Hindawi Publishing Corporation
International Journal of Analysis
Volume 2014, Article ID 393618, 5 pages
http://dx.doi.org/10.1155/2014/393618

*Research Article*

# Some Properties of $(1,2)^*$-Locally Closed Sets

**Baby Bhattacharya, Arnab Paul, and Sudip Debnath**

*Department of Mathematics, NIT Agartala, Jirania, West Tripura 799055, India*

Correspondence should be addressed to Baby Bhattacharya; babybhatt75@gmail.com

Received 23 November 2013; Accepted 22 January 2014; Published 31 March 2014

Academic Editor: Rodica D. Costin



A new kind of generalization of $(1,2)^*$-closed set, namely, $(1,2)^*$-locally closed set, is introduced and using $(1,2)^*$-locally closed sets we study the concept of $(1,2)^*$-LC-continuity in bitopological space. Also we study $(1,2)^*$-contracontinuity and lastly investigate its relationship with $(1,2)^*$-LC-continuity.

## 1. Introduction and Preliminaries

It is established that generalization of closed set plays an important role in developing the various concepts in both topological and bitopological spaces. The difference of two closed subsets of an $n$-dimensional Euclidean space was considered by Kuratowski and Sierpinski [1] in 1921 and the fundamental tool in their work is the notion of a locally closed subset of a topological space $(X,\tau)$. In 1963 Kelly [2] initiated the systematic study of bitopological spaces. Then in 1989 Ganster and Reilly [3] used locally closed sets to define LC-continuity in a topological space. According to them a function $f$ is said to be LC-continuous if the inverse image of every open set in $Y$ is locally closed set in $X$. In 1990 Jelić [4] introduced (1, 2)-locally closed sets and (1, 2) LC-continuity in bitopological space. In 1991 Lellis Thivagar introduced the open set in bitopological space which is called (1, 2) open set [5]. In general we know that a $(1,2)^*$-$\alpha$-open set [6] may not be a $\tau_{1,2}$-open set in $X$, but in this present paper we have a necessary and sufficient condition for the requirement that an $(1,2)^*$-$\alpha$-open set is a $\tau_{1,2}$-open set. Bourbaki [7] defined that a subset $A$ of a topological space $X$ is said to be locally closed if it is the intersection of an open and a closed subsets of $X$ in the year 1966. In 2004 M. Lellis Thivagar and O. Ravi introduced a generalized concept of (1, 2) open sets which is called $(1,2)^*$-open sets [8] in bitopological space. Using the $(1,2)^*$-open set and its complement in this paper we introduce $(1,2)^*$-locally closed set and $(1,2)^*$-separated set that is defined to obtain an improved result which gives that union of any two $(1,2)^*$-locally closed sets is again a $(1,2)^*$-locally closed set. We also established a relationship of $(1,2)^*$-regular open set [9] with $(1,2)^*$-Locally closed set. As an application of $(1,2)^*$-locally closed set we study $(1,2)^*$-LC-continuity. Lastly we introduce $(1,2)^*$-contracontinuous function and we investigate its relationship with $(1,2)^*$-LC-continuity.

Throughout this paper $X$, $Y$, and $Z$ denote the bitopological spaces $(X,\tau_1,\tau_2)$, $(Y,\sigma_1,\sigma_2)$, and $(Z,\eta_1,\eta_2)$, respectively, on which no separation axioms are assumed. The concept of $(1,2)^*$-continuous function from a bitopological space $X$ into another bitopological space $Y$ was defined as follows.

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be $(1,2)^*$-continuous if and only if inverse image of every $\sigma_{1,2}$-open set in $Y$ is $\tau_{1,2}$-open set in $X$. Now we study the following definitions for ready references.

*Definition 1.* A subset $A$ of a bitopological space $(X,\tau_1,\tau_2)$ is called a (1, 2)-locally closed [4] if $A = G\cap F$ where $G$ is a $\tau_1$-open set in $X$ and $F$ is $\tau_2$-closed in $X$.

*Definition 2.* Let $A$ be a subset of $X$. Then $A$ is called $\tau_{1,2}$-open [8] if $A = A_i \cup B_i$ where $A_i \in \tau_1$ and $B_i \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed set.

*Definition 3.* Let $A$ be a subset of a bitopological space $X$. Then

(i) $(1,2)^*$-$\alpha$-open [6] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int($A$))),

(ii) $\tau_{1,2}$-closure of $A$ denoted by $\tau_{1,2}$-cl($A$) [8] is defined as the intersection of all $\tau_{1,2}$-closed sets containing $A$,

(iii) $\tau_{1,2}$-interior of $A$ denoted by $\tau_{1,2}$-int($A$) [8] is defined as the union of all $\tau_{1,2}$-open sets contained in $A$,

(iv) $(1,2)^*$-preopen [8] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($A$)),

(v) $(1,2)^*$-regular open [9] if $A = \tau_{1,2}$-int($\tau_{1,2}$-cl($A$)).

The family of all $\tau_{1,2}$-open (resp., $\tau_{1,2}$-closed, $(1,2)^*$-regular open, $(1,2)^*$-preopen, and $(1,2)^*$-$\alpha$-open) set of $X$ is denoted by $(1,2)^*$-$O(X)$ (resp., $(1,2)^*$-$C(X)$, $(1,2)^*$-RO($X$), $(1,2)^*$-PO($X$), and $(1,2)^*$-$\alpha$-$O(X)$).

*Remark 4.* Note that the collection of all $\tau_{1,2}$-open subsets of $X$ need not necessarily form a topology.

Now using $\tau_{1,2}$-open set and its complement in Section 2 we define $(1,2)^*$-locally closed sets and study their properties. Then we study a new form of $(1,2)^*$-continuity, namely, $(1,2)^*$-LC-continuity, and study the interrelationship between $(1,2)^*$-contracontinuity and $(1,2)^*$-LC-continuity.

## 2. $(1,2)^*$-Locally Closed Sets

*Definition 5.* A subset $A$ of a bitopological space $(X,\tau_1,\tau_2)$ is called $(1,2)^*$-locally closed if $A = U \cap V$ where $U \in (1,2)^*$-$O(X)$ and $V \in (1,2)^*$-$C(X)$, or equivalently if $A = U \cap \tau_{1,2}$-cl($A$), for some $U \in (1,2)^*$-$O(X)$.

*Note 1.* The collection of all $(1,2)^*$-locally closed set of $(X,\tau_1,\tau_2)$ is denoted by $(1,2)^*$-LC($X$).

*Example 6.* Let $X = \{a,b,c\}$, $\tau_1 = \{\{a\},\{a,b\},\varphi,X\}$ and $\tau_2 = \{\varphi,X\}$. Then $(1,2)^*$-$O(X) = \{\{a\},\{a,b\},\varphi,X\}$ and $(1,2)^*$-$C(X) = \{\{b,c\},\{c\},\varphi,X\}$.

Hence $(1,2)^*$-LC($X$) $= \{\{a\},\{b\},\{a,b\},\{c\},\{b,c\},\varphi,X\}$.

*Remark 7.* Every $(1,2)^*$-open set and $(1,2)^*$-closed set is $(1,2)^*$-locally closed, but the converse may not be true as seen in the following example.

*Example 8.* Let $X = \{a,b,c,d\}$, $\tau_1 = \{\{a,b\},\varphi,X\}$, and $\tau_2 = \{\{a,d\},\{a,b,d\},\varphi,X\}$. Then $(1,2)^*$-$O(X) = \{\{a,b\},\{a,d\},\{a,b,d\},\varphi,X\}$ and $(1,2)^*$-$C(X) = \{\{c,d\},\{b,c\},\{c\},\varphi,X\}$. Now $\{b\}$ is neither $\tau_{1,2}$-open set nor a $\tau_{1,2}$-closed set even though $\{b\}$ is $(1,2)^*$-locally closed set as $\{b\} = \{a,b\} \cap \{b,c\}$.

**Proposition 9.** *Every $\tau_1$-open (resp., $\tau_1$-closed) set and $\tau_2$-open (resp., $\tau_2$-closed) sets is $(1,2)^*$-locally closed set.*

*Proof.* Since every $\tau_1$-open (resp., $\tau_1$-closed) set and $\tau_2$-open (resp., $\tau_2$-closed) set is $(1,2)^*$-open (resp., $(1,2)^*$-closed) set, hence they are $(1,2)^*$-locally closed set.

We can note that intersection of a $(1,2)^*$-locally closed set and $\tau_1$-open (or $\tau_1$-closed or $\tau_2$-open or $\tau_2$-closed) set is again a $(1,2)^*$-locally closed set. □

*Remark 10.* Converse of Proposition 9 may not be true in general as shown in the following example.

*Example 11.* Let $X = \{a,b,c,d\}$, $\tau_1 = \{\{a\},\{a,b\},\varphi,X\}$, and $\tau_2 = \{\{a,b\},\{a,b,c\},\{a,b,d\},\varphi,X\}$.

We get $(1,2)^*$-$O(X) = \{\{a\},\{a,b\},\{a,b,c\},\{a,b,d\},\varphi,X\}$ and $(1,2)^*$-$C(X) = \{\{b,c,d\},\{c,d\},\{d\},\{c\},\varphi,X\}$.

Here $\{a,b,d\}$ is a $(1,2)^*$-locally closed set, but $\{a,b,d\}$ is not a $\tau_1$-open (resp., $\tau_1$-closed) sets. Again $\{b,c,d\}$ is a $(1,2)^*$-locally closed set, but $\{b,c,d\}$ is not a $\tau_2$-open (resp., $\tau_2$-closed) sets.

**Theorem 12.** *For a subset $A$ of $(X,\tau_1,\tau_2)$ the following conditions are equivalent:*

(i) *$A \in (1,2)^*$-$LC(X)$,*

(ii) *$A = P \cap \tau_{1,2}$-$cl(A)$ for some $\tau_{1,2}$-open set $P$,*

(iii) *$\tau_{1,2}$-$cl(A)$–$A$ is $\tau_{1,2}$-closed,*

(iv) *$A \cup (X - \tau_{1,2}$-$cl(A))$ is $\tau_{1,2}$-open.*

*Proof.* (i) $\Rightarrow$ (ii). Let $A \in (1,2)^*$-LC($X$). Then there exist a $\tau_{1,2}$-open set $P$ and a $\tau_{1,2}$-closed set $F$ of $(X,\tau_1,\tau_2)$ such that $A = P \cap F$. Clearly $A \subseteq P \cap \tau_{1,2}$-cl($A$). Since $\tau_{1,2}$-cl($A$) $\subseteq F$, $P \cap \tau_{1,2}$-cl($A$) $\subseteq P \cap F = A$.

Thus $A = P \cap \tau_{1,2}$-cl($A$).

(ii) $\Rightarrow$ (iii). Let $F = \tau_{1,2}$-cl($A$) $- A$

Now, $X - F = X - \{\tau_{1,2}\text{-cl}(A) - A\} = (A \cup X) - \{\tau_{1,2}\text{-cl}(A) - A\} = (A \cup X) \cap \{(\tau_{1,2}\text{-cl}(A))^C \cup A\}$

(Where, $(\tau_{1,2}\text{-cl}(A))^C$ is denoted by complement of $\tau_{1,2}$-cl($A$)) $= A \cup (\tau_{1,2}\text{-cl}(A))^C = (P \cap \tau_{1,2}\text{-cl}(A)) \cup (\tau_{1,2}\text{-cl}(A))^C = (P \cup (\tau_{1,2}\text{-cl}(A))^C) \cap X = \tau_{1,2}$-open set. Hence $F$ is a $\tau_{1,2}$-closed set.

(iii) $\Rightarrow$ (iv). Since $F$ is $\tau_{1,2}$-closed,

we get $X - F = X - (\tau_{1,2}\text{-cl}(A) - A) = X \cap (\tau_{1,2}\text{-cl}(A))^C A \cup (X - \tau_{1,2}\text{-cl}(A))$.

Thus $A \cup (X - \tau_{1,2}$-cl($A$)) is $\tau_{1,2}$-open.

(iv) $\Rightarrow$ (i). Since $A \cup (X - \tau_{1,2}$-cl($A$)) is $\tau_{1,2}$-open, this implies $A \cup (\tau_{1,2}\text{-cl}(A))^C$ is $\tau_{1,2}$-open set. Thus $A$ is $\tau_{1,2}$-open. Hence $A$ is $(1,2)^*$-locally closed set (Since every $\tau_{1,2}$-open set is $(1,2)^*$-locally closed set). □

**Proposition 13.** *Every (1, 2) locally closed set is $(1,2)^*$-locally closed.*

*Proof.* Proof is obvious. □

*Remark 14.* Converse of the above proposition may not be true as seen in the following example.

*Example 15.* Let $X = \{p, q, r\}$, $\tau_1 = \{\{p\}, \varphi, X\}$, and $\tau_2 = \{\{r\}, \varphi, X\}$. We get $(1, 2)^*$-$O(X) = \{\{p\}, \{r\}, \{p, r\}, \varphi, X\}$, $(1, 2)$-$O(X) = \{\{p\}, \{r\}, \varphi, X\}$, $(1, 2)^*$-$C(X) = \{\{q, r\}, \{p, q\}, \{q\}, \varphi, X\}$, and $(1, 2)$-$C(X) = \{\{q, r\}, \{p, q\}, \varphi, X\}$.

Here $\{p, r\}$ is $(1, 2)^*$-locally closed but not $(1, 2)$-locally closed set.

**Theorem 16.** *A subset $A$ of $(X, \tau_1, \tau_2)$ is $(1, 2)^*$-locally closed if and only if $A^C$ is the union of a $\tau_{1,2}$-open set and a $\tau_{1,2}$-closed set.*

*Proof.* Proof follows from the definition. □

*Remark 17.* The union of any two $(1, 2)^*$-locally closed sets may not be a $(1, 2)^*$-locally closed set as shown in the following example.

*Example 18.* Let $X = \{a, b, c, d\}$, $\tau_1 = \{\{a\}, \{a, b\}, \varphi, X\}$, and $\tau_2 = \{\{a, b\}, \{a, b, c\}, \{a, b, d\}, \varphi, X\}$.

We get $(1, 2)^*$-$O(X) = \{\{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \varphi, X\}$ and $(1, 2)^*$-$C(X) = \{\{b, c, d\}, \{c, d\}, \{d\}, \{c\}, \varphi, X\}$.

Here $\{a\}$ and $\{c, d\}$ are two $(1, 2)^*$-locally closed sets, but $\{a, c, d\}$ is not a $(1, 2)^*$-locally closed set.

*Definition 19.* Any two subsets $A$ and $B$ in a bitopological space $X$ are said to be $(1, 2)^*$-separated sets if $A \cap \tau_{1,2}\text{-cl}(B) = \varphi = \tau_{1,2}\text{-cl}(A) \cap B$.

**Proposition 20.** *Let $A$ and $B$ be any two subsets of $(X, \tau_1, \tau_2)$. Suppose that the collection of all $(1, 2)^*$-open sets is closed under finite intersection. Let $A, B \in (1, 2)^*$-$LC(X)$, and if $A$ and $B$ are $(1, 2)^*$-separated set in $(X, \tau_1, \tau_2)$, then $A \cup B \in (1, 2)^*$-$LC(X)$.*

*Proof.* By Theorem 12 there exist $\tau_{1,2}$-open sets $G$ and $H$ of $(X, \tau_1, \tau_2)$ such that $A = G \cap \tau_{1,2}\text{-cl}(A)$ and $B = H \cap \tau_{1,2}\text{-cl}(B)$. Put $U = G \cap (X - \tau_{1,2}\text{-cl}(B))$ and $V = H \cap (X - \tau_{1,2}\text{-cl}(A))$ as $\tau_{1,2}$-open subsets of $X$. Then $A = U \cap \tau_{1,2}\text{-cl}(A)$, $B = V \cap \tau_{1,2}\text{-cl}(B)$, $U \cap \tau_{1,2}\text{-cl}(B) = \varphi$, and $V \cap \tau_{1,2}\text{-cl}(A) = \varphi$.

Consequently $A \cup B = (U \cup V) \cap (\tau_{1,2}\text{-cl}(A \cup B))$, as $(U \cup V)$ is $\tau_{1,2}$-open set in $X$, showing that $A \cup B \in (1, 2)^*$-LC$(X)$. □

**Proposition 21.** *In any bitopological space $X$ intersection of any two $(1, 2)^*$- locally closed is a $(1, 2)^*$-locally closed set.*

*Proof.* The proof is straightforward. □

Studying the properties of $(1, 2)^*$-locally closed set we have the idea to define a new concept in a bitopological space $(X, \tau_1, \tau_2)$, namely, infra bitopology, and we also introduce $(1, 2)^*$-LC-continuity using $(1, 2)^*$-locally closed sets.

*Definition 22.* Let $(X, \tau_1, \tau_2)$ be a bitopological space. Any collection $T_{1,2}$ of subsets of $X$ (strongly related with $\tau_{1,2}$-open set) is called an infra bitopology on $X$, if

(i) $\varphi, X \in T_{1,2}$;

(ii) $T_{1,2}$ is closed under finite intersection.

Then the space $(X, T_{1,2})$ is called an infra bitopological space.

**Theorem 23.** *Let $(X, \tau_1, \tau_2)$ be a bitopological space. Then the collections of all $(1, 2)^*$-locally closed sets of $(X, \tau_1, \tau_2)$ denoted by $(1, 2)^*$-$LC(X)$ are forms an infra bitopology on $X$.*

*Proof.* Proposition 21 serves the purpose. □

*Definition 24.* The infra bitopology defined above is called $(1, 2)^*$-LC infra bitopology and the space $(X, (1, 2)^*\text{-LC}(X))$ is called LC-infra bitopological space.

*Example 25.* Let $X = \{a, b, c, d\}$, $\tau_1 = \{\{a\}, \{a, b\}, \varphi, X\}$, and $\tau_2 = \{\{a, b\}, \{a, b, c\}, \{a, b, d\}, \varphi, X\}$. We get $(1, 2)^*$-$O(X) = \{\{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \varphi, X\}$, $(1, 2)^*$-$C(X) = \{\{b, c, d\}, \{c, d\}, \{d\}, \{c\}, \varphi, X\}$, and $(1, 2)^*$-LC$(X) = \{\{b, c, d\}, \{c, d\}, \{d\}, \{c\}, \{b\}, \{a\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \varphi, X\}$.

Here $\varphi, X \in (1, 2)^*$-LC$(X)$ and collection of all $(1, 2)^*$-locally closed set is closed under finite intersection. Hence the collection of all $(1, 2)^*$-locally closed forms an LC-infra bitopological space on $X$.

**Theorem 26.** *If $A \in (1, 2)^*$-$LC(X)$ and $B \in (1, 2)^*$-$C(X)$, then $A \cap B \in (1, 2)^*$-$LC(X)$.*

*Proof.* It is obvious because every $(1, 2)^*$-closed set is $(1, 2)^*$-locally closed set and intersection of two $(1, 2)^*$-locally closed sets is also a $(1, 2)^*$-locally closed. □

*Definition 27.* A subset $A$ of $(X, \tau_1, \tau_2)$ is called $(1, 2)^*$-dense (resp., $(1, 2)^*$-nowhere dense) set if and only if $\tau_{1,2}\text{-cl}(A) = X$ (resp., $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) = \varphi$).

**Theorem 28.** *Any $(1, 2)^*$-dense subset of $(X, \tau_1, \tau_2)$ is $(1, 2)^*$-locally closed if and only if it is $\tau_{1,2}$-open.*

*Proof.* Let us assume that $A$ be any $(1, 2)^*$-dense set in $X$.

Now by the given hypothesis $A = U \cap \tau_{1,2}\text{-cl}(A)$ for some $U \in (1, 2)^*$-$O(X)$, this implies that $A$ is a $\tau_{1,2}$-open set.

Conversely let $A$ be any $\tau_{1,2}$-open set in $X$. Now since every $\tau_{1,2}$-open set is $(1, 2)^*$-locally closed set, therefore $A$ is $(1, 2)^*$-locally closed set. □

**Theorem 29.** *Any $(1, 2)^*$-preopen set is $(1, 2)^*$-locally closed if and only if it is $\tau_{1,2}$-open.*

*Proof.* Suppose that $A$ is a $(1, 2)^*$-preopen set.

Now by the given hypothesis $A = U \cap \tau_{1,2}\text{-cl}(A)$ for some $U \in (1, 2)^*$-$O(X)$, since $A$ is a $(1, 2)^*$-preopen set, therefore $A = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) \cap \tau_{1,2}\text{-cl}(A) = \tau_{1,2}$-open set.

Converse the part that directly follows from Theorem 28. □

**Proposition 30.** *Any $(1, 2)^*$-$\alpha$-open set is $(1, 2)^*$-locally closed if and only if it is $\tau_{1,2}$-open.*

*Proof.* Since every $(1, 2)^*$-$\alpha$-open set is $(1, 2)^*$-preopen set and hence the proof is followed from Theorem 29. □

**Theorem 31.** *If any $(1, 2)^*$-dense subset of $X$ is $\tau_{1,2}$-open, then every subset $A$ of $X$ is $(1, 2)^*$-locally closed.*

*Proof.* Let $B_1$ be any $(1,2)^*$-dense subset in $X$. Therefore $B_1 = A \cup (\tau_{1,2}\text{-cl}(A))^C$.

Again let $B_2$ be the $\tau_{1,2}$-cl$(A)$; that is, $B_2 = A \cup \tau_{1,2}$-cl$(A)$.

Now $B_1 \cap B_2 = A \cup (\tau_{1,2}\text{-cl}(A))^C \cap (A \cup \tau_{1,2}\text{-cl}(A)) = A$.

This implies that $A$ is $(1,2)^*$-locally closed set. □

**Theorem 32.** *Any (1, 2)$^*$-regular open set $A$ in $(X, \tau_1, \tau_2)$ is a (1, 2)$^*$-locally closed set.*

*Proof.* Let $A$ be a $(1,2)^*$-regular open set in $X$. Then $A = \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$.

Therefore $A = \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A) \cap \tau_{1,2}$-cl$(A) = U \cap \tau_{1,2}$-cl$(A)$; $U$ is a $\tau_{1,2}$-open set.

Hence $A \in (1,2)^*$-LC(X). □

The converse is not true in general as seen in the following example.

*Example 33.* Let $X = \{a, b, c\}$, $\tau_1 = \{\{a\}, \{a, b\}, \varphi, X\}$, and $\tau_2 = \{\{b\}, \varphi, X\}$.

We get $(1, 2)^*$-$O(X) = \{\{a\}, \{a, b\}, \{b\}, \varphi, X\}$, $(1,2)^*$-$C(X) = \{\{b, c\}, \{a, c\}, \{c\}, \varphi, X\}$, $(1,2)^*$-LC$(X) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \varphi, X\}$, and $(1,2)^*$-RO$(X) = \{\{a\}, \{b\}, \varphi, X\}$.

*Definition 34.* A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be $(1,2)^*$-LC continuous if and only if the inverse image of every $\sigma_{1,2}$-open set in $Y$ is $(1,2)^*$-locally closed in $X$.

**Proposition 35.** *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1, 2)$^*$-LC continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is (1, 2)$^*$-continuous, then* (gof) *is (1, 2)$^*$-LC continuous.*

*Proof.* Proof is obvious. □

**Theorem 36.** *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1, 2)$^*$-continuous if it is (1, 2)$^*$-LC continuous and the preimage of every $\sigma_{1,2}$-open set is (1, 2)$^*$-dense in $X$.*

*Proof.* Let $A$ be any $\sigma_{1,2}$-open set in $Y$. Since $f$ is $(1,2)^*$-LC continuous, then $f^{-1}(A)$ is $(1,2)^*$-locally closed in $X$. By hypothesis $\tau_{1,2}$-cl$(f^{-1}(A)) = X$. Then $f^{-1}(A)$ is $\tau_{1,2}$-open in $X$. Hence $f$ is $(1,2)^*$-continuous (by Theorem 28). □

*Definition 37.* A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$-LC- irresolute if and only if inverse image of every $(1,2)^*$-locally closed set in $Y$ is $(1,2)^*$-locally closed set in $X$.

**Theorem 38.** *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be any two functions; then*

(i) *(gof) is (1, 2)$^*$-LC-irresolute if $g$ is (1, 2)$^*$-LC-irresolute and $f$ is (1, 2)$^*$-LC-irresolute;*

(ii) *(gof) is (1, 2)$^*$-LC continuous if $g$ is (1, 2)$^*$-LC-continuous and $f$ is (1, 2)$^*$-LC-irresolute.*

*Proof.* (i) Let $V$ be $(1,2)^*$-locally closed set in $(Z, \eta_1, \eta_2)$. Since $g$ is $(1,2)^*$-LC-irresolute, then $g^{-1}(V)$ is $(1,2)^*$-locally closed set in $(Y, \sigma_1, \sigma_2)$. As $f$ is $(1,2)^*$-LC-irresolute so $f^{-1}(g^{-1}(V))$ is $(1,2)^*$-locally closed set in $(X, \tau_1, \tau_2)$. Hence (gof) is $(1,2)^*$-LC-irresolute.

(ii) Let $V$ be $\eta_{1,2}$-open set in $(Z, \eta_1, \eta_2)$. Since $g$ is $(1,2)^*$-LC-continuous, so $g^{-1}(V)$ is $(1,2)^*$-locally closed set in $(Y, \sigma_1, \sigma_2)$. Also $f$ is $(1,2)^*$-LC-irresolute so $f^{-1}(g^{-1}(V)) = (\text{gof})^{-1}(V)$ is $(1,2)^*$-locally closed set in $(X, \tau_1, \tau_2)$. Therefore (gof) is $(1,2)^*$-LC- continuous. □

*Remark 39.* Every $(1,2)^*$-LC-continuity may not be $(1,2)^*$-LC-irresolute as seen in the following example.

*Example 40.* Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\{a\}, \{a, b\}, \varphi, X\}$, and $\tau_2 = \{\varphi, X\}$.

Then $(1,2)^*$-$O(X) = \{\{a\}, \{a, b\}, \varphi, X\}$.

Let $\sigma_1 = \{\{b\}, \varphi, Y\}$ and $\sigma_2 = \{\{a\}, \{a, b\}, \varphi, Y\}$.

Then $(1,2)^*$-$O(Y) = \{\{a\}, \{b\}, \{a, b\}, \varphi, Y\}$, $(1,2)^*$-LC$(X)$= $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \varphi, X\}$, and $(1,2)^*$-LC$(Y)$= $\{\{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}, \varphi, Y\}$.

Define $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ as an identity map. Then $f$ is a $(1,2)^*$-LC-continuity but not a $(1,2)^*$-LC-irresolute since $f^{-1}\{a, c\} = \{a, c\} \notin (1,2)^*$-LC$(X)$.

## 3. $(1,2)^*$-Contracontinuity

*Definition 41.* A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be $(1,2)^*$-contracontinuous if the inverse image of every $\sigma_{1,2}$-open set in $Y$ is $\tau_{1,2}$-closed set in $X$.

**Theorem 42.** *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ the following conditions are equivalent:*

(i) *$f$ is (1, 2)$^*$-contracontinuous,*

(ii) *for each $x \in X$ and each $\sigma_{1,2}$-closed set $V$ in $Y$ with $f(x) \in V$ there exists a $\tau_{1,2}$-open set $U$ in $X$ such that $x \in U$ and $f(U) \subset V$,*

(iii) *the inverse image of each $\sigma_{1,2}$-closed set in $Y$ is $\tau_{1,2}$-open set in $X$.*

*Proof.* Proof is obvious. □

**Proposition 43.** *Every (1, 2)$^*$-contracontinuous function is (1, 2)$^*$-LC-continuous.*

*Proof.* Proof is obvious from the definitions. □

*Remark 44.* Converse of the above proposition may not be true in general as shown in the following example.

*Example 45.* Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r, s\}$.

Let $\tau_1 = \{\{a, b\}, \varphi, X\}$ and $\tau_2 = \{\{a, d\}, \{a, b, d\}, \varphi, X\}$.

Also let $\sigma_1 = \{\{p\}, \{q\}, \{p, q\}, \varphi, Y\}$ and $\sigma_2 = \{\{r\}, \{q, r\}, \{p, r\}, \{p, q, r\}, \varphi, Y\}$.

We get $(1, 2)^*$-$O(X) = \{\{a, b\}, \{a, d\}, \{a, b, d\}, \varphi, X\}$, $(1,2)^*$-$C(X) = \{\{c, d\}, \{b, c\}, \{c\}, \varphi, X\}$, and $(1,2)^*$-$O(Y) = \{\{p\}, \{q\}, \{r\}, \{p, q\}, \{q, r\}, \{p, r\}, \{p, q, r\}, \varphi, Y\}$.

If $f : X \to Y$ is defined by $f(a) = f(d) = s$, $f(b) = q$, and $f(c) = r$, then any one can prove that $f$ is $(1,2)^*$-LC-continuous, but it is not a $(1,2)^*$-contracontinuous function as $\{b\}$ is $(1,2)^*$-locally closed but it is not $\tau_{1,2}$-closed set.

We know that in a topological space $X$ the concept of locally indiscrete space is defined as follows.

A topological space $(X, \tau)$ is called locally indiscrete space if every open set in $X$ is closed set in $X$. Following the above definition we now define the $(1, 2)^*$-locally indiscrete space in a bitopological space $(X, \tau_1, \tau_2)$ as follows.

A bitopological space $(X, \tau_1, \tau_2)$ is said to be $(1, 2)^*$- locally indiscrete space if every $\tau_{1,2}$-open set in $X$ is $\tau_{1,2}$-closed set in $X$.

**Theorem 46.** *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1, 2)$^*$-continuous and $X$ is (1, 2)$^*$-locally indiscrete space, then $f$ is (1, 2)$^*$-contracontinuous.*

*Proof.* Let us consider an arbitrary $\sigma_{1,2}$-open set $U$ in $Y$; then $f^{-1}(U)$ is $\tau_{1,2}$-open set in $X$ as $f$ is $(1, 2)^*$-continuous function. Again $X$ is $(1, 2)^*$-locally indiscrete so $f^{-1}(U)$ is $\tau_{1,2}$-closed set in $X$. Hence $f$ is $(1, 2)^*$-contracontinuous. □

**Theorem 47.** *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be two functions; then*

(i) *(gof) is (1, 2)$^*$-contracontinuous if $g$ is (1, 2)$^*$-continuous and $f$ is (1, 2)$^*$-contracontinuous,*

(ii) *(gof) is (1, 2)$^*$-contracontinuous if $g$ is (1, 2)$^*$-contracontinuous and $f$ is (1, 2)$^*$-continuous,*

(iii) *(gof) is (1, 2)$^*$-contracontinuous if $g$ and $f$ is (1, 2)$^*$-continuous and $Y$ is (1, 2)$^*$-locally indiscrete.*

*Proof.* (i) Let $V$ be any $\eta_{1,2}$-open set in $Z$. Since $g$ is $(1, 2)^*$-continuous, then $g^{-1}(V)$ is $\sigma_{1,2}$-open set in $Y$ and since $f$ is $(1, 2)^*$-contracontinuous, so $f^{-1}(g^{-1}(V))$ is $\tau_{1,2}$-closed in $X$. Thus $(\text{gof})^{-1}(V)$ is $\tau_{1,2}$-closed set in $X$ for an arbitrary $\eta_{1,2}$-open set $V$ in $Z$ implying that (gof) is $(1, 2)^*$-contracontinuous.

(ii) Proof is the same as (i).

(iii) Let $U$ be a $\eta_{1,2}$-open set in $Z$. Since $g$ is $(1, 2)^*$-continuous so $g^{-1}(U)$ is $\sigma_{1,2}$-open set in $Y$ and since $Y$ is $(1, 2)^*$-locally indiscrete, so $g^{-1}(U)$ is $\sigma_{1,2}$-closed set in $Y$. Again since $f$ is $(1, 2)^*$-continuous, so $f^{-1}(g^{-1}(U))$ is $\tau_{1,2}$-closed set in $X$. Thus $(\text{gof})^{-1}(U)$ is $\tau_{1,2}$-closed set in $X$ for an arbitrary $\eta_{1,2}$-open set $U$ in $Z$ which is implying that (gof) is $(1, 2)^*$-contracontinuous. □

**Theorem 48.** *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be two (1, 2)$^*$-contracontinuous functions; then*

(i) *(gof) is (1, 2)$^*$-continuous function;*

(ii) *(gof) is (1, 2)$^*$-contracontinuous function if $X$ is (1, 2)$^*$-locally indiscrete.*

*Proof.* (i) Let $U$ be an arbitrary $\eta_{1,2}$-open set in $Z$. Since $g$ is $(1, 2)^*$-contracontinuous so $g^{-1}(U)$ is $\sigma_{1,2}$-closed set in $Y$. Again $f$ is $(1, 2)^*$-contracontinuous; thus $f^{-1}(g^{-1}(U))$ is $\tau_{1,2}$-open set in $X$. Hence $(\text{gof})^{-1}(U)$ is $\tau_{1,2}$-open in $X$ for any $\eta_{1,2}$-open set in $Z$. So (gof) is $(1, 2)^*$-continuous function.

(ii) From the above proof (i) we see that $(\text{gof})^{-1}(U)$ is $\tau_{1,2}$-open in $X$ for any $\eta_{1,2}$-open set in $Z$. But $X$ is $(1, 2)^*$-locally indiscrete and consequently $(\text{gof})^{-1}(U)$ is $\tau_{1,2}$-closed set in $X$ for any $\eta_{1,2}$-open set in $Z$. Hence (gof) is $(1, 2)^*$-contracontinuous function. □

## 4. Conclusion

From the above study it is clear that in general $(1, 2)^*$-dense set may not be a $(1, 2)^*$-locally closed set, but a $(1, 2)^*$-dense set is a $(1, 2)^*$-locally closed set if and only if it contains the neighbourhood of all its points. Also it is proved that every $(1, 2)^*$-locally closed set is $\tau_{1,2}$-closed set if it is $(1, 2)^*$-nowhere dense set in $X$. There is a scope to study the concept of strongly and perfectly continuous mapping in a bitopological space and interrelationship between stronger form and weaker form of $(1, 2)^*$-continuity and $(1, 2)^*$-LC-continuous mapping.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## Acknowledgment

The authors are thankful to the learned reviewer for his appreciation and valuable comments.

## References

[1] C. Kuratowski and W. Sierpinski, "Sur les differences de deux ensembles fermes," *Tohoku Mathematical Journal*, vol. 20, pp. 22–25, 1921.

[2] J. C. Kelly, "Bitopological spaces," *Proceedings of the London Mathematical Society*, vol. 13, pp. 71–89, 1963.

[3] M. Ganster and I. L. Reilly, "Locally closed sets and LC-continuous functions," *International Journal of Mathematics and Mathematical Sciences*, vol. 12, no. 3, pp. 417–424, 1989.

[4] M. Jelić, "A decomposition of pairwise continuity," *Journal of Institute of Mathematics and Computer Sciences*, vol. 3, no. 1, pp. 25–29, 1990.

[5] M. L. Thivagar, "Generalization of pairwise $\alpha$-continuous functions," *Pure and Applied Mathematika Sciences*, vol. 33, no. 1-2, pp. 55–63, 1991.

[6] M. Lellis Thivagar, O. Ravi, and M. E. Abd El-Monsef, "Remarks on bitopological $(1, 2)^*$-quotient mappings," *Journal of the Egyptian Mathematical Society*, vol. 16, no. 1, pp. 17–25, 2008.

[7] N. Bourbaki, *General Topology, Part 1*, Addison-Wesley and Hermann, Paris, France, 1966.

[8] M. Lellis Thivagar and O. Ravi, "On stronger forms of $(1, 2)^*$ quotient mappings in bitopological spaces," *International Journal of Mathematics, Game Theory, and Algebra*, vol. 14, no. 6, pp. 481–492, 2004.

[9] O. Ravi, M. Lellis Thivagar, and E. Ekici, "On $(1, 2)^*$-sets and decompositions of bitopological $(1, 2)^*$-continuous mappings," *Kochi Journal of Mathematics*, vol. 3, pp. 181–189, 2008.