	3.80	3.26	3.91
	4	3.15	3.17	3.04	3.05	2.86	3.02	2.70	3.24
0.75	5	2.78	2.83	2.65	2.75	2.83	2.56	3.20	2.83
	6	2.53	2.47	2.57	2.54	2.43	2.27	2.83	2.56
	7	2.36	2.32	2.36	2.39	2.45	2.48	2.57	2.37
	8	2.22	2.22	2.20	2.12	2.21	2.26	2.37	2.22
	9	2.17	2.13	2.08	2.05	2.02	2.10	2.22	2.10
	10	2.08	2.06	1.98	2	2.07	1.97	2.09	2.01
	0	45.16	42.38	45.23	42.45	42.54	56.72	35.68	42.81
	1	11.05	10.46	10.69	11.32	10.83	10.30	12.87	9.10
	2	6.43	6.46	6.48	6.32	5.44	5.41	6.77	5.25
	3	4.83	4.70	4.56	4.63	4.52	4.92	4.75	3.91
	4	3.88	3.84	3.93	3.80	3.44	3.82	3.77	3.24
0.90	5	3.44	3.33	3.32	3.31	3.27	3.18	3.20	3.84
	6	3.15	3	3.11	2.99	2.79	2.77	2.83	3.40
	7	2.87	2.87	2.80	2.77	2.74	2.88	2.57	3.08
	8	2.66	2.68	2.58	2.60	2.46	2.61	2.37	2.84
	9	2.55	2.53	2.51	2.46	2.45	2.40	2.62	2.66
	10	2.41	2.41	2.37	2.36	2.26	2.24	2.46	2.51
	0	56.43	59.29	56.50	56.54	63.67	56.72	70.90	42.81
	1	12.68	12.91	12.32	13.36	10.83	10.30	12.87	9.10
	2	7.49	7.52	7.19	7.21	6.79	7.25	6.77	8.12
	3	5.47	5.33	5.41	5.16	5.33	4.92	4.75	5.70
	4	4.47	4.50	4.23	4.17	4.01	3.82	3.77	4.53
0.95	5	3.89	3.83	3.77	3.59	3.70	3.77	3.20	3.84
	6	3.50	3.40	3.29	3.22	3.13	3.24	3.46	3.40
	7	3.16	3.09	3.10	3.14	3.03	2.88	3.09	3.08
	8	2.96	2.96	2.83	2.91	2.70	2.61	2.83	2.84
	9	2.81	2.77	2.73	2.74	2.66	2.70	2.62	2.66
	10	2.65	2.63	2.56	2.60	2.44	2.50	2.46	2.51
	0	84.61	84.65	79.05	84.72	84.81	84.90	70.90	85.08
	1	17.56	17.79	17.21	17.44	16.97	14.44	12.87	15.45
	2	9.97	9.65	9.32	8.99	9.48	9.06	9.06	8.12
	3	6.95	6.92	6.69	6.76	6.14	6.02	6.15	5.70
	4	5.65	5.60	5.41	5.28	5.14	4.59	4.77	4.53
0.99	5	4.77	4.66	4.66	4.43	4.55	4.35	3.97	3.84
	6	4.21	4.19	4	4.11	3.81	3.71	3.46	4.15
	7	3.88	3.75	3.68	3.69	3.59	3.27	3.60	3.71
	8	3.58	3.52	3.45	3.38	3.18	3.27	3.26	3.39
	9	3.35	3.26	3.16	3.14	3.08	2.98	3	3.14
	10	3.12	3.05	3.03	2.96	2.81	2.76	2.80	2.95

want to establish that the mean lifetime is at least $m_0 = 1000$ hours with probability $P^* = 0.90$. The life test is terminated at $t = 400$ hours.

Table 1 shows the minimum sample size n required to ascertain that the mean life μ exceeds μ_0 with probability at least P^*, the corresponding acceptance number c. When $P^* = 0.90$, $a = t_0/\mu_0 = 400/1000 = 0.4$, $c = 2$, the corresponding

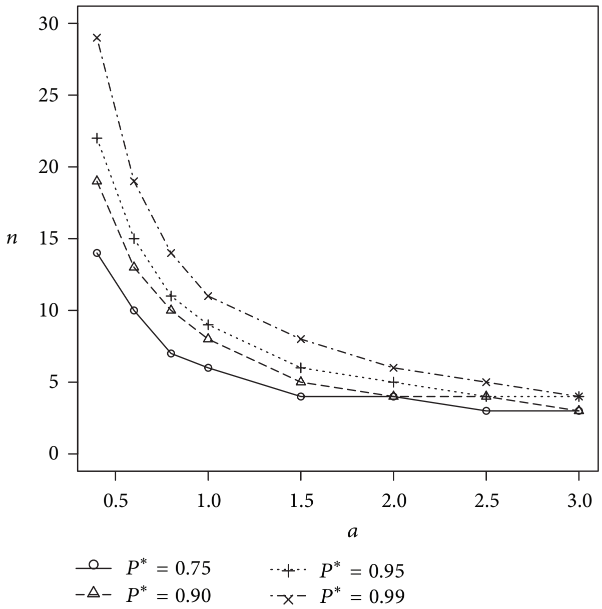

Figure 1: The minimum sample size versus $a = t_0/\mu_0$ with $\theta = 2$, $c = 2$.

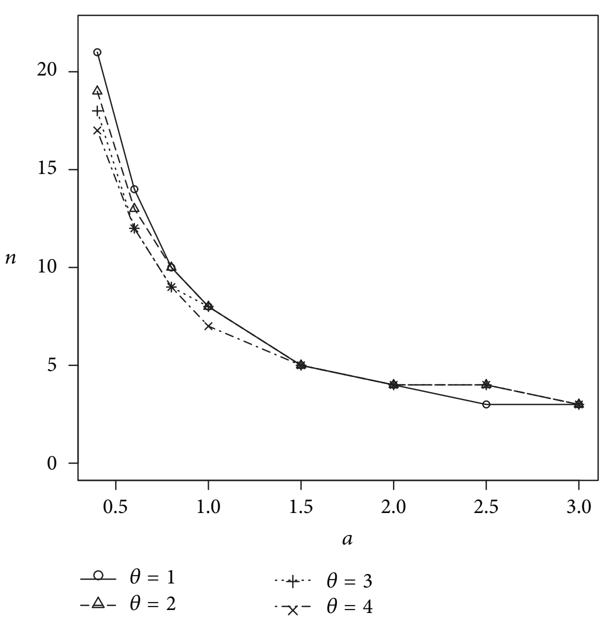

Figure 2: The minimum sample size versus $a = t_0/\mu_0$ with $P^* = 0.90$, $c = 2$.

entry in Table 1 is 21. That is, out of 21 items, if no more than 2 items fail during 400 hours, then the experimenter can assert that the true mean lifetime μ of the items is at least 1000 hours with a confidence level of 0.90.

Table 3 displays operating characteristic function values for the time truncated acceptance sampling plan adopted from Table 1, for various values of P^* and for different values of μ/μ_0 when $c = 2$. For example, when $P^* = 0.90$, $a = t_0/\mu_0 = 0.4$, $c = 2$, $\mu/\mu_0 = 6$, the corresponding entry in Table 3 is 0.951. It implies that if we accept the above acceptance sampling plan, that is, the lot is accepted if, out of 21 items, less than or equal to 2 items fail before time point $t_0 = 400$ hours, then if $\mu \geq 6 \times t_0/0.4$ or $\mu \geq 15t_0 = 6000$ hours, then the lot will be accepted with probability at least 0.951.

Table 5 shows the minimum ratio of the true mean lifetime to the specified one for the acceptance of a lot with the producer's risk $\gamma = 0.05$. For example, when the consumer's risk is 0.10 or $P^* = 0.90$, $c = 2$, and $a = t_0/\mu_0 = 0.4$, the table entry is $\mu/\mu_0 = 5.95$, which indicates that if $\mu \geq 5.95 \times t_0/0.4 = 14.875t_0 = 5950$ hours, then, with sample size $n = 21$ and $c = 2$, the lot will be rejected with probability less than or equal to 0.05.

4. Concluding Remarks

In this paper, a time truncated acceptance sampling plan is developed when the lifetime follows the Gompertz distribution. The table for the minimum sample size required to guarantee a certain mean lifetime of the test items is presented. The operating characteristic function values and the associated producer's risks are also discussed. The proposed plans can be used conveniently.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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