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Research Article

H-Matrices in Fuzzy Linear Systems

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We consider a class of fuzzy linear system of equations and demonstrate some of the existing challenges. Furthermore, we explain the efficiency of this model when the coefficient matrix is an H-matrix. Numerical experiments are illustrated to show the applicability of the theoretical analysis.

1. Introduction

In the field of scientific and technical computation, various equations which describe realistic problems like natural phenomena or engineering problems such as computational fluid dynamics, finite differences methods, finite element methods, statistics, time/frequency domain circuit simulation, dynamic and static modeling of chemical processes, cryptography, magnetohydrodynamics, electrical power systems, differential equations, quantum mechanics, structural mechanics (buildings, ships, aircraft, and human body parts…), heat transfer, MRI reconstructions, vibroacoustics, linear and nonlinear optimization, financial portfolios, semiconductor process simulation, economic modeling, oil reservoir modeling, astrophysics, crack propagation, Google page rank, Gene page rank, 3D computer vision, cell phone tower placement, tomography, model reduction, nanotechnology, acoustic radiation, density functional theory, quadratic assignment, elastic properties of crystals, natural language processing, DNA electrophoresis, and so forth must be solved numerically. These problems can lead to solving a system of linear equations. There are many methods for solving linear systems; see [1–7] and the references therein.

Nevertheless, when coefficients of a system are ambiguous and there is some inexplicit information about the exact amount of parameters, one can solve a linear equation system by fuzzy logic. In 1965 [8], fuzzy logic was proposed by Zadeh and, following his work, many papers and books were published in fuzzy system theory. In particular, the solutions of fuzzy linear systems have been considered by many researchers, for example, [8–14]. Some investigations on the numerical solution of the fuzzy linear systems have also been reported; see [15–23] and references therein. Most of these studies use the same expansion of [12], where the coefficient matrix is crisp and the right-hand side is an arbitrary fuzzy number vector. Friedman et al. [12], using the embedding method, presented a general model for solving an $n \times n$ fuzzy linear system and replaced the fuzzy linear system by $2n \times 2n$ crisp linear system. They studied also the uniqueness of the fuzzy solution for this model [12].

Dehghan and Hashemi [16] investigated the existence of a solution for this model under the condition that the coefficient matrix is a strictly diagonally dominant matrix with positive diagonal entries. Hashemi et al. [17], using the Schur complement, studied existence of the solution to this model when the coefficient matrix is M-matrix and all diagonal entries are positive.

In this paper we establish the existence of the solution to Friedman et al.'s model under general class of the coefficient matrix called H-matrix. It is well known that every strictly diagonally dominant matrix is an M-matrix and every M-matrix is an H-matrix [2]. We demonstrate effectiveness of some conditions in our theorems (Section 3) relative to fuzzy linear systems.

2. Preliminaries

2.1. Fuzzy Numbers. An arbitrary fuzzy number is represented, in parametric form, by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$, $0 \le r \le 1$, which satisfy the following conditions (see [24]):

(i) $\underline{u}(r)$ is a bounded monotonic increasing left continuous function over $[0,1]$;

(ii) $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function over $[0,1]$;

(iii) $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

A crisp number α can be simply expressed as $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \le r \le 1$. The addition and scalar multiplication of fuzzy numbers $x = (\underline{x}(r), \overline{x}(r))$ and $y = (\underline{y}(r), \overline{y}(r))$ can be described as follows:

(i) $x = y$ if and only if $\underline{x}(r) = \underline{y}(r)$ and $\overline{x}(r) = \overline{y}(r)$,

(ii) $x + y = (\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r))$,

(iii) $\forall K \in R$,

$$Kx = \begin{cases} (K\underline{x}, K\overline{x}), & K \ge 0, \\ (K\overline{x}, K\underline{x}), & K < 0. \end{cases} \tag{1}$$

By appropriate definitions the fuzzy number space $\{(\underline{u}(r), \overline{u}(r))\}$ becomes a convex cone E^1 which is then embedded isomorphically and isometrically into a Banach space [12]. An alternative definition which yields the same E^1 is given by [10].

2.2. Fuzzy Linear System (FLS)

Definition 1. Consider the $n \times n$ linear system of equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2, \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n, \end{aligned} \tag{2}$$

where the coefficient matrix $A = (a_{ij})$, $1 \le i, j \le n$, is a crisp matrix and $b_i \in E^1$, $1 \le i \le n$, is called a fuzzy linear system (FLS).

Definition 2. A fuzzy number vector $X = (x_1, x_2, \ldots, x_n)^T$, given by parametric form $x_i = (\underline{x_i}(r), \overline{x_i}(r))$, $1 \le i \le n$, $0 \le r \le 1$, is called a solution of the fuzzy linear system (2) if

$$\begin{aligned} \underline{\sum_{j=1}^{n} a_{ij}x_j} &= \sum_{j=1}^{n} \underline{a_{ij}x_j} = \underline{b_i}, \\ \overline{\sum_{j=1}^{n} a_{ij}x_j} &= \sum_{j=1}^{n} \overline{a_{ij}x_j} = \overline{b_i}. \end{aligned} \tag{3}$$

In order to solve the system given by (3), Friedman et al. [12] have solved a $2n \times 2n$ crisp linear system as

$$SX = B, \tag{4}$$

where $S = (s_{ij})$ are determined as follows:

$$\begin{aligned} a_{ij} \ge 0 &\Longrightarrow s_{ij} = a_{ij}, \quad s_{i+n,j+n} = a_{ij}, \\ a_{ij} < 0 &\Longrightarrow s_{i,j+n} = -a_{ij}, \quad s_{i+n,j} = -a_{ij} \end{aligned} \tag{5}$$

and any (s_{ij}) which is not determined by (5) is zero. Then referring to Friedman et al. [12], we have

$$\begin{gathered} S = \begin{bmatrix} s_1 & s_2 \\ s_2 & s_1 \end{bmatrix}, \\ X = \begin{bmatrix} \underline{x} \\ -\overline{x} \end{bmatrix}, \\ B = \begin{bmatrix} \underline{b} \\ -\overline{b} \end{bmatrix} \\ \Longrightarrow \begin{cases} s_1\underline{x} - s_2\overline{x} = \underline{b} \\ s_2\underline{x} - s_1\overline{x} = -\overline{b}, \end{cases} \end{gathered} \tag{6}$$

or

$$\begin{gathered} S = \begin{bmatrix} s_1 & -s_2 \\ -s_2 & s_1 \end{bmatrix}, \\ X = \begin{bmatrix} \underline{x} \\ \overline{x} \end{bmatrix}, \\ B = \begin{bmatrix} \underline{b} \\ \overline{b} \end{bmatrix} \\ \Longrightarrow \begin{cases} s_1\underline{x} - s_2\overline{x} = \underline{b} \\ -s_2\underline{x} + s_1\overline{x} = \overline{b}, \end{cases} \end{gathered} \tag{7}$$

where $s_1, s_2 \in \mathbb{R}^{n \times n}$, $A = s_1 - s_2$.

Theorem 3 (see [12]). *Matrix S is nonsingular if and only if matrices A and $s_1 + s_2$ are both nonsingular.*

Theorem 4 (see [12]). *Let S be nonsingular. Then the unique solution X is always a fuzzy vector for arbitrary vector B if and only if S^{-1} is nonnegative.*

Beside Theorem 3, the following theorem is also appropriate for proving the nonsingularity of matrix S.

Theorem 5. *Let s_1 be nonsingular. Then matrix S in (6) is nonsingular if and only if Schur complement of A, that is, $(T_A = s_1 - s_2 s_1^{-1} s_2)$, is nonsingular.*

Proof. After some manipulations, the proof is obtained. □

2.3. H-Matrices and Their Subclasses. Let A be an $n \times n$ matrix, $N = \{1, 2, \ldots, n\}$; and for each nonempty subset Q of indices N, we denote

$$r_i^Q(A) = \sum_{k \in Q \setminus \{i\}} |a_{ik}|, \qquad r_i(A) = r_i^N(A),$$
$$l_i^Q(A) = \sum_{\substack{k \in Q \\ k < i}} |a_{ik}|, \qquad l_i(A) = l_i^N(A). \tag{8}$$

Definition 6. For any matrix A the comparison matrix $M(A) = (m_{ij}) \in \mathbb{R}^{n \times n}$ is defined by

$$m_{ii} = |a_{ii}|, \quad m_{ij} = -|a_{ij}|, \quad i \neq j, \ i, j \in N. \tag{9}$$

A real square matrix whose off-diagonal elements are all nonpositive is called an L-matrix. Let A be an L-matrix; if $A^{-1} \geq 0$ then A is said to be an M-matrix.

A complex matrix A is an H-matrix if and only if $M(A)$ is an M-matrix.

Definition 7. Matrix A is called an SDDM (strictly diagonally dominant matrix) if

$$|a_{ii}| > r_i(A), \quad i = 1 : n. \tag{10}$$

Definition 8. Matrix A is called a GDDM (generalized diagonally dominant matrix) if there exists a positive diagonal matrix W such that AW is an SDDM matrix [3].

Theorem 9 (see [3]). *Matrix A is a GDDM if and only if A is an H-matrix.*

Definition 10 (see [25]). Matrix A is called a DDDM (doubly diagonally dominant matrix) if

$$|a_{ii}| |a_{jj}| > r_i(A) r_j(A), \quad i, j = 1 : n, \ i \neq j. \tag{11}$$

Definition 11 (see [25]). For a given nonempty proper subset Q of indices N matrix A is called Q-SDDM (Q-strictly diagonally dominant matrix) if

$$|a_{ii}| > r_i^Q(A), \quad i \in Q,$$
$$\left(|a_{ii}| - r_i^Q(A)\right)\left(|a_{jj}| - r_j^{\overline{Q}}(A)\right) > r_i^{\overline{Q}}(A) r_j^Q(A), \tag{12}$$
$$i \in Q, \quad j \in \overline{Q} = N \setminus Q.$$

Definition 12 (see [26]). L-matrix A is a *Stieltjes matrix* if A is SPDM (symmetric and positive definite matrix).

The following are well known classes of nonsingular matrices, introduced by Ostrowski.

Definition 13. Matrix A is called an α_1 matrix if there exists $\alpha \in [0, 1]$, such that, for each $i \in N$, it holds that $|a_{ii}| > \alpha r_i(A) + (1 - \alpha) r_i(A^T)$.

Definition 14. Matrix A is called an α_2 matrix if there exists $\alpha \in [0, 1]$, such that, for each $i \in N$, it holds that $|a_{ii}| > r_i(A)^{\alpha} + r_i(A^T)^{1-\alpha}$.

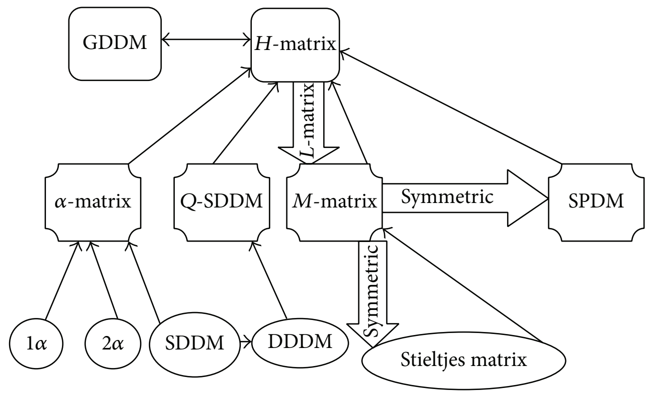

Figure 1: Relations between H-matrices and some of their subclasses.

Theorem 15 (see [27]). *If a matrix A is α_1 or α_2 matrix, then it is nonsingular. Moreover it is an H-matrix.*

3. Some Theorems

In [12] the following points are dominant.

(1) "S may be singular even if the original matrix A is not."

(2) "The solution vector is unique but may still not be an appropriate fuzzy vector."

In this part we provide some sufficient conditions to avoid the above problems. We let the diagonal elements of A be all positive.

In Figure 1, we describe the relations between H-matrices and some of their subclasses. For more details please see [1, 25–27]. These relations are very important for our discussions and in Section 2.3 all items are completely defined.

The following theorems present the conditions that matrix S needs to be an H-matrix.

Theorem 16. *Matrices S in (6) and (7) are H-matrixif and only if A in (2) is an H-matrix.*

Proof. Let A be an H-matrix; then by Theorem 9, there exists a positive diagonal matrix W such that AW is strictly diagonally dominant matrix. Without loss of generality, let AW be row strictly diagonally dominant; that is,

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij} w_{jj}| < |a_{ii} w_{ii}|. \tag{13}$$

Now, let

$$j^+ = \{a_{ij} \mid a_{ij} \geq 0\},$$
$$j^- = \{a_{ij} \mid a_{ij} < 0\}, \tag{14}$$
$$1 \leq i, j \leq n;$$

then we have

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij} w_{jj}| = \sum_{\substack{j \in j^+ \\ j \neq i}} |a_{ij} w_{jj}| + \sum_{\substack{j \in j^- \\ j \neq i}} |a_{ij} w_{jj}| < |a_{ii} w_{ii}|. \tag{15}$$

By considering the structure of S and since, for all $i = 1, \ldots, n$, $a_{ii} \geq 0$, we have

$$\begin{gathered}\sum_{\substack{j=1\\ j\neq i}}^{n} |s_{ij}| = \sum_{\substack{j\in j^+\\ j\neq i}} |a_{ij}|,\\ \sum_{j=n+1}^{2n} |s_{ij}| = \sum_{j\in j^-} |a_{ij}|,\\ 1 \leq i \leq n,\\ \sum_{j=1}^{n} |s_{ij}| = \sum_{j\in j^-} |a_{ij}|,\\ \sum_{\substack{j=n+1\\ j\neq i}}^{2n} |s_{ij}| = \sum_{\substack{j\in j^+\\ j\neq i}} |a_{ij}|,\\ n+1 \leq i \leq 2n.\end{gathered} \tag{16}$$

Therefore,

$$\sum_{\substack{j=1\\ j\neq i}}^{n} |s_{ij}w_{jj}| = \begin{cases} \sum_{\substack{j\in j^+\\ j\neq i}} |a_{ij}w_{jj}| + \sum_{\substack{j\in j^-\\ j\neq i}} |a_{ij}w_{jj}| < |a_{ii}w_{ii}| \\ \qquad\qquad 1 \leq i \leq n, \\ \sum_{\substack{j\in j^-\\ j\neq i}} |a_{ij}w_{jj}| + \sum_{\substack{j\in j^+\\ j\neq i}} |a_{ij}w_{jj}| < |a_{ii}w_{ii}| \\ \qquad\qquad n+1 \leq i \leq 2n \end{cases} \tag{17}$$

$$\Longrightarrow \sum_{\substack{j=1\\ j\neq i}}^{2n} |s_{ij}w_{jj}| < |a_{ii}w_{ii}| = |s_{ii}w_{ii}|.$$

Then by choice of $T = \left(\begin{smallmatrix} W & 0\\ 0 & W\end{smallmatrix}\right)_{2n\times 2n}$, ST is row SDDM. Therefore, by Theorem 9, S is an H-matrix too. Conversely, if S is an H-matrix, then by reasoning similar to that above, one can see that A is an H-matrix too. □

Theorem 17. *Matrices S in (6) and (7) are H-matrix if and only if A in (2) is subclasses of H-matrix.*

Proof. By Section 2.3, Figure 1, and Theorem 16 proof is completed. □

Theorem 18. *For any $A = [a_{ij}] \in R^{n\times n}$, $n \geq 2$, for which there exists an index $i \in N$such that*

$$|a_{ii}| \cdot \left(|a_{jj}| - r_j(A) + |a_{ji}|\right) > r_i(A) \cdot |a_{ji}|, \quad \forall j \neq i,\ j \in N. \tag{18}$$

It follows that S in (6) and (7) is an H-matrix.

Proof. Based on Theorem 16 and applying Dashnic-Zusmanovich's result [27, Theorem 5], the proof is completed. □

Corollary 19. *The unique solution $X = S^{-1}B$ is always a fuzzy vector for arbitrary vector Y, if A is M-matrix or subclasses of it.*

Proof. By Definition 6, Theorem 4, Figure 1, and Theorems 16-17 proof is completed. □

4. Numerical Example

In this section, we give some examples of FLS to illustrate the results obtained in the previous sections.

Example 1. Consider the 2×2 fuzzy system

$$\begin{gathered} 3x_1 - x_2 = (3r, 5 - 2r), \\ -x_1 + 2x_2 = (1 + r, 7 - 5r). \end{gathered} \tag{19}$$

Since A is an M-matrix, by Theorem 17, S is M-matrix too. Therefore, by Theorem 4 and Corollary 19, we can solve the above FLS by Friedman et al.'s model. Now we solve this FLS:

$$\begin{gathered} A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \\ \longrightarrow A^{-1} = \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.6 \end{bmatrix} \\ \xrightarrow{(5)} S = \begin{bmatrix} 3 & 0 & 0 & -1 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 3 & 0 \\ -1 & 0 & 0 & 2 \end{bmatrix} \\ \Longrightarrow S^{-1} = \begin{bmatrix} 0.4 & 0 & 0 & 0.2 \\ 0 & 0.6 & 0.2 & 0 \\ 0 & 0.2 & 0.4 & 0 \\ 0.2 & 0 & 0 & 0.6 \end{bmatrix}. \end{gathered} \tag{20}$$

Therefore

$$X = \begin{bmatrix} \underline{x}_1(r) \\ \underline{x}_2(r) \\ \overline{x}_1(r) \\ \overline{x}_2(r) \end{bmatrix} = S^{-1}B = \begin{bmatrix} 1.4 + 0.2r \\ 1.6 + 0.2r \\ 2.2 - 0.6r \\ 4.2 - 2.4r \end{bmatrix}. \tag{21}$$

The exact solution is

$$\begin{gathered} x_1 = \left(\underline{x}_1(r), \overline{x}_1\right) = (1.4 + 0.2r, 2.2 - 0.6r), \\ x_2 = \left(\underline{x}_2(r), \overline{x}_2\right) = (1.6 + 0.2r, 4.2 - 2.4r). \end{gathered} \tag{22}$$

Example 2. Consider the $n \times n$ fuzzy system

$$\begin{gathered} 3x_1 - x_3 = (2 + r, 4 - r), \\ x_2 + 3x_3 - x_4 = (2 + r, 4 - r), \\ x_3 + 3x_4 - x_5 = (2 + r, 4 - r), \\ \vdots \\ x_{n-3} + 3x_{n-2} = (2 + r, 4 - r), \\ x_{n-2} + 3x_{n-1} = (2 + r, 4 - r), \\ x_{n-1} + 3x_n - x_1 = (2 + r, 4 - r). \end{gathered} \tag{23}$$

The extended $2n \times 2n$ matrix is

$$S = \begin{bmatrix} s_1 & -s_2 \\ -s_2 & s_1 \end{bmatrix}, \tag{24}$$

where

$$s_1 = \begin{bmatrix} 3 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 1 & 3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \cdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 3 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & 3 & 0 & 0 \\ 0 & \cdots & \ddots & 0 & 1 & 3 & 0 \\ 0 & \cdots & \cdots & 0 & 0 & 1 & 3 \end{bmatrix},$$
$$s_2 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & -1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 0 & \ddots & -1 \\ 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ -1 & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{25}$$

Evidently, A is an H-matrix and then, by Theorem 16, S is also an H-matrix. Therefore, we can solve this FLS by Friedman et al.'s model.

5. Conclusion

In this paper, we have studied a class of fuzzy linear system of equations, called Friedman et al.'s model. Furthermore, we proposed some theorems and effectiveness of some conditions in our theorems relative to fuzzy linear systems. Finally, from numerical experiment, we can see that our theorems are applicable and true.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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