

Review Article

Macroscopic Theory of Dark Sector

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A simple Lagrangian with squared covariant divergence of a vector field as a kinetic term turned out to be an adequate tool for macroscopic description of the dark sector. The zero-mass field acts as the dark energy. Its energy-momentum tensor is a simple additive to the cosmological constant. Massive fields describe two different forms of dark matter. The space-like massive vector field is attractive. It is responsible for the observed plateau in galaxy rotation curves. The time-like massive field displays repulsive elasticity. In balance with dark energy and ordinary matter it provides a four-parametric diversity of regular solutions of the Einstein equations describing different possible cosmological and oscillating nonsingular scenarios of evolution of the Universe. In particular, the singular big bang turns into a regular inflation-like transition from contraction to expansion with the accelerated expansion at late times. The fine-tuned Friedman-Robertson-Walker singular solution is a particular limiting case at the lower boundary of existence of regular oscillating solutions in the absence of vector fields. The simplicity of the general covariant expression for the energy-momentum tensor allows displaying the main properties of the dark sector analytically. Although the physical nature of dark sector is still unknown, the macroscopic theory can help analyze the role of dark matter in astrophysical phenomena without resorting to artificial model assumptions.

1. Introduction

Currently there are two most intriguing long standing problems in astrophysics pointing to the existence of so called “hidden sector,” containing “dark energy” and “dark matter.” So far their interaction with the ordinary matter (baryons and leptons) is observed only via gravitation.

The first problem, named “galaxy rotation curves,” appeared in 1924, after Oort discovered the galactic halo, a group of stars orbiting the Milky Way outside the main disk [1]. In 1933, Zwicky [2] postulated “missing mass” to account for the orbital velocities of galaxies in clusters.

The second problem is the accelerated expansion of the Universe discovered through observations of distant supernovae by Riess and Perlmutter and their colleagues in 1998 [3, 4].

At first glance, these two problems have little to do with one another. The accelerated expansion of the Universe indicates the existence of a hidden mechanism of repulsion [5], while the plateau of the galaxy rotation curves is the result of additional attraction caused by the dark matter [6]. Macroscopic approach to the dark sector problems, based on

the analysis of vector fields in general relativity, provides an appropriate universal tool for the theoretical description of both these phenomena. The space-like massive vector field is attractive. It is responsible for the observed plateau in galaxy rotation curves. The time-like massive vector field displays repulsive elasticity. In the scale of the whole Universe it is the source of accelerated expansion. Naturally, the previous solutions of the Einstein equations, describing the expansion of the Universe filled with the mutually attracting matter only, inevitably contained a singularity. Inclusion of the repulsive dark matter into consideration allows the existence of nonsingular solutions describing various possible regular scenarios of evolution of the Universe.

This paper contains the macroscopic theory of dark sector, based on the analysis of vector fields in general relativity. The step by step derivations are accompanied by the references to the benchmark achievements of the predecessors. The main attention is paid to clarify the validity of basing assumptions.

Vector fields are used to describe quantum particles of the ordinary matter [7]. A zero-mass particle—photon—is a quantum of electromagnetic field obeying Maxwell

equations. Massive bosons obey Proca equations [8–11]. The relation of spinors and vectors ([7], page 88) facilitates establishing Dirac equations for fermions.

Field equations for quantum particles are easily established in accordance with the properties of their free motion in plane geometry. If necessary (for the secondary quantization, for instance), the Lagrangian of a particle is then constructed in such a way that the field equations minimize the functional of action. This approach is convenient for description of already known particles. However, it does not help to describe the unknown substance of the dark sector.

In general relativity, the standard approach starting from a general form of the Lagrangian of a vector field is capable to describe not only the already known particles. It is reasonable to start from a general form of the Lagrangian of a vector field in general relativity and derive the vector field equations and the energy-momentum tensor. Excluding the terms associated with the ordinary matter, one can separate the Lagrangian having a chance to describe the dark sector. The separation of the Lagrangian of the dark sector is necessary, especially if the ordinary matter in the Universe is considered as a continuous medium with the macroscopic energy-momentum tensor (56). Otherwise the ordinary matter would be taken into account twice: as a medium with the energy-momentum tensor (56) and as quantum particles described by the vector field.

It turns out that the most simple Lagrangian of a vector field (58) (with the squared covariant divergence as a kinetic term) allows to describe the main observed manifestations of the dark sector completely within the frames of the minimal general relativity. In this case, the massless field corresponds to the dark energy, the massive space-like field ($\phi^K \phi_K < 0$) is responsible for a plateau in the galaxy rotation curves, and the massive time-like vector field ($\phi^K \phi_K > 0$) displays the repulsive elasticity. The competition of repulsive dark matter and attractive ordinary matter leads to a variety of possible regular scenarios of evolution of the Universe.

According to the NASA “sliced cake” diagram [12] (see Figure 1) today there is only 4.6% of the ordinary matter among the stuff of the Universe. All other 95% is the unknown substance, referred to as the dark matter and dark energy. For this reason it is natural to start with the analysis of the properties of dark sector and add the ordinary matter into consideration after the role of dark sector is clarified.

The main properties of vector fields in general relativity are recalled in Section 2 in order to clarify the specifics of the approach based on the principle of regularity. The features of the Proca equations allow separating the terms in the Lagrangian, which are not connected with the ordinary matter. It turns out that a simple Lagrangian (with the squared covariant divergence as a kinetic term) is a proper tool for macroscopic description of main observed properties of the dark sector. The necessary conditions of regularity for a space-like and a time-like vectors are different. Nevertheless, the field equations and the energy-momentum tensor have the same covariant form for both kinds of vector fields. The simplicity of equations allows getting analytical solutions in the most interesting cases. The galaxy rotation curves,

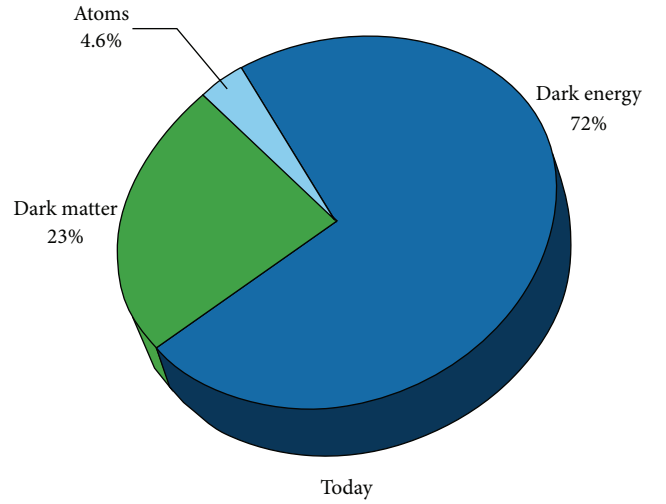


FIGURE 1: Stuff of the Universe [12].

driven by the space-like vector fields, are derived in Section 3. Various uniform and isotropic scenarios of evolution of the Universe under the joint influence of the zero-mass vector field, the time-like massive vector field, and the ordinary matter are analyzed in Section 4. The current situation with the dark sector is summarized in Section 5. Some major astrophysical problems are specified, where the macroscopic theory can be applied, helping to avoid the unnecessary model assumptions. Unraveling the still unknown physical nature of the dark energy and dark matter remains the most pressing issue.

2. Vector Fields in General Relativity

2.1. Lagrangian of a Vector Field. Within the frames of minimal general relativity (field equations no higher than of the second order), the Lagrangian of a vector field ϕ_I is a scalar consisting of bilinear combinations of the covariant derivatives $\phi_{I;K}$ and a scalar potential $V(\phi^K \phi_K)$. A bilinear combination of the covariant derivatives is a 4-index tensor

$$S_{IKLM} = \phi_{I;K} \phi_{L;M}. \quad (1)$$

A general form of the scalar S , formed via contractions of S_{IKLM} , is

$$S = (a g^{IK} g^{LM} + b g^{IL} g^{KM} + c g^{IM} g^{KL}) S_{IKLM}, \quad (2)$$

where a , b , and c are arbitrary constants. Therefore, a general form of the Lagrangian L of a vector field ϕ_I ,

$$L\left(\phi_I, \frac{\partial \phi_I}{\partial x^K}, g^{IK}, \frac{\partial g_{IK}}{\partial x^L}\right) = a(\phi_{;M}^M)^2 + b\phi_{;M}^L \phi_{;L}^M + c\phi_{;M}^L \phi_{;L}^M - V(\phi_M \phi^M), \quad (3)$$

contains three kinetic terms with the arbitrary coefficients a , b , and c . Applying the least action principle, in view of

$$\begin{aligned}\phi^K &= g^{IK}\phi_I, & \phi_{I;K} &= \frac{\partial\phi_I}{\partial x^K} - \Gamma_{IK}^L\phi_L, \\ \Gamma_{IK}^L &= \frac{1}{2}g^{LM}\left(\frac{\partial g_{MI}}{\partial x^K} + \frac{\partial g_{MK}}{\partial x^I} - \frac{\partial g_{IK}}{\partial x^M}\right), \dots,\end{aligned}\quad (4)$$

it is convenient to consider the Lagrangian (3) as a function of ϕ_I , $\partial\phi_I/\partial x^K$, g^{IK} , and $\partial g_{IK}/\partial x^L$ as independent variables.

2.1.1. Bumblebee Models. Strictly speaking, (3) is not yet the most general form of the Lagrangian of a vector field. Scalars can be made out of S_{IKLM} not only via contractions, but also by convolutions with participation of ϕ^I , like $g^{IK}\phi^L\phi^MS_{IKLM}$, and $R^{IK}\phi^L\phi^MS_{IKLM}$. In principle, the number of independent constants can exceed 3.

The direction of a vector is specified, and solutions can be less symmetric than the initial Lagrangian. In this case the symmetry of a system is considered as spontaneously broken. Spontaneous breaking of Lorentz symmetry by a vector field ϕ^I , having a nonzero expectation value in vacuum, is a subject of so called “bumblebee theories;” see [13–16] and references therein. Within the frames of field equations no higher than of the second order, the most general form of the action is

$$S = \int d^4x \sqrt{-g} \left[R + J^{IKLM}R_{IKLM} + K^{IKLM}\phi_{I;K}\phi_{L;M} - V(g_{IK}\phi^I\phi^K) \right]. \quad (5)$$

Here R_{IKLM} is the Riemann tensor of curvature and $R = g^{IL}g^{KM}R_{IKLM}$, J^{IKLM} and K^{IKLM} are arbitrary tensors formed out of the vector ϕ_I and the metric tensor g_{IK} . In practice people restrict themselves by simplified models. For instance, Seifert [17] considered recently the particular case $J^{IKLM} = 0$, $K^{IKLM}\phi_{I;K}\phi_{L;M} = \alpha\tilde{g}^{IK}\tilde{g}^{LM}(\phi_{I;L} - \phi_{L;I})(\phi_{K;M} - \phi_{M;K})$, where $\tilde{g}^{IK} = g^{IK} + \beta\phi^I\phi^K$ and α and β are constants. The potential $V(\phi^L\phi_L)$ is taken to have a minimum at some nonzero value of its argument $\phi^L\phi_L$. In this model, a perturbation of a Lorentz-violating vector field can be interpreted as a photon field.

Why “bumblebee”? Perhaps, there was something looking as strange as the ability of the insect bumblebee to fly successfully despite being sometimes questioned on theoretical grounds [18]. By the way, a possibility of existence of macroscopic objects, moving at the speed of light and having zero mass due to gravitational mass-defect, had been mentioned by Andreev back in 1973 [19, 20].

While a vector ϕ_I remains small compared to its vacuum expectation value at a minimum of $V(\phi^K\phi_K)$, the Lagrangian (3) is sufficient. However, in case of a small mass (108) the longitudinal massive field $\phi_0 \sim m^{-1}$; see (110). If in the process of compression the field comes close to its vacuum expectation value, then a phase transition with spontaneous symmetry breaking occurs. The bumblebee approach looks promising for future consideration of a symmetry breaking phase transition in the state of maximum compression.

2.1.2. Specificity of Curved Space-Time. The third kinetic term $c\phi_{;M}^L\phi_{;L}^M$ in (3) can be transformed via differentiation by parts to

$$\begin{aligned}\phi_{;M}^L\phi_{;L}^M &= \phi_{;L}^L\phi_{;M}^M + (\phi_{;L}^M\phi^L - \phi_{;L}^L\phi^M)_{;M} \\ &\quad + (\phi_{;L;M}^L - \phi_{;M;L}^L)\phi^M.\end{aligned}\quad (6)$$

Here $(\phi_{;L}^M\phi^L - \phi_{;L}^L\phi^M)_{;M}$ is a total differential. It does not change the integral of action, and one can use the equivalent Lagrangian

$$\begin{aligned}L &= (a+c)(\phi_{;M}^M)^2 + b\phi_{;M}^L\phi_{;L}^M \\ &\quad + c(\phi_{;L;M}^L - \phi_{;M;L}^L)\phi^M - V(\phi_M\phi^M),\end{aligned}\quad (7)$$

instead of (3).

In general relativity the second covariant derivative of a vector is not invariant against the replacement of the order of differentiation:

$$\phi_{;L;M}^I - \phi_{;M;L}^I = R_{KML}^I\phi^K. \quad (8)$$

R_{IKLM} is the Riemann tensor of curvature. Hence

$$\phi_{;L;M}^L - \phi_{;M;L}^L = R_{KML}^L\phi^K = R_{KM}\phi^K, \quad (9)$$

where R_{KM} is the Ricci tensor. In a curved space-time $R_{KM} \neq 0$, and the term $c(\phi_{;L;M}^L - \phi_{;M;L}^L)\phi^M$ affects the integral of action. From the point of view of general relativity all three kinetic terms in (3) are equally important. If we adhere the view that in the quantum physics each elementary particle is a quantum of some field and vice versa each field corresponds to its own quantum particle [21], then, in principle, any linear combination of the three kinetic terms in the Lagrangian (3) could be associated with some sort of matter.

In the plane geometry $R_{KM} = 0$, the term $c(\phi_{;L;M}^L - \phi_{;M;L}^L)\phi^M$ drops out, a covariant derivative $\phi_{;M}^L$ reduces to the ordinary one $\phi_{,M}^L$, and there are only two arbitrary constants (b , and $\tilde{c} = a+c$) in the Lagrangian:

$$\begin{aligned}L_{\text{plane}} &= b\phi_{,M}^L\phi_{,L}^M + \tilde{c}\phi_{,M}^L\phi_{,L}^M - V(\phi_M\phi^M), \\ R_{KM} &= 0.\end{aligned}\quad (10)$$

It is convenient to classify the vector fields ϕ_I according to their properties of invariance and symmetry.

The sign of the scalar $\phi_K\phi^K$ is invariant against the arbitrary transformations of coordinates. Therefore, if there is no interaction other than via gravitation, there can be three different independent vector fields with $\phi_K\phi^K < 0$, $\phi_K\phi^K = 0$, and $\phi_K\phi^K > 0$. If $\phi_K\phi^K \neq 0$, then in general relativity one can choose a reference frame where either $\phi_0 = 0$ when $\phi_K\phi^K < 0$ (space-like vector) or $\phi_{I>0} = 0$ if $\phi_K\phi^K > 0$ (time-like vector). $\phi_K\phi^K = 0$ is a separate case. The field equations for an ordinary massive particle are easily derived basing on the statement that in the plain space-time there is a reference frame where the particle is at rest [7]. Hence, the ordinary

massive particles are described by the space-like fields. For the zero-mass particles (such as photons) there is no reference system, where they are at rest. The photons are associated with the massless vector field $V(\phi_M \phi^M) = \text{const}$. The time-like vector fields can not be associated with the ordinary massive particles because in the case $\phi_K \phi^K > 0$ there is no frame where $\phi_0 = 0$. Nevertheless, one can not deny the existence of some substance corresponding to a time-like field. From the general relativity view point all three kinetic terms in the Lagrangian (3) are equally important, as well as any of the three types of vector fields could describe some sort of matter.

The covariant derivative $\phi_{I;K}$ can be presented as a sum of a symmetric G_{IK} and an antisymmetric F_{IK} parts:

$$\begin{aligned}\phi_{I;K} &= G_{IK} + F_{IK}, \\ G_{IK} &= \frac{1}{2} (\phi_{I;K} + \phi_{K;I}), \\ F_{IK} &= \frac{1}{2} (\phi_{I;K} - \phi_{K;I}).\end{aligned}\quad (11)$$

In view of $G_K^L F_L^K = 0$ the scalar (2) can be presented in the form

$$S = a(G_K^K)^2 + (b+c)G_K^L G_L^K + (b-c)F_K^L F_L^K. \quad (12)$$

The last term with antisymmetric derivatives is identical to electromagnetism. It becomes clear in common notations $A_I = \phi_I/2$, $F_{IK} = A_{I;K} - A_{K;I}$. The bilinear combination of the antisymmetric derivatives $F_{IK} F^{IK}$ is the same as in electrodynamics. In view of the symmetry of Christoffel symbols $\Gamma_{IK}^L = \Gamma_{KI}^L$,

$$A_{I;K} - A_{K;I} = \frac{\partial A_I}{\partial x^K} - \frac{\partial A_K}{\partial x^I}, \quad (13)$$

and the scalar $F_{IK} F^{IK}$ does not depend on the derivatives of the metric tensor. On the contrary, the two first terms in (12) with symmetric covariant derivatives contain not only the components of the metric tensor g^{IK} , but also the derivatives $\partial g_{IK}/\partial x^L$. The difference between the two terms with symmetric tensors is caused by the curvature of space-time.

2.2. Regularity in General Relativity. In the notations

$$a = A, \quad b + c = B, \quad b - c = C, \quad (14)$$

the Lagrangian is

$$L = A(G_M^M)^2 + BG_{MN}G^{MN} + CF_{MN}F^{MN} - V(\phi_M \phi^M). \quad (15)$$

In plane space-time it is necessary to require $C < 0$. Otherwise for a space-like vector ($\phi_0 = 0$, $F_{MN}F^{MN} = (1/2)[(\text{rot } \vec{\phi})^2 - (1/c^2)(\partial \vec{\phi}/\partial t)^2]$) the action could not have a minimum as required by the least action principle. If $-C(\partial \vec{\phi}/\partial t)^2$ is negative, then it is possible to make the action

negative with an arbitrarily large absolute value via fairly rapid change of $\vec{\phi}$ with time (within the considered time interval); see [22], page 98.

In the regular solutions of the Einstein equations all invariants of the Riemann curvature tensor are finite. Hence, the invariants of the Ricci tensor R_{IK} are finite too. By virtue of Einstein equations the requirement of regularity automatically excludes a possibility to achieve an infinite value for all the invariants of the energy-momentum tensor T_{IK} . In general relativity the distribution/motion of matter and the curvature of space-time are mutually balanced. Practically, there is no need to require $C < 0$ in advance. Necessary restrictions, if any, on the signs of the constants a, b , and c arise as a consequence of the condition of regularity.

The requirement that all the invariants of the Riemann curvature tensor are finite is a necessary condition of regularity in general relativity.

2.3. Vector Field Equations. The vector field ϕ_I obeys the Euler-Lagrange equations

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^L} \left(\sqrt{-g} \frac{\partial L}{\partial (\partial \phi_I / \partial x^L)} \right) = \frac{\partial L}{\partial \phi_I}. \quad (16)$$

In terms of a, b , and c the variational derivative $\partial L / \partial (\partial \phi_I / \partial x^L)$ is

$$\frac{\partial L}{\partial (\partial \phi_I / \partial x^L)} = 2(a g^{IL} \phi_{;K}^K + b \phi^{I;L} + c \phi^{L;I}). \quad (17)$$

For $\partial L / \partial \phi_I$ we have

$$\begin{aligned}\frac{\partial L}{\partial \phi_I} &= (a g^{NK} g^{LM} + b g^{NL} g^{KM} + c g^{NM} g^{KL}) \frac{\partial}{\partial \phi_I} \phi_{N;K} \phi_{L;M} \\ &\quad - \frac{\partial}{\partial \phi_I} V(g^{NK} \phi_N \phi_K).\end{aligned}\quad (18)$$

ϕ_I and $\partial \phi_I / \partial x^K$ are considered as independent variables in the Lagrangian (3). In a locally geodesic reference system (where the Christoffel symbols together with the derivatives $\partial g_{IK} / \partial x^L$ are zeros) $(\partial / \partial \phi_I) \phi_{N;K} \phi_{L;M} = 0$ and $\partial L / \partial \phi_I = -2V' \phi^I$. Here

$$V' = \frac{dV}{d(\phi_L \phi^L)}. \quad (19)$$

The vector field equations (16), having a covariant form

$$a \phi_{;L;I}^L + b \phi_{;I;L}^L + c \phi_{;I;L}^L = -V' \phi_I, \quad (20)$$

in a locally geodesic system, remain the same in all other reference frames.

In terms of A, B , and C

$$A G_{L;I}^L + B G_{I;L}^L - C F_{I;L}^L = -V' \phi_I. \quad (21)$$

There are two independent terms with the symmetric tensor G in (21) and one with the antisymmetric tensor F .

The physical origin of the two symmetric terms is connected with the curvature of space-time. It becomes clear if we set $F_{IK} = 0$. Then $\phi_{I;K;L} = \phi_{K;I;L}$, $G_{I;L}^L = g^{KL}\phi_{K;I;L} = \phi_{I;L}^L$, and (21) reduces to

$$A\phi_{I;L}^L + B\phi_{I;L}^L = -V'\phi_I. \quad (22)$$

The two left terms differ by the order of differentiation. In accordance with (9) the difference between the two terms in (22) exists only in the curved space-time:

$$\phi_{I;L;I}^L - \phi_{I;I;L}^L = R_{IK}\phi^K. \quad (23)$$

In the flat space-time the Ricci tensor $R_{IK} = 0$, and in case $F_{IK} = 0$ there is no physical difference between the two left terms in (22).

If the vector field is weak, so that the second and higher derivatives of the potential $V(\phi_L\phi^L)$ can be neglected, then (20) are linear,

$$a\phi_{I;L}^L + b\phi_{I;L}^L + c\phi_{I;L}^L = -V'_0\phi_I, \quad V'_0 = V'(0), \quad (24)$$

and the principle of superposition takes place, as it should be in the case of free (noninteracting) fields.

2.3.1. Proca Equations. In the particular case $a = 0$, $c = -b$ the field equations (20) reduce to the Proca equations [8–11] in the case of no sources:

$$(F^{IL})_{;L} = -\frac{V'_0}{2b}\phi^I, \quad a = 0, \quad c = -b. \quad (25)$$

Usually $m = \sqrt{-V'_0/b}$ is referred to as the “mass” of a field. Proca equations are used to describe a free massive spin-1 particle. In the case of a massless field $m = 0$ (25) reduce to the Maxwell equations.

For any tensor Q^{IL} the scalar $(Q^{IL})_{;L;I}$ is symmetric with respect to the lower indexes. Renaming the blind indexes, in view of the symmetric properties of the Riemann and Ricci tensors

$$\begin{aligned} R_{KQTL} &= R_{TLKQ} = -R_{LTKQ}, \\ R_{KT} &= R_{TK}, \end{aligned} \quad (26)$$

we have

$$\begin{aligned} Q_{I;K;L}^{LK} - Q_{I;K;L}^{LK} &= g^{QL}R_{TQLK}Q^{TK} + g^{PK}R_{TPLK}Q^{LT} \\ &= (R_{KT} - R_{TK})Q^{TK} = 0. \end{aligned} \quad (27)$$

The scalar

$$(F^{IL})_{;L;I} = 0, \quad (28)$$

because in accordance with (27) $(F^{IL})_{;L;I} = (F^{IL})_{;I;L}$, while for the antisymmetric tensor $(F^{IL})_{;L;I} = -(F^{LI})_{;L;I} = -(F^{IL})_{;I;L}$. Thus, it follows from the Proca equations (25) that in the particular case $a = 0$, $c = -b$ the covariant divergence of the vector ϕ_I is zero:

$$\phi_{I;I}^I = 0, \quad a = 0, \quad c = -b. \quad (29)$$

In electrodynamics $\phi_{I;I}^I = 0$ is referred to as Lorentz gauge. The fact that $\phi_{I;I}^I = 0$ does not mean that the Proca equations are gauge invariant. (29) is the consequence of the particular choice $a = 0$, $c = -b$.

If still $c = -b$, but $a \neq 0$, we have

$$a(\phi_{I;L}^L)^{;I} + 2b(F^{IL})_{;L} = -V'_0\phi^I. \quad (30)$$

The Lorentz gauge restriction (29) would automatically exclude the case $a \neq 0$ from the consideration. Applying $(\cdot)_{;I}$ to (30), the term $-b$ drops out in view of (28), and, instead of the Lorentz condition (29), the scalar $\phi_{I;I}^I$ obeys the Klein-Gordon equation

$$(\phi_{I;I}^I)_{;L} \equiv \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^L} \left(\sqrt{-g} g^{KL} \frac{\partial \phi_{I;I}^I}{\partial x^K} \right) = -\frac{V'_0}{a} \phi_{I;I}^I, \quad (31)$$

$$a \neq 0, \quad c = -b.$$

In plain space-time “the additional condition (29) excludes the part of ϕ^I belonging to spin 0” [7], page 72. In general relativity the condition (29) excludes also longitudinal vectors, existing in curved space-time. As it turns out, these longitudinal vector fields are just suitable for the description of dark matter and energy.

In the particular case $c = -b$ the field equation (31) for the divergence $\phi_{I;I}^I$ does not contain b . Though the scalar $\phi_{I;I}^I$ does not depend on b , the field itself ϕ_I still obeys (30) containing b .

In the most simple case

$$a \neq 0, \quad b = c = 0, \quad (32)$$

the field equations (24) reduce to $\partial\phi_{I;L}^L/\partial x^I = 0$ if $V'_0 = 0$ (massless field) and to

$$\phi_I = -\frac{a}{V'_0} \frac{\partial \phi_{I;L}^L}{\partial x^I}, \quad a \neq 0, \quad b = c = 0, \quad (33)$$

if $V'_0 \neq 0$ (massive field).

The covariant divergence $\phi_{I;L}^L$ of a zero-mass field remains constant through the whole space-time:

$$\phi_{I;L}^L \equiv \phi'_0 = \text{const}, \quad a \neq 0, \quad b = c = 0, \quad V' = 0. \quad (34)$$

The fact that “the gauge-fixing term exactly behaves as a cosmological constant throughout the history of the Universe, irrespective of the background evolution” had been mentioned by Beltran Jimenez and Antonio [23]. The massive field ϕ_I has a potential: it is a gradient of the scalar $\phi_{I;L}^L$ obeying (31).

2.3.2. Einstein-Aether Models. Vector field ϕ_K considered as the gradient of a scalar field Φ , ($\phi_K = \partial\Phi/\partial x^K$), was used in a scalar variant of the Einstein-aether theory; see a recent paper by Haghani et al. [24], providing a brief comprehensive review of the topic. Einstein-aether theories [25, 26] consider phase transitions with spontaneous violation of Lorentz symmetry by a vector field whose nonzero vacuum expectation

value plays the role of the order parameter. “Einstein-aether” is a kind of bumblebee models oriented mainly on time-like vector fields. It is not clear how to associate a time-like vector field with a massive quantum particle of ordinary matter, because there is no reference frame where such a particle could be at rest. The word “aether” in a title reflects the situation that a time-like vector field should correspond to something different from the ordinary matter.

A typical action in the Einstein-aether theory [27–30] is a particular case of (5) with $J^{IKLM} = 0$ and

$$K^{IKLM} = c_0 g^{IK} g^{LM} + c_1 g^{IM} g^{KL} + c_2 g^{IL} g^{KM} + c_3 \phi^I \phi^K g^{LM}. \quad (35)$$

In [24] the potential $V(\phi_I \phi^I)$ is taken as $V = \lambda(\phi_I \phi^I \pm 1)$, where λ is a Langrange-Euler multiplier. The sign being chosen to enforce the vector field ϕ_I to be time-like. Another commonly used example for the potential is a smooth quadratic function $V = (1/2)\lambda(\phi_I \phi^I \pm 1)^2$.

If the symmetry breaking vector ϕ_I is chosen as a gradient of a scalar, $\phi_I = \partial\Phi/\partial x^I$, then it is possible to consider the potential V as a function of the scalar Φ only, $V = V(\Phi)$. In this case a Lagrangian contains not only Φ and $\partial\Phi/\partial x^I$, but also second derivatives of Φ . From my point of view, a vector field approach is more convenient than the scalar one. The equations become more simple, while their solutions are more general. Though, in the particular case $a \neq 0$, $b = c = 0$ the vector ϕ_I is a gradient of a scalar (see (33)), this scalar is a covariant divergence $\phi_{;L}^L$. The Lagrangian contains only ϕ_I and $\phi_{;K}^I$.

In the state of broken symmetry there are 4 constants (c_0, c_1, c_2, c_3) in (35) instead of 3 constants (a, b, c) in (3). If a vector field is small compared to its vacuum expectation value and $V'_0 \equiv (dV(\phi^K \phi_K)/d(\phi^K \phi_K))|_{\phi^K \phi_K=0} \neq 0$, then $V(\phi^K \phi_K) = V(0) + V'_0 \phi^K \phi_K$ and the second and higher derivatives of $V(\phi^K \phi_K)$ can be omitted. While the field is small, the symmetry remains unbroken, and there is no need in the fourth term in (35). On the contrary, the full scale bumblebee models, including the Einstein-aether theory, provide a solid basis for analyzing phase transitions with spontaneous symmetry breaking in a strongly compressed state.

2.3.3. Nongauge Longitudinal Vector Field in Plane Geometry.

In the plain centrally symmetric metric

$$ds^2 = dx^{02} - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (36)$$

$$g = -r^4 \sin^2 \theta$$

$g^{rr} = -1$, and (31) for a static longitudinal vector field, depending only on the distance r from the center, is

$$\frac{1}{r^2} \frac{d}{dr} r^2 \frac{d\phi_{;L}^L}{dr} = \frac{V'_0}{a} \phi_{;L}^L. \quad (37)$$

The center $r = 0$ is a singular point of the spherical coordinate system. Like a pole on the globe, the singularity

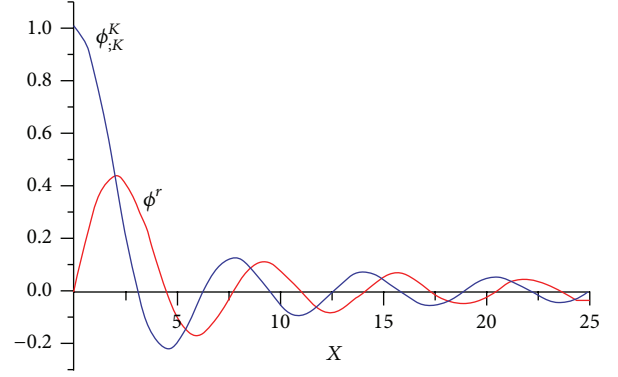


FIGURE 2: $\phi^r \sim (1/x)((1/x) \sin x - \cos x)$ and $\phi_{;M}^M \sim (1/x) \sin x$.

of a coordinate system has nothing to do with physical properties of matter. The finite, regular at $r \rightarrow 0$, solution of (37)

$$\phi_{;K}^K = \phi'_0 \frac{\sin(mr)}{mr}, \quad m = \sqrt{-\frac{V'_0}{a}} \quad (38)$$

(blue curve in Figure 2) exists if

$$\frac{V'_0}{a} < 0. \quad (39)$$

Restriction (39) is a necessary condition of regularity for a space-like vector field.

According to (33) the nonzero component of the vector ϕ^I is ϕ^r :

$$\phi^r = \frac{\phi'_0}{mr} \left[\frac{1}{mr} \sin(mr) - \cos(mr) \right]. \quad (40)$$

Radial dependences of $\phi_{;K}^K$ and ϕ^r are shown in Figure 2. The field is space-like and longitudinal: it is directed along and depends upon the same coordinate r .

Boundary conditions for (37), separating the regular solution at $r \rightarrow 0$, are

$$\phi_{;L}^L(0) \equiv \phi'_0, \quad \left. \frac{d\phi^r}{dr} \right|_{r=0} = \frac{1}{3} \phi'_0. \quad (41)$$

(37) is a linear uniform equation, and the constant of integration ϕ'_0 remains arbitrary.

In plain geometry $g^{00} = 1$, and in the case of a longitudinal time-like vector field the Klein-Gordon equation (31) is simply

$$\frac{d^2 \phi_{;L}^L}{(dx^0)^2} = -\frac{V'_0}{a} \phi_{;L}^L. \quad (42)$$

Its solution $\phi_{;L}^L = \phi'_0 \sin(\sqrt{V'_0/a}(x^0 - x_0^0))$ is finite if

$$\frac{V'_0}{a} > 0. \quad (43)$$

ϕ'_0 and x_0^0 are constants of integration.

Inequality (43) is a necessary condition of regularity for a time-like massive vector field. It is just the opposite to the one for a space-like massive field (39).

In plain space-time conditions of regularity determine the sign of the ratio V'_0/a , which appears different for space-like and time-like fields. In plain space-time the specific sign of a is not restricted by the requirement of regularity. However, there is no reason why the sign of a should be different for space-like and time-like vector fields. Actually it turns out that the sign of a ,

$$a < 0, \quad (44)$$

is restricted by the requirement of regularity in a curved space-time see Section 4.2 below. Then, as it follows from (39), (43), and (44), the regular solutions exist, if V'_0 is positive for a space-like field and negative for a time-like one.

2.4. Energy-Momentum Tensor. Using the identity

$$\delta g_{IK} = -g_{KM}g_{IN}\delta g^{NM}, \quad (45)$$

the energy-momentum tensor can be expressed as

$$T_{IK} = \frac{2}{\sqrt{-g}} \times \left[\frac{\partial \sqrt{-g} L}{\partial g^{IK}} + g_{MI}g_{NK} \frac{\partial}{\partial x^L} \left(\sqrt{-g} \frac{\partial L}{\partial (\partial g_{MN}/\partial x^L)} \right) \right]. \quad (46)$$

It differs from (94.4) in [22], where the Lagrangian is a function of g^{IK} and $\partial g^{IK}/\partial x^L$. The form (46) is more convenient when g^{IK} and $\partial g_{IK}/\partial x^L$ are considered as independent variables in the Lagrangian (3). In view of

$$\frac{2}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial g^{IK}} = -g_{IK}, \quad (47)$$

in a locally geodesic system (where $\partial g^{IK}/\partial x^L = 0$) the energy-momentum tensor can be written as follows:

$$T_{IK} = -g_{IK}L + 2 \frac{\partial L}{\partial g^{IK}} + 2g_{MI}g_{NK} \left(\frac{\partial L}{\partial (\partial g_{MN}/\partial x^L)} \right)_{;L}. \quad (48)$$

It is worth mentioning that $\partial g^{IK}/\partial x^L$ should be set to zero after the variational differentiations $\partial L/\partial g^{IK}$ and $\partial L/\partial (\partial g_{MN}/\partial x^L)$ are done. In terms of symmetric and anti-symmetric tensors (11) we find

$$\frac{\partial L}{\partial g^{IK}} = 2 \left(AG_L^L G_{IK} + BG_K^L G_{IL} + CF_I^L F_{KL} \right) - V' \phi_I \phi_K, \quad (49)$$

$$\begin{aligned} \frac{\partial L}{\partial (\partial g_{MN}/\partial x^L)} = & -A \phi_{;P}^P \left(g^{LN} \phi^M + g^{LM} \phi^N - g^{NM} \phi^L \right) \\ & - B \left(G^{LN} \phi^M + G^{LM} \phi^N - G^{MN} \phi^L \right). \end{aligned} \quad (50)$$

The tensor (50) is presented in a symmetric form against the indexes M, N .

Substituting (49) and (50) into (48), we find the following covariant expression for the energy-momentum tensor:

$$\begin{aligned} T_{IK} = & -g_{IK}L + 2V' \phi_K \phi_I + 2Ag_{IK} \left(G_M^M \phi^L \right)_{;L} \\ & + 2B \left[\left(G_{IK} \phi^L \right)_{;L} - G_K^L F_{IL} - G_I^L F_{KL} \right] \\ & + 2C \left(2F_I^L F_{LK} - F_{K;L}^L \phi_I - F_{I;L}^L \phi_K \right). \end{aligned} \quad (51)$$

The vector field equations (21) were used to reduce T_{IK} to a rather simple form (51).

2.4.1. Checking the Zero of the Covariant Divergence $T_{I;K}^K=0$. The correctness of the energy-momentum tensor (51) is confirmed by demonstration that the covariant divergence

$$\begin{aligned} T_{I;K}^K = & -L_{;I} + 2 \left(V' \phi^K \phi_I \right)_{;K} + 2A \left(G_M^M \phi^L \right)_{;L;I} \\ & + 2B \left[\left(G_{IK} \phi^L \right)_{;L} - G^{LK} F_{IL} - G_I^L F_{LK} \right]_{;K} \\ & + 2C \left(2F_I^L F_{LK} - F_{L;K}^L \phi_I - F_{I;L}^L \phi^K \right)_{;K} \end{aligned} \quad (52)$$

is zero [31].

Using the vector field equations (21), $T_{I;K}^K$ can be presented as

$$T_{I;K}^K = Aa_I + Bb_I + Cc_I. \quad (53)$$

The coefficients A, B , and C are arbitrary constants. However, it does not mean that the three vectors a_I, b_I , and c_I in (53) are zeros separately. These vectors are reduced to a similar form (see [31] for details):

$$\begin{aligned} a_I = & 2 \left(\phi_{;K;I}^K \phi_L - \phi_{;K;L}^K \phi_I \right)^{;L}, \\ b_I = & 2 \left(G_{I;K}^K \phi_L - G_{L;K}^K \phi_I \right)^{;L}, \\ c_I = & 2 \left(F_{L;K}^K \phi_I - F_{I;K}^K \phi_L \right)^{;L}. \end{aligned} \quad (54)$$

The covariant divergence of the energy-momentum tensor (53) with a_I, b_I , and c_I , given by (54) is evidently zero due to the vector field equations (21).

The covariant field equations (20) and the energy-momentum tensor (51) describe the behavior of vector fields in the background of any arbitrary given metric g_{IK} [31]. If the back reaction of the field on the curvature of space-time is essential, then the metric obeys the Einstein equations

$$R_{IK} - \frac{1}{2} g_{IK} R + \Lambda g_{IK} = \kappa T_{IK}, \quad (55)$$

with (51) added to T_{IK} . Here Λ and κ are the cosmological and gravitational constants, respectively. With account of back reaction the field equations (20) are not independent. They follow from the Einstein equations (55) with T_{IK} (51) due to the Bianchi identities.

2.4.2. Ordinary Matter and Dark Sector. The ordinary matter enters the Einstein equations via the well known energy-momentum tensor of macroscopic objects. Consider

$$T_{\text{om } IK} = (\varepsilon + p) u_I u_K - p g_{IK}. \quad (56)$$

The energy ε , pressure p , and, generally speaking, temperature T of the ordinary matter obey the equation of state. So far there is no evidence of any direct interaction between dark and ordinary matter other than via gravitation. The gravitational interaction is described by Einstein equations (55) with

$$T_{IK} = T_{\text{dark } IK} + T_{\text{om } IK}. \quad (57)$$

The general expression (51) for the energy-momentum tensor of a vector field describes (but is not limited to) the vector particles, which are also the ordinary matter. In order to describe the dark sector via vector fields it is reasonable to separate from T_{IK} (51) the part $T_{\text{dark } IK}$ which does not relate to the ordinary matter.

The Proca equations (25) are associated with a spin-1 particle—a quantum of the ordinary matter. The particular choice of the parameters $b = -c$ ensures the Lorentz condition (29), which allows avoiding “the difficulties of negative contribution to the energy” [32]. However, in curved space-time the energy is not a scalar, and its sign is not invariant against the arbitrary coordinate transformations. As a result of the Lorentz gauge restriction the terms $\sim a$ drop out from the Lagrangian (3), from the vector field equations (20) and from the energy-momentum tensor (51).

To avoid the double contribution of particles of the ordinary matter, it is reasonable to consider $a \neq 0$ and set $b = c = 0$. As simple a Lagrangian as possible

$$L_{\text{dark}} = a(\phi_{;M}^M)^2 - V(\phi^L \phi_L) \quad (58)$$

turns out to be an adequate tool for macroscopic description of the dark sector. Accordingly, (20) and the energy-momentum tensor (51) reduce to

$$a \frac{\partial \phi_{;M}^M}{\partial x^I} = -V' \phi_I, \quad (59)$$

$$T_{\text{dark } IK} = g_{IK} \left[a(\phi_{;M}^M)^2 + V \right] + 2V' (\phi_I \phi_K - g_{IK} \phi^M \phi_M). \quad (60)$$

Gravity is currently considered the basic interaction of cosmic objects in the scale of galaxies and larger. The three types of vector fields (massless, massive space-like, and massive time-like) can be of different physical nature.

The energy-momentum tensor (60) of a zero-mass ($V' = 0$) vector field reduces to

$$T_{(0)IK} = g_{IK} (a\phi_0'^2 + V_{(0)}). \quad (61)$$

$V_{(0)}$ is the constant value of the potential of the massless field. $T_{(0)IK}$ acts in the Einstein equations (55) as a simple addition to the cosmological constant, changing Λ to

$$\tilde{\Lambda} = \Lambda - \kappa (a\phi_0'^2 + V_{(0)}). \quad (62)$$

ϕ_0' is the constant divergence of the zero-mass vector field (34).

It is convenient to consider the values $V(0)$ of the potentials of massive fields as already included into $\tilde{\Lambda}$:

$$V_0 = V_{(0)} + V_{(s)}(0) + V_{(t)}(0), \quad (63)$$

so that the power series of the potentials $V(\phi^M \phi_M)$ of massive fields start with $V_0' \phi^M \phi_M$:

$$V(\phi^M \phi_M) = V_0' \phi^M \phi_M + O((\phi^M \phi_M)^2). \quad (64)$$

In the case of weak vector fields the second and higher derivatives of the potentials $V(\phi_L \phi^L)$ can be neglected, and the energy-momentum tensor of a massive field is

$$T_{\text{dark } I}^K = a(\phi_{;M}^M)^2 \delta_I^K + V_0' (2\phi_I \phi^K - \delta_I^K \phi^M \phi_M), \quad (65)$$

$$V_0' \equiv \left. \frac{dV(\phi^M \phi_M)}{d(\phi^M \phi_M)} \right|_{\phi^M \phi_M=0}.$$

In general, it could be necessary to consider both independent vectors— $\phi_{(s)}^M$ for a space-like and $\phi_{(t)}^M$ for a time-like—massive fields with different potentials $V_{(s)}(\phi_{(s)}^M \phi_{(s)M})$ and $V_{(t)}(\phi_{(t)}^M \phi_{(t)M})$. As far as the energy-momentum tensor of a massless field (dark energy) is included into $\tilde{\Lambda}$ (62), the remaining energy-momentum tensor $T_{\text{dark } IK}$ of massive fields is the sum of two tensors

$$T_{\text{dark } IK} = T_{(s)IK} + T_{(t)IK}, \quad (66)$$

corresponding to $\phi_{(s)}^M$ and $\phi_{(t)}^M$, respectively.

In the scale of a galaxy (~ 10 kpc) the space-like vector field ($\phi_{(s)}^L \phi_{(s)L} < 0$) dominates. It is responsible for the plateau in galaxy rotation curves; see Section 3. The time-like field ($\phi_{(t)}^L \phi_{(t)L} > 0$) dominates at the scales much larger than the distance between the galaxies, where the Universe can be considered as uniform and isotropic. The time-like field displays the repulsive elasticity. Together with the zero-mass vector field (dark energy) and the ordinary matter it gives rise to a variety of possible regular scenarios of evolution of the Universe and rules out the problem of fine tuning; see Section 4. In particular, the singular Big Bang turns into a regular inflation-like bounce with accelerated expansion at late times.

It would be interesting to trace how the additional attraction of the space-like dark matter, dominating in the galaxy scale, transforms into the elastic repulsion of the time-like dark matter, dominating in the scale of the Universe. Both types of massive fields $\phi_{(s)}^M$ and $\phi_{(t)}^M$ are supposed to be active in the intermediate region, so the energy-momentum tensor of the dark sector should be the sum (66).

The study of the structure of the Universe in the intermediate range (Mpc to hundred Mpc) had been initiated in the pioneering papers by Zel'dovich [33]. Continuous research by his followers shows that dark energy and dark matter significantly affect the structural dynamics of galaxies and clusters in this range; see a review by Gurbatov et al. [34]. Utilizing the energy-momentum tensor ((66), (60)) in the analysis of the large scale structure of the Universe would allow avoiding unnecessary model assumptions.

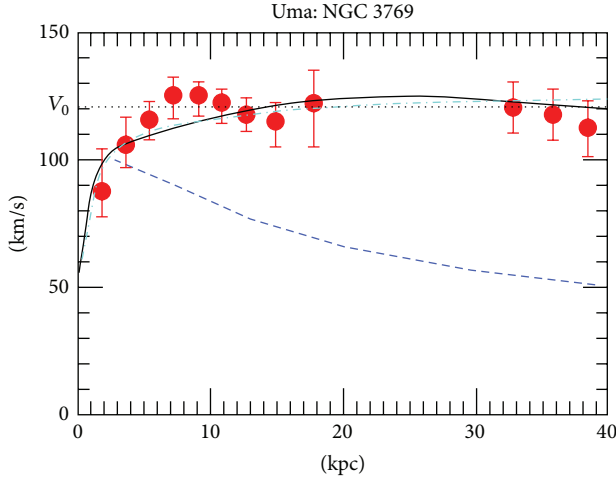


FIGURE 3: Rotation curve $V(r)$ of a spiral galaxy in the Ursa Major cluster (UMA). In the New General Catalogue of Nebulae and Clusters of Stars (NGC) [46] its number is 3769. The vertical axis is the velocity V in km/sec, and the horizontal axis is the distance r from the center of the galaxy in kpc. Dots with error bars are observations. Solid curve is fitting by empirical MOND (modified Newton's dynamics) [37, 41]. Dashed line is Newton's $V(r) \sim 1/\sqrt{r}$.

3. Galaxy Rotation Curves

The description of the dark sector via vector fields allows deriving the galaxy rotation curves directly from the first principles within the minimal Einstein's general relativity [6].

The velocity V of a star, orbiting around the center of a galaxy and satisfying the balance between the centrifugal V^2/r and centripetal $GM(r)/r^2$ accelerations, should decrease with the radius r of its orbit as $V(r) \sim 1/\sqrt{r}$ at $r \rightarrow \infty$. However, the numerous observed dependences $V(r)$, named galaxy rotation curves, practically remain constant at far periphery of a galaxy. An example is presented in Figure 3. It was a fundamental problem, because the general relativity reduces to Newton's theory in the limit of nonrelativistic velocities and weak gravitation.

3.1. Benchmarks in History. The “galaxy rotation curves” problem appeared after Oort discovered the galactic halo, a group of stars orbiting the Milky Way outside the main disk [1]. In 1933, Zwicky [2] postulated “missing mass” to account for the orbital velocities of galaxies in clusters. Persistent investigations by Rubin and colleagues [35, 36] in seventies practically dispelled the skepticism about the existence of dark matter on the periphery of the galaxies.

Among numerous attempts to solve the problem of galaxy rotation curves the most discussed one is the empirical explanation named MOND (modified Newtonian dynamics), proposed by Milgrom back in 1983 [37, 38]. For a relativistic justification of MOND Bekenstein [39], Sanders [40], and Brownstein and Moffat [41–43] introduce additional scalar, vector, or tensor fields. Though these (and many others) empirical improvements of MOND are able to fit a large number of samples for about a hundred galaxies, the concern still remains. So far we had neither self-consistent description

of the dark sector as a whole nor direct derivation of MOND from the first principles within Einstein's general relativity. The survey [44] by Famaey and McGaugh and a recent review article [45] by Bernabei et al. reflect the current state of research and contain the comprehensive lists of references.

3.2. Rotation Curve Driven by a Massive Vector Field. Applying general relativity to the galaxy rotation problem it is reasonable to consider a static centrally symmetric metric

$$ds^2 = g_{IK} dx^I dx^K = e^{\nu(r)} (dx^0)^2 - e^{\lambda(r)} dr^2 - r^2 d\Omega^2, \quad (67)$$

with two functions $\nu(r)$ and $\lambda(r)$ depending on only one coordinate—circular radius r . Real distribution of stars and planets in a galaxy is neither static nor centrally symmetric. However this simplification facilitates analyzing the problem and allows displaying the main results analytically. If a galaxy is concentrated around a supermassive black hole, the deviation from the central symmetry caused by the peripheral stars is small.

In the background of the centrally symmetric metric (67) the vector ϕ^I is longitudinal. In accordance with the field equation (59) its only nonzero component ϕ^r depends on r . The covariant divergence is

$$\phi_{;M}^M = \frac{1}{\sqrt{-g}} \frac{\partial (\sqrt{-g} \phi^M)}{\partial x^M} = \frac{d\phi^r}{dr} + \left(\frac{2}{r} + \frac{\lambda' + \nu'}{2} \right) \phi^r. \quad (68)$$

In the “dust matter” approximation $p = 0$ and the only nonzero component of the energy-momentum tensor (56) is $T_{om\,00} = \varepsilon g_{00}$. Whatever the distribution of the ordinary matter $\varepsilon(r)$ is, the covariant divergence $T_{om\,I;K}^K$ is automatically zero. In the dust matter approximation the curving of space-time by ordinary matter is taken into account, but the back reaction of the gravitational field on the distribution of matter is ignored. If $p = 0$ the energy $\varepsilon(r)$ is considered as a given function.

In the space-time with metric (67) the energy-momentum tensor (65) of a weak space-like longitudinal vector field is

$$T_{\text{dark}\,I}^K = \delta_I^K \begin{cases} a(\phi_{;M}^M)^2 - V_0' e^\lambda \phi^{r2}, & I = r, \\ a(\phi_{;M}^M)^2 + V_0' e^\lambda \phi^{r2}, & I \neq r. \end{cases} \quad (69)$$

In the scale of galaxies the role of expansion of the Universe as a whole is negligible, and one can omit $\tilde{\Lambda}$ (62) in the Einstein

equations. In the dust matter approximation the Einstein equations are

$$\begin{aligned} -e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2} &= \kappa T_0^0 \\ &= \kappa \left[a(\phi_{;M}^M)^2 + V_0' e^\lambda (\phi^r)^2 + \varepsilon \right], \end{aligned} \quad (70)$$

$$\begin{aligned} -e^{-\lambda} \left(\frac{\nu'}{r} + \frac{1}{r^2} \right) + \frac{1}{r^2} &= \kappa T_r^r \\ &= \kappa \left[a(\phi_{;M}^M)^2 - V_0' e^\lambda (\phi^r)^2 \right], \end{aligned} \quad (71)$$

$$\begin{aligned} -\frac{1}{2} e^{-\lambda} \left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu' \lambda'}{2} \right) \\ = \kappa \left[a(\phi_{;M}^M)^2 + V_0' e^\lambda (\phi^r)^2 \right], \quad I, K \neq 0, r. \end{aligned} \quad (72)$$

See [22], page 382 for the derivation of the left-hand sides. The prime ' stands for d/dr , except $V' = dV(\phi_M \phi^M)/d(\phi_M \phi^M)$. Among the four equations (59), ((70)–(72)) for the unknowns ϕ^r , λ , and ν any three are independent.

Extracting (71) from (70) we get a relation

$$\nu' + \lambda' = \kappa r e^\lambda \left[2e^\lambda (\phi^r)^2 V_0' + \varepsilon \right]. \quad (73)$$

With account of (68) and (73) the vector field equation (59) takes the form

$$\left[(\phi^r)' + \left(\frac{2}{r} + \kappa r e^{2\lambda} (\phi^r)^2 V_0' + \frac{1}{2} \kappa r e^\lambda \varepsilon \right) \phi^r \right]' = \frac{V'}{a} e^\lambda \phi^r. \quad (74)$$

Equations (73) and (74) are derived with no assumptions concerning the strength of the gravitational field.

Omitting the second and higher derivatives of the potential $V(\phi_M \phi^M)$, in the dust matter approximation ($p = 0$), we get from (70) the following expression for ν' :

$$\nu' = -a \kappa r e^\lambda \left[m^2 e^\lambda (\phi^r)^2 + (\phi_{;M}^M)^2 \right] + \frac{e^\lambda - 1}{r}. \quad (75)$$

Here $m^2 = -V_0'/a$ is the squared mass of the vector field. The sign minus is in conjunction with the necessary condition of regularity (39) for a space-like vector field.

In a static centrally symmetric gravitational field ν' determines the centripetal acceleration of a particle ([22], page 323). Without dark matter $\phi^r = 0$ (75) gives Newton's attractive potential far from the center:

$$\varphi_N(r) = \frac{1}{2} c^2 \nu(r) \sim -r^{-1}, \quad r \rightarrow \infty. \quad (76)$$

The first term in the r.h.s. of (75) appears due to the dark matter.

The constant a in the Lagrangian (58) is considered the same for any vector, be it space-like or time-like, massive or massless. Regularity condition for a space-like vector (39)

does not determine the sign of a . The negative sign $a < 0$ (107) is dictated by the self-consistent requirement of regularity for massless and massive time-like vectors acting together; see Section 4.2 below. In view of (39) and (107) the requirement of regularity for a space-like vector field is satisfied if V_0' is positive:

$$V_{0 \text{ space-like}}' > 0. \quad (77)$$

The first term with square brackets in (75) with $a < 0$ is positive, and both terms in the r.h.s. are of the same sign, so the presence of dark matter increases the attraction to the center.

The curvature of space-time caused by a galaxy is small. In the linear approximation the influence of dark and ordinary matter can be separated from one another. For $\lambda \ll 1$ (75) reduces to

$$\nu' = \kappa |a| r \left[m^2 (\phi^r)^2 + (\phi_{;M}^M)^2 \right] + \frac{\lambda}{r}, \quad (78)$$

where the first term does not contain ε . However, the contribution of dark matter comes from both additives. The vector field equation (74) and the Einstein equation (70) at $\lambda \ll 1$ are simplified:

$$(\phi^r)'' + \left\{ \left[\frac{2}{r} + \kappa r \left(|a| m^2 (\phi^r)^2 + \frac{1}{2} \varepsilon \right) \right] \phi^r \right\}' = -m^2 \phi^r, \quad (79)$$

$$\lambda' + \frac{\lambda}{r} = \kappa r \left[|a| \left(m^2 \phi^{r2} - \phi_{;M}^{M2} \right) + \varepsilon \right]. \quad (80)$$

The boundary conditions for these equations,

$$\phi^r = \frac{1}{3} \phi_0' r, \quad \lambda = \frac{1}{3} \kappa (\varepsilon_0 - |a| \phi_0'^2) r^2, \quad r \rightarrow 0, \quad (81)$$

are determined by the requirement of regularity in the center. Here $\varepsilon_0 = \varepsilon(0)$, $\phi_0' = \phi_{;M}^M(0)$.

The term $(1/2) \kappa r \varepsilon$ in (79) reflects the interaction of dark and ordinary matter via gravitation. If the curvature of space-time caused by the ordinary matter is small, this term is negligible compared to $2/r$. The nonlinear term $\kappa m^2 r (\phi^r)^2$ is small compared to $2/r$ at $r \rightarrow 0$, but at $r \rightarrow \infty$, despite being small, it decreases a little bit quicker than $2/r$. Neglecting both nonlinear terms in square brackets, the field equation (79) reduces to the one in the plane space-time. $\phi_{;M}^M$ obeys the Klein-Gordon equation (37). The regular solution is ((38), (40)):

$$\phi_{;M}^M = \phi_0' \frac{\sin mr}{mr}, \quad (82)$$

$$\phi^r = \frac{\phi_0'}{m^3 r^2} (\sin mr - mr \cos mr),$$

where $\phi_0' \equiv \phi_{;M}^M(0)$.

Substitution of (82) into (78) results in

$$\nu'(r) = \frac{\kappa |a| (\phi_0')^2}{m^2 r} f(mr) + \frac{\lambda}{r}, \quad \lambda \ll 1, \quad (83)$$

where

$$f(x) = 1 - \frac{\sin 2x}{x} + \frac{\sin^2 x}{x^2}. \quad (84)$$

The balance of the centripetal $c^2 v' / 2$ and centrifugal V^2 / r accelerations determines the velocity V of a rotating object as a function of the radius r of its orbit:

$$V(r) = \sqrt{V_{\text{pl}}^2 f(mr) + \frac{c^2}{2} \lambda(r)}, \quad (85)$$

$$V_{\text{pl}} = c \sqrt{\frac{\kappa |a| \phi'_0}{2m}}. \quad (86)$$

c is the velocity of light. Far from the center $\lambda(r)$ decreases as $1/r$, while $f(mr) \rightarrow 1$. The dependence $V(r)$ (85) turns at $r \geq m^{-1}$ from a linear to a plateau (86) with damping oscillations. The plateau appears entirely due to the vector field. At the same time the vector field contributes to $\lambda(r)$ as well. Substituting (82) into (80) we get a regular at $r \rightarrow 0$ solution for $\lambda(r)$:

$$\lambda(r) = 2 \left(\frac{V_{\text{pl}}}{c} \right)^2 \left(\frac{\sin 2mr}{2mr} - \frac{\sin^2 mr}{m^2 r^2} \right) + \frac{\kappa}{r} \int_0^r \varepsilon(r) r^2 dr, \quad (87)$$

$\lambda \ll 1,$

where the last term in (87) gives Newton's potential. Substituting (87) into (85) we finally have

$$V(r) = \sqrt{V_{\text{pl}}^2 \left(1 - \frac{\sin 2mr}{2mr} \right) + c^2 \frac{\kappa}{2r} \int_0^r \varepsilon(r) r^2 dr}, \quad \lambda \ll 1. \quad (88)$$

Without dark matter ($\phi'_0 = 0$) (88) would give Newton's $V(r) \sim 1/\sqrt{r}$ at $r \rightarrow \infty$. In the presence of dark matter ($\phi'_0 \neq 0$) the velocity of rotation $V(r)$ tends to V_{pl} at $r \rightarrow \infty$ with damping oscillations. If the contribution of stars and planets to the total mass of a galaxy is small compared to the mass of the black hole in the center, then outside the black hole $r \gg r_{\text{Sch}}$

$$\kappa \int_0^r \varepsilon(r) r^2 dr = r_{\text{Sch}}, \quad (89)$$

r_{Sch} is the Schwarzschild radius of a black hole. Outside a black hole $r \gg r_{\text{Sch}}$, and the condition $\lambda \ll 1$ is fulfilled for all galaxies. Far outside the Schwarzschild radius the velocity of rotation around a black hole is

$$V(r) = \sqrt{V_{\text{pl}}^2 \left(1 - \frac{\sin 2mr}{2mr} \right) + \frac{c^2}{2} \frac{r_{\text{Sch}}}{r}}, \quad r \gg r_{\text{Sch}}. \quad (90)$$

The deviation from Newton's law due to the dark matter takes place at $r \geq r_{\text{Sch}} (c^2/V_{\text{pl}}^2)$. At $r \gg r_{\text{Sch}} (c^2/V_{\text{pl}}^2)$ the curve of rotation around a black hole is a universal function. In dimensionless units there are no parameters; see Figure 4.

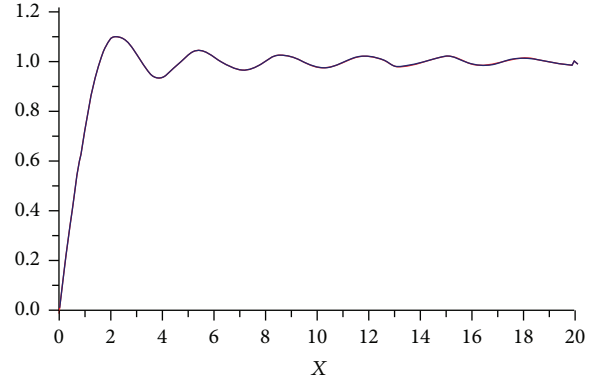


FIGURE 4: Universal function $\sqrt{1 - (\sin 2x/2x)}$ (90).

While the contribution of dark matter to the rotation curve is described by the universal function shown in Figure 4, the distribution of stars and planets, circulating around a black hole, differs from one galaxy to another. The rotation curves of different galaxies look different. However, the deviation from Newton's $V(r) \sim 1/\sqrt{r}$ on the periphery of a galaxy is their common feature. Dark matter manifests itself most clearly in the periphery of galaxies. Therefore, in order to compare with (90), among the numerous available rotation curves, it is natural to choose those having the stars outside the main disk.

Fitting the rotation curves of two such galaxies via the universal function (90) is presented in Figure 5. These two spiral galaxies are located in the Ursa Major cluster (UMa). Their numbers are from "The New General Catalogue of Nebulae and Clusters of Stars" (abbreviated as NGC). It is a catalogue of deep-sky objects in astronomy compiled by Dreyer in 1888 [46], as a new version of John Herschel's Catalogue of Nebulae and Clusters of Stars.

Agreement with the oscillations is hardly accidental. Actually, damping oscillations of a rotation curve in the far periphery of a galaxy can be considered as a "signature of dark matter." I would strongly recommend this observational test as confirming the existence of dark matter, along with its adequate description by a longitudinal nongauge vector field.

The fact of small deviations from the universal curve indicates that the main contribution comes from the dark matter. It is in agreement with the modern concept that there is only some 5% of ordinary matter in the Universe, while the amount of dark matter is about five times as much; see Figure 1. The fitting in Figure 5 also testifies that the deviation from the central symmetry by a disk of circulating stars and planets is small. It confirms the existence of a heavy object, like a black hole, in the center of a galaxy. In general, the observed deviations of rotation curves from the universal curve could clarify the average distribution of the ordinary matter within other galaxies.

The contribution of dark matter to a rotation curve (90) is expressed via two observable parameters: the limiting plateau value V_{pl} and mass m . They allow restoring the value of the parameter $\phi'_0 \equiv \phi'_{i,M}(0)$ at $r \rightarrow 0$ in the boundary conditions (81): $\phi'_0 = \sqrt{2/\kappa|a|m} V_{\text{pl}}$ (86). As far as there is no evidence of

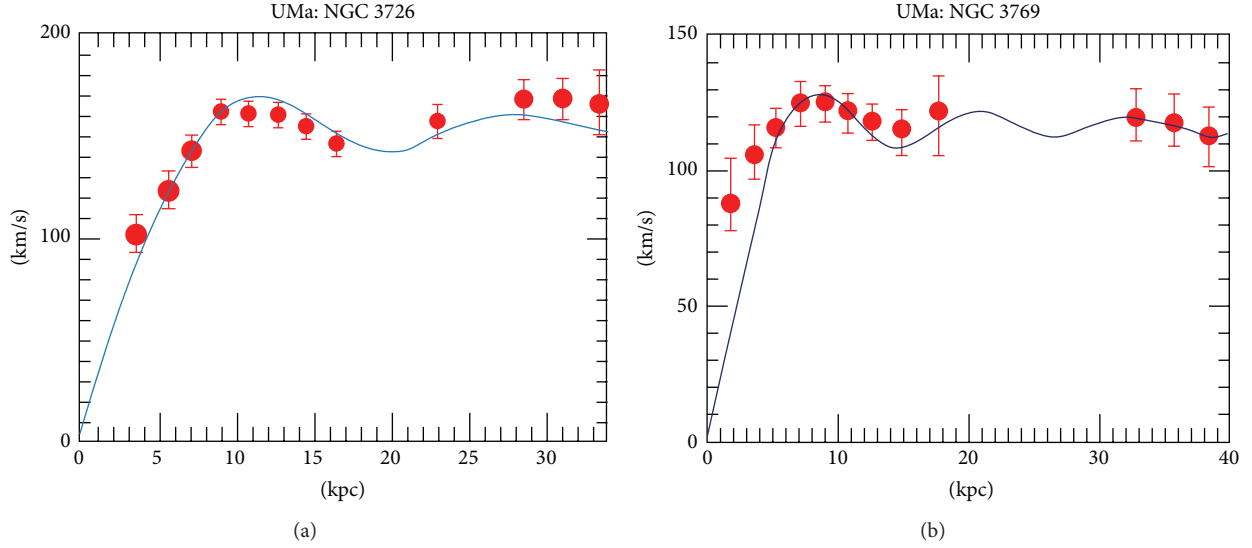


FIGURE 5: Rotation curves of two spiral galaxies in the Ursa Major cluster (UMa). Abbreviation NGC stands for “The New General Catalogue of Nebulae and Clusters of Stars.” The vertical axis is the velocity V in km/sec, and the horizontal axis is the distance r from the center of a galaxy in kpc. Dots with error bars are observations. Solid curve is fitting by $V(r) = V_{\text{pl}} \sqrt{1 - (\sin 2mr/2mr)} \quad (90)$.

any direct interaction of dark and ordinary matter, the origin of specific values ϕ'_0 and m of a particular galaxy depends on what happens in the center. The values V_{pl} and m differ from one galaxy to another. It looks like for each galaxy these values are driven by a heavy object (may be a black hole, may be a neutron star) located in the center and, by the way, supporting the central symmetry of the gravitational field.

Dark matter, described by a vector field with the Lagrangian (58), actually justifies the empirical Milgrom’s hypothesis of MOND—the modified Newton’s dynamics [37]. Newton’s dynamics really get modified by the vector field so that the rotation curve flattens out at the far periphery of a galaxy. This is because the perturbation of the gravitational field due to a massive longitudinal vector field decreases slower than the perturbation caused by the ordinary matter. The empirical Milgrom’s hypothesis of MOND was a real breakthrough in the 80s. Naturally, basing only on the intuitive arguments, it was scarcely possible to guess that the transition to a plateau is accompanied by damping oscillations.

However, the question of the physical origin of dark matter remains open. In other words, what makes $\phi'_0 = \phi'_{;K}(0)$ different from zero? Solutions of the linearized Einstein equations do not answer this question. Within the approximation of weak fields ϕ'_0 and m remain free parameters. The energy $\varepsilon(r)$ is an arbitrary function in the dust matter approximation, so the parameters ϕ'_0 and m of dark matter are in no way connected with the ordinary matter.

According to the empirical MOND prediction the limiting plateau value V_{plMOND} is connected with the mass M of a galaxy:

$$V_{\text{plMOND}} = (\kappa M a_0)^{1/4}. \quad (91)$$

Milgrom postulates the existence of a very small acceleration a_0 , and that at $a \lesssim a_0$ the violation of Newton’s law takes place [37]. To answer the question “what the empirical MOND relation (91) should be replaced by?” one has to find out the reason why the divergence $\phi'_{;K}$ is not zero at $r = 0$. The gravitational field of a collapsing black hole is neither static nor weak. In the close vicinity of a black hole the velocities of circulating stars and planets are relativistic. It is necessary to get a self-consistent solution of the nonlinear Einstein equations. At $r \lesssim r_{\text{Sch}}$ strong interaction via gravitation should affect the dynamic balance of the ordinary and dark matter. For nonlinear equations the requirement of regularity can impose an additional restriction on the parameters ε_0 and ϕ'_0 in the boundary conditions (81). It is very likely that it will fix the connection between ε_0 and ϕ'_0 , providing the dependence of the limiting plateau value V_{pl} on the mass M of a galaxy.

Actually it is a revision of equilibrium [47] and collapse [48] of supermassive bodies with the dark matter taken into account. The ordinary matter could be still considered as a degenerate relativistic Fermi gas (see [49], problem 3 in the end of the paragraph 61, page 207). The dark matter should be included into Einstein equations via the energy-momentum tensor (66). Considering a collapsing system, there is no reason to ignore a time-like vector. It is possible that the repulsive ability of a time-like vector field can dynamically balance the collapse. In this case there will be a regular solution of Einstein’s equations describing the internal structure of a black hole without a singularity in the center. This is a worthy task for future.

One can trace two main trends in the literature in trying to unravel the puzzle of a plateau in the rotation curves of galaxies—to “improve” general relativity and to compose

a mixture of fields able to fit the observations without a “mysterious dark matter.”

For instance, Sanders argues that “...the correct theory may well be one in which MOND reflects the influence of cosmology on local particle dynamics and arises only in a cosmological setting” and concludes: “It goes without saying that this theory is not general relativity, because in the context of general relativity local particle dynamics is immune to the influence of cosmology” [40]. Obviously, I do not share Sanders’ conclusion. I have presented above the complete derivation from the Einstein equations ((70)–(72)) to the galaxy rotation curve (85). The formulae ((88), (90)) are derived completely within the Einstein’s theory. For the time being, there is no need in any modifications of the general relativity to explain the observable plateau in galaxy rotation curves.

An example of opposing fields and dark matter is the Moffat’s attempt of applying a mixture of scalar, vector, and tensor fields in order “to explain the flat rotation curves of galaxies and cluster lensing without postulating exotic dark matter” [42, 43]. It is a question of terminology. In quantum physics there always is a quantum particle corresponding to the field describing a material substance. From my point of view, the fields are convenient mathematical instruments that we utilize to describe the physical phenomena, no matter how we name them.

According to observations the period of oscillations π/m (see Figure 5) is around 15 kpc. If in quantum mechanics it is the de Broglie wavelength $\lambda = \hbar/mc$, then the rest energy of a quantum particle, corresponding to the space-like vector field, should be $mc^2 \sim 10^{-27}$ eV. The lightest particles as candidates for the cosmological nonbaryonic dark matter are discussed by Khlopov [50] in connection with spontaneously symmetry breaking in phase transitions in the early Universe.

The theory predicts the oscillating features with no baryonic counterparts in the rotation curves of the outer regions of galaxies. As this would be the main observational signature of existence of the dark matter, I persistently recommend this observational test.

The “unprecedented constraints on the stellar and dark matter mass distribution within our Milky Way” are reported by Bovy and Rix [51]. Continuous progress in the accuracy of observations would be able to provide the values of the main parameters ϕ'_0 and m for Milky Way and other galaxies and clusters.

Having the energy-momentum tensor (60), it is worth considering a possible role of dark matter in the “pioneer anomaly” in the scale of the solar system. It appeared that a very small unexpected force caused an approximately constant additional acceleration of $(8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2$ directed towards the Sun for both spacecraft Pioneer 10 and Pioneer 11 [52, 53]. It is interesting to trace the dependence $\nu'(r)$ (78) along the two spacecraft hyperbolic orbits at distances between 20–70 astronomical units ($\text{AU} = 1.5 \cdot 10^{13} \text{ cm}$) from the Sun. Manifestation of dark matter on the periphery of the solar system would be a great surprise!

4. Regular Cosmology

From the standpoint of general relativity the matter curves the space-time, giving rise to mutual attraction between the bodies. However, according to modern observations, the Universe is expanding as a whole, despite the gravitational attraction between material objects. The expanding solution of Einstein’s equations due to the cosmological constant belongs to De Sitter [54, 55]. The expanding solutions of the Einstein equations without the cosmological constant (Friedman [56], Robertson [57, 58], and Walker [59] (FRW)) inevitably contained the singularity. The unknown origin of expansion of the Universe, containing only mutually attracting objects, was supposed to be hidden within the singularity. For a long time the singularity was considered a general property of the Universe. The singular point, referred to as “Big Bang,” is still widely recognized as the “date of birth” of the Universe.

Discovery of the accelerated expansion of the Universe [3, 4] shows that the source of acceleration continues to exist for a long time after the Big Bang. Naturally, the fact of accelerated expansion gave rise to the assumption that the physical vacuum is not just the absence of the ordinary matter. The existence of dark energy and dark matter, as the unknown source of the Universe’s expansion, is widely discussed in modern literature and Internet [60, 61].

The macroscopic approach to the theory of evolution of the Universe driven by vector fields plays the central role among numerous attempts to guess the riddle of accelerated expansion. It allows avoiding unnecessary model assumptions (like “ $f(R)$ ”, quintessence, phantom-like cosmologies, ..., see a review [62]) and remains in the classical frames of Einstein’s general relativity. Utilization of vector fields in general relativity shows undoubtable advantages in comparison with scalar fields and with multiplets of scalar fields. The equations appear to be more simple, while their solutions are more general. The solutions have additional parametric freedom, allowing forgetting the fine-tuning problem [63]. However, starting from the pioneer paper by Dolgov [64], people considered mostly gauge vector fields [65–69] in applications to the dark sector. The Lorentz gauge restriction allowed avoiding the difficulty of negative contribution to the energy. But at the same time it does not allow utilizing all the advantages of vector fields. In general relativity (in curved space-time) the energy is not a scalar, and its sign is not invariant against the arbitrary coordinate transformations. Considering the vector fields in general relativity, it is worth rejecting the gauge restriction, using instead a more weak condition of regularity. Step by step, people are now getting rid of the Lorentz gauge restrictions [70–73].

Today it is generally accepted that among the stuff of the Universe only 4.5% is the ordinary matter; see Figure 1 [12]. It is reasonable to analyze the role of vector fields in cosmology step by step. First step—dark energy only (zero-mass field): about 72%. Second step is adding massive vector field (dark matter, about 23%) into consideration. The final step is to include the ordinary matter, after the main role of vector fields is clarified.

According to observations the Universe expands, and its large scale structure remains homogeneous and isotropic. Consider the space-time with the metric

$$ds^2 = g_{IK} dx^I dx^K = (dx^0)^2 - e^{2F(x^0)} \sum_{I=1}^3 (dx^I)^2, \quad (92)$$

depending on only one time-like coordinate $x^0 = ct$ (In the derivations below the units with $c = 1$, and $\hbar = 1$ are used.). The metric tensor g_{IK} is diagonal. The uniform and isotropic expansion is characterized by the single metric function $F(x^0)$, and the rate of expansion is $dF/dx^0 \equiv F'$. The Ricci tensor is also diagonal:

$$\begin{aligned} R_{00} &= -3(F'^2 + F''), \\ R_{II} &= e^{2F} (F'' + 3F'^2), \quad I > 0. \end{aligned} \quad (93)$$

4.1. Massless Field as Dark Energy. The energy-momentum tensor (61) of a massless field acts in the Einstein equations (55) as a simple addition to the cosmological constant (62):

$$\begin{aligned} R_{IK} - \frac{1}{2} g_{IK} R + \tilde{\Lambda} g_{IK} &= 0, \\ \tilde{\Lambda} &= \Lambda - \kappa (a\phi_0'^2 + V_0). \end{aligned} \quad (94)$$

The contribution of the zero-mass field to the curvature of space-time remains constant in the process of the Universe evolution. The fact that “the gauge-fixing term exactly behaves as a cosmological constant throughout the history of the Universe, irrespective of the background evolution” had been mentioned by Beltran Jimenez and Antonio [23].

The metric function

$$F(x^0) = \pm \sqrt{-\frac{1}{3} \tilde{\Lambda}} (x^0 - x_0^0) \quad (95)$$

is the self-consistent regular solution of the Einstein equations (94), provided that

$$\tilde{\Lambda} < 0. \quad (96)$$

$F(x^0)$ is a linear function; x_0^0 is a constant of integration. The metric (92) with the metric function (95) is called de Sitter (or anti-de Sitter, depending on the sign definition of the Ricci tensor). It describes either expansion (sign +) or contraction (sign -) of the Universe at a constant rate; see Figure 6.

In the case of sign + the rate of expansion

$$F' = H = \sqrt{-\frac{\tilde{\Lambda}}{3}} \quad (97)$$

is called Hubble constant.

In general relativity the requirement of regularity for a massless field (96) replaces the artificially imposed Lorentz gauge restriction (29) allowing to avoid the negative energy problem [32].

The zero-mass vector field determines the constant rate of expansion. Available today properties of the so called dark

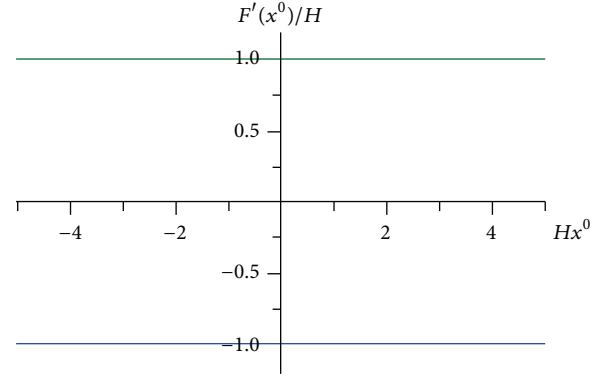


FIGURE 6: De Sitter solution $F'(x^0)/H = \pm 1$. Upper green horizontal line is expansion, and lower blue horizontal line is compression.

energy (presently unknown form of matter providing the major contribution to the uniform isotropic expansion of the Universe) can be described macroscopically by the zero-mass vector field with a simple Lagrangian

$$L = a(\phi_{;M}^M)^2 - V_0. \quad (98)$$

As long as the physical nature of vacuum is not known, the “geometrical” origin of the cosmological constant Λ and the “material” contribution to $\tilde{\Lambda}$ (62) by the zero-mass vector field can not be separated from one another. The combined action of the massless field and/or the cosmological constant is described by the single parameter—Hubble constant H (97).

4.2. Massless Field + Massive Field. Over the scales much larger than the distances between the galaxies the Universe is uniform and isotropic. In the scale of the whole Universe the massive vector field is time-like and longitudinal: the only nonzero component is ϕ_0 , and it depends upon the time coordinate x^0 .

The energy-momentum tensor (65) for the longitudinal massive time-like field is

$$\begin{aligned} T_{\text{dark } 0}^0 &= a(\phi_{;M}^M)^2 + V_0\phi_0^2, \\ T_{\text{dark } I}^K &= \delta_I^K [a(\phi_{;M}^M)^2 - V_0\phi_0^2], \quad I > 0. \end{aligned} \quad (99)$$

The massless field enters the Einstein equations

$$3F'^2 + \tilde{\Lambda} = \kappa a [(\phi_{;M}^M)^2 + m^2\phi_0^2], \quad (100)$$

$$2F'' + 3F'^2 + \tilde{\Lambda} = \kappa a [(\phi_{;M}^M)^2 - m^2\phi_0^2], \quad (101)$$

only via the cosmological constant $\tilde{\Lambda}$ (62). $\tilde{\Lambda}$ remains constant throughout the whole history of the Universe. The massive field is described by the function $\phi_0(x^0)$, which enters Einstein equations (100)-(101) directly [74]. In accordance with the necessary condition of regularity for a time-like massive vector field (43) $V_0'/a > 0$, and the squared mass

of the field is $m^2 = V'_0/a$. The applicability of (100)-(101) is restricted by the condition that the field ϕ_0 is small, so that the second and higher derivatives of the potential $V(\phi_L \phi^L)$ can be ignored.

The field equations (59) for a longitudinal time-like field reduce to the only one equation

$$(\phi'_0 + 3F'\phi_0)' + m^2\phi_0 = 0, \quad (102)$$

which is a consequence of the Einstein equations ((100), (101)).

4.2.1. Asymptotic Behavior at Large x^0 . As it is confirmed below, set (100)-(101) has regular solutions with the rate $F'(x^0)$ changing from $F'(-\infty) = -H$ in the past to $F'(\infty) = H$ in future. Far back in the past and in the late future the temporal evolution of the massive field is described by (102) with $F' = \mp H$. Its solution

$$\begin{aligned} \phi_0(x^0) &= C_+ e^{\lambda_+ x^0} + C_- e^{\lambda_- x^0}, \\ \lambda_{\pm} &= \begin{cases} \frac{3H}{2} \pm \sqrt{\left(\frac{3H}{2}\right)^2 - m^2}, & x^0 \rightarrow -\infty, \\ -\frac{3H}{2} \pm \sqrt{\left(\frac{3H}{2}\right)^2 - m^2}, & x^0 \rightarrow \infty, \end{cases} \end{aligned} \quad (103)$$

is a linear combination of two functions, vanishing at $x^0 \rightarrow \pm\infty$. The functions are monotonic if $m < 3H/2$ or oscillating with a decreasing magnitude if $m > 3H/2$. If m/H is small, the field decreases very slowly:

$$\phi_0(x^0) = C_+ \exp\left(-\frac{2m^2}{9H^2}mx^0\right), \quad x^0 \rightarrow \infty, \quad \frac{m}{H} \ll 1. \quad (104)$$

In the limit $m/H \rightarrow 0$ the term with C_- disappears as $\exp(-3Hx^0)$, while the term with C_+ becomes indistinguishable from the massless field, which remains constant during the whole process of evolution. In dimensional units the ratio m/H is $mc^2/\hbar H$.

4.2.2. Regular Bounce. Extracting (100) from (101), we have

$$F'' = -axm^2\phi_0^2. \quad (105)$$

Without massive field, that is, if $\phi_0 = 0$, the second derivative $F'' = 0$, and we return to the de Sitter metric with the metric function (95) describing the two isolated solutions—compression and expansion—at a constant rate $F' = H = \text{const}$ (97).

The second order set of Einstein equations ((100), (101)) for the unknowns F' and ϕ_0 looks more complicated than the equivalent set ((102), (105)). At the same time the set (102), (105) is of the third order. Hence, it has extra solutions. So, working with the set (102), (105), it is necessary to eliminate extra solutions that are not the solutions of the Einstein equations ((100), (101)).

The time coordinate x^0 is a cyclic variable, and it is convenient to set the origin $x^0 = 0$ at a moment when $F' = 0$. Initial conditions for the Einstein equations contain only $\phi_0(0)$. The derivative $\phi'_0(0)$ is strictly fixed by the solutions of (100), (101). As for the set (102), (105), the value $\phi'_0(0)$ in the initial conditions is a free parameter, independent of $\phi_0(0)$. The connection between $\phi_0(0)$ and $\phi'_0(0)$, eliminating extra solutions, follows from (100) at $x^0 = 0$:

$$\phi_0'^2(0) + m^2\phi_0^2(0) = \frac{\tilde{\Lambda}}{\kappa a}, \quad F'(0) = 0. \quad (106)$$

The l.h.s. of (106) is positive. The initial conditions (106) are self-consistent if both $\tilde{\Lambda}$ and a are of the same sign. According to the requirement of regularity of the de Sitter metric (96) $\tilde{\Lambda}$ is negative. Hence a is negative too:

$$a < 0. \quad (107)$$

Then F'' (105) is positive, $F'' > 0$. We conclude that the massive time-like vector field makes the rate of evolution $F'(x^0)$ a monotonically growing function from $-H$ in the past to $+H$ in future. The Universe contracts at $x^0 < 0$, and expands at $x^0 > 0$. $x^0 = 0$ is the moment of maximum compression.

One of the two constants ϕ_0 and ϕ'_0 at $x^0 = 0$ remains arbitrary within the initial conditions (106). The Einstein equations ((100), (101)) are $x^0 \rightarrow -x^0$ invariant. In the case $\phi'_0(0) = 0$ the field $\phi_0(x^0)$ is a symmetric function, and if $\phi_0(0) = 0$ it is an antisymmetric one. In both cases $F'(x^0)$ is antisymmetric. If both constants ϕ_0 and ϕ'_0 at $x^0 = 0$ are not zeroes, a regular solution still exists, but there is no symmetry with respect to $x^0 \rightarrow -x^0$. The scale factor $R = e^F$ decreases with time while $F' < 0$, reaches its minimum, and grows when F' becomes positive.

In the case of a small mass,

$$m \ll H, \quad (108)$$

(in dimensional units $mc^2 \ll \hbar H$) the compression-to-expansion transition is described by the analytical solution for the symmetric configuration as follows:

$$F'(x^0) = H \tanh(3Hx^0), \quad (109)$$

$$\phi_0(x^0) = \sqrt{\frac{\tilde{\Lambda}}{\kappa a}} \frac{1}{m \cosh(3Hx^0)}, \quad m \ll H. \quad (110)$$

The rate of evolution $F'(x^0)$ (109) is shown in Figure 7. With no ordinary matter the time interval of transition is of the order of Hubble time $\sim 1/3H$. At $m \ll H$ it does not depend on the mass m of the massive field. The scale factor is

$$R(x^0) = e^{F(x^0)} = e^{F_0} [\cosh(3Hx^0)]^{1/3}, \quad m \ll H. \quad (111)$$

The metric function enters the Einstein equations ((100), (101)) only via the derivatives of $F(x^0)$ but not directly. Without ordinary matter $F_0 = F(0)$ is a free parameter, and

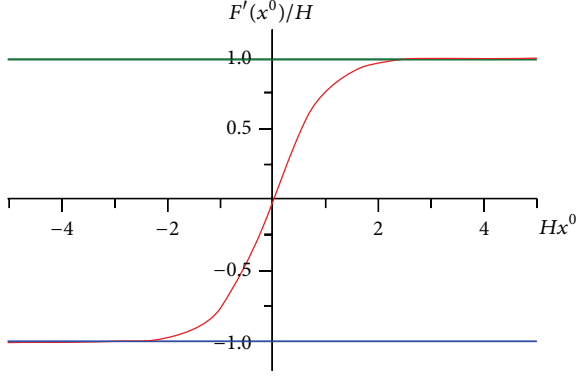


FIGURE 7: Compression-to-expansion transition. Red line is the rate of evolution $F'(x^0)/H = \tanh(3Hx^0)$ (109).

$R(x^0)$ is defined up to an arbitrary constant factor e^{F_0} . The acceleration F'' is positive:

$$F''(x^0) = \frac{3H^2}{\cosh^2(3Hx^0)} > 0, \quad m \ll H. \quad (112)$$

Like an elastic spring, the longitudinal vector field enables the transition from compression to expansion. The kinetic energy of contraction completely converts at $x^0 = 0$ into potential energy of the compressed vector field, and at $x^0 > 0$ the energy is being released back in the form of the kinetic energy of expansion.

A time-like longitudinal massive vector field displays repulsive elasticity. It can hardly be attributed to a particle of ordinary matter, because there is no reference frame where such particle could be at rest. However, a time-like vector field can be associated with a topological defect, inevitably arising in a phase transition with spontaneous symmetry breaking [50, 72]. It is worth mentioning that in the case (108) in the state of maximum compression the field $\phi_0(0)$ (110) is proportional to $H/m \gg 1$. Depending on the parameters of the potential $V(\phi^M \phi_M)$ for very small m the field $\phi_0(0)$ can be too big to omit the second and higher derivatives of $V(\phi^M \phi_M)$. Then in the state of maximum compression a phase transition with spontaneous symmetry breaking can take place. The idea to consider the topological defect as aether looks nice. Spontaneous breaking of Lorentz symmetry caused by a time-like vector field is a subject of research entitled “Einstein-aether model;” see [24–30] and references there in.

In the opposite case of a large mass,

$$m \gg H, \quad (113)$$

the field $\phi_0(x^0)$ is a rapidly oscillating function as compared with $\bar{F}'(x^0)$. The solution is

$$\bar{F}'(x^0) = H \tanh\left(\frac{3}{2}Hx^0\right), \quad (114)$$

$$\phi_0(x^0) = \sqrt{\frac{\bar{\Lambda}}{\kappa a m}} \frac{\cos(mx^0 + \varphi)}{\cosh((3/2)Hx^0)}, \quad m \gg H. \quad (115)$$

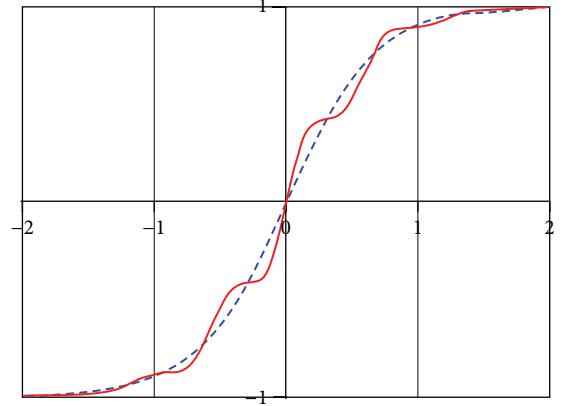


FIGURE 8: The rate of evolution in the case $m \gg H$. $F'(x^0)/H$ —red solid curve, found numerically for $m/H = 5$, $\phi'(0) = 0$. The averaged rate $\bar{F}'(x^0)/H = \tanh((3/2)Hx^0)$ —blue dashed line.

$\bar{F}'(x^0)$ is the rate of expansion, averaged over the rapid oscillations.

Oscillations of $\phi_0(x^0)$ at large m initiate weak vibrations of the rate $F'(x^0)$ around the averaged value $\bar{F}'(x^0)$; see Figure 8. Red curve is the numerical solution for $m/H = 5$, $\phi'(0) = 0$. Blue dashed line is the analytical solution $\bar{F}'(x^0)$ (114). The phase φ depends on the relation between the initial values $\phi_0(0)$ and $\phi'_0(0)$.

In Figures 9(a) and 9(b) $\phi_0(x^0)$, found numerically for $m/H = 5$, practically coincide with found analytically (115). The initial condition $\phi'_0(0) = 0$ for the symmetric solution in Figure 9(a) corresponds to $\varphi = 0$ in (115). The antisymmetric solution in Figure 9(b) (the initial condition $\phi_0(0) = 0$) coincides with (115) at $\varphi = -\pi/2$.

See [5] for the details of analytical and numerical solutions.

4.2.3. Acceleration and “Acceleration Coefficient”. In modern literature the uniform and isotropic evolution of the Universe is described by the rate of expansion (Hubble parameter) $H(x^0) = R'/R = F'(x^0)$ and by the artificially introduced so called “acceleration coefficient” q ,

$$q = \frac{R''}{R} = F'' + F'^2. \quad (116)$$

$R(x^0) = e^{F(x^0)}$ is the scale factor. This way defined q remains positive even if the evolution goes at a constant rate $F' = H = \text{const}$ (as under the action of the massless field):

$$q = H^2 > 0, \quad F'' = 0. \quad (117)$$

The commonly spread statement that the dark energy is the source of accelerated expansion is based on the definition (116). It is worth clarifying that the term “acceleration” is used here for F'' —the velocity of variation of the rate F' : $F'' = (R'/R)'$, not to be confused with the “acceleration coefficient” $q = R''/R$.

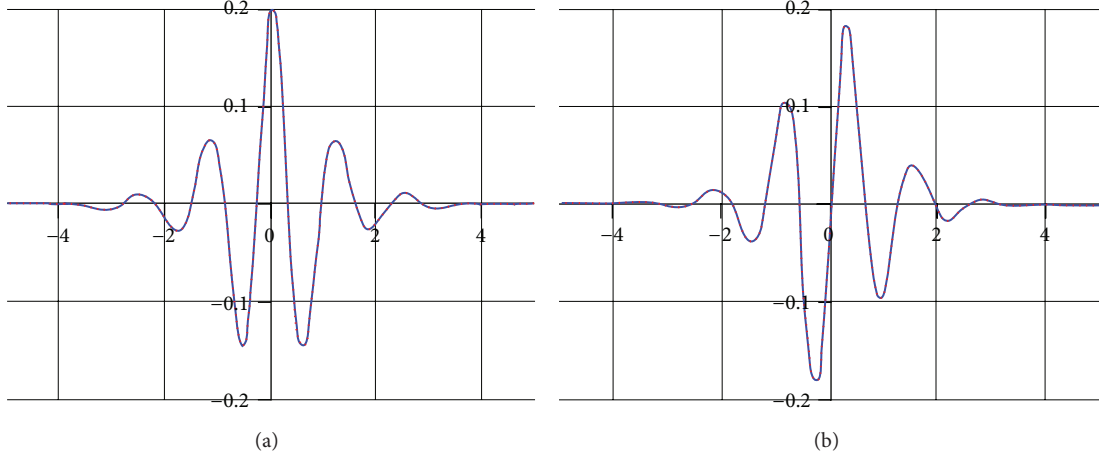


FIGURE 9: Symmetric $\varphi = 0$ (a) and antisymmetric $\varphi = -\pi/2$ (b) time-like massive vector fields $\sqrt{\kappa a/\Lambda}\phi_0(x^0)$ (115), $m/H = 5$.

4.3. Scenarios of Evolution of the Universe Driven by Vector Fields and Ordinary Matter. The above analysis of the set (102), (105) facilitates clarifying the solutions of the Einstein equations with the ordinary matter taken into account.

Applying the general relativity to the Universe as a whole it is natural to consider the ordinary matter (stars, galaxies, ...) as separate subsystems located at large distances from one another. Averaged over the distances much larger than the distance between the objects, the ordinary matter can be considered macroscopically as a uniformly distributed dust. In the dust matter approximation all components of the macroscopic energy-momentum tensor (56), except $T_{00} = \varepsilon g_{00}$, are zeros. In the cosmological metric (92) $g_{00} = 1$.

In the process of uniform and isotropic evolution the average energy density of matter ε depends only on x^0 . Due to Bianchi identities the covariant divergence of the energy-momentum tensor is zero: $T_{I;K}^K = e^{-3F}(d(e^{3F}\varepsilon)/dx^0)\delta_{I0} = 0$. Thus $\varepsilon e^{3F} = \text{const}$, the energy density of the ordinary matter is inversely proportional with the cube of scale factor $R = e^F$. If we accept that $\varepsilon(x^{0*}) = \varepsilon_0$ is the averaged energy density of the ordinary matter now, then $\varepsilon(x^0) = \varepsilon_0 e^{-3F(x^0)}$, and the present moment x^{0*} is defined by

$$F(x^{0*}) = 0. \quad (118)$$

In the process of expansion the metric function is negative in the past: $F(x^0) < 0$ at $x^0 < x^{0*}$.

4.3.1. Einstein Equations and Initial Conditions. With the ordinary dust matter taken into account, instead of the Einstein equation (100) we have

$$3F'^2 + \tilde{\Lambda} = \kappa a \left[(\phi_{,M}^M)^2 + m^2 \phi_0^2 \right] + \kappa \varepsilon_0 e^{-3F}, \quad I = 0. \quad (119)$$

The second equation (101) remains the same. The vector field equation (102) also remains the same, and (105) is changed to

$$F'' = \kappa |a| m^2 \phi_0^2 - \frac{1}{2} \kappa \varepsilon_0 e^{-3F}. \quad (120)$$

Equation (120) resembles Newton's law: acceleration F'' is proportional to the "repulsing force" $\kappa |a| m^2 \phi_0^2$ minus the "attracting force" $(1/2) \kappa \varepsilon_0 e^{-3F}$. Equations (102) and (120) and the initial conditions

$$\begin{aligned} \frac{\kappa a}{\Lambda} [\phi_0'^2(0) + m^2 \phi_0^2(0)] &= 1 + \Omega e^{-3F_0}, \\ F'(0) &= 0, \quad F(0) = F_0, \\ \tilde{\Lambda} &< 0, \quad a < 0, \end{aligned} \quad (121)$$

following from (119) at $F' = 0$, contain five dimensionless parameters: m/H , Ω , F_0 , $\sqrt{\kappa a/\Lambda}\phi_0(0)$, and $\sqrt{\kappa a/\Lambda}\phi_0'(0)$. In view of the connection (121) four of them are independent. As usual, the parameter Ω ,

$$\Omega = -\frac{\kappa \varepsilon_0}{\Lambda} = \frac{\kappa \varepsilon_0}{3H^2}, \quad (122)$$

denotes the ratio of today's energy density of the ordinary matter to the density of kinetic energy of expansion at a constant rate H . According to the NASA's "sliced cake" diagram (Figure 1)

$$\Omega \sim 0.06. \quad (123)$$

Actually the exact value of Ω becomes important in the vicinity of $\Omega e^{-3F_0} = 1$: regular oscillating solutions appear at $\Omega e^{-3F_0} > 1$ in addition to the cosmological ones; see Section 4.3.3 below.

4.3.2. Regular Cosmological Solutions. Equations (102), (120) with initial conditions (121) are easily integrated numerically. Regular solutions are free from any fine tuning. Moreover, the existing parametric freedom leads to a great variety of possible regular scenarios of evolution.

Numerical analysis (see details in [5]) shows that if both $\phi_0(0)$ and $\phi_0'(0)$ are not zeros at a turning point $F'(0) = 0$, then there can be other even more sharp turning points with

ϕ'_0 more close to zero. With the ordinary matter taken into account the metric function $F(x^0)$ enters (120) directly. The parameter $F_0 = F(0) < 0$ determines the degree of maximum compression at the turning point $F'(0) = 0$. The peak value of the rate of expansion grows exponentially with the increasing negative value of F_0 , $F' \sim e^{-3F_0}$, while the width of the transition decreases exponentially. It resembles inflation, except that there is no singularity. The regular contraction-to-expansion transition is often referred to as “nonsingular bounce” [75–77]. In the literature there are attempts to construct a self-consistent model in order to explain from a unified viewpoint the inflation in the early Universe and the late-time accelerated expansion [78]. However, one should keep in mind that the dust matter approximation is not applicable until the galaxies become noninteracting systems located at far distances from one another.

In the most interesting case of small m (108) the transition from contraction to expansion, resembling inflation, can be described analytically [5]. Utilizing the fact that at $m \rightarrow 0$ the antisymmetric term acts as a cosmological constant, it is natural to consider its contribution as already included into $\bar{\Lambda}$ (94), so that at $m \rightarrow 0$, $\bar{\Lambda}$ corresponds to the observable Hubble constant H (97), and $\phi'_0(0) = 0$ in the initial conditions (121). At $m \ll H$ one can neglect the term $m^2\phi_0$ in the field equation (102) and express $\phi_0(x^0)$ via $F(x^0)$:

$$\phi_0(x^0) = \phi_0(0) \exp \left\{ -3 \left[F(x^0) - F_0 \right] \right\}, \quad m \ll H. \quad (124)$$

Substituting (124) into (120), we exclude the field ϕ_0 and come to the single equation for F :

$$F'' = 3H^2 \left[\left(e^{6F_0} + \Omega e^{3F_0} \right) e^{-6F} - \frac{1}{2} \Omega e^{-3F} \right], \quad m \ll H. \quad (125)$$

Its regular solution with initial conditions $F'(0) = 0$, $F(0) = F_0$ is

$$F(x^0) = F_0 + \frac{1}{3} \ln \left[\left(1 + \frac{\Omega}{2} e^{-3F_0} \right) \cosh(3Hx^0) - \frac{\Omega}{2} e^{-3F_0} \right], \quad m \ll H. \quad (126)$$

In the limit $m \ll H$ the metric function $F(x^0)$ does not depend on the mass m of the vector field. $F_0 = F(0) < 0$ remains a free parameter. It determines the strength of maximum compression at $x^0 = 0$.

For the rate of evolution $F'(x^0)$ and for the scale factor $R(x^0)$ we get

$$F'(x^0) = H \frac{\sinh(3Hx^0)}{\cosh(3Hx^0) - (1 + (2/\Omega) e^{3F_0})^{-1}}, \quad (127)$$

$$R(x^0) = \left[\left(e^{3F_0} + \frac{1}{2} \Omega \right) \cosh(3Hx^0) - \frac{1}{2} \Omega \right]^{1/3}, \quad m \ll H. \quad (128)$$

Analytical solutions (126)–(128), derived for $m \ll H$, are as well applicable for $m \sim H$ in the vicinity of the turning

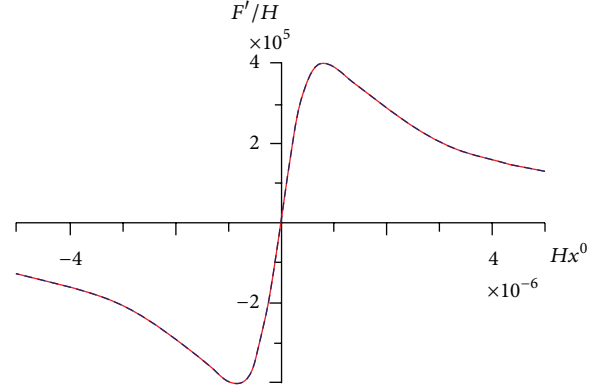


FIGURE 10: F'/H in the vicinity of the turning point. Blue dashed curve is the numerical result for $F_0 = -10$, $m/H = 10$, and $\Omega = 0.06$. It coincides with the red solid curve—analytical solution (127).

point, if $|F_0| \gg 1$. See Figure 10, where the variation of $F'(x^0)$, found numerically for $\Omega = 0.06$, $m/H = 10$, and $F_0 = -10$, practically coincides with (127) in the vicinity of the turning point $x^0 = 0$. It is because of very large negative F_0 the width of the contraction-to-expansion transition Δx^0 is very narrow:

$$H\Delta x^0 \sim \frac{2}{3\sqrt{\Omega}} e^{-3|F_0|/2}, \quad F_0 < 0, \quad |F_0| \gg 1. \quad (129)$$

In the process of compression the repulsing term $\sim e^{-6F}$ in (120) increases faster than the compressing term $\sim e^{-3F}$. It is the reason why a regular bounce replaces the singularity independent of how big the negative F_0 is. After the bounce the repulsing term decreases faster than the compressing one, leading to matter domination over the field at late times.

The limits of applicability of (120) are connected with the dust matter approximation and with omitting the second and higher derivatives of the potential $V(\phi^K \phi_K)$. The symmetric field at the bounce $\phi^2(0) \sim \kappa \epsilon_0 e^{-6F_0}/m^2$ can be very large, and it should lead to a phase transition with spontaneous symmetry breaking. Naturally, in this case the solution (127) would be unstable. The analysis of possible symmetry breaking in the limit $m/H \ll 1$ deserves a separate consideration.

According to the analysis of the Hubble space telescope data [79], the expansion of the Universe switched from deceleration to acceleration at about a half of the age of the Universe. In the analytical solution (127) the second derivative $F''(x^0)$ is negative on the slope of $F'(x^0)$ after the peak (blue curve in Figure 11). The expansion continues ($F' > 0$), but in the case $m \ll H$ it goes with deceleration ($F'' < 0$) all the time after the peak.

The red solid curve in Figure 11 is the numerical solution for $m/H = 1$, $F_0 = -2$, and $\Omega = 0.06$. In the case $m/H = 1$ the expansion switches from deceleration to acceleration at about half the time that passed from the turning point $x^0 = 0$ to the present moment $Hx^{0*} \approx 2$ (where $F(x^{0*}) = 0$; see the “calendar” (118)). It is in accordance with the Hubble space telescope data [79]. The particular value $F_0 = -2$ in Figure 11 is taken not big for better clarity. The peak value

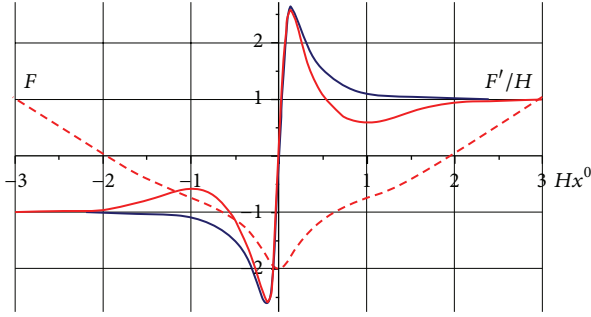


FIGURE 11: Solid curves are the rates of evolution F'/H for $\Omega = 0.06$, $F_0 = -2$. Blue curve is the analytical solution (127) for $m/H \ll 1$, and the red solid curve is the numerical solution for $m/H = 1$. The dashed red curve is the metric function F . Horizontal axis is the dimensionless time Hx^0 . In accordance with the “calendar” (118) today’s date is $Hx^{0*} \approx 2$. The moment of switching from deceleration to acceleration ($F'' = 0$) is $Hx^0 \approx 1$.

of F' grows exponentially with increasing $|F_0|$. However at a fixed value $m/H \geq 1$ the qualitative picture remains the same: the transition from deceleration to acceleration does not disappear.

4.3.3. Regular Oscillating Solutions (with Positive $\tilde{\Lambda}$). There is an important difference between the initial conditions (106) and (121). The relation (106) can be satisfied only if $\tilde{\Lambda} < 0$, provided that $a < 0$. Appearance of the term Ωe^{-3F_0} in (121) admits the solutions with positive $\tilde{\Lambda}$. If $\tilde{\Lambda}$ changes sign, then H (97) becomes imaginary. Equations (102), (120) are invariant against $H \rightarrow iH$, but the initial conditions (121) are not:

$$\frac{\kappa|a|}{\tilde{\Lambda}} [\phi_0'^2(0) + m^2 \phi_0^2(0)] = -1 + \Omega e^{-3F_0}, \quad (130)$$

$$F'(0) = 0, \quad F(0) = F_0, \quad \tilde{\Lambda} > 0.$$

A necessary condition for regular solutions with $\tilde{\Lambda} > 0$ is the existence of an extremum moment ($F'(0) = 0$) with the energy density of ordinary matter exceeding the kinetic energy of expansion:

$$\Omega e^{-3F_0} = \frac{\kappa \varepsilon(0)}{\tilde{\Lambda}} > 1, \quad F'(0) = 0, \quad \tilde{\Lambda} > 0. \quad (131)$$

In the case $\tilde{\Lambda} > 0$, $m \ll H$ the symmetric analytical solution of (102), (120) with the initial conditions (130) is expressed in terms of trigonometric functions. The metric function $F(x^0)$, the scale factor $R(x^0)$, and the rate of evolution $F'(x^0)$,

$$F(x^0) = F_0 + \frac{1}{3} \ln \left[\left(1 - \frac{1}{2} \Omega e^{-3F_0} \right) \cos(3Hx^0) + \frac{1}{2} \Omega e^{-3F_0} \right], \quad (132)$$

$$R(x^0) = e^{F_0} \left[\left(1 - \frac{1}{2} \Omega e^{-3F_0} \right) \cos(3Hx^0) + \frac{1}{2} \Omega e^{-3F_0} \right]^{1/3}, \quad (133)$$

$$F'(x^0) = H \frac{\sin(3Hx^0)}{(1 - (2/\Omega e^{-3F_0}))^{-1} - \cos(3Hx^0)}, \quad (134)$$

$$\tilde{\Lambda} > 0, \quad m \ll H,$$

are periodic functions with no singularity; see red curves in Figures 12(a) and 12(b). In the case $\tilde{\Lambda} > 0$ the origin $x^0 = 0$ is a point of maximum of the scale factor $R(x^0)$. The points of minimum (where $\cos(3Hx^0) = -1$) are

$$x^0 = x_n^0 = \frac{\pi}{3H} (1 + 2n), \quad n = 0, \pm 1, \pm 2, \dots \quad (135)$$

For the values of the parameters $m/H = 0.02$ and $\Omega e^{-3F_0} = 1.032$ (barely exceeding the boundary $\Omega e^{-3F_0} = 1$) there is no difference in Figures 12(a) and 12(b) between the curves found numerically and analytically.

As it follows from (130) without the massive field ($\phi_0 = 0$) the solutions with positive $\tilde{\Lambda}$ are possible only if the parameters are fine tuned:

$$\Omega e^{-3F_0} = 1, \quad (136)$$

$$F'(x^0) = -H \tan \frac{3Hx^0}{2}, \quad (137)$$

$$R(x^0) = e^{F_0} \left(\cos^2 \frac{3Hx^0}{2} \right)^{1/3}. \quad (138)$$

The scale factor $R/R(0)$ (138) and the rate of expansion F'/H (137) are the blue curves in Figures 12(a) and 12(b). These fine tuned solutions have a periodic singularity at $x^0 = x_n^0$ (135). In the vicinity of each singular point x_n^0 , as well as at $H \rightarrow 0$, the Hubble constant H drops out, and the scale factor (138) reduces to

$$R(x^0) = \left(\frac{3\kappa \varepsilon_0}{4} \right)^{1/3} |x^0 - x_n^0|^{2/3}, \quad (139)$$

$$\left| \frac{x^0}{x_n^0} - 1 \right| \ll 1.$$

(139) is the scale factor of the Friedman-Robertson-Walker [56–59] cosmology with dust matter in the plane space geometry. The longitudinal time-like vector field $\phi_0 \neq 0$ removes the singularity; see red curves in the Figures 12(a) and 12(b).

(a) Domain of Regular Oscillating Solutions. The maximum value F_0 of the metric function (132) corresponds to $\cos(3Hx^0) = 1$ and the minimum value $(1/3) \ln(\Omega - e^{3F_0})$ —to $\cos(3Hx^0) = -1$. According to the “calendar” (118) today’s date x^{0*} is determined by $F(x^{0*}) = 0$. This means that $F_0 \geq 0$ and $\ln(\Omega - e^{3F_0}) \leq 0$. Otherwise, if today’s zero value of the metric function F is outside the interval of its variation, the oscillating solutions (132)–(134) are not related

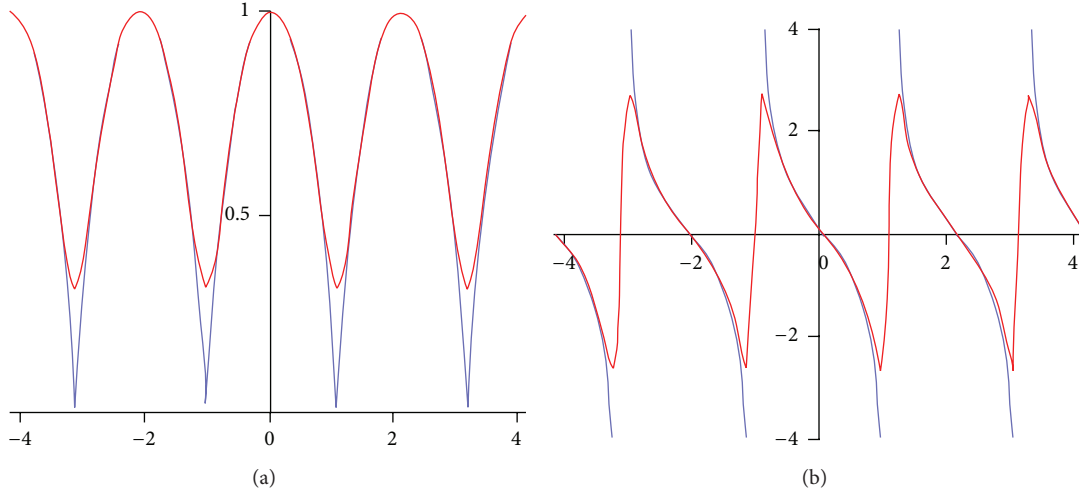


FIGURE 12: Scale factor $R/R(0)$ (a) and rate of evolution F'/H (b). The horizontal axis is “time” Hx^0 . Red curves—regular numerical solution (coinciding with (133) and (134)) for $m/H = 0.02$, $\Omega \exp(-3F_0) = 1.032$. Blue curves with periodic singularity are the fine tuned solutions (138) and (137) at the lower boundary of the area of regular oscillating solutions (with $\tilde{\Lambda} > 0$).

to the evolution of the Universe. Hence, on the map of the parameters $\Omega(F_0)$ the domain of regular oscillating solutions is limited by $F_0 \geq 0$, $e^{3F_0} < \Omega \leq 1 + e^{3F_0}$. The oscillating scenario of evolution is possible if $\Omega > 1$. It is one of the two: either the estimate (123) is understated, or the oscillating scenario has nothing to do with evolution of the Universe. The same applies to the Friedman-Robertson-Walker scenario [56–59], existing on the lower boundary $\Omega = e^{3F_0}$ (136).

The positivity of the energy $T_{00} > 0$ (see (94.10) in [22]), supported by the Lorentz gauge restriction (29) [32], promotes the mutual attraction between material objects and excludes the possibility of repulsion. In thirties the dark matter had not been taken seriously. Under the action of only contracting forces the observed expansion of the Universe as a whole was considered as an explosion of some extremely small highly compressed source. Accordingly, the Friedman-Robertson-Walker solutions [56–59] inevitably contained a singularity and existed only under the fine-tuning restriction (136).

The discovery of accelerated expansion confirms that in addition to the ordinary matter there is some medium named dark matter with repulsive properties. This source of acceleration exists for a long time after the mysterious big bang. The longitudinal time-like vector field with a simple Lagrangian (58) turns out an appropriate tool for macroscopic description of the dark matter, including its repulsive ability. Abandoning the Lorentz gauge restrictions we get a variety of regular solutions with no need of any fine tuning.

The idea of oscillating Universe has been proposed earlier by Lessner [80] as an alternative approach to cosmology. Meanwhile, I would better call it additional, for both kinds of regular scenarios—cosmological and oscillating—are derived completely within the frames of the Euler-Lagrange approach and general relativity.

5. Summary

The nongauge vector field with the most simple Lagrangian (58) turns out to be an appropriate tool for macroscopic description of the dark sector. The dark substance is described via the covariant vector field equations (59) and the energy-momentum tensor (60). It no longer needs to invent its own model of dark matter for understanding each observed astrophysical phenomenon.

In the galaxy scale 10 to 100 Kpc the dark matter, described by the space-like ($\phi^K \phi_K < 0$) vector field, is responsible for a plateau in galaxy rotation curves. In the scale of the whole Universe the time-like ($\phi^K \phi_K > 0$) vector field is “the missing link in the chain,” necessary to understand the main features of evolution of the Universe and avoid, better say—resolve, the Big Bang singularity. A mysterious Big Bang is no longer the inevitable property of the Universe evolution, provided that the dark matter is taken into account. The reason, why the singularity was considered inevitable, is the impossibility of such a regular solution, where mutually attracting objects fly away from each other.

The macroscopic description of the dark sector is applicable for studying the structure of the Universe in the intermediate range Mpc to hundred Mpc [34]. The field equations (59) and the energy-momentum tensor (60) of dark substance allow avoiding unnecessary model assumptions. It would be interesting to trace how additional attraction by a space-like field, dominating in the galaxy scale, transforms into elastic repulsion of a time-like field, dominating in the scale of the whole Universe. In the intermediate range all three components of the dark sector (described by zero-mass field, space-like field, and time-like field) should be taken into account altogether.

Most likely, the manifestation of dark matter in the scale of the solar system is a fantastic, but still it is worth it to trace

the acceleration $\nu'(r)$ (78) along the two spacecrafts Pioneer 10 and Pioneer 11 hyperbolic orbits at distances between 20–70 AU from the Sun. Who knows?

It is rather involuntarily but the modern interpretations of the observational data are mostly based on the idea of the Big Bang birth of the Universe. The cosmic background radiation, among other phenomena, definitely testifies that the Universe had been strongly compressed in the past. But how strongly? The information from the past, coming to us with electromagnetic waves, tells us only about the phenomena that happened after the Universe became transparent. The far extrapolation to the Planck's era is based on the assumption that the singularity is an inevitable property of cosmological solutions of Einstein equations. The discovery of accelerated expansion strictly pointed on the existence of hidden sector, able to resist compression. The macroscopic theory does not restrict the strength of compression. The degree of maximum compression is determined by the parameter F_0 . At large negative F_0 the regular expansion after the turning point resembles inflation. I draw attention to Figure 10, where for $F_0 = -10$ ($R \approx 1/22000$) there is a 10-order difference in the horizontal and vertical scales. However, one should keep in mind that the dust matter approximation is applicable if the galaxies are located at far distances from one another.

In accordance with the Section 4.3.2, the observed [79] point of minimum of F' , where the deceleration turns back to acceleration, corresponds to $m/H \sim 1$ (see Figure 11). A time-like vector field can hardly be associated with a massive quantum particle. Nevertheless it is worth it to try to detect an extremely light quasiparticle—elementary excitation of some nonconventional medium like “ghost condensate,” or “aether”—with a quantum of ground energy $mc^2 \sim \hbar H \sim 10^{-33}$ eV.

On the contrary, for a space-like field there is a frame of reference where $\phi_0 = 0$. The estimate for the rest energy of a particle associated with a space-like field with the wavelength $\lambda = \hbar/mc \sim 15$ kpc (see Figure 5) is $mc^2 \sim 10^{-27}$ eV.

Though the macroscopic theory describes the galaxy rotation curves and various scenarios of the Universe evolution, the physical origin of dark matter and dark energy still remains unknown. There is a hope that, by applying the energy-momentum tensor of the dark sector (60) and considering the ordinary matter as a degenerate relativistic Fermi gas, it would be possible not only to find the connection between the parameters of dark and ordinary matter, but also to describe the internal structure of a heavy black hole with no singularity in the center.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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