

Research Article

Extended Games Played by Managerial Firms with Asymmetric Costs

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Received 5 May 2014; Accepted 26 June 2014; Published 13 July 2014

Academic Editor: X. Henry Wang

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Both demand and cost asymmetries are considered in oligopoly model with managerial delegation. It shows that (i) both efficient and inefficient firms with delegation have second move advantage under quantity setting and first move advantage under price competition; (ii) the extended games under both quantity and price competition have subgame equilibria. Lastly, the social welfare of all strategy combinations is considered to find that when the efficient firm moves first and the inefficient firm moves second under price competition, the social welfare can be higher than Bertrand case, if the efficiency gap between the two firms is huge.

1. Introduction

Industrial organization analysis in oligopoly and duopoly usually assumes that firms move either simultaneously or sequentially. Technically speaking, it is Cournot (Bertrand) game or Stackelberg game. In a seminal paper, Singh and Vives [1] show that, in a differentiated duopoly, Cournot competition entails higher prices and profits than Bertrand competition, whereas both firms' output and social welfare are higher under Bertrand competition. López and Naylor [2] show that the standard result that Cournot equilibrium profits exceed those under Bertrand competition—when the differentiated duopoly game is played in imperfect substitutes—is reversible. Whether equilibrium profits are higher under Cournot or Bertrand competition is shown to depend upon the nature of the upstream agents' preferences and on the distribution of bargaining power over the input price. Hsu and Wang [3] show that both consumer surplus and total surplus are higher under price competition than under quantity competition, regardless of whether goods are substitutes or complements. Zanchettin [4] shows that both the efficient firm's profits and industry profits are higher under Bertrand competition when asymmetry is strong and/or products are weakly differentiated. (See Mukherjee [5] for comparison of equilibrium outcome under quantity and price competition in free entry, and in Mukherjee et al. [6], comparison of

equilibrium outcome under quantity and price competition is analyzed in a vertical structure with production efficiency. The productivity difference between the downstream firms is the key to their analyses, which enriches Zanchettin [4] because in spite of the homogenous products Mukherjee et al. [6] utilize the productivity differences to justify the cost asymmetry assumed in Zanchettin [4]. Correa-López [7] also makes similar comparisons in a vertical structure but allows the downstream firm to choose either price or quantity to compete. However, Zanchettin [4] and Correa-López [7] did not consider the sequential game, which is one key component to this work.) Hence, Singh and Vives's ranking of profits between the two modes of competition is reversed in a sizeable portion of the relevant parameter space. From the above papers' demonstration, we see that the conventional wisdom on the ranking of profit and welfare under Cournot or Bertrand competition has been challenged on the ground of product substitutability and cost asymmetry. Furthermore, in most Stackelberg duopoly games, which firm is the leader and which firm is the follower are often assigned at random or not even considered.

There is literature which starts to research the strategy choice of timing: when do the duopolies move simultaneously and when do they move sequentially? Hamilton and Slutsky [8] first endogenize the choice of roles in noncooperative duopoly games by adding one more stage to a traditional

duopoly game. At the adding stage, firms decide the timing of taking actions. If the timing choices of the firms are different, then a sequential subgame occurs. Contrarily, if the firms choose to act at the same time, a simultaneous subgame is derived. The choices of timing are made in a preplay stage. Lambertini [9] analyzes endogenous timing in symmetric duopoly game with managerial schemes. He adds a delegation contract stage to Hamilton and Slutsky's [8] extended game with observable delay. Lambertini shows that when the market decisions are delegated to managers and the owners decide the timing of move, the extended game with observable delay exhibits the same subgame-perfect equilibria that would arise if both decisions are delegated to managers. (There is another strand of literature on mixed market under endogenous timing with or without managerial incentive. See, e.g., Wang et al. [10] on optimal tariff and privatization; Nakamura and Inoue [11] on price competition with weighted welfare and managerial delegation and Bárcena-Ruiz and Sedano [12] on price competition with capacity choice, resp.; Tomaru et al. [13] on the model with capacity choice and managerial delegation.)

The strategic use of managers' contracts in competition games was introduced and being widely used from the 1980s. Fershtman and Judd [14], Sklivas [15], Bárcena-Ruiz and Espinoza [16], Jansen et al. [17], and Ritz [18] offer a game-theoretic explanation for managers' nonprofit-maximizing behavior. They set up a two-stage game, where in the first stage (contract stage) the owner announces publicly a contract that delegates the market decision to the manager, which is before the competition that happens in the next stage. They provide several different types of delegation, such as sales revenue, market share, and relative profit. (See recent works of Jansen et al. [19] on finding dominant delegation equilibrium and Manasakis et al. [20] on the issue of commitment.) Recently, Chirco et al. [21] analyze the case of relative performance and show that the distortion from a profit-maximizing rule decreases as market becomes less contracted, while it increases as demand becomes more elastic.

Though there is horizontal product differentiation in Hamilton and Slutsky [8] and Lambertini [9], the two firms in duopoly are symmetric. Therefore, in their conclusions, the roles played by the two firms are the same. Thus they actually failed to provide decided explanations for endogenous determination of firms' move order. It is because the desirability of one firm to act first does not necessarily indicate that the other symmetric firm wants to be the follower. Hence the question that counts now becomes the following: under what circumstances does a firm want to move first or second and what determinates the decisions of the firms?

To explore the question, asymmetry between the two firms with sales revenue delegation is considered in this paper while demand and cost asymmetry were not considered by Lambertini [9]. A model with asymmetric demand was first introduced into duopoly competition by Singh and Vives [1] and used by Häckner [22] and Zanchettin [4]. Like the model in Zanchettin [4], both demand and cost asymmetries are allowed for. (Wang [23] shows that while profit ranking between price and quantity competition can be

(partially) reversed, the celebrated result by Singh and Vives that firms always choose a quantity contract in a two-stage game continues to hold in the enlarged parameter space.) As in Häckner [22], demand asymmetry is interpreted as a quality difference between the two kinds of products. Then we reduce both demand and cost asymmetries of one firm to be more efficient than the other in terms of per unit cost of quality supplied. However, in this paper, the assumption of positive primary outputs holds; that is, the parameter space discussed here is also used in Singh and Vives [1], which is a subset of that in Zanchettin [4]. It is because the inefficient firm will be confronted with zero demand in the limit-pricing region, which is not the purpose of this study.

The conclusions of this paper are threefold: if products are substitute and the assumption of positive primary outputs holds, firstly, both efficient and inefficient firms with delegation have second move advantage under quantity setting and first move advantage under price competition; secondly, the extended games under quantity competition and under price competition have subgame equilibria; thirdly, the social welfare can be higher when the efficient firm moves first and the inefficient firm moves second under price competition than the Bertrand case, if the efficiency gap between the two firms is huge.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes the extended games with observable delay under quantity and price competition over the relevant parameter space. In Section 3, the equilibrium outcomes under Cournot and Bertrand cases are compared. Section 4 analyzes the two extended games and presents the main results of the game equilibria. Lastly, Section 5 gives some concluding remarks.

2. The Model

Consider a linear duopoly model used in Singh and Vives [1]. Two firms in the industry produce two differentiated products. The inverse demand function is thus given by

$$p_i = \alpha_i - q_i - \gamma q_j \quad (i, j = 1, 2; i \neq j), \quad (1)$$

where parameter $\gamma \in [-1, 1]$ measures the degree of product differentiation. When γ goes to 1, the two goods become perfect substitute; when γ approaches 0, they become independent; and if $\gamma < 0$, the two goods are complements. This paper focuses on the substitute case. The other parameter $\alpha_1 \neq \alpha_2$ denotes the quality difference between the products supplied by the two firms, as is the same in Häckner [22].

On the supply side, these two firms are assumed to have zero fixed cost competing either in quantities or in prices with asymmetric constant marginal costs, c_1, c_2 . Moreover, the firms' owners may employ managers to operate their firms, and with delegation the considered firms play a duopoly game, which is extended by the firms' move timing decisions. In the first stage (time stage), the profit-maximizing owners decide and announce the timing of move in the market stage: if they move simultaneously, a Cournot or Bertrand Nash equilibrium obtains; conversely, if they decide to move sequentially, then a Stackelberg game with price or quantity

competition plays. Hence the choices of timing are in fact a preplay stage that happens before the delegation and the competition occurs. In the second stage (contract stage), the owners set up managerial incentive schemes to motivate the managers' behavior. The third stage (market stage) is divided into two periods: managers first see the announcement of the move timing and the contract of managerial incentive schemes and then follow the announcement to decide the output levels. (See the generalized extended game details in Hamilton and Slutsky [8] or Lambertini [9].) Following Sklivas [15], the bonus given to the manager of firm i is defined as

$$G_i = \lambda_i \pi_i + (1 - \lambda_i) p_i q_i = p_i q_i - \lambda_i c_i q_i, \quad (2)$$

where parameter λ_i denotes the weight of profit attached to the manager's bonus. (See Sklivas [15] in a two-stage game showing that $\lambda_i < 1$ under quantity competition while $\lambda_i > 1$ under price competition.) Equation (2) implies that managerial delegation definitely affects the effective marginal cost which is perceived by the manager. It is useful to explain the later results.

Recall that both demand asymmetry ($\alpha_1 \neq \alpha_2$) and cost asymmetry ($c_1 \neq c_2$) are allowed for, and then a single index can be introduced to capture both demand and cost asymmetry: $a = (\alpha_1 - c_1) - (\alpha_2 - c_2)$. Without loss of generality, letting $\alpha_1 - c_1 = 1$ makes the index of asymmetry reduce to $a = 1 - (\alpha_2 - c_2)$. For the comparing convenience, we constrain $a \geq 0$ indicating that firm 1 is more efficient than firm 2 in terms of per unit cost of quality supplied if $a > 0$, whereas the firms are symmetric if $a = 0$.

For simplicity, we focus on the case of substitute goods and assume that both firms are active in the market. (It is known that the competition modes, the property of goods (substitutes), and the strategic relationship (strategic substitute or complement) in oligopolistic competition cause the difference of results. See Sklivas [15] for details.) This restricts the parameter space to $S = \{0 < \gamma < 1; 0 \leq a < 1 - \gamma\}$, where $0 \leq a < 1 - \gamma$ is following the assumption of positive primary outputs in Singh and Vives [1].

Therefore, when $0 < \gamma < 1$, the demand functions can be derived from (1) as follows:

$$q_i = \frac{1}{1 - \gamma^2} (\alpha_i - \gamma \alpha_j - p_i + \gamma p_j) \quad (i, j = 1, 2; i \neq j), \quad (3)$$

and the profit of firm i is given by

$$\pi_i = (\alpha_i - q_i - \gamma q_j - c_i) q_i = \frac{(p_i - c_i) (\alpha_i - p_i - \gamma \alpha_j + \gamma p_j)}{(1 - \gamma^2)} \quad (i, j = 1, 2; i \neq j). \quad (4)$$

2.1. Quantity Competition. Cournot case is investigated in this subsection. The payoff function of owner i is $\pi_i^k(C)$, where C denotes quantity competition and $k \in \{n, l, f\}$ denotes the role of the firm: simultaneous move, leader, and follower. (Mujumdar and Pal [24] examined the nature of

optimal managerial incentives in the context of a duopoly marked by competition between the firm's managers in a dynamic production environment. It is shown that if the marginal cost of production falls moderately over time or remains unchanged, there exists only one equilibrium where one owner requires his/her manager to maximize profit, whereas the rival-owner requires his/her manager to maximize sales revenue. The profit-maximizing manager turns his/her firm into a Stackelberg-leader, while the sales-revenue-maximizing manager turns his/her firm into a Stackelberg-follower. Furthermore, the profit-maximizing manager may generate a larger firm profit relative to the sales-revenue-maximizing manager.) (5) illustrates this extended game in the reduced form as follows:

$$i \begin{array}{c} \text{F} \\ \text{S} \end{array} \begin{array}{c} j \\ \text{F} \quad \text{S} \end{array} \begin{array}{|c|c|} \hline \pi_1^n(C), \pi_2^n(C) & \pi_1^l(C), \pi_2^f(C) \\ \hline \pi_1^f(C), \pi_2^l(C) & \pi_1^n(C), \pi_2^n(C) \\ \hline \end{array}. \quad (5)$$

If the owners of these two firms decide to move simultaneously, no matter $\{F, F\}$ or $\{S, S\}$, a Cournot-Nash equilibrium is obtained. Using the method in Sklivas [15], the following results can be shown.

The incentive schemes:

$$\lambda_1^n(C) = 1 - \frac{\gamma^2 [-\gamma^2 - 2(1-a)\gamma + 4]}{c_1(\gamma^4 - 12\gamma^2 + 16)}; \quad (6)$$

$$\lambda_2^n(C) = 1 - \frac{\gamma^2 [-(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{c_2(\gamma^4 - 12\gamma^2 + 16)}.$$

Output:

$$q_1^n(C) = \frac{2[-\gamma^2 - 2(1-a)\gamma + 4]}{\gamma^4 - 12\gamma^2 + 16}; \quad (7)$$

$$q_2^n(C) = \frac{2[-(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{\gamma^4 - 12\gamma^2 + 16}.$$

Price:

$$p_1^n(C) = c_1 + \frac{(2 - \gamma^2) [-\gamma^2 - 2(1-a)\gamma + 4]}{\gamma^4 - 12\gamma^2 + 16}; \quad (8)$$

$$p_2^n(C) = c_2 + \frac{(2 - \gamma^2) [-(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{\gamma^4 - 12\gamma^2 + 16}.$$

Profit:

$$\pi_1^n(C) = \frac{2(2 - \gamma^2) [-\gamma^2 - 2(1-a)\gamma + 4]^2}{(\gamma^4 - 12\gamma^2 + 16)^2}; \quad (9)$$

$$\pi_2^n(C) = \frac{2(2 - \gamma^2) [-(1-a)\gamma^2 - 2\gamma + 4(1-a)]^2}{(\gamma^4 - 12\gamma^2 + 16)^2}.$$

If the owner of firm 1 chooses to move first and firm 2 opts to take action later, that is, {F, S}, a Stackelberg subgame is yielded.

The incentive schemes:

$$\begin{aligned}\lambda_1^l(C) &= 1; \\ \lambda_2^f(C) &= 1 - \frac{\gamma^2 [-(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{c_2(3\gamma^4 - 16\gamma^2 + 16)}.\end{aligned}\quad (10)$$

Output:

$$\begin{aligned}q_1^l(C) &= \frac{(1-a)\gamma^3 - 4\gamma^2 - 4(1-a)\gamma + 8}{3\gamma^4 - 16\gamma^2 + 16}; \\ q_2^f(C) &= \frac{-(1-a)\gamma^2 - 2\gamma + 4(1-a)}{2(4 - 3\gamma^2)}.\end{aligned}\quad (11)$$

Profit:

$$\begin{aligned}\pi_1^l(C) &= \frac{(2 - \gamma^2) [(1-a)\gamma^3 - 4\gamma^2 - 4(1-a)\gamma + 8]^2}{2(3\gamma^4 - 16\gamma^2 + 16)^2}; \\ \pi_2^f(C) &= \frac{[-(1-a)\gamma^2 - 2\gamma + 4(1-a)]^2}{4(3\gamma^4 - 16\gamma^2 + 16)}.\end{aligned}\quad (12)$$

Similarly, if the owner of firm 1 wants to move later and firm 2 wishes to move earlier, that is, {S, F}, a symmetric Stackelberg subgame is produced.

The incentive schemes:

$$\lambda_1^f(C) = 1 - \frac{\gamma^2 [-\gamma^2 - 2(1-a)\gamma + 4]}{c_1(3\gamma^4 - 16\gamma^2 + 16)}; \quad \lambda_2^l(C) = 1.\quad (13)$$

Output:

$$\begin{aligned}q_1^f(C) &= \frac{-\gamma^2 - 2(1-a)\gamma + 4}{2(4 - 3\gamma^2)}; \\ q_2^l(C) &= \frac{\gamma^3 - 4(1-a)\gamma^2 - 4\gamma + 8(1-a)}{3\gamma^4 - 16\gamma^2 + 16}.\end{aligned}\quad (14)$$

Profit:

$$\begin{aligned}\pi_1^f(C) &= \frac{[-\gamma^2 - 2(1-a)\gamma + 4]^2}{4(3\gamma^4 - 16\gamma^2 + 16)}; \\ \pi_2^l(C) &= \frac{(2 - \gamma^2) [\gamma^3 - 4(1-a)\gamma^2 - 4\gamma + 8(1-a)]^2}{2(3\gamma^4 - 16\gamma^2 + 16)^2}.\end{aligned}\quad (15)$$

It is worth mentioning that in quantity competition with delegation, both firms' incentive schemes under four strategy combinations are less than or equal to 1, especially in the two sequential subgames; the leaders' incentive schemes are equal to 1. This is interesting because when the owner decides to move first, it is already an aggressive strategy, which makes it needless to set managerial schemes to make the manager aggressive as well because the manager's bonus which he maximizes is just the firm's profit.

2.2. Price Competition. Followed with the Cournot case in the former subsection, Bertrand case is analyzed within this subsection. Similarly, the payoff function of owner i becomes $\pi_i^k(B)$, where B stands for price competition and $k \in \{n, l, f\}$ denotes the role of the firm: simultaneous move, leader, and follower. Likewise, (16) depicts the extended game in the reduced form as follows:

$$i \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{cc} & j \\ & \begin{array}{cc} F & S \end{array} \\ \begin{array}{c} F \\ S \end{array} & \begin{array}{|c|c|} \hline \pi_1^n(B), \pi_2^n(B) & \pi_1^l(B), \pi_2^f(B) \\ \hline \pi_1^f(B), \pi_2^l(B) & \pi_1^n(B), \pi_2^n(B) \\ \hline \end{array} \end{array}.\quad (16)$$

If the owners of these two firms decide to move simultaneously, no matter earlier or later, that is, {F, F} or {S, S}, Bertrand-Nash equilibrium is realized.

The incentive schemes:

$$\begin{aligned}\lambda_1^n(B) &= 1 + \frac{\gamma^2 [(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4]}{c_1(\gamma^4 - 12\gamma^2 + 16)}; \\ \lambda_2^n(B) &= 1 + \frac{\gamma^2 [\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{c_2(\gamma^4 - 12\gamma^2 + 16)}.\end{aligned}\quad (17)$$

Output:

$$\begin{aligned}q_1^n(B) &= \frac{(2 - \gamma^2) [(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4]}{(1 - \gamma^2)(\gamma^4 - 12\gamma^2 + 16)}; \\ q_2^n(B) &= \frac{(2 - \gamma^2) [\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{(1 - \gamma^2)(\gamma^4 - 12\gamma^2 + 16)}.\end{aligned}\quad (18)$$

Price:

$$\begin{aligned}p_1^n(B) &= c_1 + \frac{2 [(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4]}{\gamma^4 - 12\gamma^2 + 16}; \\ p_2^n(B) &= c_2 + \frac{2 [\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{\gamma^4 - 12\gamma^2 + 16}.\end{aligned}\quad (19)$$

Profit:

$$\begin{aligned}\pi_1^n(B) &= \frac{2(2 - \gamma^2) [(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4]^2}{(1 - \gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2}; \\ \pi_2^n(B) &= \frac{2(2 - \gamma^2) [\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)]^2}{(1 - \gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2}.\end{aligned}\quad (20)$$

If the owner of firm 1 prefers to move first and firm 2 chooses to move later, that is, {F, S}, a Stackelberg subgame is acquired.

The incentive schemes:

$$\begin{aligned}\lambda_1^l(B) &= 1; \\ \lambda_2^f(B) &= 1 + \frac{\gamma^2 [\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{c_2(3\gamma^4 - 16\gamma^2 + 16)}.\end{aligned}\quad (21)$$

Price:

$$p_1^l(B) = c_1 + \frac{\gamma^4 + 3(1-a)\gamma^3 - 8\gamma^2 - 4(1-a)\gamma + 8}{3\gamma^4 - 16\gamma^2 + 16};$$

$$p_2^f(B) = c_2 + \frac{\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)}{2(4 - 3\gamma^2)}.$$
(22)

Profit:

$$\pi_1^l(B) = \frac{(2 - \gamma^2) [\gamma^4 + 3(1-a)\gamma^3 - 8\gamma^2 - 4(1-a)\gamma + 8]^2}{2(1 - \gamma^2)(3\gamma^4 - 16\gamma^2 + 16)^2};$$

$$\pi_2^f(B) = \frac{[\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)]^2}{4(1 - \gamma^2)(3\gamma^4 - 16\gamma^2 + 16)}.$$
(23)

Similarly, if the owner of firm 1 is in favor of moving later and firm 2 determines to move earlier, that is, {S, F}, a symmetric Stackelberg subgame is fulfilled.

The incentive schemes:

$$\lambda_1^f(B) = 1 + \frac{\gamma^2 [(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4]}{c_1(3\gamma^4 - 16\gamma^2 + 16)};$$
(24)

$$\lambda_2^l(B) = 1.$$

Price:

$$p_1^f(B) = c_1 + \frac{(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4}{2(4 - 3\gamma^2)};$$

$$p_2^l(B) = c_2 + \frac{(1-a)\gamma^4 + 3\gamma^3 - 8(1-a)\gamma^2 - 4\gamma + 8(1-a)}{3\gamma^4 - 16\gamma^2 + 16}.$$
(25)

Profit:

$$\pi_1^f(B) = \frac{[(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4]^2}{4(1 - \gamma^2)(3\gamma^4 - 16\gamma^2 + 16)};$$

$$\pi_2^l(B) = \left((2 - \gamma^2) \times [(1-a)\gamma^4 + 3\gamma^3 - 8(1-a)\gamma^2 - 4\gamma + 8(1-a)]^2 \right) \times \left(2(1 - \gamma^2)(3\gamma^4 - 16\gamma^2 + 16) \right)^{-1}.$$
(26)

Note that in price competition with delegation, both firms' incentive schemes under four strategy combinations are more than or equal to 1, especially in two sequential

subgames; the leaders' incentive schemes are always equal to 1. The reason is similar to quantity competition. (In essence, an owner of a managerial firm hiring an aggressive manager may lead the firm to enjoying a dominant position replicating the performance of a Stackelberg-leader, provided that rivals remain entrepreneurial. One can refer to Basu [25] for explaining the role of managerial incentive which leads to Stackelberg equilibrium in oligopoly.)

3. The Comparison between Cournot and Bertrand Cases

In contrast with Singh and Vives [1], the Cournot and Bertrand equilibrium with delegation under both demand and cost asymmetry are compared within this section.

Lemma 1. *When parameters $(\gamma, a) \in S = \{0 < \gamma < 1, 0 \leq a < 1 - \gamma\}$ (the assumption of positive primary outputs holds),*

- (1) *both efficient and inefficient firms charge higher prices under Cournot equilibrium than under Bertrand competition;*
- (2) *both firms produce less under Cournot than under Bertrand competition;*
- (3) *both firms make more profits under Cournot than under Bertrand competition.*

Proof. See the appendix. □

Conventional wisdom suggests that a decrease in the degree of product differentiation always reduces firms' profits by increasing the intensity of product market competition, irrespective of the competition mode, that is, irrespective of the fact that firms compete à la Cournot or à la Bertrand in the product market. The theoretical reason behind this result can be understood by referring to the standard differentiated duopoly model, in which a decrease in the degree of product differentiation diminishes total demand and induces firms to compete more aggressively [1]. Under both quantity and price competition, this unambiguously leads to lower firms' profits. Zanchettin [4, section 4] modifies the Singh and Vives [1] original framework by allowing for a wider range of cost and demand asymmetry between firms and finds that, under both modes of competition, the efficient firm's profit and industry profits as a whole can decrease with the degree of product differentiation. (Zanchettin's [4] main purpose is to compare in a differentiated duopoly with asymmetric firms, Cournot and Bertrand equilibria.)

From Lemma 1, it can be understood that the conclusion of Singh and Vives [1] still holds under delegation with both demand and cost asymmetry. The more differentiated the products are, the smaller the difference between the Cournot and Bertrand prices is. Just because managers are more aggressive under Bertrand than Cournot case, both firms produce more output under Bertrand; thus the firms make less profits. Nonetheless, it is important to perceive from the proof that our conclusions hold if and only if the assumption of positive primary outputs holds. Our results suggest that both firms make more profits under Cournot than under

Bertrand competition irrespective of the degree of product differentiation but essentially generated by the interplay of the market competition effect and the delegation incentive effect. To be more specific, the delegation incentive effect developed within this work augments the market competition effect, that is, the traditional cause of the mentioned finding, whilst in Zanchettin [4], also with differentiated products, this phenomenon only occurs with regard to the most efficient firm.

4. The Equilibria of Endogenous Timing Game: Profit and Welfare Ranking

After realizing the results of the game, the endogenous timing problem is able to be resolved. First, the payoffs of simultaneous Nash subgame and two sequential subgames under quantity competition and price competition should be, respectively, compared.

Proposition 2. *When $(\gamma, a) \in S = \{0 < \gamma < 1, 0 \leq a < 1 - \gamma\}$,*

- (1) *in the quantity-setting extended game, each firm's profit under simultaneous game is lower than the profit when being a follower in the sequential game, but higher than the leadership payoff; that is, $\pi_1^l(C) < \pi_1^n(C) < \pi_1^f(C)$; $\pi_2^l(C) < \pi_2^n(C) < \pi_2^f(C)$; this point indicates that both owners' preferences for the order of the roles their firms play are $f > n > l$;*
- (2) *in the price-setting extended game, each firm makes more profit when being a follower under sequential game than under simultaneous game, but less when being a leader; that is, $\pi_1^n(B) < \pi_1^f(B) < \pi_1^l(B)$; $\pi_2^n(B) < \pi_2^f(B) < \pi_2^l(B)$.*

Thus both owners' preferences for the order of the roles their firms play are $l > f > n$.

Proof. See the appendix. \square

Realizing the preference order of the owners under the considered game structures makes it possible to figure out the owners' timing choice. We have the following proposition.

Proposition 3. (1) *When firms compete in quantity, the extended game has unique pure-strategy subgame-perfect equilibrium.*

(2) *When firms compete in price, the extended game has two pure-strategy subgame-perfect equilibria, and mixed strategy equilibrium exists.*

Proof. (1) From Proposition 2, it can be understood that both profit-maximizing owners' preferences for the role their firms play are $f > n > l$ when both firms compete in quantity. For owner 1, {S} is the dominant strategy within its action space {F, S} meaning that no matter what owner 2's choice is, {S} is firm 1's best response, which also fits for the owner 2. Therefore, there is one and only one pure-strategy subgame-perfect equilibrium {S, S}. In contrary, Gal-Or [26] shows that, without the consideration of delegation, the player that

moves first earns higher profit than the player that moves second. Clearly, delegation contracts make the difference on the role chosen by the firms.

(2) Proposition 2 shows that both profit-maximizing owners' preferences for the role their firms play are $l > f > n$ when both firms are price competitors. Recall that (16) illustrates four strategy combinations: {F, F}, {F, S}, {S, F}, and {S, S}. Consider {F, F} first that makes both firms obtain the simultaneous-game payoffs; given rival's timing choice, both owners intend to deviate to strategy {S} to be better off. {S, S} is likewise. On the contrary, consider {F, S} and {S, F}; both owners cannot be better off from deviation. Sklivas [15] shows that, without cost asymmetry, Bertrand players with managerial delegation lead to collusion in game and then raise both firms' profits. Interestingly, when taking cost asymmetry into account, we find that two Bertrand players will become more aggressive and then choose to be leader in game. Furthermore, our finding is quite different from Gal-Or's [26] result. She clearly shows that the follower receives higher profit than the leader, when goods are substitutes and the strategic complement prevails. \square

As a result, both {F, S} and {S, F} are equilibria indicating that both owners possess no dominant strategy. Accordingly, there must be mixed strategy equilibrium in which owners randomize over {F} and {S}.

Though the equilibrium of the two extended games is being found out here, the most Pareto-efficient outcome is not determined yet, which can be figured out through the comparison of the welfare under all strategy combinations.

The total social surplus is defined as follows and can be derived through the substitution of the equilibrium outcomes back to the model under each case:

$$\begin{aligned} TS &= CS + \pi_1 + \pi_2 \\ &= q_1 + (1 - a)q_2 - \frac{1}{2}(q_1 + q_2)^2 + (1 - \gamma)q_1q_2. \end{aligned} \quad (27)$$

When both firms are simultaneous quantity competitors, that is, {F, F} or {S, S},

$$\begin{aligned} TS^n(C) &= 2 \left[a^2 (-\gamma^6 + 11\gamma^4 - 44\gamma^2 + 48) \right. \\ &\quad + a (2\gamma^6 + 6\gamma^5 - 22\gamma^4 - 48\gamma^3 + 88\gamma^2 + 64\gamma - 96) \\ &\quad \left. - 2\gamma^6 - 6\gamma^5 + 22\gamma^4 + 48\gamma^3 - 88\gamma^2 - 64\gamma + 96 \right] \\ &\quad \times (\gamma^4 - 12\gamma^2 + 16)^{-2}. \end{aligned} \quad (28)$$

When both firms are quantity competitors and firm 1 is the leader, that is, {F, S},

$$\begin{aligned} TS^{lf}(C) &= \left[a^2 (7\gamma^8 - 100\gamma^6 + 512\gamma^4 - 1088\gamma^2 + 768) \right. \\ &\quad + a (-14\gamma^8 - 52\gamma^7 + 200\gamma^6 + 464\gamma^5 \\ &\quad \left. - 1024\gamma^4 - 1280\gamma^3 + 2176\gamma^2) \right] \end{aligned}$$

$$\begin{aligned}
& + 1024\gamma - 1536) \\
& + 7\gamma^8 + 52\gamma^7 - 168\gamma^6 - 464\gamma^5 + 992\gamma^4 \\
& + 1280\gamma^3 - 2176\gamma^2 - 1024\gamma + 1536] \\
& \times \left(8(3\gamma^4 - 16\gamma^2 + 16)^2\right)^{-1}.
\end{aligned} \tag{29}$$

When both firms are quantity competitors and firm 2 is the leader, that is, {S, F},

$$\begin{aligned}
TS^{fl}(C) = & \left[a^2 (-68\gamma^6 + 480\gamma^4 - 1088\gamma^2 + 768) \right. \\
& + a (-52\gamma^7 + 136\gamma^6 + 464\gamma^5 - 960\gamma^4 \\
& \quad - 1280\gamma^3 + 2176\gamma^2 + 1024\gamma - 1536) \\
& + 7\gamma^8 + 52\gamma^7 - 168\gamma^6 - 464\gamma^5 + 992\gamma^4 \\
& \left. + 1280\gamma^3 - 2176\gamma^2 - 1024\gamma + 1536 \right] \\
& \times \left(8(3\gamma^4 - 16\gamma^2 + 16)^2\right)^{-1}.
\end{aligned} \tag{30}$$

When both firms are simultaneous price competitors, that is, {F, F} or {S, S},

$$\begin{aligned}
TS^n(B) = & (2 - \gamma^2) \\
& \times \left[a^2 (-\gamma^6 + 50\gamma^4 - 136\gamma^2 + 96) \right. \\
& + a (-2\gamma^7 + 2\gamma^6 + 52\gamma^5 - 100\gamma^4 - 160\gamma^3 \\
& \quad + 272\gamma^2 + 128\gamma - 192) + 2\gamma^7 - 2\gamma^6 \\
& \quad - 52\gamma^5 + 100\gamma^4 + 160\gamma^3 - 272\gamma^2 \\
& \quad \left. - 128\gamma + 192 \right] \\
& \times \left(2(1 - \gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2\right)^{-1}.
\end{aligned} \tag{31}$$

When both firms are price competitors and firm 1 is the leader, that is, {F, S},

$$\begin{aligned}
TS^{lf}(B) = & \left[a^2 (63\gamma^8 - 564\gamma^6 + 1600\gamma^4 - 1856\gamma^2 + 768) \right. \\
& + a (66\gamma^9 - 126\gamma^8 - 604\gamma^7 + 1128\gamma^6 \\
& \quad + 1840\gamma^5 - 3200\gamma^4 - 2304\gamma^3 + 3712\gamma^2 \\
& \quad \left. + 1024\gamma - 1536) - 5\gamma^{10} - 66\gamma^9 \right.
\end{aligned}$$

$$\begin{aligned}
& + 163\gamma^8 + 604\gamma^7 - 1192\gamma^6 - 1840\gamma^5 \\
& + 3232\gamma^4 + 2304\gamma^3 - 3712\gamma^2 \\
& \left. - 1024\gamma + 1536 \right] \\
& \times \left(8(1 - \gamma^2)(3\gamma^4 - 16\gamma^2 + 16)^2\right)^{-1}.
\end{aligned} \tag{32}$$

When both firms are price competitors and firm 2 is the leader, that is, {S, F},

$$\begin{aligned}
TS^{fl}(B) & = \left[a^2 (-5\gamma^{10} + 100\gamma^8 - 628\gamma^6 + 1632\gamma^4 - 1856\gamma^2 + 768) \right. \\
& + a (10\gamma^{10} + 66\gamma^9 - 200\gamma^8 - 604\gamma^7 + 1256\gamma^6 \\
& \quad + 1840\gamma^5 - 3264\gamma^4 - 2304\gamma^3 + 3712\gamma^2 \\
& \quad + 1024\gamma - 1536 + 66\gamma^9 - 200\gamma^8) \\
& - 5\gamma^{10} - 66\gamma^9 + 163\gamma^8 + 604\gamma^7 - 1192\gamma^6 - 1840\gamma^5 \\
& \left. + 3232\gamma^4 + 2304\gamma^3 - 3712\gamma^2 - 1024\gamma + 1536 \right] \\
& \times \left(8(1 - \gamma^2)(3\gamma^4 - 16\gamma^2 + 16)^2\right)^{-1}.
\end{aligned} \tag{33}$$

Comparing the total social surplus obtained under different competition modes, we have the following proposition.

Proposition 4. For any $(\gamma, a) \in S = \{0 < \gamma \leq 1, 0 \leq a < 1 - \gamma\}$,

- (1) it is more Pareto-efficient under price competition than all other cases under quantity competition; that is, for any $r \in \{n, lf, fl\}$, $TS^r(C) < TS^r(B)$;
- (2) when both firms act as quantity-setters, the welfare ranking of simultaneous and sequential games is $TS^n(C) < TS^{lf}(C) < TS^{fl}(C)$;
- (3) when firms compete in price, the welfare ranking of the three cases with different move orders is

$$TS^{fl}(B) < TS^{lf}(B), \quad TS^{fl}(B) < TS^{lf}(B). \tag{34}$$

For any degree of product differentiation $\gamma \in (0, 1)$, there exists a critical level of efficiency gap $a^*(\gamma) \in [0, 1 - \gamma)$, such that when $0 \leq a < a^*(\gamma)$, we have $TS^{lf}(B) < TS^n(B)$; when $a^*(\gamma) \leq a < 1 - \gamma$, we have $TS^n(B) \leq TS^{lf}(B)$.

Proof. See the appendix. \square

Generally, price competition is more Pareto-efficient than quantity competition, no matter the firms are symmetric or not. Under quantity competition, the Nash equilibrium is {S, S}, so both firms will act as a Cournot-player, but the society will suffer the most under this condition. Under price

competition, the subgame equilibrium is {F, S} or {S, F}. It can be seen that {S, F} is the most inefficient case, and when the efficiency gap between the two firms is huge, the welfare of {F, S} case will even exceed the Bertrand case. The intuition behind this may be simple. The leader in a Stackelberg price competition game has much advantage, which produces much more than the follower; then the society will benefit from the efficient firm if it acts as a leader. When the efficiency gap between the two firms is huge, the benefit resulted from the efficient firm will even exceed the Bertrand case.

5. Conclusions

Many papers have discussed the endogenous timing game, but those papers usually assumed duopoly market with symmetric firms. Due to the specification they had, the roles of the firms in the games they concluded are often the same. Thus they did not really answer the question of endogenously determined move order. It is because when one firm chooses to act as a leader, it is not sufficient for the desirability of the other symmetric firm to act as a follower.

Therefore, to solve the question and figure out the reason, this paper extends the Hamilton and Slutsky [8] model with asymmetric demand, asymmetric cost, and managerial delegation over the whole relevant parameter space. It shows the following: firstly, both the efficient and the inefficient firms with delegation will have second move advantage under quantity setting and first move advantage under price competition; secondly, the extended games under quantity competition and under price competition have subgame equilibria. Furthermore, the welfare analysis of the extended game points out that the most Pareto-efficient of all strategy combinations is mostly the Bertrand case with one exception that when the efficiency gap between the two firms is huge, the case that efficient firm moves first and inefficient firm moves second is even more welfare beneficial than the Bertrand case.

Appendix

Proof of Lemma 1. When $(\gamma, a) \in S = \{0 < \gamma < 1, 0 \leq a < 1 - \gamma\}$, consider the following.

Price:

$$p_1^n(C) - p_1^n(B) = \frac{\gamma^4}{\gamma^4 - 12\gamma^2 + 16} > 0; \quad (\text{A.1})$$

$$p_2^n(C) - p_2^n(B) = \frac{(1-a)\gamma^4}{\gamma^4 - 12\gamma^2 + 16} > 0.$$

Output:

$$q_1^n(C) - q_1^n(B) = \frac{\gamma^4[\gamma(1-a) - 1]}{(1-\gamma^2)(\gamma^4 - 12\gamma^2 + 16)} < 0; \quad (\text{A.2})$$

$$q_2^n(C) - q_2^n(B) = \frac{\gamma^4(\gamma - 1 + a)}{(1-\gamma^2)(\gamma^4 - 12\gamma^2 + 16)} < 0.$$

Incentive scheme:

$$\lambda_1^n(C) - 1 = -\frac{\gamma^2[-\gamma^2 - 2(1-a)\gamma + 4]}{c_1(\gamma^4 - 12\gamma^2 + 16)} < 0;$$

$$\lambda_2^n(C) - 1 = -\frac{\gamma^2[-(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{c_2(\gamma^4 - 12\gamma^2 + 16)} < 0;$$

$$\lambda_1^n(B) - 1 = \frac{\gamma^2[(1-a)\gamma^3 - 3\gamma^2 - 2(1-a)\gamma + 4]}{c_1(\gamma^4 - 12\gamma^2 + 16)} > 0;$$

$$\lambda_2^n(B) - 1 = \frac{\gamma^2[\gamma^3 - 3(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{c_2(\gamma^4 - 12\gamma^2 + 16)} > 0. \quad (\text{A.3})$$

So $\lambda_1^n(C) < 1 < \lambda_1^n(B)$; $\lambda_2^n(C) < 1 < \lambda_2^n(B)$.

Profit:

$$\begin{aligned} \pi_1^n(C) - \pi_1^n(B) &= \frac{2\gamma^5(2-\gamma^2)[- \gamma a^2 + 2a(\gamma-1) - 2\gamma + 2]}{(1-\gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2} > 0, \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \pi_2^n(C) - \pi_2^n(B) &= \frac{2\gamma^5(2-\gamma^2)[- \gamma a^2 + 2a(\gamma-1) - 2\gamma + 2]}{(1-\gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2} > 0. \end{aligned}$$

□

Proof of Proposition 2. When $(\gamma, a) \in S = \{0 < \gamma < 1, 0 \leq a < 1 - \gamma\}$,

$$\begin{aligned} \pi_1^n(C) - \pi_1^f(C) &= -\frac{\gamma^8[-\gamma^2 - 2\gamma(1-a) + 4]^2}{4(\gamma^4 - 12\gamma^2 + 16)^2(3\gamma^4 - 16\gamma^2 + 16)} < 0; \\ \pi_1^n(C) - \pi_1^l(C) &= \frac{\gamma^5(\gamma^2 - 2)[- \gamma^2(1-a) - 2\gamma + 4(1-a)]}{2(\gamma^4 - 12\gamma^2 + 16)^2(3\gamma^4 - 16\gamma^2 + 16)^2} \\ &\quad \times [\gamma^7(a-1) + 10\gamma^6 + 28\gamma^5(1-a) - 112\gamma^4 \\ &\quad - 128\gamma^3(1-a) + 320\gamma^2 + 128\gamma(1-a) - 256] > 0; \end{aligned}$$

$$\begin{aligned} \pi_2^n(C) - \pi_2^l(C) &= -\frac{\gamma^5(\gamma^2 - 2)[- \gamma^2 - 2\gamma(1-a) + 4]}{2(\gamma^4 - 12\gamma^2 + 16)^2(3\gamma^4 - 16\gamma^2 + 16)^2} \\ &\quad \times [\gamma^7 - 10\gamma^6(1-a) - 28\gamma^5 + 112\gamma^4(1-a) \\ &\quad + 128\gamma^3 - 320\gamma^2(1-a) - 128\gamma + 256(1-a)] \gg 0; \end{aligned}$$

$$\begin{aligned}
& \pi_2^n(C) - \pi_2^f(C) \\
&= -\frac{\gamma^8[-\gamma^2(1-a) - 2\gamma + 4(1-a)]^2}{4(\gamma^4 - 12\gamma^2 + 16)^2(3\gamma^4 - 16\gamma^2 + 16)} < 0; \\
& \pi_1^n(B) - \pi_1^f(B) \\
&= -\frac{\gamma^8[\gamma^3(1-a) - 3\gamma^2 - 2\gamma(1-a) + 4]^2}{4(1-\gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2(3\gamma^4 - 16\gamma^2 + 16)} \\
&< 0; \\
& \pi_1^f(B) - \pi_1^l(B) \\
&= \frac{\gamma^5}{4(1-\gamma^2)(3\gamma^4 - 16\gamma^2 + 16)^2} \\
&\quad \times [a^2(3\gamma^5 - 10\gamma^3 + 8\gamma) \\
&\quad + a(-6\gamma^5 + 6\gamma^4 + 20\gamma^3 - 20\gamma^2 - 16\gamma + 16) \\
&\quad + 5\gamma^5 - 6\gamma^4 - 19\gamma^3 + 20\gamma^2 + 16\gamma - 16] < 0; \\
& \pi_2^n(B) - \pi_2^f(B) \\
&= -\frac{\gamma^8[\gamma^3(1-a) - 3\gamma^2(1-a) - 2\gamma + 4(1-a)]^2}{4(1-\gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2(3\gamma^4 - 16\gamma^2 + 16)} \\
&< 0; \\
& \pi_2^f(B) - \pi_2^l(B) \\
&= \frac{\gamma^5}{4(1-\gamma^2)(3\gamma^4 - 16\gamma^2 + 16)^2} \\
&\quad \times [a^2(2\gamma^5 - 9\gamma^3 + 8\gamma) \\
&\quad + a(-4\gamma^5 + 6\gamma^4 + 18\gamma^3 - 20\gamma^2 - 16\gamma + 16) \\
&\quad + 5\gamma^5 - 6\gamma^4 - 19\gamma^3 + 20\gamma^2 + 16\gamma - 16] < 0. \tag{A.5}
\end{aligned}$$

Then the inequalities of Proposition 2 are derived. \square

Proof of Proposition 4. If $(\gamma, a) \in S = \{0 < \gamma < 1, 0 \leq a < 1 - \gamma\}$,

$$\begin{aligned}
& TS^{lf}(C) - TS^n(C) \\
&= \frac{\gamma^4[-(1-a)\gamma^2 - 2\gamma + 4(1-a)]}{8(3\gamma^4 - 16\gamma^2 + 16)^2(\gamma^4 - 12\gamma^2 + 16)^2} \\
&\quad \times [a(7\gamma^{10} - 96\gamma^8 + 640\gamma^6 - 2240\gamma^4 + 3584\gamma^2 - 2048) \\
&\quad - 7\gamma^{10} - 38\gamma^9 + 96\gamma^8 + 504\gamma^7 - 640\gamma^6 - 2144\gamma^5 \\
&\quad + 2240\gamma^4 + 3584\gamma^3 - 3584\gamma^2 - 2048\gamma + 2048] > 0;
\end{aligned}$$

$$TS^{fl}(C) - TS^{lf}(C) = \frac{a\gamma^4(7\gamma^4 - 32\gamma^2 + 32)(2-a)}{(3\gamma^4 - 16\gamma^2 + 16)^2} > 0;$$

$$\begin{aligned}
& TS^{fl}(B) - TS^{fl}(C) \\
&= -\frac{\gamma^4}{8(1-\gamma^2)(3\gamma^4 - 16\gamma^2 + 16)^2} \\
&\quad \times [a^2(5\gamma^6 - 32\gamma^4 + 80\gamma^2 - 64) \\
&\quad + a(-10\gamma^6 - 14\gamma^5 + 64\gamma^4 + 88\gamma^3 \\
&\quad - 160\gamma^2 - 96\gamma + 128) - 2\gamma^6 + 14\gamma^5 \\
&\quad + 12\gamma^4 - 88\gamma^3 + 32\gamma^2 + 96\gamma - 64] > 0;
\end{aligned}$$

$$\begin{aligned}
& TS^n(B) - TS^{fl}(B) \\
&= \frac{-\gamma^4[\gamma^3(1-a) - 3\gamma^2 - 2\gamma(1-a) + 4]}{8(1-\gamma^2)(3\gamma^4 - 16\gamma^2 + 16)^2(\gamma^4 - 12\gamma^2 + 16)^2} \\
&\quad \times [a(5\gamma^{11} - 174\gamma^9 + 1304\gamma^7 - 3744\gamma^5 \\
&\quad + 4608\gamma^3 - 2048\gamma) - 5\gamma^{11} - 9\gamma^{10} + 174\gamma^9 \\
&\quad - 72\gamma^8 - 1304\gamma^7 + 1120\gamma^6 + 3744\gamma^5 - 3648\gamma^4 \\
&\quad - 4608\gamma^3 + 4608\gamma^2 + 2048\gamma - 2048] > 0,
\end{aligned}$$

$$TS^{lf}(B) - TS^{fl}(B) = \frac{a\gamma^4(5\gamma^4 - 32\gamma^2 + 32)(2-a)}{8(3\gamma^4 - 16\gamma^2 + 16)^2} > 0,$$

$$\begin{aligned}
& TS^n(B) - TS^{lf}(B) \\
&= \frac{\gamma^4[\gamma^3 - 3\gamma^2(1-a) - 2\gamma + 4(1-a)]}{8(1-\gamma^2)(\gamma^4 - 12\gamma^2 + 16)^2(3\gamma^4 - 16\gamma^2 + 16)^2} X, \tag{A.6}
\end{aligned}$$

where

$$\begin{aligned}
X = & [a(-9\gamma^{10} - 72\gamma^8 + 1120\gamma^6 - 3648\gamma^4 + 4608\gamma^2 - 2048) \\
& + 5\gamma^{11} + 9\gamma^{10} - 174\gamma^9 + 72\gamma^8 + 1304\gamma^7 - 1120\gamma^6 \\
& - 3744\gamma^5 + 3648\gamma^4 + 4608\gamma^3 - 4608\gamma^2 \\
& - 2048\gamma + 2048]. \tag{A.7}
\end{aligned}$$

The sign of last formula is uncertain. For $[\gamma^3 - 3\gamma^2(1-a) - 2\gamma + 4(1-a)] > 0$, so when $X > 0$, we have $TS^n(B) > TS^{lf}(B)$; when $X \leq 0$, we obtain $TS^n(B) \leq TS^{lf}(B)$. Moreover, $X = 0 \Leftrightarrow a = a^*(\gamma)$, where

$$\begin{aligned}
& a^*(\gamma) \\
&= ((1-\gamma)(-\gamma^2 + 2\gamma + 4))
\end{aligned}$$

$$\begin{aligned} & \times (5\gamma^8 + 24\gamma^7 - 92\gamma^6 - 176\gamma^5 + 496\gamma^4 \\ & \quad + 384\gamma^3 - 896\gamma^2 - 256\gamma + 512)) \\ & \times ((4 - 3\gamma^2)(-3\gamma^8 - 28\gamma^6 + 336\gamma^4 - 768\gamma^2 + 512))^{-1} \end{aligned} \quad (\text{A.8})$$

and we can easily show that, when $0 < \gamma \leq 1$, $0 \leq a^*(\gamma) < 1 - \gamma$. \square

Disclosure

The author declares that this paper is not supported by any funding institutions or any government's divisions.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The author wishes to thank the assistance provided by Zhigang Zhang, Tien-Der Han, and Tai-Liang Chen and constructive suggestions received from two reviewers and academic editor Henry X. Wang.

References

- [1] N. Singh and X. Vives, "Price and quantity competition in a differentiated duopoly," *The RAND Journal of Economics*, vol. 15, no. 4, pp. 546–554, 1984.
- [2] M. C. López and R. A. Naylor, "The Cournot-Bertrand profit differential: a reversal result in a differentiated duopoly with wage bargaining," *European Economic Review*, vol. 48, no. 3, pp. 681–696, 2004.
- [3] J. Hsu and X. H. Wang, "On welfare under Cournot and Bertrand competition in differentiated oligopolies," *Review of Industrial Organization*, vol. 27, no. 2, pp. 185–191, 2005.
- [4] P. Zanchettin, "Differentiated duopoly with asymmetric costs," *Journal of Economics and Management Strategy*, vol. 15, no. 4, pp. 999–1015, 2006.
- [5] A. Mukherjee, "Price and quantity competition under free entry," *Research in Economics*, vol. 59, no. 4, pp. 335–344, 2005.
- [6] A. Mukherjee, U. Broll, and S. Mukherjee, "Bertrand versus cournot competition in a vertical structure: a note," *Manchester School*, vol. 80, no. 5, pp. 545–559, 2012.
- [7] M. Correa-López, "Price and quantity competition in a differentiated duopoly with upstream suppliers," *Journal of Economics and Management Strategy*, vol. 16, no. 2, pp. 469–505, 2007.
- [8] J. H. Hamilton and S. M. Slutsky, "Endogeneous timing in duopoly games: stackelberg or Cournot equilibria," *Games and Economic Behavior*, vol. 2, no. 1, pp. 29–46, 1990.
- [9] L. Lambertini, "Extended games played by managerial firms," *The Japanese Economic Review*, vol. 51, no. 2, pp. 274–283, 2000.
- [10] L. F. S. Wang, Y.-C. Wang, and L. Zhao, "Privatization and efficiency gain in an international mixed oligopoly with asymmetric costs," *The Japanese Economic Review*, vol. 60, no. 4, pp. 539–559, 2009.
- [11] Y. Nakamura and T. Inoue, "Endogenous timing in a mixed duopoly: price competition with managerial delegation," *Managerial and Decision Economics*, vol. 30, no. 5, pp. 325–333, 2009.
- [12] J. C. Bárcena-Ruiz and M. Sedano, "Endogenous timing in a mixed duopoly: weighted welfare and price competition," *The Japanese Economic Review*, vol. 62, no. 4, pp. 485–503, 2011.
- [13] Y. Tomaru, Y. Nakamura, and M. Saito, "Strategic managerial delegation in a mixed duopoly with capacity choice: partial delegation or full delegation," *The Manchester School*, vol. 79, no. 4, pp. 811–838, 2011.
- [14] C. Fershtman and K. Judd, "Equilibrium incentives in oligopoly," *American Economic Review*, vol. 77, pp. 927–940, 1987.
- [15] S. D. Sklivas, "The strategy choice of managerial incentives," *Rand Journal of Economics*, vol. 18, pp. 452–458, 1987.
- [16] J. C. Bárcena-Ruiz and M. B. Espinoza, "Long-term or short-term managerial contracts," *Journal of Economics and Management Strategy*, vol. 5, pp. 343–359, 1996.
- [17] T. Jansen, A. van Lier, and A. van Witteloostuijn, "A note on strategic delegation: the market share case," *International Journal of Industrial Organization*, vol. 25, no. 3, pp. 531–539, 2007.
- [18] R. A. Ritz, "Strategic incentives for market share," *International Journal of Industrial Organization*, vol. 26, no. 2, pp. 586–597, 2008.
- [19] T. Jansen, A. V. Lier, and A. van Witteloostuijn, "On the impact of managerial bonus systems on firm profit and market competition: the cases of pure profit, sales, market share and relative profits compared," *Managerial and Decision Economics*, vol. 30, no. 3, pp. 141–153, 2009.
- [20] C. Manasakis, E. Mitrokostas, and E. Petrakis, "Endogenous managerial incentive contracts in a differentiated duopoly, with and without commitment," *Managerial and Decision Economics*, vol. 31, no. 8, pp. 531–543, 2010.
- [21] A. Chirco, M. Scrimatore, and C. Colombo, "Competition and the strategic choice of managerial incentives: the relative performance case," *Metroeconomica*, vol. 62, no. 4, pp. 533–547, 2011.
- [22] J. Häckner, "A note on price and quantity competition in differentiated oligopolies," *Journal of Economic Theory*, vol. 93, no. 2, pp. 233–239, 2000.
- [23] X. H. Wang, "Price and quantity competition revisited," *Economics Bulletin*, vol. 4, no. 8, pp. 1–7, 2008.
- [24] S. Mujumdar and D. Pal, "Strategic managerial incentives in a two-period Cournot duopoly," *Games and Economic Behavior*, vol. 58, no. 2, pp. 338–353, 2007.
- [25] K. Basu, "Stackelberg equilibrium in oligopoly: an explanation based on managerial incentives," *Economics Letters*, vol. 49, no. 4, pp. 459–464, 1995.
- [26] E. Gal-Or, "First mover and second mover advantages," *International Economic Review*, vol. 26, no. 3, pp. 649–653, 1985.



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