Hindawi Publishing Corporation
ISRN High Energy Physics
Volume 2014, Article ID 535010, 11 pages
http://dx.doi.org/10.1155/2014/535010

Research Article

A Dark Energy Model with Higher Order Derivatives of H in the $f(R,T)$ Modified Gravity Model

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Received 20 September 2013; Accepted 23 October 2013; Published 29 January 2014

Academic Editors: B. Bilki and C. A. D. S. Pires

We consider a model of dark energy (DE) which contains three terms (one proportional to the squared Hubble parameter, one to the first derivative, and one to the second derivative with respect to the cosmic time of the Hubble parameter) in the light of the $f(R,T) = \mu R + \nu T$ modified gravity model, with μ and ν being two constant parameters. R and T represent the curvature and torsion scalars, respectively. We found that the Hubble parameter exhibits a decaying behavior until redshifts $z \approx -0.5$ (when it starts to increase) and the time derivative of the Hubble parameter goes from negative to positive values for different redshifts. The equation of state (EoS) parameter of DE and the effective EoS parameter exhibit a transition from $\omega < -1$ to $\omega > -1$ (showing a quintom-like behavior). We also found that the model considered can attain the late-time accelerated phase of the universe. Using the statefinder parameters r and s, we derived that the studied model can attain the ΛCDM phase of the universe and can interpolate between dust and ΛCDM phase of the universe. Finally, studying the squared speed of sound v_s^2, we found that the considered model is classically stable in the earlier stage of the universe but classically unstable in the current stage.

1. Introduction

The late-time accelerated expansion of the universe (which is well-established from different cosmological observations) [1, 2] is a major challenge for cosmologists. The universe underwent two phases of accelerated expansion: the inflationary stage in the very early universe and a late-time acceleration in which our universe entered only recently. Models trying to explain this late-time acceleration are dubbed as dark energy (DE) models. An important step toward the comprehension of the nature of DE is to understand whether it is produced by a cosmological constant Λ or it is originated from other sources dynamically changing with time [3]. For good reviews on DE see [4–6].

In a recent paper, Nojiri and Odintsov [7] described the reasons why modified gravity approach is extremely attractive in the applications for late accelerating universe and DE. Another good review on modified gravity was made by Clifton et al. [8]. Many different theories of modified gravity have been recently proposed: some of them are $f(R)$ (with R being the Ricci scalar curvature) [9, 10], $f(T)$ (with T being the torsion scalar) [11–14], Hořava-Lifshitz [15, 16], and Gauss-Bonnet [17–20] theories.

In this paper, we concentrate on $f(R,T)$ gravity, with f being in this case a function of both R and T, manifesting a coupling between matter and geometry. Before going into the details of $f(R,T)$ gravity, we describe some important features of the $f(R)$ gravity. The recent motivation for studying $f(R)$ gravity came from the necessity to explain the apparent late-time accelerating expansion of the universe. Detailed reviews on $f(R)$ gravity can be found in [21–24]. Thermodynamic aspects of $f(R)$ gravity have been investigated in the works of [25, 26]. A generalization of the $f(R)$ modified theory of gravity that includes an explicit coupling of an arbitrary function of R with the matter Lagrangian density L_m leads to a non-geodesic motion of massive particles and an extra force, orthogonal to the four-velocity, arises. [27]. Harko et al. [28] recently suggested an extension of standard general

relativity, where the gravitational Lagrangian is given by an arbitrary function of R and T and called this model $f(R,T)$. The $f(R,T)$ model depends on a source term, representing the variation of the matter stress-energy tensor with respect to the metric. A general expression for this source term can be obtained as a function of the matter Lagrangian L_m. In a recent paper, Myrzakulov [29] proposed $f(R,T)$ gravity model and studied its main properties of FRW cosmology. Moreover, Myrzakulov [30] recently derived exact solutions for a specific $f(R,T)$ model which is a linear combination of R and T, that is, $f(R,T) = \mu R + \nu T$, where μ and ν are two free constant parameters. Moreover, it was demonstrated that, for some specific values of μ and ν, the expansion of universe results to be accelerated without the necessity to introduce extra dark components. Recently, Chattopadhyay [31] studied the properties of interacting Ricci DE considering the model $f(R,T) = \mu R + \nu T$. Pasqua et al. [32] recently considered the modified holographic Ricci dark energy (MHRDE) model in the context of the specific $f(R,T)$ model we are considering in this work. Moreover, Alvarenga et al. [33] studied the evolution of scalar cosmological perturbations in the metric formalism in the framework of $f(R,T)$ modified theory of gravity.

In this work, we consider a DE model proposed in the recent paper of Chen and Jing [34]. The DE model considered contains three different terms, one proportional to the squared Hubble parameter, one to the first derivative with respect to the cosmic time of the Hubble parameter, and one proportional to the second derivative with respect to the cosmic time of the Hubble parameter:

$$\rho_{\mathrm{DE}} = \varepsilon \frac{\ddot{H}}{H} + \lambda \dot{H} + \theta H^2, \tag{1}$$

where ε, λ, and θ are three positive constant parameters. The first term is divided by the Hubble parameter H in order that all the three terms have the same dimensions. The energy density given in (1) can be considered as an extension and generalization of other two DE models widely studied in recent time, that is, the Ricci DE (RDE) model and the DE energy density with Granda-Oliveros cut-off. In fact, in the limiting case corresponding to $\varepsilon = 0$, we obtain the energy density of DE with Granda-Oliveros cut-off, and in the limiting case corresponding to $\varepsilon = 0$, $\lambda = 1$, and $\theta = 2$, we recover the RDE model for flat universe (i.e., with curvature parameter k equal to zero).

In this work we are considering DE interacting with pressureless DM which has energy density ρ_m. Various forms of interacting DE models have been constructed in order to fulfil the observational requirements. Many different works are presently available where the interacting DE have been discussed in detail. Some examples of interacting DE are presented in [35–40].

This work aims to reconstruct the DE model considered under $f(R,T)$ gravity and it is organized as follows. In Section 2, we describe the main features of the $f(R,T) = \mu R + \nu T$ model. In Section 3, we consider the energy density of DE given in (1) in the context of $f(R,T)$ gravity considering the particular model considered. In Section 4, we study the statefinder parameters r and s for the energy density model we are considering in this work. In Section 5, we write a detailed discussion about the results found in this work. Finally, in Section 6, we write the Conclusions of this work.

2. The $f(R,T) = \mu R + \nu T$ Model

The metric of a spatially flat, homogeneous, and isotropic universe in Friedmann-Lemaitre-Robertson-Walker (FLRW) model is given by

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \tag{2}$$

where $a(t)$ represents a dimensionless scale factor (which gives information about the expansion of the universe), t indicates the cosmic time, r represents the radial component, and θ and φ are the two angular coordinates.

We also know that the tetrad orthonormal components $e_i(x^\mu)$ are related to the metric through the following relation:

$$g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu. \tag{3}$$

The Einstein field equations are given by

$$\begin{aligned} H^2 &= \frac{1}{3}\rho, \\ \dot{H} &= -\frac{1}{2}(\rho + p), \end{aligned} \tag{4}$$

where ρ and p indicate (choosing units of $8\pi G = c = 1$) the total energy density and the total pressure, respectively. The conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{5}$$

where

$$\begin{aligned} \rho &= \rho_{\mathrm{DE}} + \rho_m, \\ p &= p_{\mathrm{DE}}. \end{aligned} \tag{6}$$

We must emphasize here that we are considering pressureless DM ($p_m = 0$). Since the components do not satisfy the conservation equation separately in presence of interaction, we reconstruct the conservation equation by introducing an interaction term Q which can be expressed in any of the following forms [41]: $Q \propto H\rho_{\mathrm{DE}}$, $Q \propto H\rho_m$, and $Q \propto H(\rho_m + \rho_{\mathrm{DE}})$.

In this paper, we consider as interaction term the second of the three forms mentioned above. Accordingly, the conservation equation is reconstructed as

$$\dot{\rho}_{\mathrm{DE}} + 3H(\rho_{\mathrm{DE}} + p_{\mathrm{DE}}) = 3H\delta\rho_m, \tag{7}$$

$$\dot{\rho}_m + 3H\rho_m = -3H\delta\rho_m, \tag{8}$$

where δ indicates an interaction constant parameter which gives information about the strength of the interaction between DE and DM. The present day value of δ is still not known exactly and it is under debate.

One of the most interesting models of $f(R,T)$ gravity is the so-called M_{37}-model, whose action S is given by [29]

$$S = \int f(R,T)\, e d^4x + \int L_m e d^4x, \tag{9}$$

where e is defined as $e = \det(e^i_\mu) = \sqrt{-g}$ (with g being the determinant of the metric tensor $g_{\mu\nu}$), L_m is the matter Lagrangian, R is the curvature scalar, and T is the torsion scalar.

In this paper, we consider the following expressions for the curvature scalar R and for the torsion scalar T given, respectively, by

$$\begin{gathered} R = u + 6\left(\dot{H} + 2H^2\right), \\ T = v - 6H^2. \end{gathered} \tag{10}$$

We now consider the particular case corresponding to $u = u(a,\dot{a})$ and $v = v(a,\dot{a})$, where $\dot{a}$ is the derivative of the scale factor with respect to the cosmic time t. Moreover, the scale factor $a(t)$, the torsion scalar T, and the curvature scalar R are considered as independent dynamical variables. Then, after some algebraic calculations, the action given in (9) can be rewritten as

$$S_{37} = \int dt L_{37}, \tag{11}$$

where the Lagrangian L_{37} is given by

$$\begin{gathered} L_{37} = a^3\left(f - Tf_T - Rf_R + vf_T + uf_R\right), \\ -6\left(f_R + f_T\right)a\dot{a}^2 - 6\left(f_{RR}\dot{R} + f_{RT}\dot{T}\right)a^2\dot{a} - a^3 L_m. \end{gathered} \tag{12}$$

The quantities f_R, f_T, f_{RR}, and f_{RT} are, respectively, the first derivative of f with respect to R, the first derivative of f with respect to T, the second derivative of f with respect to R, and the second derivative of f with respect to R and T.

The equations of $f(R,T)$ gravity are usually more complicated with respect to the equations of Einstein's theory of general relativity even if the FLRW metric is considered. For this reason, as stated before, we consider the following simple particular model of $f(R,T)$ gravity:

$$f(R,T) = \mu R + \nu T, \tag{13}$$

with μ and ν being two constant parameters.

The equations system of this model of $f(R,T)$ gravity is given by

$$\begin{gathered} \mu D_1 + \nu E_1 + K\left(\mu R + \nu T\right) = -2a^3\rho, \\ \mu A_1 + \nu B_1 + M\left(\mu R + \nu T\right) = 6a^2 p, \\ \dot{\rho} + 3H\left(\rho + p\right) = 0, \end{gathered} \tag{14}$$

where

$$\begin{aligned} D_1 &= -6a\dot{a}^2 + a^3 u_{\dot{a}}\dot{a} - a^3\left(u - R\right) \\ &= 6a^2\ddot{a} + a^3\dot{a}u_{\dot{a}} = a^3\left(6\frac{\ddot{a}}{a} + \dot{a}u_{\dot{a}}\right), \\ E_1 &= -6a\dot{a}^2 + a^3\dot{a}v_{\dot{a}} - a^3\left(v - T\right) \\ &= -12a\dot{a}^2 + a^3\dot{a}v_{\dot{a}} = a^3\left(-12\frac{\dot{a}^2}{a^2} + \dot{a}v_{\dot{a}}\right), \\ K &= -a^3, \\ A_1 &= 12\dot{a}^2 + 6a\ddot{a} + 3a^2\dot{a}u_{\dot{a}} + a^3 u_{\dot{a}} - a^3 u_a, \\ B_1 &= -24\dot{a}^2 - 12a\ddot{a} + 3a^2\dot{a}v_{\dot{a}} + a^3 v_{\dot{a}} - a^3 v_a, \\ M &= -3a^2. \end{aligned} \tag{15}$$

We get from (14)

$$\begin{gathered} -6\left(\mu + \nu\right)\frac{\dot{a}^2}{a^2} + \mu\dot{a}u_{\dot{a}} + \nu\dot{a}v_{\dot{a}} - \mu u - \nu v = -2\rho, \\ -2\left(\mu + \nu\right)\left(\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a}\right) + \mu\dot{a}u_{\dot{a}} + \nu\dot{a}v_{\dot{a}} - \mu u \\ - \nu v + \frac{\mu}{3}a\left(\dot{u}_{\dot{a}} - u_a\right) + \frac{\nu}{3}a\left(\dot{v}_{\dot{a}} - v_a\right) = 2p, \\ \dot{\rho} + 3H\left(\rho + p\right) = 0. \end{gathered} \tag{16}$$

Then (16) can be rewritten as follows:

$$\begin{gathered} 3\left(\mu + \nu\right)\frac{\dot{a}^2}{a^2} - \frac{1}{2}\left(\mu\dot{a}u_{\dot{a}} + \nu\dot{a}v_{\dot{a}} - \mu u - \nu v\right) = \rho, \\ \left(\mu + \nu\right)\left(\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a}\right) - \frac{1}{2}\left(\mu\dot{a}u_{\dot{a}} + \nu\dot{a}v_{\dot{a}} - \mu u - \nu v\right) \\ - \frac{\mu}{6}a\left(\dot{u}_{\dot{a}} - u_a\right) - \frac{\nu}{6}a\left(\dot{v}_{\dot{a}} - v_a\right) = -p, \\ \dot{\rho} + 3H\left(\rho + p\right) = 0, \end{gathered} \tag{17}$$

or equivalently

$$\begin{gathered} 3\left(\mu + \nu\right)H^2 - \frac{1}{2}\left(\mu\dot{a}u_{\dot{a}} + \nu\dot{a}v_{\dot{a}} - \mu u - \nu v\right) = \rho, \\ \left(\mu + \nu\right)\left(2\dot{H} + 3H^2\right) - \frac{1}{2}\left(\mu\dot{a}u_{\dot{a}} + \nu\dot{a}v_{\dot{a}} - \mu u - \nu v\right) \\ - \frac{\mu}{6}a\left(\dot{u}_{\dot{a}} - u_a\right) - \frac{\nu}{6}a\left(\dot{v}_{\dot{a}} - v_a\right) = -p, \\ \dot{\rho} + 3H\left(\rho + p\right) = 0. \end{gathered} \tag{18}$$

The above system has two equations and five unknown functions, which are a, ρ, p, u, and v.

We now assume the following expressions for u and, v:

$$\begin{gathered} u = \alpha a^n, \\ v = \beta a^m, \end{gathered} \tag{19}$$

where m, n, α, and β are real constants. We also have that u and v can be expressed as

$$u = \alpha\left(\frac{v}{\beta}\right)^{n/m}, \qquad v = \beta\left(\frac{u}{\alpha}\right)^{m/n}. \tag{20}$$

Then, the system made by (18) leads to

$$3(\mu+\nu)H^2 + \frac{1}{2}(\mu\alpha a^n + \nu\beta a^m) = \rho, \tag{21}$$

$$(\mu+\nu)\left(2\dot{H} + 3H^2\right) + \frac{\mu\alpha(n+3)}{6}a^n + \frac{\nu\beta(m+3)}{6}a^m = -p, \tag{22}$$

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{23}$$

Finally, we have that the EoS parameter ω for this model is given by the relation

$$\omega = \frac{p}{\rho} = -1 - \frac{2(\mu+\nu)\dot{H} - (\mu/6)a(\dot{u}_{\dot{a}} - u_a) - (\nu/6)a(\dot{v}_{\dot{a}} - v_a)}{3(\mu+\nu)H^2 - (1/2)(\mu\dot{a}u_{\dot{a}} + \nu\dot{a}v_{\dot{a}} - \mu u - \nu v)}. \tag{24}$$

3. Interacting DE in $f(R,T)$ Gravity

Solving the differential equation for ρ_m given in (8), we derive the following expression for ρ_m:

$$\rho_m = \rho_{m0}a^{-3(1+\delta)}, \tag{25}$$

where ρ_{m0} indicates the present day value of ρ_m.

Using (1) and (25) in the right-hand side of (21), we obtain the following expression of H^2 as function of the scale factor:

$$\begin{aligned} H^2 = {} & C_1 a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + C_2 a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + \frac{\alpha\mu a^n}{n^2\varepsilon + n\lambda + 2[\theta - 3(\mu+\nu)]} \\ & + \frac{\beta\nu a^m}{m^2\varepsilon + m\lambda + 2[\theta - 3(\mu+\nu)]} \\ & - \left(2a^{-3(1+\delta)}\rho_{m0}\right) \\ & \times \left(9(1+\delta)^2\varepsilon + 2\theta \right. \\ & \left. -3[\lambda(1+\delta) + 2(\mu+\nu)]\right)^{-1}, \end{aligned} \tag{26}$$

where C_1 and C_2 are two constants of integration.

In order to have a real and definite expression of H^2 given in (26), the following conditions must be satisfied:

(1) $\varepsilon \neq 0$,

(2) $\lambda^2 - 8\varepsilon[\theta - 3(\mu+\nu)] \geq 0$,

(3) $n^2\varepsilon + n\lambda + 2[\theta - 3(\mu+\nu)] \neq 0$,

(4) $m^2\varepsilon + m\lambda + 2[\theta - 3(\mu+\nu)] \neq 0$,

(5) $9(1+\delta)^2 + 2\theta - 3[\lambda(1+\delta) + 2(\mu+\nu)] \neq 0$.

We can now derive the expressions of the first and the second time derivative of the Hubble parameter H, that is, $\dot{H}$ and $\ddot{H}$, as functions of the scale factor a differentiating (26) with respect to the cosmic time t:

$$\begin{aligned} \dot{H} = {} & -\frac{C_1}{2}\left(\frac{\lambda + \sqrt{\lambda^2 - 8\varepsilon[\theta - 3(\mu+\nu)]}}{2\varepsilon}\right) \\ & \times a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + \frac{C_2}{2}\left(\frac{-\lambda + \sqrt{\lambda^2 - 8\varepsilon[\theta - 3(\mu+\nu)]}}{2\varepsilon}\right) \\ & \times a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + \frac{n\alpha\mu a^n}{2\{n^2\varepsilon + n\lambda + 2[\theta - 3(\mu+\nu)]\}} \\ & + \frac{m\beta\nu a^m}{2\{m^2\varepsilon + m\lambda + 2[\theta - 3(\mu+\nu)]\}} \\ & + \left(3(1+\delta)a^{-3(1+\delta)}\rho_{m0}\right) \\ & \times \left(9(1+\delta)^2\varepsilon + 2\theta \right. \\ & \left. - 3[\lambda(1+\delta) + 2(\mu+\nu)]\right)^{-1}, \end{aligned} \tag{27}$$

$$\begin{aligned} \ddot{H} = H \Bigg\{ & \frac{C_1}{2}\left(\frac{\lambda + \sqrt{\lambda^2 - 8\varepsilon[\theta - 3(\mu+\nu)]}}{2\varepsilon}\right)^2 \\ & \times a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + \frac{C_2}{2}\left(\frac{-\lambda + \sqrt{\lambda^2 - 8\varepsilon[\theta - 3(\mu+\nu)]}}{2\varepsilon}\right)^2 \\ & \times a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + \frac{n^2\alpha\mu a^n}{2\{n^2\varepsilon + n\lambda + 2[\theta - 3(\mu+\nu)]\}} \\ & + \frac{m^2\beta\nu a^m}{2\{m^2\varepsilon + m\lambda + 2[\theta - 3(\mu+\nu)]\}} \\ & - \left(9(1+\delta)^2 a^{-3(1+\delta)}\rho_{m0}\right) \\ & \times \left(9(1+\delta)^2\varepsilon + 2\theta \right. \\ & \left. - 3[\lambda(1+\delta) + 2(\mu+\nu)]\right)^{-1} \Bigg\}. \end{aligned} \tag{28}$$

Using (26), (27), and (28) in (1), we obtain the following expression of the energy density ρ_{DE}:

$$\begin{aligned}\rho_{\mathrm{DE}} = \frac{1}{2}\Bigg[& \frac{\left(n^2\varepsilon + 2\theta + n\lambda\right)\alpha\mu a^n}{\{n^2\varepsilon + n\lambda + 2\left[\theta - 3\left(\mu+\nu\right)\right]\}} \\ & + \frac{\left(m^2\varepsilon + 2\theta + 2m\lambda\right)\beta\nu a^m}{\{m^2\varepsilon + m\lambda + 2\left[\theta - 3\left(\mu+\nu\right)\right]\}} \\ & + 6C_1\left(\mu+\nu\right)a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + 6C_2\left(\mu+\nu\right)a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & - \left(2\left[9(1+\delta)^2\varepsilon + 2\theta - 3\left(1+\delta\right)\lambda\right]a^{-3(1+\delta)}\rho_{m0}\right) \\ & \times\Big(9(1+\delta)^2\varepsilon + 2\theta \\ & -3\left[\lambda\left(1+\delta\right) + 2\left(\mu+\nu\right)\right]\Big)^{-1}\Bigg].\end{aligned} \tag{29}$$

Taking into account the expression of ρ_{DE} given in (29), we derive that the expression of the pressure p_{DE} of DE is given by

$$\begin{aligned}p_{\mathrm{DE}} = {} & C_1\left(\mu+\nu\right)\left(\frac{-6\varepsilon + \lambda + \sqrt{\lambda^2 - 8\varepsilon\left[\theta - 3\left(\mu+\nu\right)\right]}}{2\varepsilon}\right) \\ & \times a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & \times C_2\left(\mu+\nu\right)\left(\frac{6\varepsilon - \lambda + \sqrt{\lambda^2 - 8\varepsilon\left[\theta - 3\left(\mu+\nu\right)\right]}}{2\varepsilon}\right) \\ & \times a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & - \left(\left[2\theta + n\left(n\varepsilon+\lambda\right)\right]\left(n+3\right)\alpha\mu a^n\right) \\ & \times\left(6\left\{n^2\varepsilon + n\lambda + 2\left[\theta - 3\left(\mu+\nu\right)\right]\right\}\right)^{-1} \\ & - \left(\left[2\theta + n\left(n\varepsilon+\lambda\right)\right]\left(m+3\right)\beta\nu a^m\right) \\ & \times\left(6\left\{m^2\varepsilon + m\lambda + 2\left[\theta - 3\left(\mu+\nu\right)\right]\right\}\right)^{-1} \\ & - \left(\left[9(1+\delta)^2\varepsilon + 2\theta - 3\left(1+\delta\right)\lambda\right]a^{-3(1+\delta)}\delta\rho_{m0}\right) \\ & \times\left(9(1+\delta)^2\varepsilon + 2\theta - 3\left[\lambda\left(1+\delta\right) + 2\left(\mu+\nu\right)\right]\right)^{-1}.\end{aligned} \tag{30}$$

Using the expressions of the energy density ρ_{DE} and the pressure p_{DE} of DE given, respectively, in (29) and (30), and the expression of ρ_m given in (25), we get the EoS parameter ω_{DE} for DE and the total EoS parameter ω_{tot} as follows:

$$\begin{aligned}\omega_{\mathrm{DE}} &= \frac{p_{\mathrm{DE}}}{\rho_{\mathrm{DE}}}, \\ \omega_{\mathrm{tot}} &= \frac{p_{\mathrm{DE}}}{\rho_{\mathrm{DE}} + \rho_m}.\end{aligned} \tag{31}$$

We must remember here that we are considering the case of pressureless DM, so that $p_m = 0$.

We now want to consider the properties of the deceleration parameter q for the model we are considering. The deceleration parameter q is generally defined as follows:

$$q = -1 - \frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}, \tag{32}$$

where the expressions of H^2 and $\dot{H}$ are given, respectively, in (26) and (27). The deceleration parameter, the Hubble parameter H, and the dimensionless energy density parameters Ω_{DE}, Ω_m, and Ω_k (which will be considered and studied in the following Sections) are a set of useful parameters if it is needed to describe cosmological observations.

4. The Statefinder Parameters

In order to have a better comprehension of the properties of the DE model taken into account, we can compare it with a model independent diagnostics which is able to differentiate between a wide variety of dynamical DE models, including the ΛCDM model. We consider here the diagnostic, also known as statefinder diagnostic, which introduces a pair of parameters $\{r, s\}$ defined, respectively, as follows:

$$\begin{aligned}r &= 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} = 1 + \frac{3\dot{H} + \ddot{H}/H}{H^2}, \\ s &= -\frac{3H\dot{H} + \ddot{H}}{3H\left(2\dot{H} + 3H^2\right)} = -\frac{3\dot{H} + \ddot{H}/H}{3\left(2\dot{H} + 3H^2\right)}.\end{aligned} \tag{33}$$

Using (26), (27), and (29), we get the statefinder parameters as

$$\begin{aligned}r &= 1 + \frac{\rho_1}{\rho_2}, \\ s &= \frac{\zeta_1}{\zeta_2},\end{aligned} \tag{34}$$

with

$$\begin{aligned}\rho_1 = {} & C_1\left(\mu+\nu\right)\left(\lambda + \sqrt{\lambda^2 - 8\varepsilon\left[\theta - 3\left(\mu+\nu\right)\right]}\right) \\ & \times\left(\frac{-6\varepsilon + \lambda + \sqrt{\lambda^2 - 8\varepsilon\left[\theta - 3\left(\mu+\nu\right)\right]}}{8\varepsilon^2}\right) \\ & \times a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon} \\ & + C_2\left(\mu+\nu\right)\left(-\lambda + \sqrt{\lambda^2 - 8\varepsilon\left[\theta - 3\left(\mu+\nu\right)\right]}\right)\end{aligned}$$

$$\times\left(\frac{6\varepsilon-\lambda+\sqrt{\lambda^2-8\varepsilon\left[\theta-3\left(\mu+\nu\right)\right]}}{8\varepsilon^2}\right)$$
$$\times a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon}$$
$$+\frac{(n+3)\,n\alpha\mu a^n}{2\left\{n^2\varepsilon+n\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]\right\}}$$
$$+\frac{(m+3)\,m\beta\nu a^m}{2\left\{m^2\varepsilon+m\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]\right\}}$$
$$-\left(9a^{-3(1+\delta)}\delta\left(1+\delta\right)\rho_{m0}\right)$$
$$\times\left(9(1+\delta)^2\varepsilon+2\theta\right.$$
$$\left.-3\left[\lambda\left(1+\delta\right)+2\left(\mu+\nu\right)\right]\right)^{-1},$$
$$\rho_2=C_1a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon}$$
$$+C_2a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon}$$
$$+\frac{\alpha\mu a^n}{n^2\varepsilon+n\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]}$$
$$+\frac{\beta\nu a^m}{m^2\varepsilon+m\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]}$$
$$-\left(2a^{-3(1+\delta)}\rho_{m0}\right)$$
$$\times\left(9(1+\delta)^2\varepsilon+2\theta\right.$$
$$\left.-3\left[\lambda\left(1+\delta\right)+2\left(\mu+\nu\right)\right]\right)^{-1},$$
$$\zeta_1=-C_1\left(\mu+\nu\right)\left(\lambda+\sqrt{\lambda^2-8\varepsilon\left[\theta-3\left(\mu+\nu\right)\right]}\right)$$
$$\times\left(\frac{-6\varepsilon+\lambda+\sqrt{\lambda^2-8\varepsilon\left[\theta-3\left(\mu+\nu\right)\right]}}{8\varepsilon^2}\right)$$
$$\times a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon}$$
$$-C_2\left(\mu+\nu\right)\left(-\lambda+\sqrt{\lambda^2-8\varepsilon\left[\theta-3\left(\mu+\nu\right)\right]}\right)$$
$$\times\left(\frac{6\varepsilon-\lambda+\sqrt{\lambda^2-8\varepsilon\left[\theta-3\left(\mu+\nu\right)\right]}}{8\varepsilon^2}\right)$$
$$\times a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon}$$
$$-\frac{(n+3)\,n\alpha\mu a^n}{2\left\{n^2\varepsilon+n\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]\right\}}$$
$$-\frac{(m+3)\,m\beta\nu a^m}{2\left\{m^2\varepsilon+m\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]\right\}}$$
$$+\left(9a^{-3(1+\delta)}\delta\left(1+\delta\right)\rho_{m0}\right)$$
$$\times\left(9(1+\delta)^2\varepsilon+2\theta\right.$$
$$\left.-3\left[\lambda\left(1+\delta\right)+2\left(\mu+\nu\right)\right]\right)^{-1},$$
$$\zeta_2=-3C_1\left(\frac{-6\varepsilon+\lambda+\sqrt{\lambda^2-8\varepsilon\left[\theta-3\left(\mu+\nu\right)\right]}}{2\varepsilon}\right)$$
$$\times a^{-(\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon}$$
$$+3C_2\left(\frac{6\varepsilon-\lambda+\sqrt{\lambda^2-8\varepsilon\left[\theta-3\left(\mu+\nu\right)\right]}}{2\varepsilon}\right)$$
$$\times a^{(-\lambda+\sqrt{\lambda^2-8\varepsilon[\theta-3(\mu+\nu)]})/2\varepsilon}$$
$$+3\frac{(n+3)\,\alpha\mu a^n}{\left\{n^2\varepsilon+n\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]\right\}}$$
$$+3\frac{(m+3)\,\beta\nu a^m}{\left\{m^2\varepsilon+m\lambda+2\left[\theta-3\left(\mu+\nu\right)\right]\right\}}$$
$$+\left(18a^{-3(1+\delta)}\delta\rho_{m0}\right)$$
$$\times\left(9(1+\delta)^2\varepsilon+2\theta\right.$$
$$\left.-3\left[\lambda\left(1+\delta\right)+2\left(\mu+\nu\right)\right]\right)^{-1}. \tag{35}$$

5. Discussion

In this Section, we discuss the behavior of the physical quantities derived in the previous sections. We have considered the following values for the parameters involved: $C_1 = 0.2$, $C_2 = 1.2$, $m = 1.2$, $n = 1.4$, $\beta = 1.2$, $\alpha = 1.5$, $\theta = 0.002$, $\lambda = 2$, $\nu = 0.5$, $\mu = 0.9$, $\delta = 0.05$, and $\rho_{m0} = 0.23$. We considered three different cases corresponding to three different values of the parameter ε, that is, $\varepsilon = 2, 3$, and 4.

In Figure 1, we plotted the expression of the Hubble parameter H, obtained from (26), as function of the redshift z. It is evident that the Hubble parameter H has a decaying behavior with varying values of the parameter ε and the redshift z going from higher to lower redshifts. However, this decaying pattern is apparent till $z \approx -0.5$. In fact, in a very late stage $z > -0.5$, it shows an increasing pattern.

In Figure 2, we have plotted the time derivative of Hubble parameter $\dot{H}$ against the redshift z. We have observed that for $\varepsilon = 3$, $\dot{H}$ transits from negative to positive side at $z \approx -0.5$. However, for $\varepsilon = 2$ and 4 this transition occurs at lower redshift $z \approx -0.1$. In Figures 3 and 4 we have plotted, respectively, the equation of state (EoS) parameter for DE, defined as $\omega_{\mathrm{DE}} = p_{\mathrm{DE}}/\rho_{\mathrm{DE}}$, and the effective EoS parameter, defined as $\omega_{\mathrm{eff}} = p_{\mathrm{DE}}/(\rho_{\mathrm{DE}} + \rho_{\mathrm{DM}})$, for the three different values of ε considered in this work. In Figure 3,

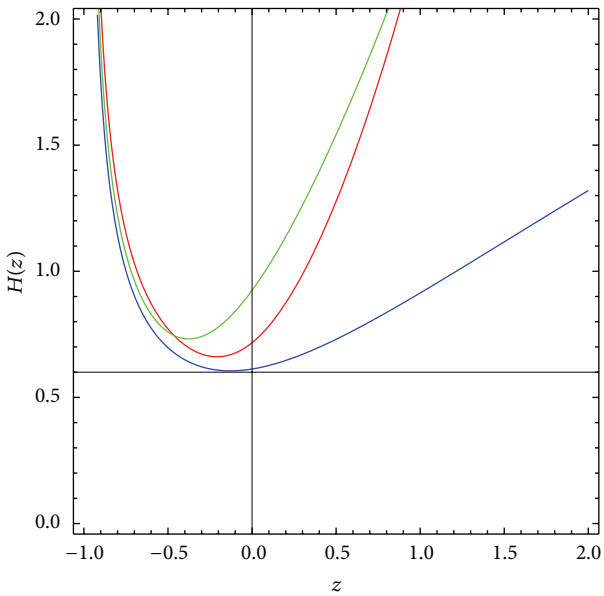

Figure 1: The Hubble parameter H, obtained from (26), as a function of redshift z. The red, green, and blue lines correspond to $\varepsilon = 2, 3$, and 4, respectively.

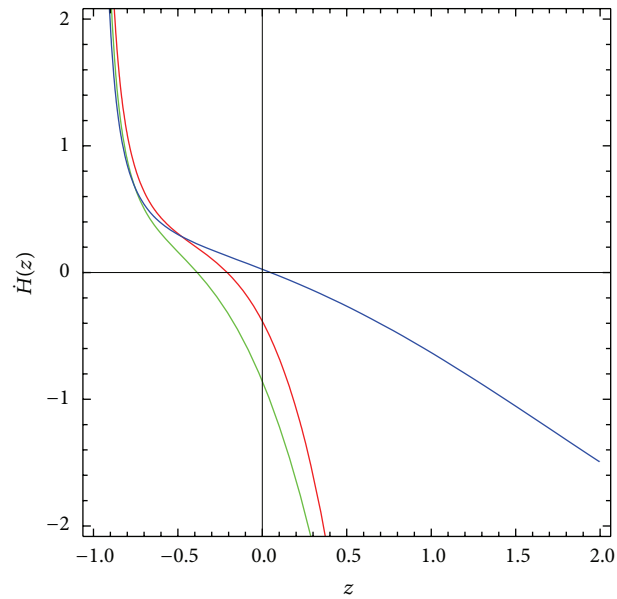

Figure 2: The time derivative of reconstructed Hubble parameter as a function of redshift z. The red, green and blue lines correspond to $\varepsilon = 2, 3$, and 4, respectively.

we have observed that for $\varepsilon = 2$, ω_{DE} crosses the phantom divide -1 at $z \approx 0$. For $\varepsilon = 3$ the phantom divide is crossed at $z \approx -0.2$. However, for $\varepsilon = 4$, the equation of state (EoS) parameter for DE stays below -1. Thus, for $\varepsilon = 2$ and 3, ω_{DE} transits from quintessence to phantom, that is, has a quintom-like behavior. Instead, for $\varepsilon = 4$, the EoS parameter has a phantom-like behavior. In Figure 4, we have plotted the effective EoS parameter ω_{eff}. In this case, for all values of ε considered, there is a crossing of phantom divide. Moreover, for $\varepsilon = 4$, ω_{eff} crosses the phantom divide earlier with respect to the other cases, in particular for $z \approx 0.2$.

The deceleration parameter q has been plotted as a function of z in Figure 5. For $\varepsilon = 2$, and 3, there is a transition from positive to negative q, that is, transition from decelerated to accelerated expansion. For $\varepsilon = 3$, the deceleration parameter changes sign at $z = 0$, and for $\varepsilon = 2$, it changes sign at $z \approx 0.1$. However, for $\varepsilon = 4$, the deceleration parameter always stays at negative level. Thus, for $\varepsilon = 4$, we are getting ever-accelerating universe.

Next, we have plotted in Figure 6 the fractional density of DE, given by $\Omega_{\mathrm{DE}} = \rho_{\mathrm{DE}}/3\widetilde{H}^2(z)$, and the fractional density of matter, given by $\Omega_m = \rho_m/3\widetilde{H}^2(z)$, against the redshift z. $\widetilde{H}^2$ is defined as $\widetilde{H}^2(z) = (\mu + \nu)H^2 + (1/6)[\alpha\mu(1+z)^{-n} + \beta\nu(1+z)^{-m}]$. The solid lines correspond to Ω_{DE} and the dashed lines correspond to Ω_{DM}. In this Figure, there is a clear indication of transition of the universe from dark matter dominated phase to the dark energy dominated phase. At very early stage of the universe $z > 1$, the dark energy density is largely dominated by dark matter density. We denote the cross-over point by z_{cross} and it comes out to be $z_{\mathrm{cross}} \approx 0.5$, that is, where $\Omega_{\mathrm{DE}} = \Omega_{\mathrm{DM}}$ for all values of ε considered in this work. Hence, the $f(R,T)$ model, based on which we have reconstructed DE density, is capable of achieving

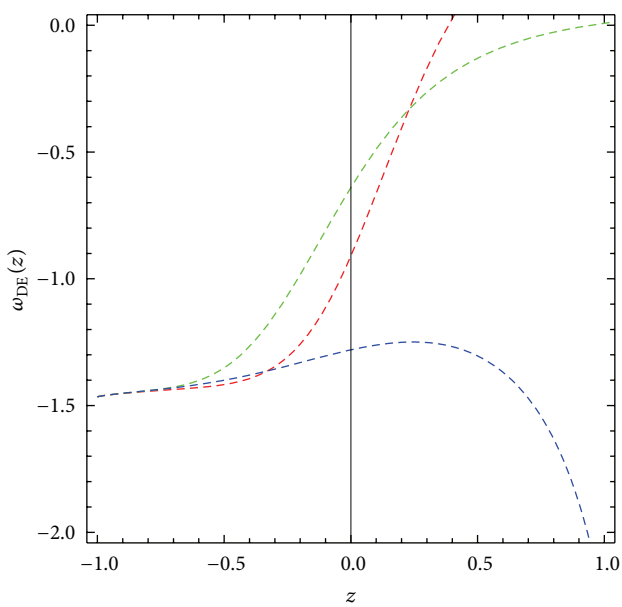

Figure 3: The EoS parameter ω_{DE} for the reconstructed DE.

the present DE dominated universe from the earlier dark matter dominated universe.

Sahni et al. [42] recently demonstrated that the statefinder diagnostic is effectively able to discriminate between different models of DE. Chaplygin gas, braneworld, quintessence, and cosmological constant models were investigated by Alam et al. [43] using the statefinder diagnostics; they observed that the statefinder pair could differentiate between these different models. An investigation on statefinder parameters for differentiating between DE and modified

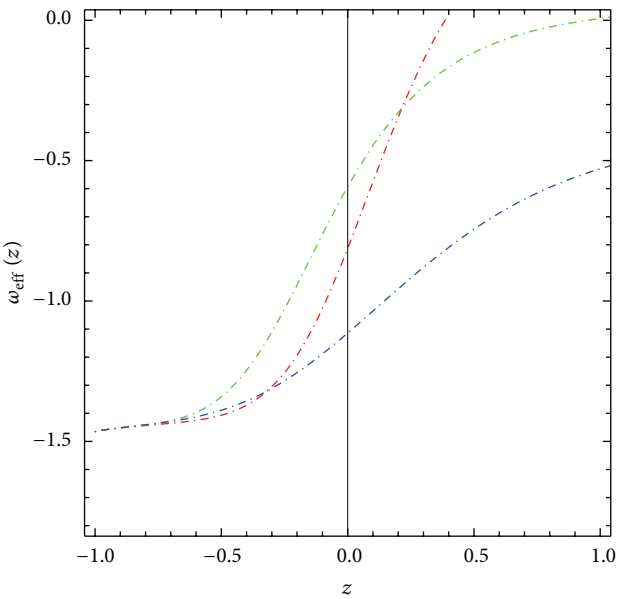

FIGURE 4: The effective EoS parameter $\omega_{\text{eff}}(z) = p_{\text{DE}}/(\rho_{\text{DE}} + \rho_{\text{DM}})$.

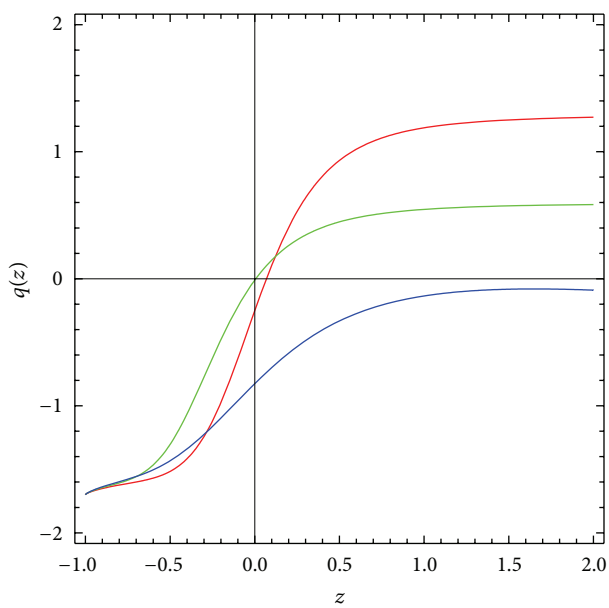

FIGURE 5: The deceleration parameter q as a function of z.

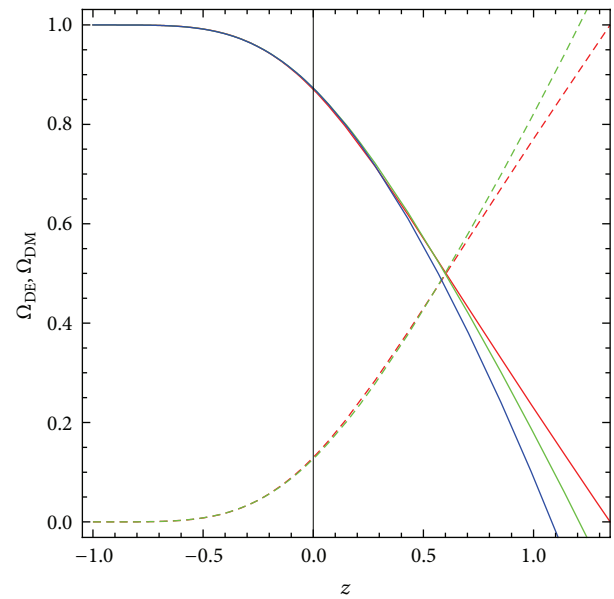

FIGURE 6: The fractional densities Ω_{DE} (smooth lines) and Ω_{DM} (dashed lines) as function of redshift z.

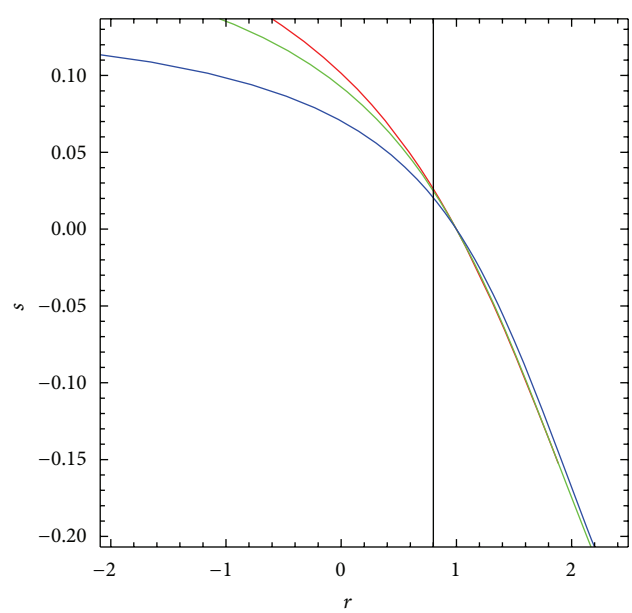

FIGURE 7: Statefinder trajectories for various choices of parameters.

gravity was carried out in [44]. Statefinder diagnostics for $f(T)$ gravity has been well studied in Wu and Yu [45]. In the $\{r, s\}$ plane, $s > 0$ corresponds to a quintessence-like model of DE and $s < 0$ corresponds to a phantom-like model of DE. Moreover, an evolution from phantom to quintessence or inverse is given by crossing of the fixed point $(r = 1, s = 0)$ in $\{r, s\}$ plane [45], which corresponds to ΛCDM scenario. The statefinder parameters $\{r, s\}$ have been plotted in Figure 7 for different values of the parameter ε. It is clearly visible that the $\{r - s\}$ trajectories are converging towards the fixed point $\{r = 1, s = 0\}|_{\Lambda\text{CDM}}$. Thus, the $f(R, T)$ model is capable of attaining the ΛCDM phase of the universe. Furthermore, for finite r, $s \to -\infty$. Thus, the model can interpolate between dust and ΛCDM phase of the universe.

Finally, in Figure 8, we plotted the squared speed of the sound, defined as $v_s^2 = \dot{p}/\dot{\rho}$, where the upper dot indicates derivative with respect to the cosmic time t for the model we are considering as a function of z. The sign of the squared speed of sound is fundamental in order to study the stability of a background evolution. A negative value of v_s^2 implies a classical instability of a given perturbation in general relativity [46, 47]. Myung [47] recently observed that the squared speed of sound for HDE stays always negative if

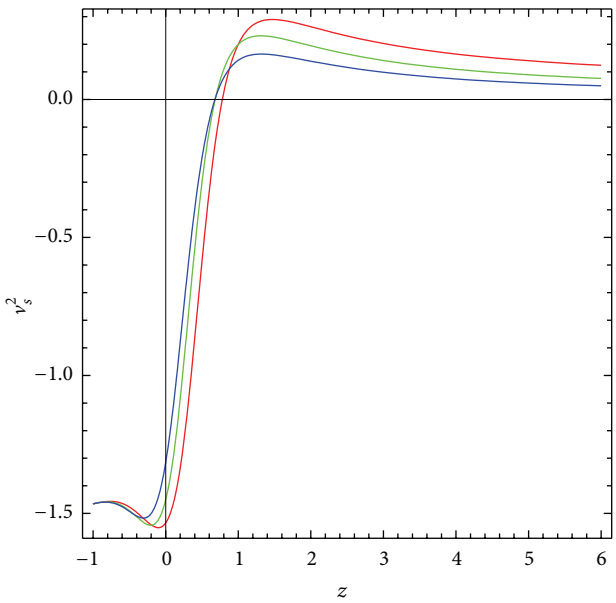

FIGURE 8: Squared speed of sound $v_s^2 = \dot{p}/\dot{\rho}$ as a function of redshift z.

the future event horizon is considered as IR cutoff, while for Chaplygin gas and tachyon v_s^2 is observed to benonnegative. Kim et al. [46] found that v_s^2 for Agegraphic DE (ADE) stays always negative, which leads to the instability of the perfect fluid for the model. Moreover, it was found that the ghost QCD [48] DE model is unstable. In a recent work, Sharif and Jawad [49] have shown that interacting new HDE is characterized by negative squared speed of the sound.

In the recent work of Pasqua et al. [32], authors observed that the interacting modified holographic Ricci DE (MHRDE) model in $f(R,T) = \mu R + \nu T$ gravity is classically stable.

Jawad et al. [50] showed that $f(G)$ model in HDE scenario with power-law scale factor is classically unstable.

Pasqua et al. [51] showed that the DE model based on generalized uncertainty principle (GUP) with power-law form of the scale factor $a(t)$ is instable.

We have observed that, for all values of ε, v_s^2 is positive till the redshift $z \approx 0.5$. However, after this stage it enters the negative region. Thus, although the model is classically stable in the early universe, for present universe it is classically unstable.

6. Concluding Remarks

In this work, we have considered a recently proposed model of energy density of DE which depends on three terms, one proportional to the squared Hubble parameter H, one proportional to the time derivative of H, and one proportional to the second time derivative of H interacting with pressureless DM in the framework of the $f(R,T)$ modified gravity theory for the special model given by $f(R,T) = \mu R + \nu T$, where μ and ν represent two constants. The DE model considered here reduces to other two well-studied DE model (the Ricci DE model and the DE energy density model with Granda-Oliveros cut-off) for some particular values of the three parameters involved, that is, ε, λ, and θ. We have derived the expressions and studied the behavior of some important physical quantities which gave useful hints about the model studied. The Hubble parameter H exhibits a decaying behavior going from higher to lower redshifts until the redshift of about $z \approx -0.5$, when it starts to increase. The time derivative of the Hubble parameter, that is, $\dot{H}$, shows a transition from negative to positive values for different values of the redshift z according to the value of ε considered. We have observed that the equation of state (EoS) parameter of DE exhibits a transition from $\omega < -1$ to $\omega > -1$, that is, transition from quintessence to phantom (i.e., quintom) with the evolution of the universe for $\varepsilon = 2$ and $\varepsilon = 3$, instead it is always negative for $\varepsilon = 4$. Moreover, the effective equation of state (EoS) parameter ω_{eff} always shows a transition from quintessence to phantom. Hence, we can conclude that the reconstructed DE model based on the $f(R,T)$ gravity model considered leads to an equation of state (EoS) parameter that has a quintom-like behavior. We have further observed that the said model is capable of attaining the dark energy dominated accelerated phase of the universe from dark matter dominated decelerated phase of the universe. Through statefinder trajectories we have shown that the $f(R,T)$ is capable of attaining the ΛCDM phase of the universe and can interpolate between dust and ΛCDM phase of the universe. Through squared speed of sound we have seen that the model under consideration is classically stable in the earlier stage of the universe but classically unstable in the current stage.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The second author acknowledges financial support from the Department of Science and Technology, Govtvernment of India, under project Grant no. SR/FTP/PS-167/2011.

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